Section 11

slide 8

Lagrangian for a massive vector field

$$\mathcal{L} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Kinetic Term}} + \underbrace{\frac{m^2}{2}A_\mu A^\mu}_{\text{Mass Term}}$$

A is our field ~~$A_\mu = (\phi, \vec{A})$ $A^\mu = (\phi, -\vec{A})$~~

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ covariant $\partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$ contravariant $\partial^\mu = \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right)$

$\partial_\mu \partial^\mu = \left(\frac{\partial^2}{\partial t^2}, -\vec{\nabla}^2\right)$

but this $\mathcal{L}$ is not invariant under gauge transformation

$A_\mu \to A_\mu - \partial_\mu X$

$F^{\mu\nu}F_{\mu\nu}$ is invariant ∴ it is due to the $A_\mu A^\mu$ term

$$A_\mu A^\mu \neq (A_\mu - \partial_\mu X)(A^\mu - \partial^\mu X)$$

Solution: new scalar field ϕ

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m_2}{2}A_\mu A^\mu + \underbrace{\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi)}_{\phi \text{ self interactions}} - \underbrace{m\phi\,\partial_\mu A^\mu}_{\phi \text{ couples to } A \text{ with strength } m}$$

Now $\mathcal{L}$ is invariant under $A_\mu \to A_\mu - \partial_\mu X$

& $\phi \to \phi - mX$

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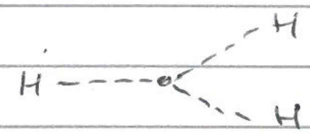
Higgs field complex doublet

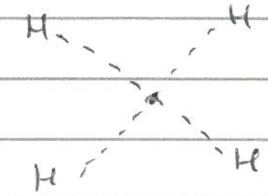
$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \qquad \phi^* = \frac{\phi_1 - i\phi_2}{\sqrt{2}}$$

self interactions $\phi\phi^*$



$\sim -b\phi^2$

$(\phi^*\phi)^2$



$\sim a\phi^4$

Higgs Potential $V = a\phi^4 - b\phi^2$

Ground/vacuum expectation by minimising V $\quad \frac{\partial V}{\partial \phi} = 4a\phi^3 - 2b\phi \equiv 0 \quad$ VEV $|\phi|^2 = \frac{b}{2a}$

if $\phi = 0$ then $\frac{\partial V}{\partial \phi} \equiv 0$ BUT then gauge bosons are massless

Need non-zero VEV

"spontaneous symmetry breaking"

Weak bosons couple to Higgs field in $\mathcal{L}$ in $m\phi\,\partial_\mu A^\mu$ term

$\hookrightarrow \sqrt{\frac{b}{2a}}$

Higgs also couples to itself in $\mathcal{L}$ in $\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)$ term

2nd derivative of V $\quad \frac{1}{2}\frac{\partial^2 V}{\partial \phi^2} = m_H^2 = \frac{12a\phi_{VEV}^2 - 2b}{2} = 4a\phi_{VEV}^2$

slide 10 Analogue of Higgs mechanism

- massless γ travelling in charged plasma

Current density

$$\vec{J} = n\,e\,\vec{v}$$

e^- density, e^- charge, e^- velocity

Equation of motion

$$e\vec{E} = m_e \frac{\partial \vec{v}}{\partial t}$$

$$F = m\,a$$

$$\frac{\partial \vec{J}}{\partial t} = \frac{\partial \vec{J}}{\partial \vec{v}}\,\frac{\partial \vec{v}}{\partial t} = ne\,\frac{e\vec{E}}{m_e} = \frac{ne^2\vec{E}}{m_e}$$

- Maxwell's equations

Waves travelling at $v \equiv c$ $\Rightarrow$ massless γ

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \nabla \times \vec{E} = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$-\nabla^2 \vec{E} = -\frac{\partial(\nabla \times \vec{B})}{\partial t} = -\frac{\partial}{\partial t}\left(\mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}\right)$$

μ_0: permeability of free space

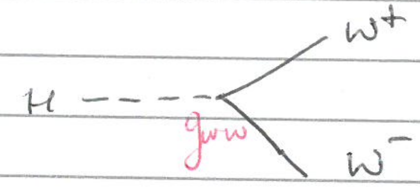
$$\nabla^2 \vec{E} = \mu_0 \frac{\partial \vec{J}}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

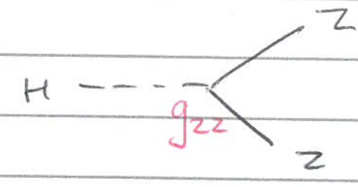
rearrange $\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\vec{E} + \frac{\mu_0 n e^2}{m_e}\vec{E} = 0$

KG eqn $\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\psi + m^2\psi = 0$

$\frac{\mu_0 n e^2}{m_e} \equiv m^2$ γ has "effective mass"

slide 12 just in case there are questions





why is $g_{WW} > g_{ZZ}$ when $m_H > 2m_Z$?

Ignoring differences in m_Z & m_W....

$$\frac{\Gamma(H \to WW)}{\Gamma(H \to ZZ)} = \frac{1}{1/2} \sim 2$$

suppression from the identical particles in the final state