SECTION 8

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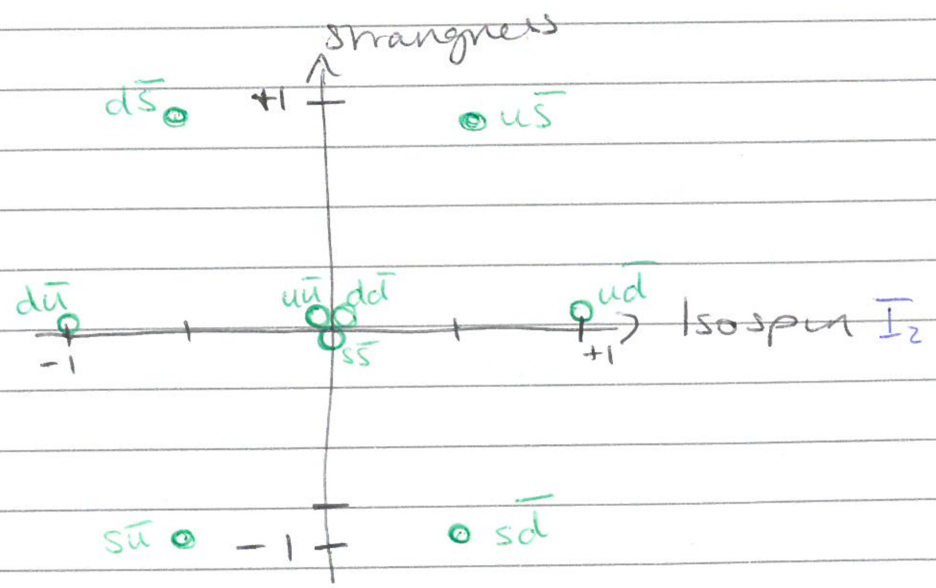
$q\bar{q}$ combinations

	Isospin I_z $= \frac{1}{2}(n_u - n_d - n_{\bar{u}} + n_{\bar{d}})$	Strangeness $= n_{\bar{s}} - n_s$
$u\bar{u}$	0	0
$u\bar{d}$	+1	0
$u\bar{s}$	+½	+1
$d\bar{d}$	0	0
$d\bar{u}$	−1	0
$d\bar{s}$	$-\frac{1}{2}$	+1
$s\bar{s}$	0	0
$s\bar{u}$	$-\frac{1}{2}$	−1
$s\bar{d}$	$+\frac{1}{2}$	−1



CONCEPT OF ISOSPIN SETS

Pion triplet: π^+ $I_z = +1$, π^0 $I_z = 0$, π^- $I_z = -1$ } $I = 1$

Kaon 2x doublets: K^{+}_{0} with $I_z = \pm\frac{1}{2}$, $\bar{K}^{0}$ with $I_z = \pm\frac{1}{2}$ } $I = \frac{1}{2}$

Eta 2x singlets: η and η' with $I_z = 0$ and $I = 0$.

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$$\rho^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}} \qquad \omega^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

$\mathcal{M}(\rho_0 \to ee)$

1 diagram with $u\bar{u}$

" $d\bar{d}$

appropriately weighted

$\mathcal{M}(\omega^0 \to ee)$

same diagrams

$$\mathcal{M} \propto \frac{Q_u - Q_d}{\sqrt{2}} \qquad \mathcal{M} \propto \frac{Q_u + Q_d}{\sqrt{2}}$$

$$\Gamma \propto \mathcal{M}^2 = \left(\frac{\frac{2}{3} - -\frac{1}{3}}{\sqrt{2}}\right)^2 \sim \frac{1}{2} \qquad \Gamma \propto \mathcal{M}^2 \sim \left(\frac{\frac{2}{3} + -\frac{1}{3}}{\sqrt{2}}\right)^2 \sim \frac{1}{18}$$

Similarly for $\phi = s\bar{s}$ $\quad \mathcal{M} \propto Q_s \quad \Gamma \sim \left(-\frac{1}{3}\right)^2 \sim \frac{1}{9}$

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$\vec{S} = \vec{S}_1 + \vec{S}_2$ square & rearrange

$$\vec{S}_1 . \vec{S}_2 = \frac{\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2}{2}$$

$$\vec{S}_1^2 = S_1(S_1+1) = \tfrac{1}{2}(\tfrac{1}{2}+1) = 3/4 \equiv \vec{S}_2^2$$

$$= \tfrac{1}{2}\vec{S}^2 - \tfrac{3}{4}$$

J=0 mesons $\vec{S}^2 = S(S+1) = 0$ $\quad \vec{S}_1\vec{S}_2 = -3/4$

J=1 mesons $\vec{S}^2 = \quad = 2$ $\quad \vec{S}_1\vec{S}_2 = +1/4$

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① must be antisymm. under exchange of 2 quarks (or violate PEP)

$$\Psi_{baryon} = \Psi_{space}\,\Psi_{flavour}\,\Psi_{spin}\,\Psi_{colour}$$

② simplify approach
Only consider $l=0$
ground states
symmetric

③ know this is asym from Ω^-
asymmetric

∴ must be
symmetric

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qqq state $J = \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2} = \frac{1}{2}$ or $\frac{3}{2}$

$J = 3/2$

$|\frac{3}{2}, \frac{3}{2}\rangle = \uparrow\uparrow\uparrow$ $\qquad |J, M_J\rangle$ notation

$|\frac{3}{2}, \frac{1}{2}\rangle$?

$|\frac{3}{2}, -\frac{1}{2}\rangle$?

$|\frac{3}{2}, -3/2\rangle = \downarrow\downarrow\downarrow$

use ladder operator on $|\frac{3}{2}, \frac{3}{2}\rangle$ to find $|\frac{3}{2}, \frac{1}{2}\rangle$

$$J_-|\tfrac{3}{2}, \tfrac{3}{2}\rangle = J_-(\uparrow\uparrow\uparrow)$$

$$\sqrt{\tfrac{3}{2}(\tfrac{3}{2}+1) - \tfrac{3}{2}(\tfrac{3}{2}-1)}\,|\tfrac{3}{2}, \tfrac{1}{2}\rangle = J_-\uparrow(\uparrow\uparrow) + \uparrow(J_-\uparrow)\uparrow + (\uparrow\uparrow)J_-\uparrow$$

$$\sqrt{3}\,|\tfrac{3}{2}, \tfrac{1}{2}\rangle = \downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow$$

$$\therefore |\tfrac{3}{2}, \tfrac{1}{2}\rangle = \tfrac{1}{\sqrt{3}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow)$$

similar for $|\frac{3}{2}, -\frac{1}{2}\rangle$ use J_+ on $|\frac{3}{2}, -\frac{3}{2}\rangle$

$$= \tfrac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow)$$

All symmetric under exchange of any two spins

ladder operator

$$J_-|j, m\rangle = \sqrt{j(j+1) - m(m-1)}\,|j, m-1\rangle$$

J_- $\qquad m+1 \qquad m+1$

$J=\frac{1}{2}$ $\quad|\frac{1}{2},\frac{1}{2}\rangle$ and $|\frac{1}{2},-\frac{1}{2}\rangle$ states

Case 1 Two quarks are in asymmetric state $\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)=|0,0\rangle$
3rd quark must be in $J=\frac{1}{2}$ state, $m_j=\pm\frac{1}{2}$

$|\frac{1}{2},\frac{1}{2}\rangle=\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$ $\qquad |\frac{1}{2},-\frac{1}{2}\rangle=\frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow-\downarrow\uparrow\downarrow)$

2 states asymmetric under exchange of $1\leftrightarrow 2$

Case 2 Two quarks are in symmetric state $\uparrow\uparrow=|1,1\rangle$
$\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)=|1,0\rangle$

for a $|\frac{1}{2},\frac{1}{2}\rangle$ baryon, there are two possible combinations
$\uparrow\uparrow$ with 3rd quark $m_j=-\frac{1}{2}$
$\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)$ with " " $+\frac{1}{2}$
→ take linear combination

$|\frac{1}{2},\frac{1}{2}\rangle=A\,|1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle+B\,|1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle$

use ladder state to find $|\frac{1}{2},\frac{3}{2}\rangle\equiv 0$ not allowed

$$J_+|\tfrac{1}{2},\tfrac{1}{2}\rangle=A\Big[\underbrace{(J_+|1,1\rangle)}_{0}|\tfrac{1}{2},-\tfrac{1}{2}\rangle+|1,1\rangle\underbrace{(J_+|\tfrac{1}{2},-\tfrac{1}{2}\rangle)}_{1\times|\frac{1}{2},\frac{1}{2}\rangle}\Big]$$
$$+B\Big[\underbrace{(J_+|1,0\rangle)}_{\sqrt{2}|1,1\rangle}|\tfrac{1}{2},\tfrac{1}{2}\rangle+|1,0\rangle\underbrace{(J_+|\tfrac{1}{2},\tfrac{1}{2}\rangle)}_{0}\Big]\equiv 0$$

$\rightarrow A=-\sqrt{2}B$
require $A^2+B^2=1$ $\quad\therefore A=\sqrt{\frac{2}{3}}\quad B=-\sqrt{\frac{1}{3}}$

$|\frac{1}{2},\frac{1}{2}\rangle=\sqrt{\frac{2}{3}}(\uparrow\uparrow\downarrow)+-\sqrt{\frac{1}{3}}\sqrt{\frac{1}{2}}(\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)$

$=\sqrt{\frac{1}{6}}(2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$

similarly $|\frac{1}{2},-\frac{1}{2}\rangle=\sqrt{\frac{1}{6}}(2\downarrow\downarrow\uparrow-\downarrow\uparrow\downarrow-\uparrow\downarrow\downarrow)$

2 states, symmetric under exchange $1\leftrightarrow 2$

$\boxed{\text{slide 20}}$

$\Psi_{spin}\Psi_{flavour}$ must be symmetric under exchange of any two quarks

uuu ddd sss $\Psi_{flavour}$ is symmetric under exchange of any two quarks $\Rightarrow$ only $J=3/2$ $\uparrow\uparrow\uparrow$ $3\times J=\frac{3}{2}$ states	uud uus ddu dds ssu ssd $\Psi_{flavour}$ is symmetric under exchange $1\leftrightarrow 2$ $\Rightarrow J=\frac{3}{2}$ or $\frac{1}{2}$ Ψ_{spin} symm $\uparrow\uparrow\uparrow$ or $(\uparrow\downarrow+\downarrow\uparrow)\uparrow$ $6\times J=\frac{3}{2}$ states $6\times J=\frac{1}{2}$ states	uds • If ud is flavour symmetric: $\frac{1}{\sqrt{2}}(ud+du)$ then Ψ_{spin} needs symm. under exchange $1\leftrightarrow 2$ $\Rightarrow J=\frac{3}{2}$ or $\frac{1}{2}$ $1\times J=\frac{3}{2}$ state $\uparrow\uparrow\uparrow$ $1\times J=\frac{1}{2}$ state $(\uparrow\downarrow+\downarrow\uparrow)\uparrow$ • If ud is flavour antisymmetric: $\frac{1}{\sqrt{2}}(ud-du)$ Ψ_{spin} must be antisymm for $1\leftrightarrow 2$ $\Rightarrow$ only $J=\frac{1}{2}$ $1\times J=\frac{1}{2}$ state $(\uparrow\downarrow-\downarrow\uparrow)\uparrow$

$\Rightarrow$ 10 $J=\frac{3}{2}$ baryons
8 $J=\frac{1}{2}$ baryons

slide 30 Magnetic moment of a proton

spin up proton $J = \frac{1}{2}$

$$\Psi^{p}_{spin} = \frac{1}{\sqrt{6}}\left(2\, u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow\right)$$

Magnetic moment

$$\mu^{p} = \langle \Psi_{spin} | \hat{\mu}_B | \Psi_{spin} \rangle$$

$$= \frac{1}{\sqrt{6}}\frac{1}{\sqrt{6}} \langle 2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \,|\, \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 \,|\, 2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow \rangle$$

$$= \frac{1}{6}\langle 2\uparrow\uparrow\downarrow | \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 | 2\uparrow\uparrow\downarrow \rangle \qquad \frac{1}{6}\left(4(\mu_1 + \mu_2 - \mu_3)\right)$$

$$+ \frac{1}{6}\langle -\uparrow\downarrow\uparrow | \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 | -\uparrow\downarrow\uparrow \rangle \qquad \frac{1}{6}\left(\mu_1 - \mu_2 + \mu_3\right)$$

$$+ \frac{1}{6}\langle -\downarrow\uparrow\uparrow | \hat{\mu}_1 + \hat{\mu}_2 + \hat{\mu}_3 | -\downarrow\uparrow\uparrow \rangle \qquad \frac{1}{6}\left(-\mu_1 + \mu_2 + \mu_3\right)$$

$$= \frac{1}{6}\left[4(\mu_1 + \mu_2) - 2\mu_3\right]$$

Can assume $m_u = m_d$ $\qquad \mu_1 = \mu_2 = \mu_u$, $\quad \mu_3 = \mu_d = -\frac{1}{2}\mu_u$

$$\mu^{p} = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

$$= \frac{3}{2}\mu_u = \frac{e\hbar}{2m_u} = \frac{m_p}{m_u}\mu_N$$

Nuclear magneton

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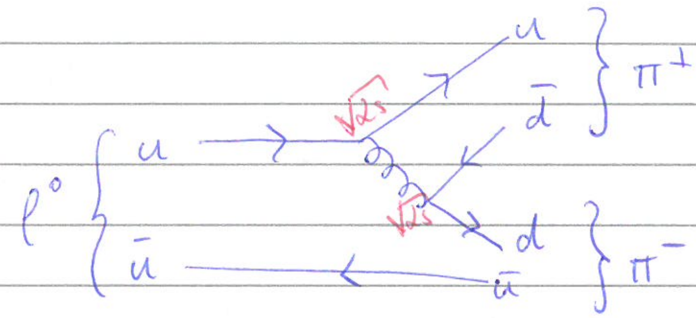
$\rho^0 \to \pi^+ \pi^-$

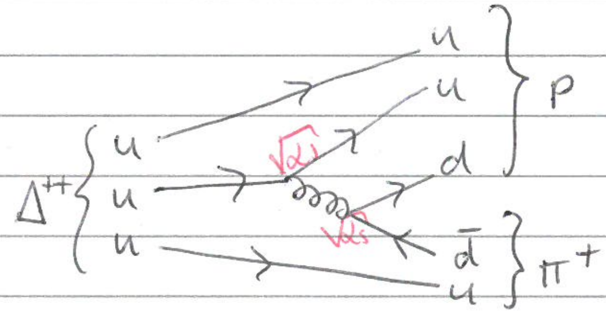


	Initial	Final	
J^P	1^-	0^- 0^-	
P	-1	$(-1)(-1)(-1)^L$	reminder $P_{q\bar{q}} = P_1 P_2 (-1)^L$
J	$+1 + 0$	$\lvert L \pm 0 \rvert$	reminder $J = \lvert L-S\rvert, \ldots, \lvert L+S\rvert$

assume ρ^0 is $L=0$

$\to L = 1$ To conserve J & P

$\Delta^{++} \to p\,\pi^+$



	Initial	Final	
J^P	$\frac{3}{2}^+$	$\frac{1}{2}^+$ 0^-	
P	$+1$	$(+1)(-1)(-1)^L$	
J	$0 + \frac{3}{2}$	$\lvert L - \frac{1}{2}\rvert, \ldots, \lvert L + \frac{1}{2}\rvert$	$\Rightarrow L = 1$

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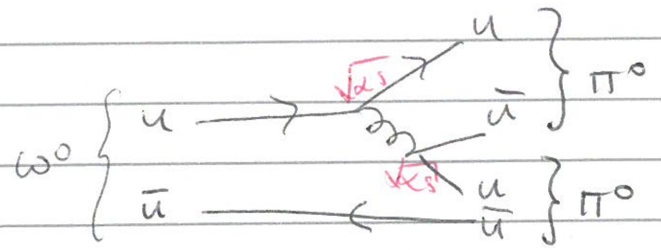
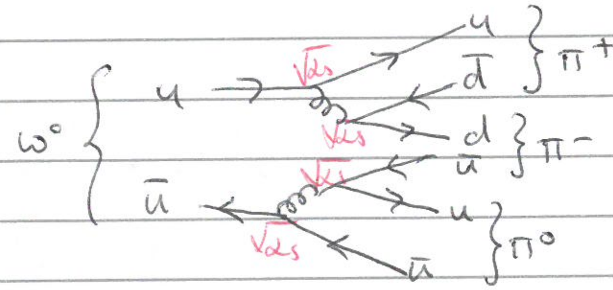
$\omega^0 \to \pi^0 \pi^0$



	Initial state	Final state	
J^P	1^-	$0^- \quad 0^-$	
P	-1	$(-1)(-1)(-1)^L$	
J	$0+1$	$\lvert L-0\rvert \ldots \lvert L+0\rvert$	$\Rightarrow L=1$

BUT identical bosons in final state
ψ must be even under exchange   need even L
$\Rightarrow$ forbidden decay

$\omega^0 \to \pi^+ \pi^- \pi^0$



	Initial	Final	
J^P	1^-	$0^- \quad 0^- \quad 0^-$	
P	-1	$(-1)(-1)(-1)(-1)^{L_{12}+L_3}$	
J	$0+1$	$\lvert L-0\rvert \ldots \lvert L+0\rvert$	$\Rightarrow L=L_{12}=L_3=1$

allowed decay ✓

BF ~ 90%

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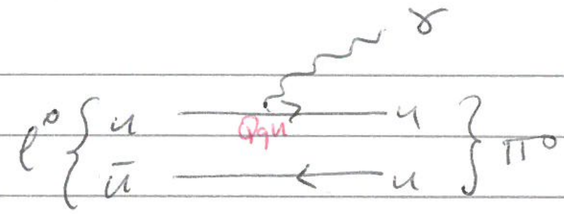
$\rho^0 \to \pi^0 \gamma$

BR ~ 8×10^{-4}

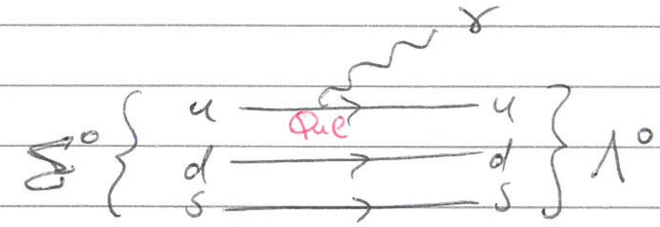


	Initial	Final	
J^P	1^-	$0^- \quad 1^-$	
P	-1	$(-1)(-1)(-1)^L$	$\to$ need $L = 1, 3 \ldots$
J	$0+1$	$\lvert L-1\rvert \ldots \lvert L+1\rvert$	$L=0$: $J=1$
			$L=1$ $J=0,1,2$

$\Rightarrow L=1$

$\Sigma^0 \to \Lambda^0 \gamma$

BR $\approx$ 100%



both uds & $\frac{1}{2}^+$: one state has $q_1 q_2$ in flavour symm state, other in antisymm state

	Initial	Final	
J^P	$\frac{1}{2}^+$	$\frac{1}{2}^+ \quad 1^-$	
P	$+1$	$(+1)(-1)(-1)^L$	need $L = 1, 3 \ldots$
J	$0+\frac{1}{2}$	$\lvert L-\frac{3}{2}\rvert \ldots \lvert L+\frac{3}{2}\rvert$	$L=1$ $J=\frac{1}{2} \ldots \frac{5}{2}$

$\Rightarrow L=1$

? check both $\frac{1}{2}^+$

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$c\bar{c}$ bound state
2 fermions $\therefore$ $S = 0, 1$

Must have same J^P as γ if produced in e^+e^- collisions
$\therefore J^P = 1^-$

can make other states from hadron colliders

Principal Quantum Number n	L ($0, \ldots, n-1$)	P ($(-1)(-1)^L$)	S	J^P ($J = \|L-S\| \ldots \|L+S\|$)
1	0	-1	0	0^-
			1	1^-
2	0	-1	0	0^-
			1	1^-
	1	+1	0	1^+
			1	2^+

only 1^- can be produced in e^+e^- colliders

All these states have different mass
$\therefore$ decay differently

- strong
- EM to lower energy states

$\Rightarrow \underline{R(e^+e^- \to \text{hadrons})}_{\mu\mu}$ is affected