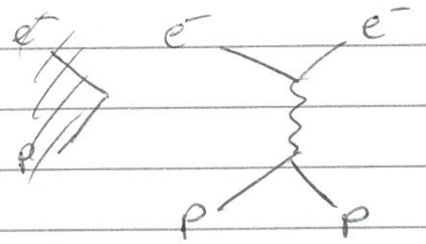
SECTION 6

slide 8



$e^- p$ scattering

- Matrix element $\mathcal{M} = \dfrac{e^2}{q^2} = \dfrac{4\pi\alpha}{q^2}$

- $q^2 = (E_f^e - E_i^e)^2 - (\vec{p}_f^{\,e} - \vec{p}_i^{\,e})^2$ t-channel

$$= E_f^2 + E_i^2 - 2E_iE_f - \vec{p}_f^{\,2} - \vec{p}_i^{\,2} + 2\vec{p}_f\vec{p}_i$$
$$= m_f^2 + m_i^2 - 2E_iE_f + 2|p_f||p_i|\cos\theta$$

assume $m_e = 0$ $\therefore E = \vec{p}$

$$q^2 = -2E_iE_f(1-\cos\theta)$$
$$= -4E_iE_f\sin^2\frac{\theta}{2}$$

- $$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}|\mathcal{M}|^2 = \frac{E^2}{(2\pi)^2}\,\frac{16\pi^2\alpha^2}{16E_i^2E_f^2\sin^4\theta/2}$$

$$= \frac{\alpha^2}{4E^2\sin^4\theta/2}$$

QED gives same result as QM
(Rutherford scattering)

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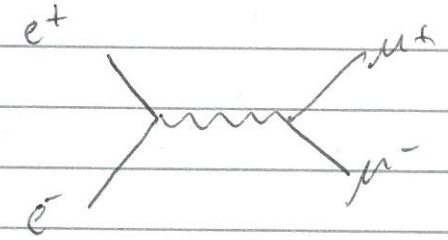

e^+e^- scattering

- Matrix element $\mathcal{M} = \dfrac{e^2}{q^2} = \dfrac{4\pi\alpha}{q^2}$

- $q^2 = (E_+ + E_-)^2 - (\vec{p}_+ + \vec{p}_-)^2$ s-channel

assume cm system $E_+ = E_- = E$ $\vec{p}_+ = -\vec{p}_-$

$$q^2 = 4E^2 \equiv s$$

- $$\frac{d\sigma}{d\Omega} = \frac{E^2}{(2\pi)^2}|\mathcal{M}|^2 = \frac{E^2}{(2\pi)^2}\,\frac{16\pi^2\alpha^2}{16E^4} = \frac{\alpha^2}{4E^2} \equiv \frac{\alpha^2}{s}$$

If spin is taken into account (Dirac eqn)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

$$\sigma = \frac{4\pi\alpha^2}{3s}$$

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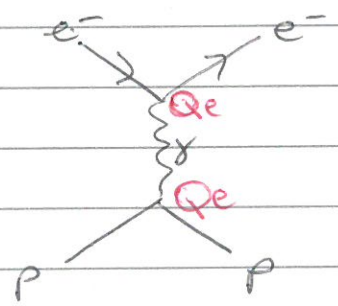
e^-p scattering vs e^+p scattering

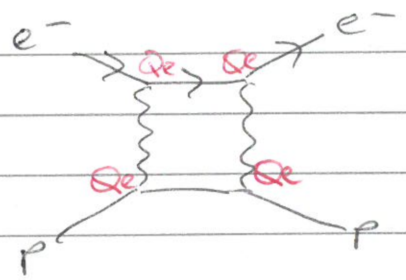
First order



e^-p scattering
$$\mathcal{M} \propto (-Qe)(+Qe) \propto -e^2$$

e^+p scattering
$$\mathcal{M} \propto (+Qe)(+Qe) \propto e^2$$

Higher order



e^-p
$$\mathcal{M} \propto (-Qe)^2(+Qe)^2 = e^4$$

e^+p
$$\mathcal{M} \propto (+Qe)^2(+Qe)^2 = e^4$$

$$\mathcal{M}_{TOT} = \sum \mathcal{M}$$

$$\sigma \propto |\mathcal{M}|^2$$

$\therefore$ difference in e^-p & e^+p scattering due to interference from higher order terms