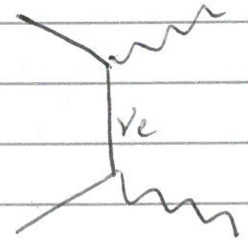
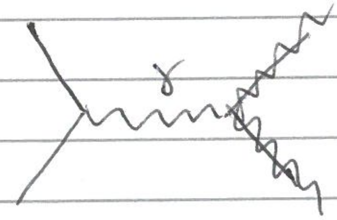
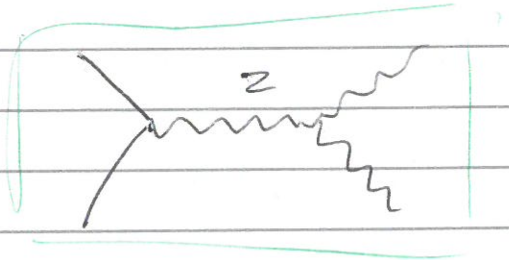
Section 10

Slide 3







Both have massless propagators

$\therefore \sigma \propto E \Rightarrow$ unitarity violation!

* Add in Z diagram *

Individual contributions do diverge

BUT now they cancel each other

σ_{TOTAL} well behaved at high $\sqrt{s}$

Can only cancel if couplings are related

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SU(2) acts on weak isospin I

U(1) acts on weak hypercharge $Y = 2(Q - I_3)$

↑ third component of isospin

Act on LH doublets $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} u \\ d \end{pmatrix}_L$

but not on RH singlets ν_{eR} e^-_R u_R d_R

	LH			RH		
	Q	I_3	Y	Q	I_3	Y
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	-1	0	0	0
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-1	-1	0	-2
u c t	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0	$+4/3$
d s b	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0	$-2/3$

↑ no weak interaction

Imposing SU(2)×U(1) symmetry

$\Rightarrow$ 4 massless gauge bosons

$W^+ \quad W^- \quad \underbrace{W_3 \quad B}_{\text{mix} \Rightarrow \gamma, Z}$

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only if time

$$\langle \psi | \hat{M}^2 | \psi \rangle = m^2$$

$$m_Z^2 = \langle Z|\hat{M}^2|Z\rangle = \langle W_3\cos\theta - B\sin\theta|\hat{M}^2|W_3\cos\theta - B\sin\theta\rangle$$
$$= m_W^2\cos^2\theta + m_B^2\sin^2\theta - 2m_{WB}^2\cos\theta\sin\theta$$

$$m_A^2 = \langle A|\hat{M}^2|A\rangle = \langle W_3\sin\theta + B\cos\theta|\hat{M}^2|W_3\sin\theta + B\cos\theta\rangle$$
$$= m_W^2\sin^2\theta + m_B^2\cos^2\theta + 2m_{WB}^2\cos\theta\sin\theta$$

$m_Z^2 + m_A^2 = m_W^2 + m_B^2$ if $A \equiv \gamma$, then $m_A = 0$

$$\Rightarrow m_Z^2 = m_W^2 + m_B^2$$

find m_B^2 from $m_A^2 = 0$

find m_{WB} from unphysical state $\langle Z|\hat{M}^2|A\rangle \equiv 0$

$$m_{ZA}^2 = (m_W^2 - m_B^2)\cos\theta\sin\theta + m_{WB}^2(\cos^2\theta - \sin^2\theta) \equiv 0$$
$$\rightarrow m_{WB}^2 = \frac{(m_B^2 - m_W^2)\cos\theta\sin\theta}{\cos^2\theta - \sin^2\theta}$$

$$m_A^2 = m_W^2\sin^2\theta + m_B^2\cos^2\theta + \frac{2(m_B^2 - m_W^2)\cos^2\theta\sin^2\theta}{\cos^2\theta - \sin^2\theta} \equiv 0$$
$$\rightarrow m_B^2 = m_W^2\frac{\sin^2\theta}{\cos^2\theta}$$

$$\Rightarrow m_Z^2 = m_W^2\frac{\sin^2\theta}{\cos^2\theta} + m_W^2 = m_W^2\left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right)$$
$$= \frac{m_W^2}{\cos^2\theta}$$

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only if time

EWK unification using $SU(3) \times U(1)$

$W^+W^-W_3$ couple to I_3 with strength g_W

B couples to Y with strength g'

Photon coupling $A = W_3\sin\theta + B\cos\theta$

$$Qe = g_W I_3\sin\theta + g' Y\cos\theta = g_W I_3\sin\theta + g'2(Q - I_3)\cos\theta$$

for $Q=0$ $\quad 0 = g_W I_3\sin\theta - 2g' I_3\cos\theta \qquad g' = \frac{g_W}{2}\tan\theta$

for $I_3=0$ $\quad Qe = 2g' Q\cos\theta \qquad g' = e/2\cos\theta$

$$\Rightarrow g_W = e/\sin\theta$$

Z coupling $Z = W_3\cos\theta - B\sin\theta$

$$g_W I_3\cos\theta - g'2(Q - I_3)\sin\theta$$

Subs $g_W = \frac{e}{\sin\theta}$ & $g' = \frac{e}{2\cos\theta}$

$$= \frac{eI_3\cos\theta}{\sin\theta} - \frac{e}{\cos\theta}(Q - I_3)\sin\theta$$
$$= \underbrace{\frac{e}{\sin\theta\cos\theta}}_{g_Z}\underbrace{\left[I_3 - Q\sin^2\theta\right]}_{\text{couples to } I_3 \,\&\, Y \text{ with strength } g_Z}$$

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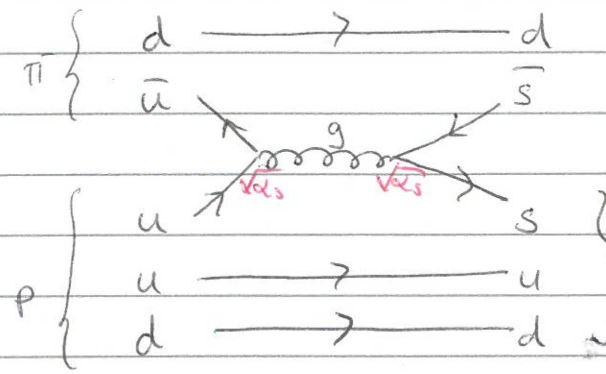
$\pi^- \; p \to K^0 \; \Lambda$

$\bar{u}d \quad uud \quad d\bar{s} \quad uds$

$\bar{u}u \to s\bar{s}$

Strong, EM or weak

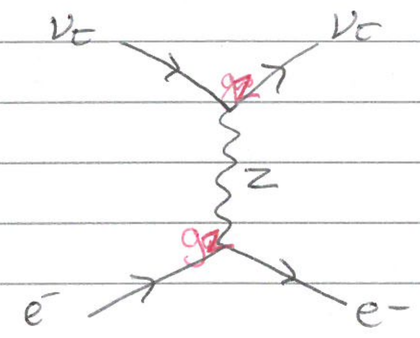


$\nu_\tau + e^- \longrightarrow \nu_\tau + e^-$

ONLY WEAK

No W as not same gen.

no s-chan as not same gen

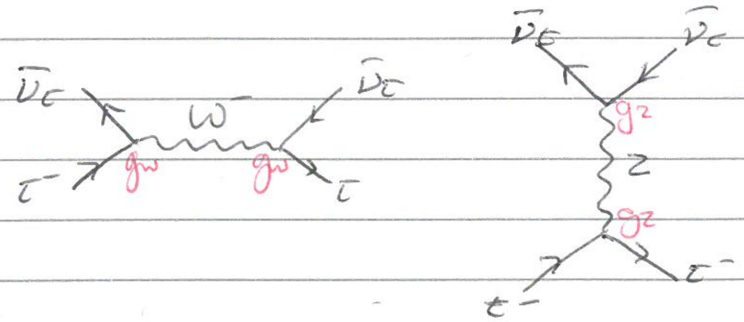


$\bar{\nu}_\tau + \tau^- \longrightarrow \bar{\nu}_\tau + \tau^-$

ONLY WEAK

Same gen: 2 diagrams

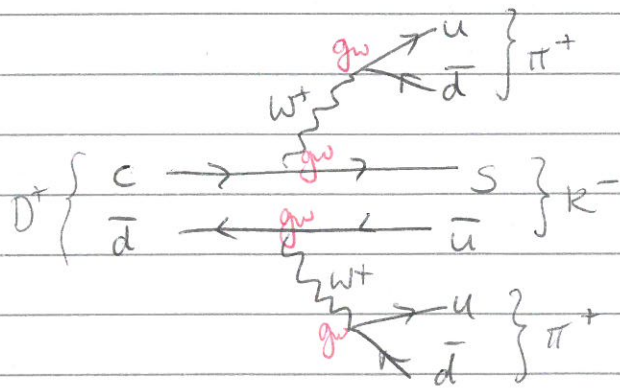
Explain why no t-chan with W



$D^+ \to K^- \pi^+ \pi^+$

$c\bar{d} \quad s\bar{u} \quad u\bar{d} \quad u\bar{d}$

Change of flavour → WEAK



~~$\Xi^0 \to \Sigma^+ \; e^- \; \bar{\nu}_e$~~

~~ν̄e ⇒ ONLY WEAK~~

~~uss dds~~

~~us → dd~~

slide 20/21/22

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{g\pi}{E_e^2}\,\frac{\Gamma_{ee}\Gamma_{ff}}{(E_{cm}-M_Z)^2 + \Gamma_Z^2/4}$$

g: spin factor: make spin 1 Z from 2x spin ½ electrons

$$g = \frac{2J_Z+1}{(2J_e+1)(2J_e+1)} = \frac{3}{4}$$

E_e^2 collide $E_{cm}^2 = (E_1+E_2)^2 - (\vec{p}_1+\vec{p}_2)^2$ head on collisions $E_1 = E_2$

$$= 4E_e^2$$

$$\Rightarrow E_e^2 = E_{cm}^2/4$$

$$\sigma = \frac{3\pi}{E_{cm}^2}\,\frac{\Gamma_{ee}\Gamma_{ff}}{(E_{cm}-M_Z)^2 + \Gamma_Z^2/4}$$

at $E_{cm} = M_Z$ $\quad \sigma = \frac{3\pi}{M_Z^2}\,\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2/4}$

Γ_Z is the total width $= \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{qq} + \Gamma_{\nu\nu}$

$\therefore \sigma(e^+e^- \to Z \to \text{anything}) = \frac{12\pi}{M_Z^2}\,\frac{\Gamma_{ee}}{\Gamma_Z}$ $\qquad \Gamma_{ff} = \Gamma_Z$

compare to

$\sigma(e^+e^- \to \gamma \to \text{anything}) = \frac{4\pi\alpha^2}{3E_{cm}^2} = \frac{4\pi\alpha^2}{3M_Z^2}$ $\qquad E_{cm} = M_Z$

$\Gamma_{ee} = 85\,\text{MeV}$

$\Gamma_Z = 2.5\,\text{GeV}$

$\alpha = 1/137$

$\frac{\sigma_Z}{\sigma_{QED}} \sim 5700$

Z diagram dominates at $\sim M_Z = \sqrt{s}$

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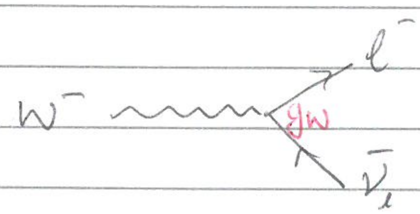

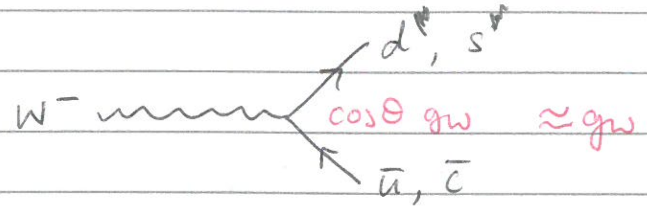

Assume $\cos\theta = 1$: only Cabibbo allowed decays.

Expect $W^- \to e^- \bar{\nu}_e$
$\mu^- \bar{\nu}_\mu$
$\tau^- \bar{\nu}_\tau$
$\bar{u}$ d ×3 for color
$\bar{c}$ s ×3 "

} Should have ~ same coupling

Cabibbo suppressed final states also contribute in a small way

$W^- \to \ell\nu = \frac{3}{9} = \frac{1}{3}$ $\quad$ $W \to q\bar{q} = \frac{6}{9} = \frac{2}{3}$

$W^+W^- \to \ell\nu\,\ell\nu \qquad \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\to \ell\nu\, qq \qquad \frac{1}{3} \times \frac{2}{3} \times 2 = 4/9$

$\to qqqq \qquad \frac{2}{3} \times \frac{2}{3} = 4/9$