SECTION 12

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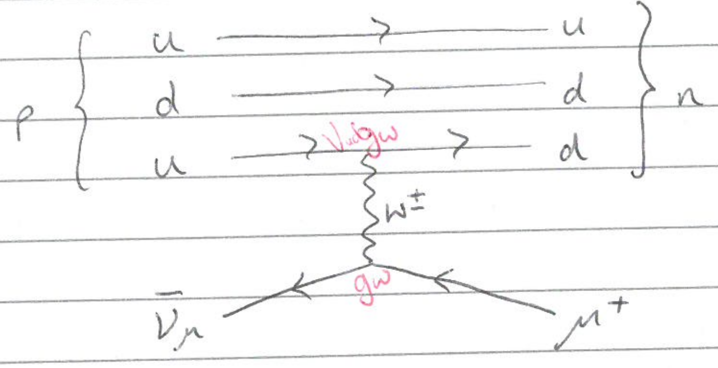

Speed of light in wate ~ 0·75c

for e/μ > 0·75c

As charged particle travels, it polarises the wate

- travels so fast it leaves a disturbance
- radiated photons are contained within the electric field of the disturbance => cone

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$\pi^+ \rightarrow \mu^+ \nu_\mu$

ν_μ is a mixture of ν_1 & ν_2

$$\Psi(\vec{p}, t=0) = \nu_2(\vec{p})\cos\theta - \nu_1(\vec{p})\sin\theta$$

if masses of ν_1 & ν_2 are different, then they must evolve differently with time

ie ν mass $\neq 0$

$$\nu_x(\vec{p}, t) = \nu_x(\vec{p})\, e^{-iE_x t}$$

← contains mass term $m^2 = E^2 - \vec{p}^{\,2}$

so at a late time

$$\Psi(\vec{p},t) = \underbrace{\nu_2(\vec{p})}_{\nu_e(\vec{p})\sin\theta + \nu_\mu(\vec{p})\cos\theta} e^{-iE_2t}\cos\theta \quad - \underbrace{\nu_1(\vec{p})}_{\nu_e(\vec{p})\cos\theta - \nu_\mu(\vec{p})\sin\theta} e^{-iE_1t}\sin\theta$$

Express as weak eigenstates

$$= \nu_e(\vec{p})\underbrace{\left[\sin\theta\cos\theta\left(e^{-iE_2t} - e^{-iE_1t}\right)\right]}_{c_e} + \nu_\mu(\vec{p})\underbrace{\left[\cos^2\theta\, e^{-iE_2t} + \sin^2\theta\, e^{-iE_1t}\right]}_{c_\mu}$$

$P(\nu_e) = |c_e|^2$

$P_\mu = |c_\mu|^2$

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$\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$

$$P(\nu_e) = |C_e|^2 = \Big|\underbrace{\sin^4\theta\cos\theta}_{\frac{1}{4}\sin^2 2\theta}\underbrace{\left(e^{-iE_2t} - e^{-iE_1t}\right)}\Big|^2$$

$$\left(e^{-iE_2t} - e^{-iE_1t}\right)\left(e^{iE_2t} - e^{iE_1t}\right)$$

$$2 - e^{-i(E_2-E_1)t} - e^{+i(E_2-E_1)t}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$2 - 2\cos(E_2-E_1)t$$

$$\sin^2\frac{x}{2} = \frac{1}{2} - \frac{1}{2}\cos\theta$$

$$4\sin^2\left(\frac{(E_2-E_1)t}{2}\right)$$

$$= \sin^2 2\theta \sin^2\left[\frac{(E_2-E_1)t}{2}\right]$$

$$E^2 = \vec{p}^{\,2} + m^2$$

$$E = \vec{p}\sqrt{1 + \frac{m^2}{\vec{p}^{\,2}}}$$

m is small

$\vec{p}$ is large

$$1 + x \sim \left(1 + \frac{x}{2}\right)^2$$

$$\sqrt{1+x} \sim 1 + \frac{x}{2}$$

$$\sim \vec{p}\left(1 + \frac{m^2}{2\vec{p}^{\,2}}\right)$$

$$\sim \vec{p} + \frac{m^2}{2\vec{p}}$$

for $E_2 - E_1$, the small $\frac{m^2}{\vec{p}}$ is important!

$$E_2 - E_1 = \vec{p}_2 - \vec{p}_1 + \frac{m_2^2}{2\vec{p}_2} - \frac{m_1^2}{2\vec{p}_1} \qquad (\vec{p}_2 \equiv \vec{p}_1)$$

$$= \frac{m_2^2 - m_1^2}{2\vec{p}} = \frac{m_2^2 - m_1^2}{2E}$$

since m is small

$$P(\nu_e) = \sin^2 2\theta \sin^2\left[\frac{(m_2^2 - m_1^2)t}{4E}\right] \sim \sin^2 2\theta \sin^2\left[\frac{\Delta m^2 L}{4E}\right]$$

$L = ct \sim t$

Proby of $\nu_\mu \to \nu_e$ oscillation depends on

- ν energy
- mixing angle
- mass^2 differences
- time from production