WORKED EXAMPES

Section 2

slide 10 — 2 body invariant mass equation

$$X \to 1 + 2$$

$$M_X^2 = \left(\sum E\right)^2 - \left(\sum \vec{p}\right)^2$$

$$= (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$

$$= E_1^2 + E_2^2 + 2E_1E_2 - \vec{p}_1^{\,2} - \vec{p}_2^{\,2} - 2\vec{p}_1\vec{p}_2$$

$m^2 = E^2 - \vec{p}^{\,2}$

$$= m_1^2 + m_2^2 + 2(E_1E_2 - \vec{p}_1\vec{p}_2)$$

$\vec{p}_1\vec{p}_2 = |\vec{p}_1||\vec{p}_2|\cos\theta$

$$= m_1^2 + m_2^2 + 2\left(E_1E_2 - |\vec{p}_1||\vec{p}_2|\cos\theta\right)$$

$$= E_{cm}^2 = s$$

slide 11 — Pion decay $\pi^- \to \mu^- \bar{\nu}_\mu$ π at rest

$$\bar{\nu}_\mu \longleftarrow \overset{\circ}{\pi^-} \longrightarrow \mu^-$$

Conservation of energy $E_\pi = E_\mu + E_\nu$

momentum $0 = \vec{p}_\mu + \vec{p}_\nu$

$E_\pi = m_\pi$ since $\vec{p}_\pi = 0$ $\qquad m_\pi = E_\mu + E_\nu$

$\vec{p}_\nu = E_\nu$ since $m_\nu = 0$ $\qquad (m_\pi - E_\mu)^2 = E_\nu^2 = |\vec{p}_\nu|^2 = |\vec{p}_\mu|^2$

$$m_\pi^2 + E_\mu^2 - 2m_\pi E_\mu = \vec{p}_\mu^{\,2} \qquad E_\mu^2 - \vec{p}_\mu^{\,2} = m_\mu$$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}$$

$$= \frac{(140\,\text{MeV})^2 + (106\,\text{MeV})^2}{2(140\,\text{MeV})}$$

$$= 110\,\text{MeV}$$

$$|\vec{p}_\mu| = \sqrt{E_\mu^2 - m_\mu^2} = \sqrt{(110\,\text{MeV})^2 - (106\,\text{MeV})^2}$$

$$= 30\,\text{MeV}$$

$$|\vec{p}_\nu| = |\vec{p}_\mu| = 30\,\text{MeV}$$

slide 40

Breit-Wigner $\sigma = \frac{g\pi}{p_i^2} \frac{\Gamma_{i\to 2}\Gamma_{2-f}}{(E-E_0)^2 + \Gamma^2/4}$

At resonance $E=E_0$

Total σ $= \frac{g\pi}{p_i^2} \frac{4\Gamma_{\pi p}}{\Gamma}$

Elastic σ $= \frac{g\pi}{p_i^2} \frac{4\Gamma_{\pi p}^2}{\Gamma^2}$

$\frac{\sigma_{elastic}}{\sigma_{total}} = \frac{\Gamma_{\pi p}}{\Gamma}$

subs into $\sigma_{total} = \frac{g\pi \, 4\sigma_{elastic}}{p_i^2 \, \sigma_{total}}$

$\Rightarrow g = \frac{p_i^2}{4\pi} \frac{\sigma_{total}^2}{\sigma_{elastic}}$

$P_{lab} = 0.3$ GeV   from graph, $E_{cm} \sim 1.25$ GeV

in cm fram $E_{cm} = E_1 + E_2 = \sqrt{m_1^2 + \vec{p}_{cm}^{\,2}} + \sqrt{m_2^2 + \vec{p}_{cm}^{\,2}}$
solve

$\Rightarrow \vec{p}_{cm} \sim 0.24$ GeV

~~$\therefore g = \frac{1}{4\pi} \frac{\sigma_{total}^2}{\sigma_{elastic}} = \frac{(72\text{ mb})^2}{28\text{ mb}} = 185\text{ mb} = 0.185\text{ b} = 0.185 \times 10^{-28}\text{ m}^2$~~

$$\vec{p}_{cm} = \sqrt{\frac{(E_{cm}^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2}{4E_{cm}^2}}$$

$g = \frac{p_{cm}^2}{4\pi} \frac{\sigma_{TOTAL}^2}{\sigma_{el}} = \frac{(0.24\text{GeV})^2}{4\pi}\left(\frac{72^2}{28}\right) \times 10^{-3} \times 10^{-28} \times \frac{1}{3.89\times10^{-32}}$ GeV^{-2}

mb   to m^2 to GeV^{-2} $(\hbar c)^2$

$= 2.2 \quad \sim 2$

now use g to determine J

$g = \frac{(2J+1)}{(2J_\pi+1)(2J_p+1)}$ $J_\pi = 0$ $J_p = \frac{1}{2}$

$\Rightarrow J = \frac{3}{2}$ resonance is actually a udd state