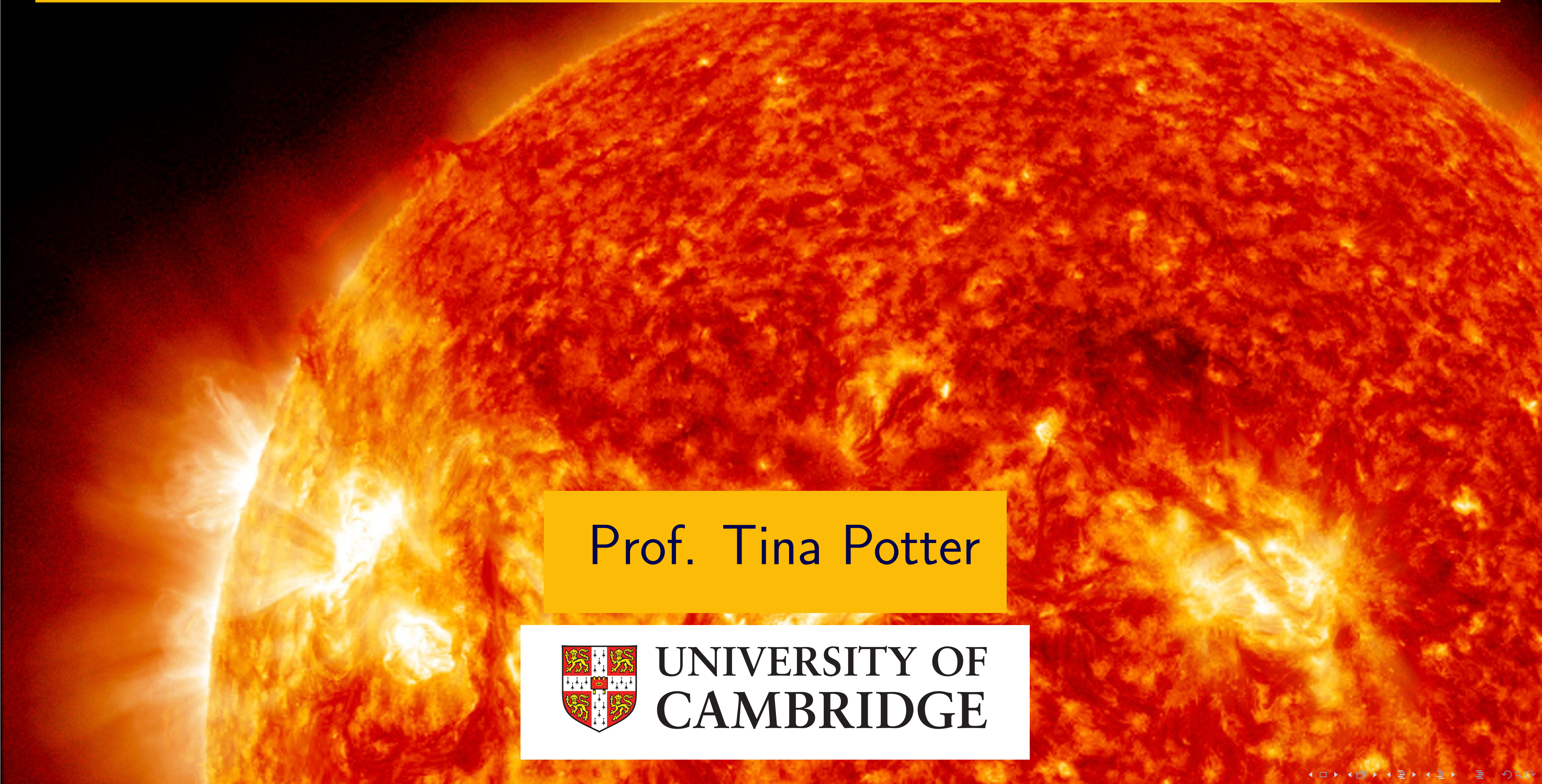
16. Fission and Fusion

Particle and Nuclear Physics



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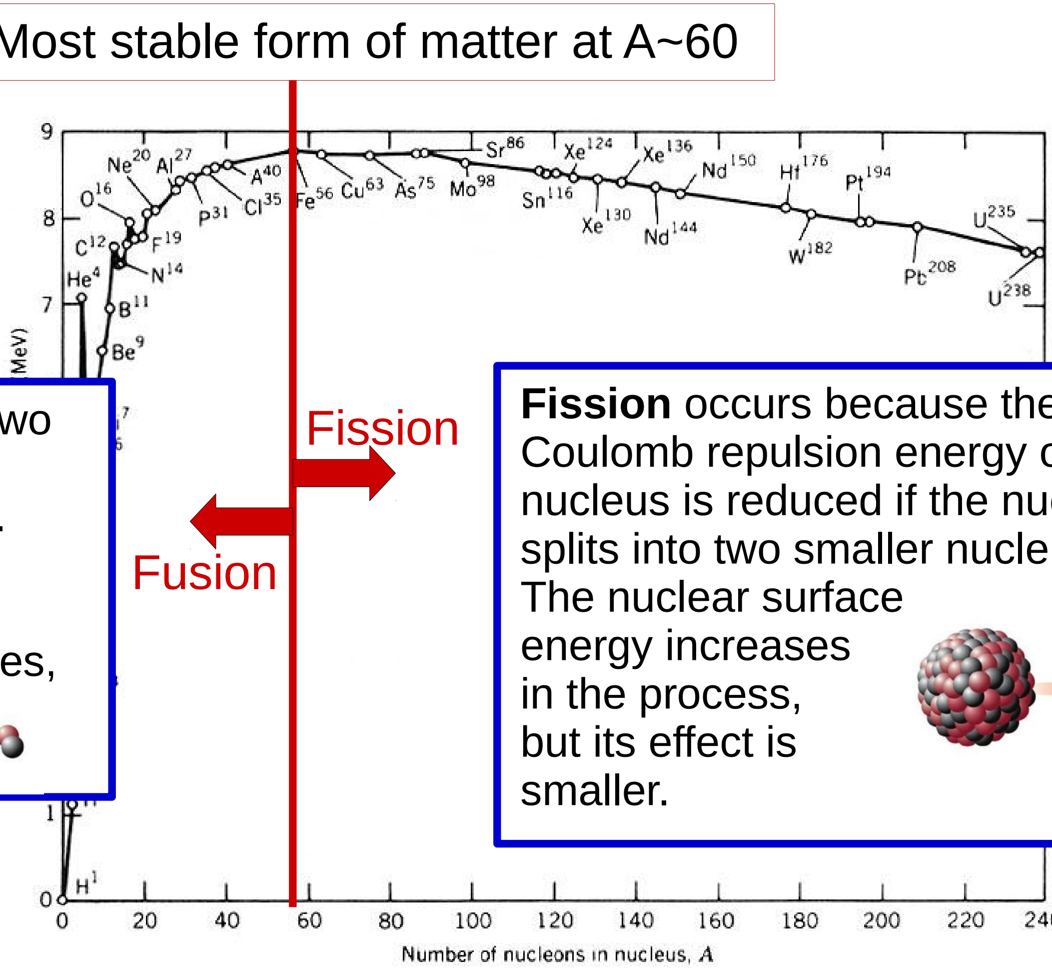
In this section...

- Fission
- Reactors
- Fusion
- Nucleosynthesis
- Solar neutrinos

Fission and Fusion



Fusion occurs because the two low A nuclei have too large a surface area for their volume. The surface area decreases when they amalgamate. The Coulomb energy increases, but its influence is smaller.

Fission occurs because the total Coulomb repulsion energy of p's in a nucleus is reduced if the nucleus splits into two smaller nuclei. The nuclear surface energy increases in the process, but its effect is smaller.

Expect a large amount of energy released in the fission of a heavy nucleus into two medium-sized nuclei or in the fusion of two light nuclei into a single medium nucleus.

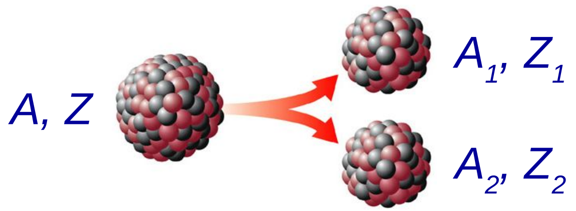
SEMF
$$B(A,Z) = a_V A - a_S A^{2/3} - \frac{a_C Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} + \delta(A)$$

Spontaneous Fission

Expect spontaneous fission to occur if energy released

$$E_0 = B(A_1, Z_1) + B(A_2, Z_2) - B(A, Z) > 0$$

Assume nucleus divides as



where $\frac{A_1}{A} = \frac{Z_1}{Z} = y$ and $\frac{A_2}{A} = \frac{Z_2}{Z} = 1 - y$

from SEMF $\quad E_0 = a_S A^{2/3}(1 - y^{2/3} - (1-y)^{2/3}) + a_C \dfrac{Z^2}{A^{1/3}}(1 - y^{5/3} - (1-y)^{5/3})$

maximum energy released when $\frac{\partial E_0}{\partial y} = 0$

$$\frac{\partial E_0}{\partial y} = a_S A^{2/3}\left(-\frac{2}{3}y^{-1/3} + \frac{2}{3}(1-y)^{-1/3}\right) + a_C \frac{Z^2}{A^{1/3}}\left(-\frac{5}{3}y^{2/3} + \frac{5}{3}(1-y)^{2/3}\right) = 0$$

solution $y = 1/2 \quad \Rightarrow$ Symmetric fission

max. $E_0 = 0.37 a_C \frac{Z^2}{A^{1/3}} - 0.26 a_S A^{2/3}$

e.g. $^{238}_{92}$U: maximum $E_0 \sim 200\ \mathrm{MeV}$ $\quad (a_S = 18.0\ \mathrm{MeV}, \quad a_C = 0.72\ \mathrm{MeV})$

$\sim 10^6 \times$ energy released in chemical reaction!

Fission Barrier

In the fission process, nuclei have to pass through an intermediate state where the surface energy is increased, but where the Coulomb energy is not yet much reduced.

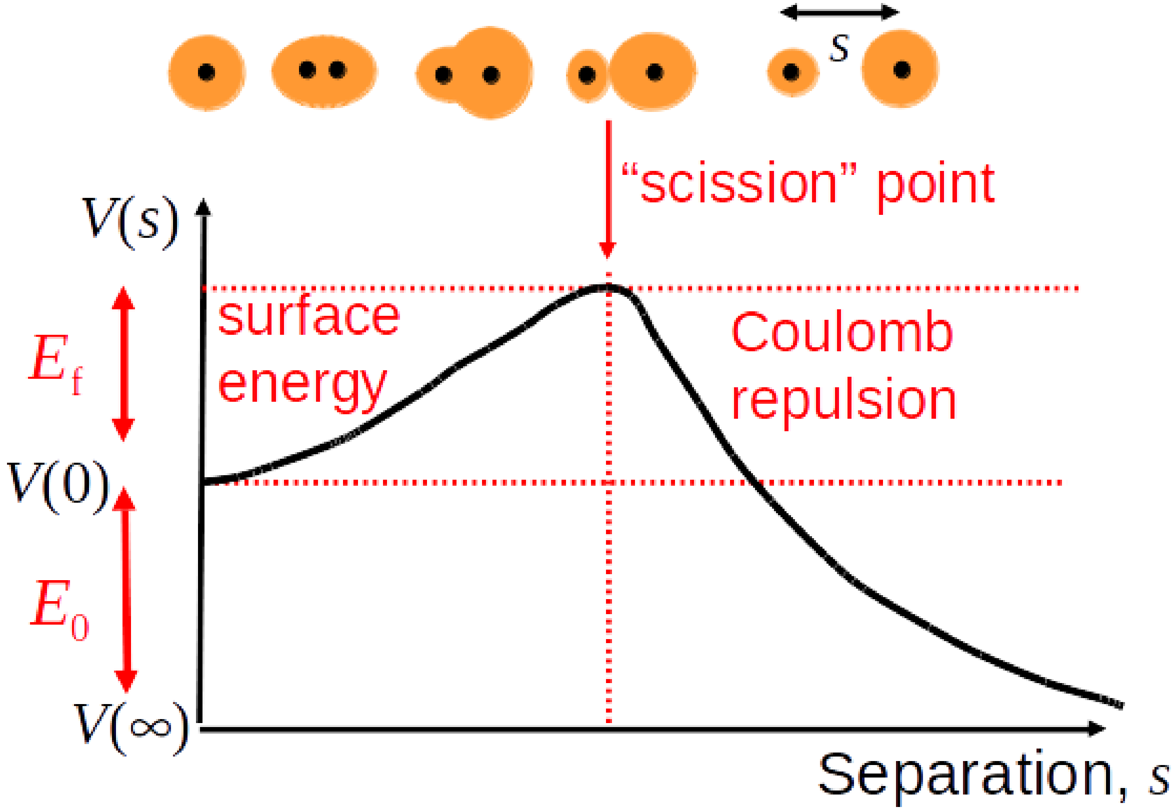
This is a tunnelling problem, similar to α decay.

E_f = fission activation energy

$E_\mathrm{f} \sim 6\ \mathrm{MeV}\ {}^{236}_{92}\mathrm{U}$

E_0 = energy released

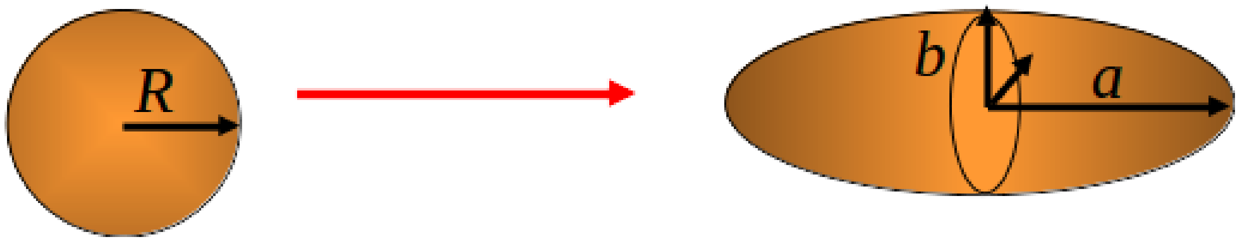
$\rightarrow$ K.E. of fragments.



Although E_0 is maximal for symmetric fission, so is the Coulomb barrier.
In fact, asymmetric fission is the norm.

Fission Barrier

Estimate mass at which nuclei become unstable to fission (i.e. point at which energy change due to ellipsoidal deformation gives a change in binding energy, $\Delta B > 0$)



$$a = R(1+\epsilon) \qquad \epsilon \ll 1$$
$$b = R(1+\epsilon)^{-1/2}$$

SEMF Volume term unchanged: $\quad \mathrm{Volume} = \mathrm{const} = \frac{4}{3}\pi ab^2 = \frac{4}{3}\pi R^3$

Change in Surface term: $\quad a_S A^{2/3} \longrightarrow a_S A^{2/3}(1 + \frac{2}{5}\epsilon^2)$

Change in Coulomb term: $\quad a_C \frac{Z^2}{A^{1/3}} \longrightarrow a_C \frac{Z^2}{A^{1/3}}(1 - \frac{\epsilon^2}{5})$

Not proved, just geometry

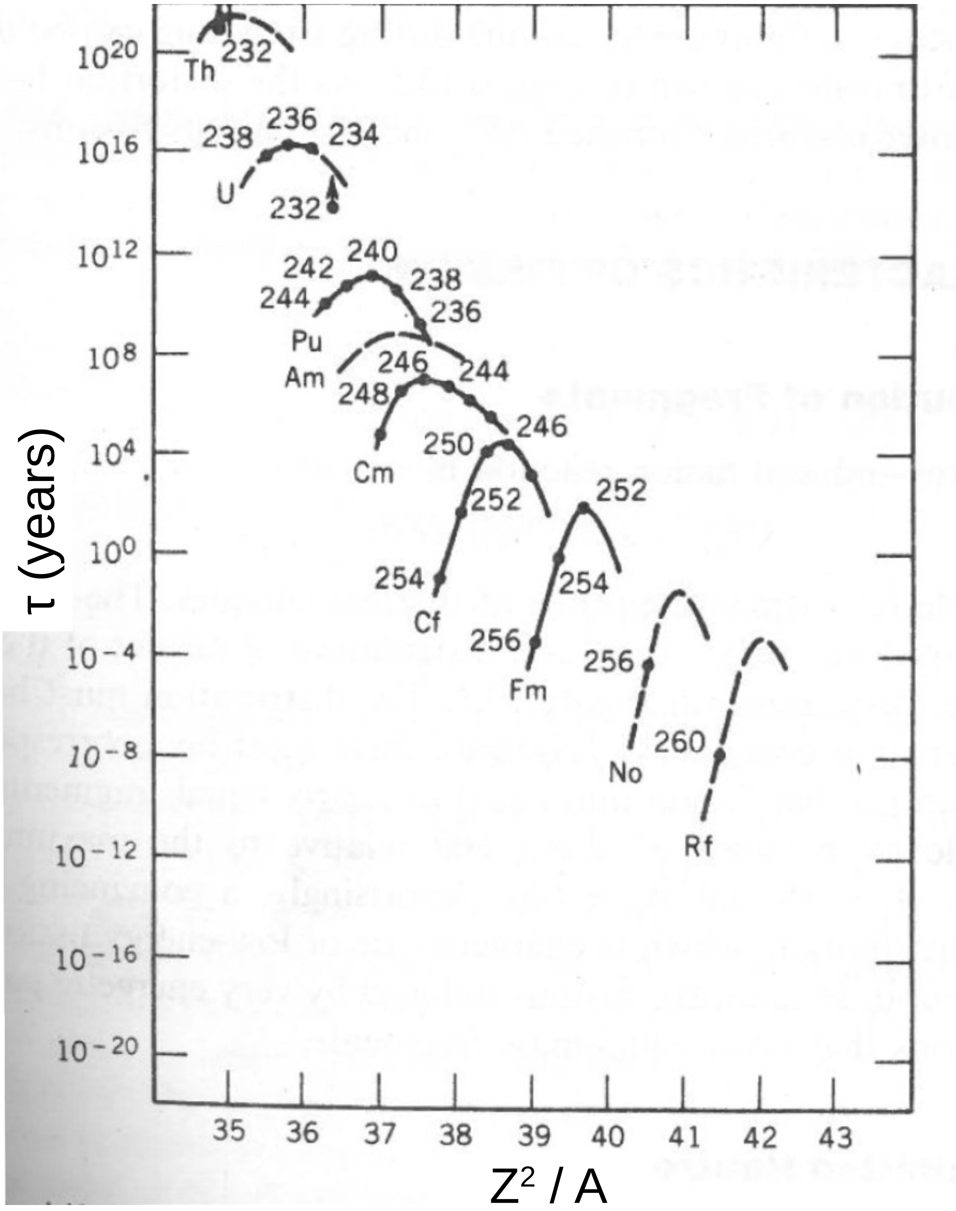
Change in Binding Energy: $\quad \Delta B = B(\epsilon) - B(0) = a_C A^{2/3}\left(\frac{Z^2}{A} - \frac{2a_S}{a_C}\right)\frac{\epsilon^2}{5}$

i.e. if $\frac{Z^2}{A} > \frac{2a_S}{a_C}$, then $\Delta B > 0$ and the nucleus unstable under deformation

$\Rightarrow \frac{Z^2}{A} > 47$ predicted point (roughly) at which the fission barrier vanishes.

Fission Barrier

And indeed we observe that spontaneous fission lifetimes fall rapidly as Z^2/A increases.



Fission Barrier

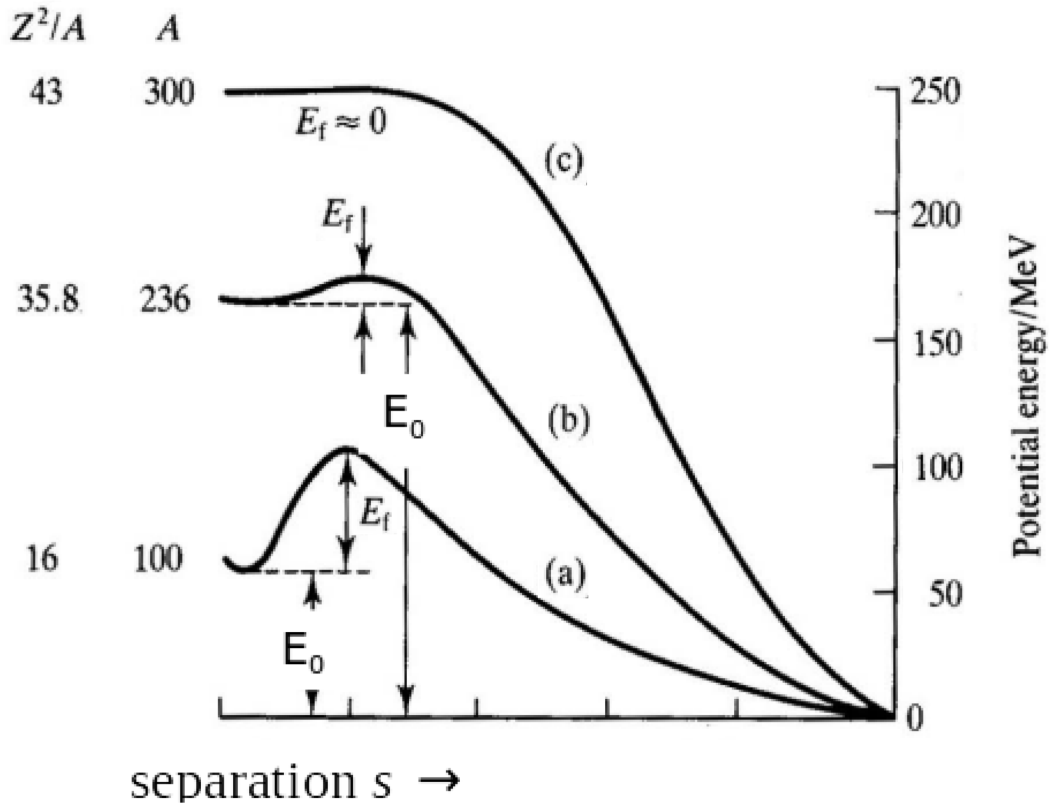
Spontaneous fission is possible if tunnelling through fission barrier occurs (c.f. α decay).

Tunnelling probability depends on height of barrier

$$E_f \propto \left(\frac{Z^2}{A}\right)^{-1}$$

and on the mass of fragment

$$P \propto e^{-2G}; \qquad G \propto m^{1/2}$$



Large mass fragment → low probability for tunnelling

e.g. fission is $\sim 10^6$ less probable than α decay for $^{238}_{92}$U

So there are naturally occurring spontaneously fissile nuclides, but it tends to be a rare decay.

Neutron Induced Fission *Low energy neutron capture*

Use neutrons to excite nuclei and overcome fission barrier.

Important for the design of thermonuclear reactors.

Low energy neutrons are easily absorbed by nuclei (no Coulomb barrier) $\rightarrow$ excited state.

Excited state may undergo $n + {}^{A}\mathrm{U} \rightarrow {}^{A+1}\mathrm{U}^* \rightarrow {}^{A+1}\mathrm{U} + \gamma$ or $\mathrm{X}^* + \mathrm{Y}^*$

γ **decay** (most likely): (n,γ) reaction

Fission (less likely): excitation energy may help to overcome E_f

(n,γ) reaction:

Breit-Wigner cross-section

$$\sigma(n,\gamma) = \frac{g\pi\bar{\lambda}^2\Gamma_n\Gamma_\gamma}{(E-E_0)^2 + \Gamma^2/4}, \qquad \Gamma_n \ll \Gamma_\gamma \sim \Gamma$$

At resonance

$$\sigma(n,\gamma) = 4\pi\bar{\lambda}^2 g\frac{\Gamma_n\Gamma_\gamma}{\Gamma^2} \sim 4\pi\bar{\lambda}^2 g\frac{\Gamma_n}{\Gamma}$$

Typically, $\Gamma_n \sim 10^{-1}$ eV, $\Gamma \sim 1$ eV; for 1 eV neutron, $\sigma \sim 10^3$ b (largest: ^{135}Xe $\sigma \sim 10^6$ b)

Far below resonance, $(E \ll E_0)$

$$\sigma(n,\gamma) = \bar{\lambda}^2\Gamma_n\left[\frac{g\pi\Gamma_\gamma}{E_0^2 + \Gamma^2/4}\right] = \bar{\lambda}^2\Gamma_n \times \mathrm{constant}$$

Γ_n dominated by phase space

$$\Gamma_n \sim \frac{p^2}{v} \sim v; \quad \bar{\lambda} = \frac{\hbar}{p} \rightarrow \bar{\lambda}^2 \sim \frac{1}{v^2}$$

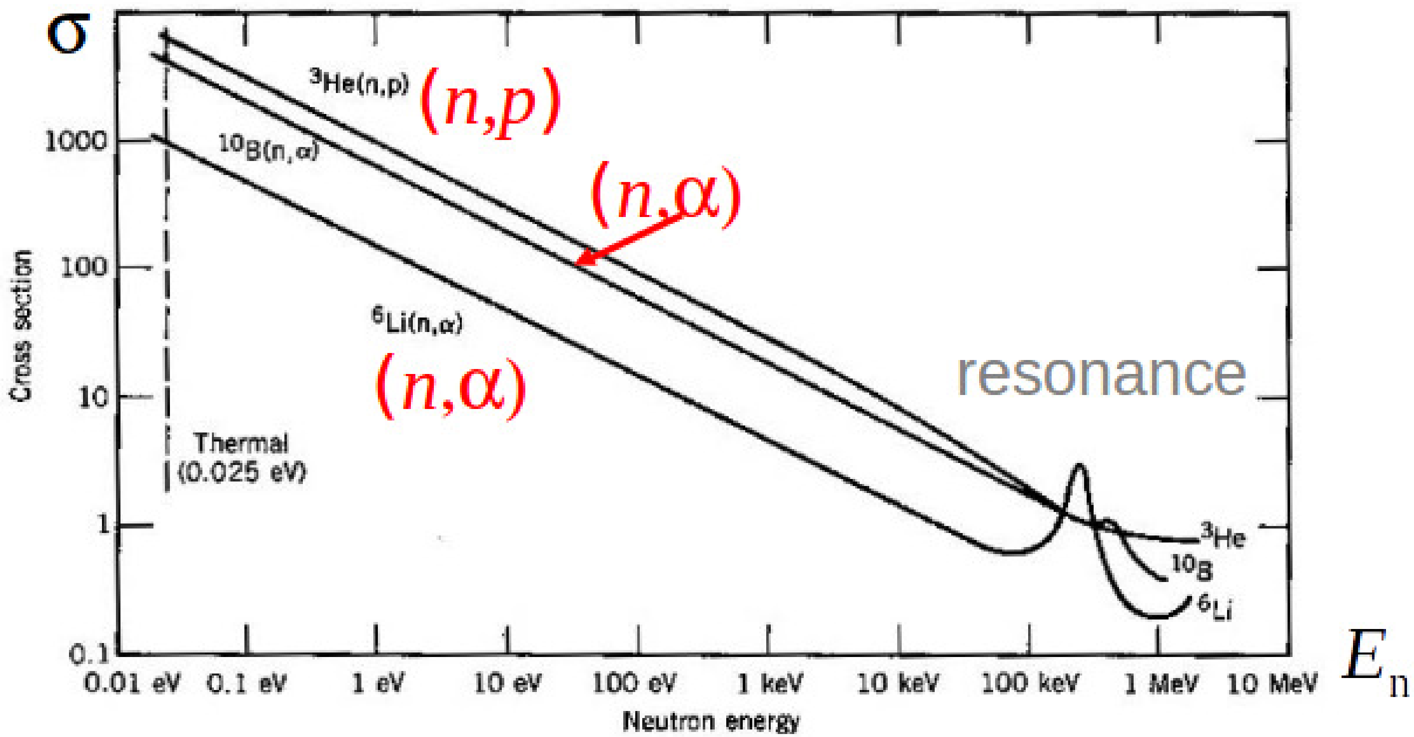
$\therefore\ \sigma(n,\gamma) \sim 1/v$

"$1/v$ law" (for low energy neutron reactions)

Neutron Induced Fission *Low energy neutron capture*

$\sigma \sim 1/v$ dependence far below resonances

$E \propto v^2 \;\Rightarrow \ln\sigma \propto -1/2 \ln E + \text{constant}.$



Low energy neutrons can have very large absorption cross-sections.

Neutron Induced Fission *Induced Fission*

Induced fission occurs when a nucleus captures a low energy neutron receiving enough energy to climb the fission barrier.

e.g. $^{235}_{92}$U $\quad n + {}^{235}_{92}\text{U} \rightarrow {}^{236}_{92}\text{U}^* \rightarrow \text{X}^* + \text{Y}^* \rightarrow \text{X} + \text{Y} + \kappa n$

$\kappa \sim 2.4$ prompt neutrons

Excitation energy of $^{236}\text{U}^* > E_f$ fission activation energy, hence fission occurs rapidly, even for zero energy neutrons

$\rightarrow$ thermal neutrons will induce fission.

Otherwise need to supply energy using K.E. of neutron.

e.g. $^{238}_{92}$U $\quad n + {}^{238}\text{U} \rightarrow {}^{239}\text{U}^* \qquad E_f \sim 6\ \text{MeV}$

$E_n = 0 \qquad E^* \sim 5\ \text{MeV}$ no thermal fission

$E_n = 1.4\ \text{MeV} \qquad E^* \sim 6.4\ \text{MeV}$ rapid fission

but neutron absorption cross-section decreases rapidly with energy.

^{235}U is the more interesting isotope for fission reactor (or bombs).

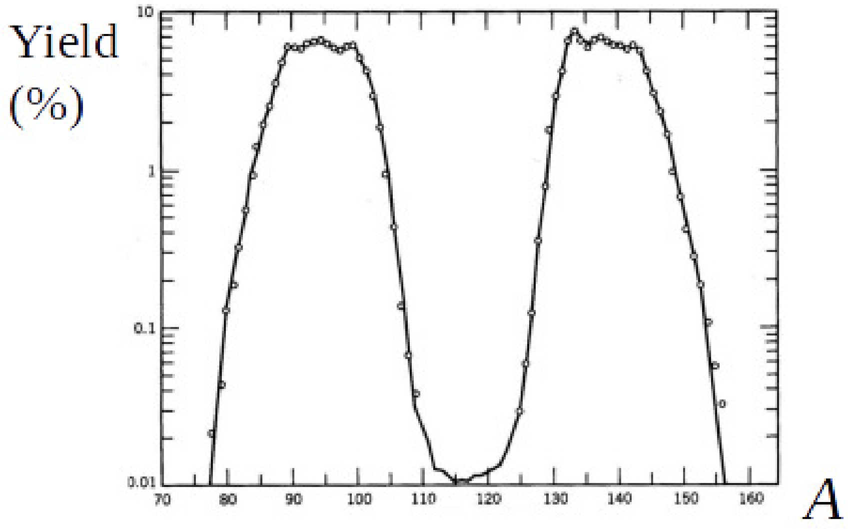
Neutron Induced Fission *Induced Fission*

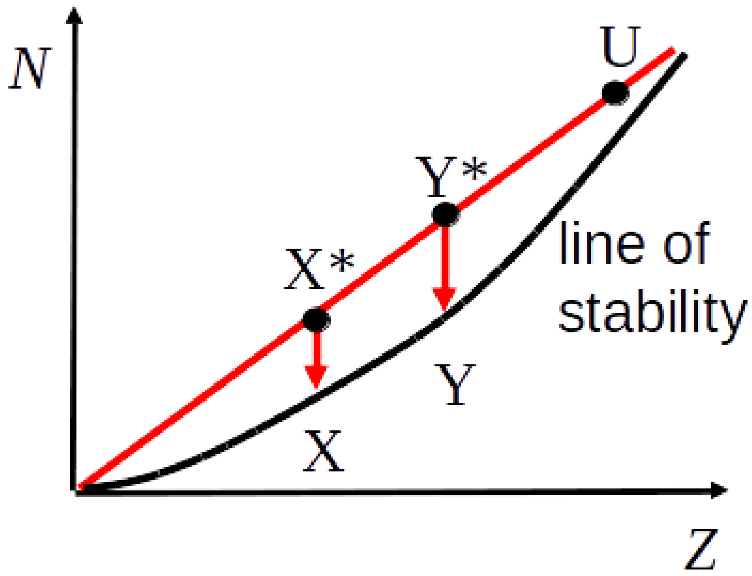
$$n + {}^{A}\mathrm{U} \rightarrow {}^{A+1}\mathbf{U}^* \rightarrow \mathrm{X}^* + \mathrm{Y}^*$$

Masses of fragments are unequal (in general). Tend to have Z, N near magic numbers.

Fragments X*, Y* tend to have same Z/N ratio as parent $\rightarrow$ neutron rich nuclei which emit prompt neutrons (10^{-16}s).

X and Y undergo β decay more slowly; may also undergo neutron emission
$\rightarrow$ delayed neutron emission
($\sim$1 delayed neutron per 100 fissions).





Note wide variety of (usually radioactive) nuclei are produced in fission; can be very useful, but potentially very nasty.

Neutron Induced Fission *Chain Reaction*

Neutrons from fission process can be used to induce further fission

$\rightarrow$ chain reaction, can be sustained if at least one neutron per fission induces another fission process.

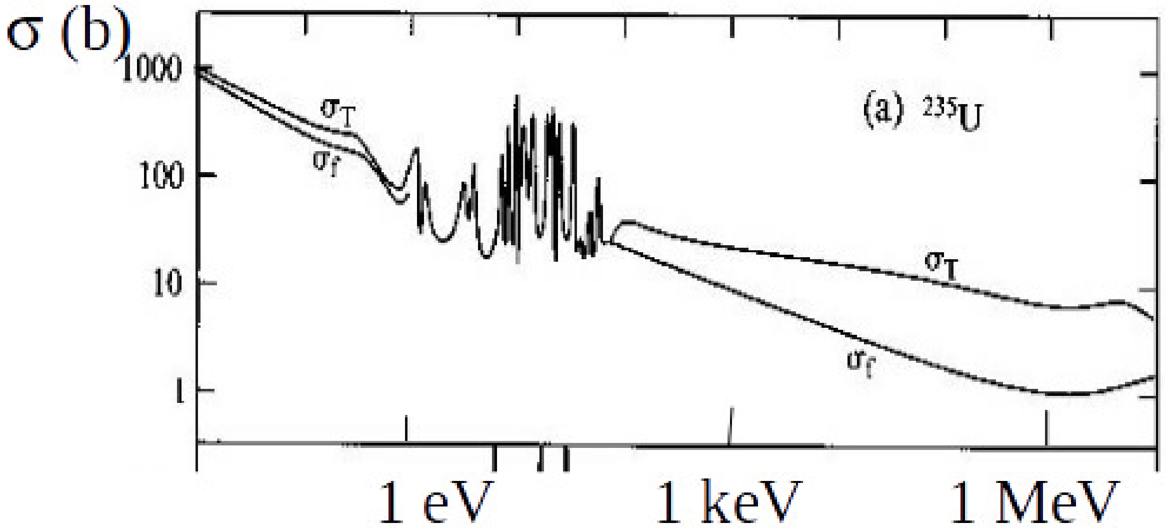
k = number of neutrons from one fission which induce another fission

$k < 1$ sub-critical,

$k = 1$ critical, $\leftarrow$ For reactors want a steady energy release, exactly critical

$k > 1$ super-critical.

Prompt neutrons are fast, $\langle E \rangle \sim 2\ \mathrm{MeV}$ and their absorption σ is small.
Need to slow down fast neutrons before they escape or get absorbed by (n,γ) process
$\rightarrow$ achieve a chain reaction.



Fission Reactors

Power reactor

e.g. Sizewell in Suffolk

KE of fission products $\rightarrow$ heat $\rightarrow$ electric power

Research reactor

e.g. ISIS at RAL in Oxfordshire

Beams of neutrons for (e.g.) condensed matter research

Breeder reactor

e.g. Springfields in Lanarkshire

Converts non-fissile to fissile isotopes, e.g.

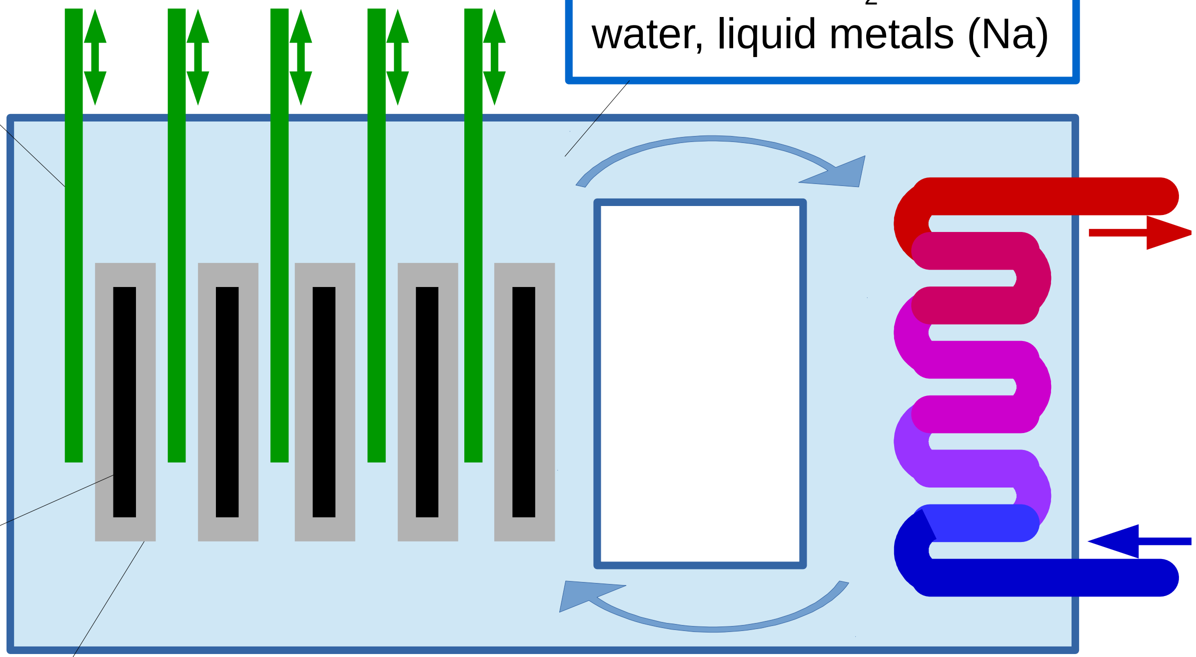
- Plutonium: $n + {}^{238}\mathrm{U} \rightarrow {}^{239}\mathrm{U} \rightarrow {}^{239}\mathrm{Np} \rightarrow {}^{239}\mathrm{Pu}$
- Uranium: $n + {}^{232}\mathrm{Th} \rightarrow {}^{233}\mathrm{Th} \rightarrow {}^{233}\mathrm{Pa} \rightarrow {}^{233}\mathrm{U}$
- Can separate fissile isotopes chemically

Fission Reactors

A simple reactor needs fuel, moderators, control rods, and a cooling system.

Cooling System:
gases (air, CO_2, He),
water, liquid metals (Na)

Control Rods: control number of neutrons by absorption (^{113}Cd)



Fuel:
Natural U (0.72% ^{235}U),
enriched U (2-3% ^{235}U),
^{239}Pu, ^{233}U

Moderator: slows neutrons via elastic collisions.
Large energy transfer requires use of a light nucleus.

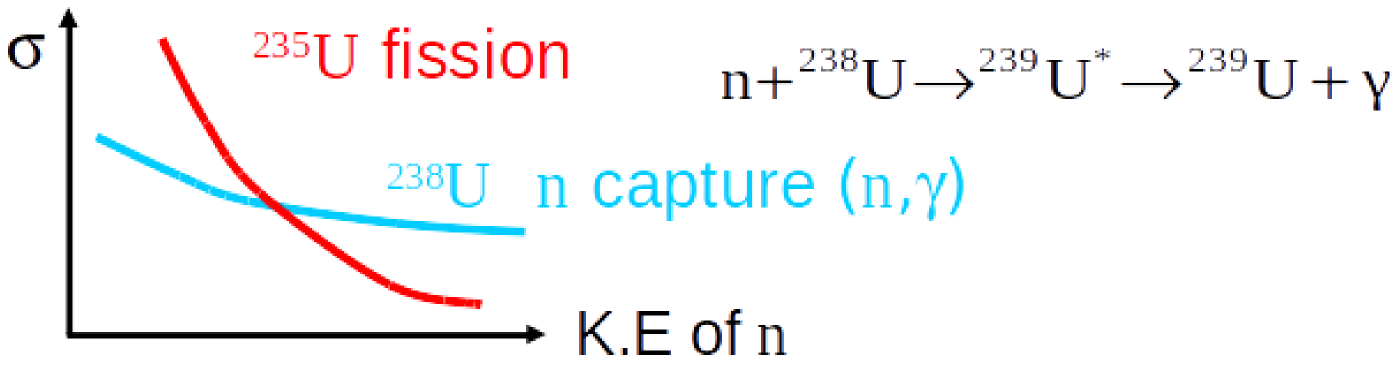
- H_2O – cheap but absorbs neutrons through $n+p \rightarrow d+\gamma$
- D_2O – extractable from seawater, but forms nasty radioactive tritium ^{3}H
- ^{13}C – graphite, larger mass → less energy transfer per collison → need more of it.

UK reactors are mainly graphite moderated, gas cooled.

Fission Reactors

The problem

Natural U is (99.3% ^{238}U, 0.7% ^{235}U) and *n* capture cross-section large for ^{238}U



Need to

1. thermalise fast neutrons away from ^{238}U to avoid capture (moderators)
2. control number of neutrons by absorption (control rods).

But

typical time between fission and daughter inducing another fission $\sim 10^{-3}$s

$\rightarrow$ mechanical control of rods in times $\ll$ seconds not possible!

Fission Reactors

The consequence – what happens if we fail to control the neutrons?

$$N(t+\mathrm{d}t) = N(t) + (k-1)N(t)\frac{\mathrm{d}t}{\tau}$$

$N(t)$ number of neutrons at time t
$(k-1)$ fractional change in number of neutrons in 1 cycle
τ mean time for one cycle $\sim 10^{-3}$s (fission $\rightarrow$ fission)

$$\mathrm{d}N = (k-1)N\frac{\mathrm{d}t}{\tau} \quad \Rightarrow \quad \int_{N(0)}^{N(t)} \frac{\mathrm{d}N}{N} = \int_0^t (k-1)\frac{\mathrm{d}t}{\tau} \quad \Rightarrow \quad N(t) = N(0)\mathrm{e}^{(k-1)t/\tau}$$

for $k > 1$ $\rightarrow$ exponential growth – bad!

e.g. $k = 1.01$, $\tau = 0.001$s, $t = 1$s

$$\frac{N(t)}{N(0)} = \mathrm{e}^{0.01/0.001} = \mathrm{e}^{10} \quad (\times 22,000 \text{ in } 1\text{s})$$

Note: Uranium reactor will not explode if it goes super-critical. As it heats up, K.E. of neutrons increases and fission cross-section drops. Reactor stabilises at a very high temperature $\Rightarrow$ MELTDOWN

Fission Reactors

The solution

Make use of delayed neutron emission (delay ∼13s).

Design reactor to be subcritical to prompt neutrons and use the delayed neutrons to take it to critical.

Thermal reactors require the following steps:

fission → fast neutrons

↓

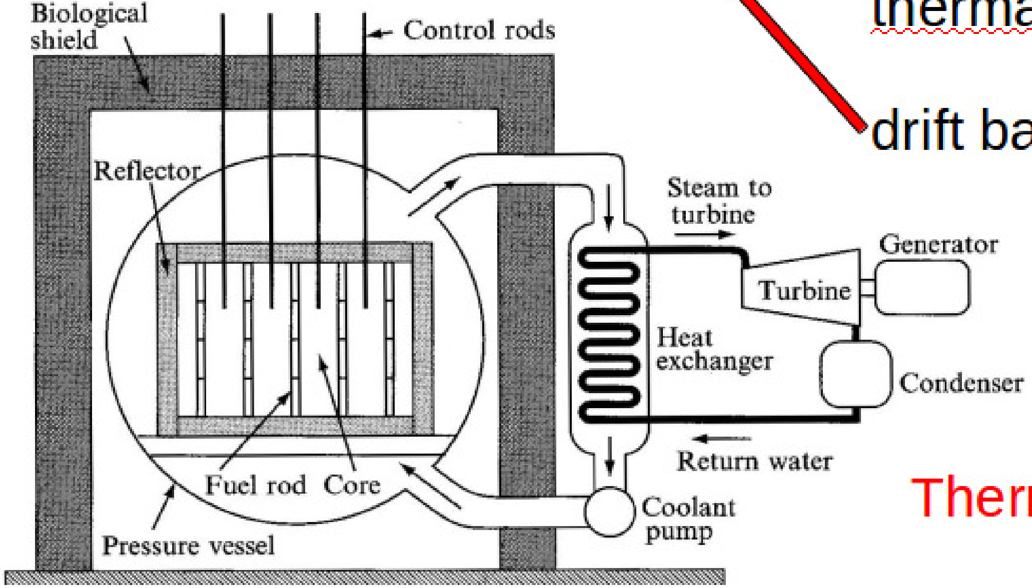
remove from fissile material

↓

thermalize them (gives heat)

↓

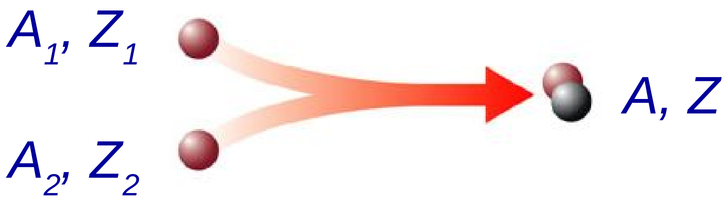
drift back into fuel → fission



Thermal Reactor

Nuclear Fusion

Energetically favourable for light nuclei to fuse and release energy.



However, nuclei need energy to overcome Coulomb barrier

e.g. most basic process: $\quad \text{p+p} \rightarrow \text{d} + e^+ + \nu_e, \quad E_0 = 0.42\ \mathrm{MeV}$

but Coulomb barrier $V = \dfrac{e^2}{4\pi\epsilon_0 R} = \dfrac{\alpha\hbar c}{R} = \dfrac{197}{137 \times 1.2} = 1.2\ \mathrm{MeV}$

Overcoming the Coulomb barrier

Accelerators: Energies above barrier easy to achieve. However, high particle densities for long periods of time very difficult. These would be required to get a useful rate of fusion reactions for power generation.

Stars: Large proton density $10^{32}\ \mathrm{m}^{-3}$. Particle K.E. due to thermal motion.

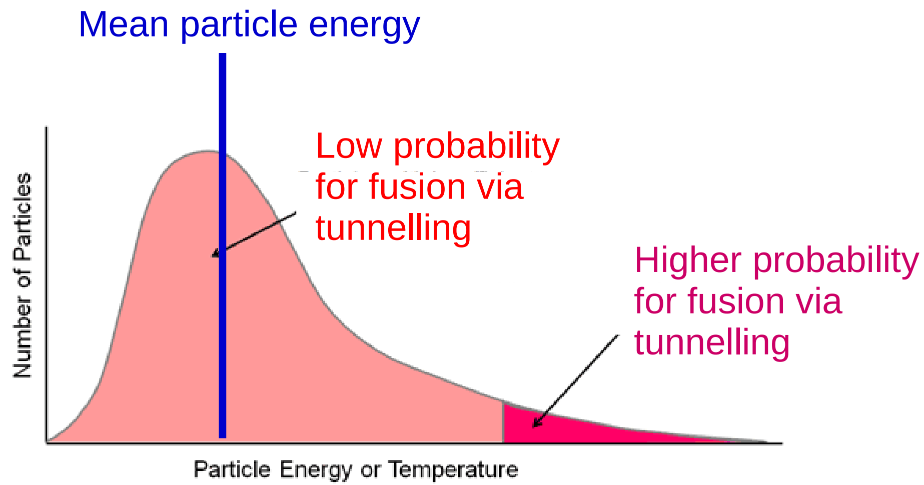
To achieve $kT \sim 1\ \mathrm{MeV}$, require $T \sim 10^{10}$K

Interior of Sun: $T \sim 10^7$K, i.e. $kT \sim 1\ \mathrm{keV}$

$\Rightarrow$ Quantum Mechanical tunnelling required.

Fusion in the Sun *Fusion rate in the Sun*

Particles in the Sun have Maxwell-Boltzmann velocity distribution with long tails – very important because tunnelling probability is a strong function of energy.



Reaction rate in unit volume for particles of velocity v: $\Gamma = \sigma(v)\Phi N$, where flux $\Phi = Nv$

σ is dominated by the tunnelling probability $P = \mathrm{e}^{-2G(v)}$

and a factor $1/v^2$ arising from the $\lambdabar^2$ in the Breit-Wigner formula.

reminder, Gamow Factor $G(v) \sim \left(\frac{2m}{E_0}\right)^{1/2} \frac{e^2}{4\pi\epsilon_0}\frac{Z_1 Z_2}{\hbar}\frac{\pi}{2} = \frac{e^2}{4\pi\epsilon_0}\frac{\pi Z_1 Z_2}{\hbar v}$

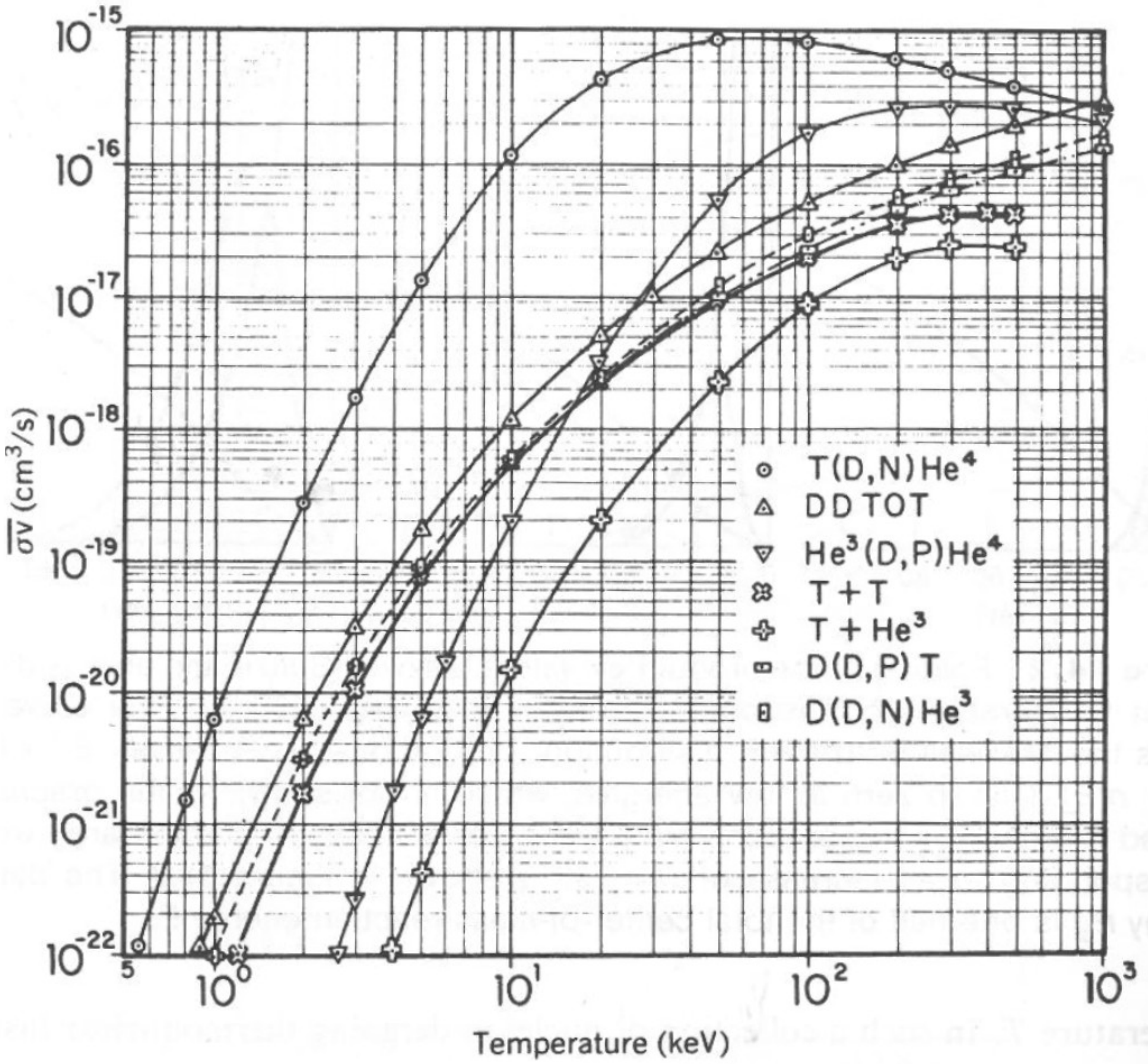
Averaged over the Maxwell-Boltzmann velocity distribution $\Gamma \sim N^2\langle\sigma v\rangle$

Probability velocity between v and $v + \mathrm{d}v = f(v)\,\mathrm{d}v \propto v^2\mathrm{e}^{-mv^2/2kT}\,\mathrm{d}v$

$$\Rightarrow \Gamma \propto \int N.Nv.\frac{1}{v^2}\mathrm{e}^{-2G}f(v)\,\mathrm{d}v \propto \int v\mathrm{e}^{-2G}\mathrm{e}^{-mv^2/2kT}\,\mathrm{d}v \propto \int \mathrm{e}^{-2G}\mathrm{e}^{-E/kT}\,\mathrm{d}E$$

Fusion in the Sun *Fusion rate in the Sun*

Typical fusion reactions peak at $kT \sim 100\ \mathrm{keV} \Rightarrow T \sim 10^9\mathrm{K}$



e.g. for $\mathrm{p+p} \to \mathrm{d} + e^+ + \nu_e$

$\sigma \sim 10^{-32}\mathrm{b}$ – tiny! weak!

but there are an awful lot of protons...

per proton, $\Gamma \sim 5 \times 10^{-18}\mathrm{s}^{-1}$

$\Rightarrow$ Mean life, $\tau = 10^{10}$ yrs.

This defines the burning rate in the Sun.

pp I chain

(1) $\boxed{p+p \rightarrow d+e^{+}+\nu}$ $E_0=0.42\,\mathrm{MeV}$ $\boxed{p+p \rightarrow d+e^{+}+\nu}$ $E_0=0.42\,\mathrm{MeV}$

(2) $\boxed{p+d \rightarrow {}^{3}\mathrm{He}+\gamma}$ $E_0=5.49\,\mathrm{MeV}$ $\boxed{p+d \rightarrow {}^{3}\mathrm{He}+\gamma}$ $E_0=5.49\,\mathrm{MeV}$

(3) $\boxed{{}^{3}\mathrm{He}+{}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He}+2p+\gamma}$ $E_0=12.86\,\mathrm{MeV}$

Net reaction ($2e^+$ annihilate with $2e^-$): $4p \rightarrow {}^4\mathrm{He} + 2e^+ + 2\nu$ $E_0 = 4m_e = 2.04\ \mathrm{MeV}$

Total energy release in fusion cycle $= 26.7\ \mathrm{MeV}$ (per proton $= 26.7/4 = 6.7\ \mathrm{MeV}$)

ν's emerge without further interaction with $\sim 2\%$ of the energy. The rest of the energy (γ-rays; KE of fission products) heats the core of the star.

Observed luminosity $\sim 4\times10^{26}$ J/s ($1\ \mathrm{MeV} = 1.6\times10^{-13}$ J)

$\Rightarrow$ Number of protons consumed $= \dfrac{4\times10^{26}}{1.6\times10^{-13}}\dfrac{1}{6.7} = 4\times10^{38}\ \mathrm{s}^{-1}$

Fusion in the Sun *Fusion processes in the Sun*

pp I chain

(1) $p+p \rightarrow d+e^{+}+\nu$ $E_0=0.42\,\mathrm{MeV}$ $p+p \rightarrow d+e^{+}+\nu$ $E_0=0.42\,\mathrm{MeV}$

(2) $p+d \rightarrow {}^{3}\mathrm{He}+\gamma$ $E_0=5.49\,\mathrm{MeV}$ $p+d \rightarrow {}^{3}\mathrm{He}+\gamma$ $E_0=5.49\,\mathrm{MeV}$

(3) ${}^{3}\mathrm{He}+{}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He}+2p+\gamma$ $E_0=12.86\,\mathrm{MeV}$

pp II chain

(1) ${}^{3}\mathrm{He}+{}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be}+\gamma$

(2) ${}^{7}\mathrm{Be}+e^{-} \rightarrow {}^{7}\mathrm{Li}+\nu$ $E_\nu=0.861/0.383\,\mathrm{MeV}$
Li vs Li*

(3) ${}^{7}\mathrm{Li}+p \rightarrow 2\,{}^{4}\mathrm{He}$

pp III chain

(1) ${}^{3}\mathrm{He}+{}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be}+\gamma$

(2) ${}^{7}\mathrm{Be}+p \rightarrow {}^{8}\mathrm{B}+\gamma$

(3) ${}^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be}+e^{+}+\nu$ $E_\nu=14.06\,\mathrm{MeV}$

(4) ${}^{8}\mathrm{Be} \rightarrow 2\,{}^{4}\mathrm{He}$

Other fusion cycles also possible e.g. C-N-O cycle.

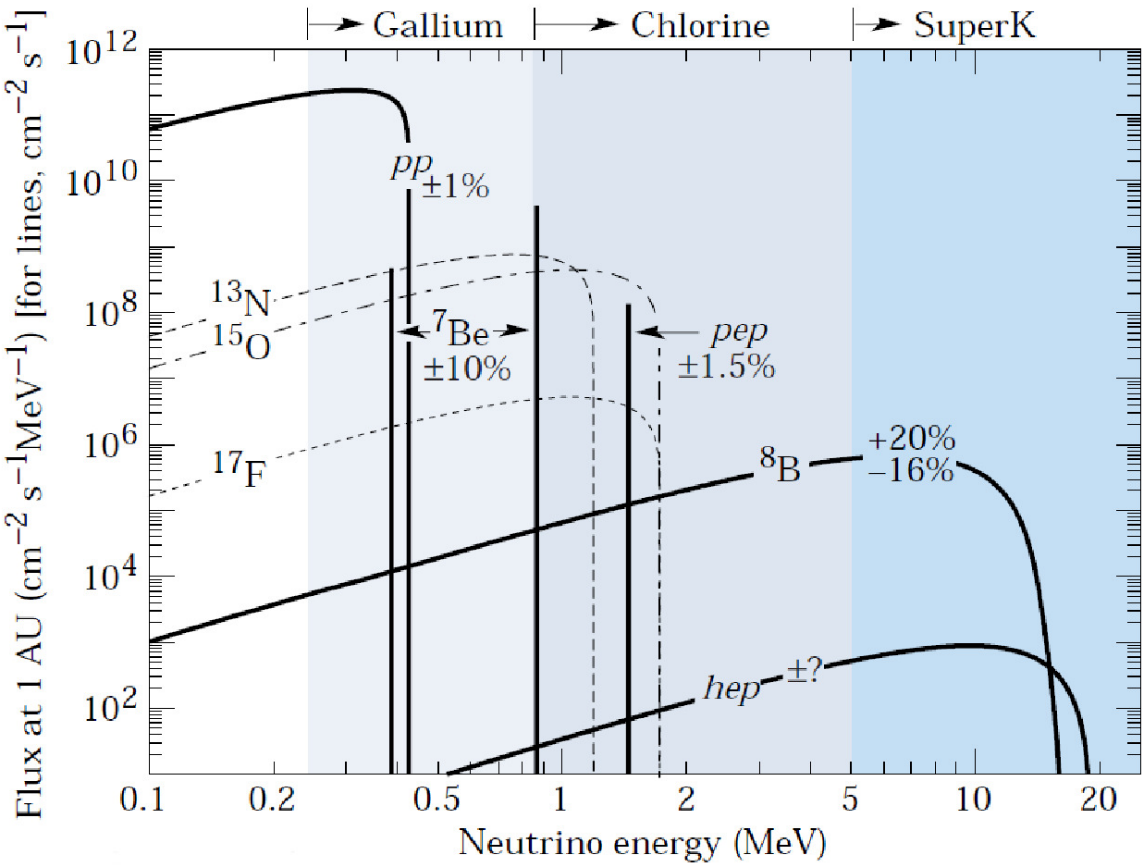
Observation of solar neutrinos from the various sources directly addresses the theory of stellar structure and evolution (Standard Solar Model). Probes the core of the Sun where the nuclear reactions are taking place.

The Sun also provides an opportunity to investigate ν properties e.g. mass, oscillations...

Also, the rare pp IV (Hep) chain: ${}^{3}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + e^{+} + \nu_e \quad (E_\nu = 18.8\,\mathrm{MeV})$

Solar Neutrinos

Many experiments have studied the solar neutrino flux



Expected flux depends on

- Standard Solar Model (temperature, density, composition vs r)
- Nuclear reaction cross-sections

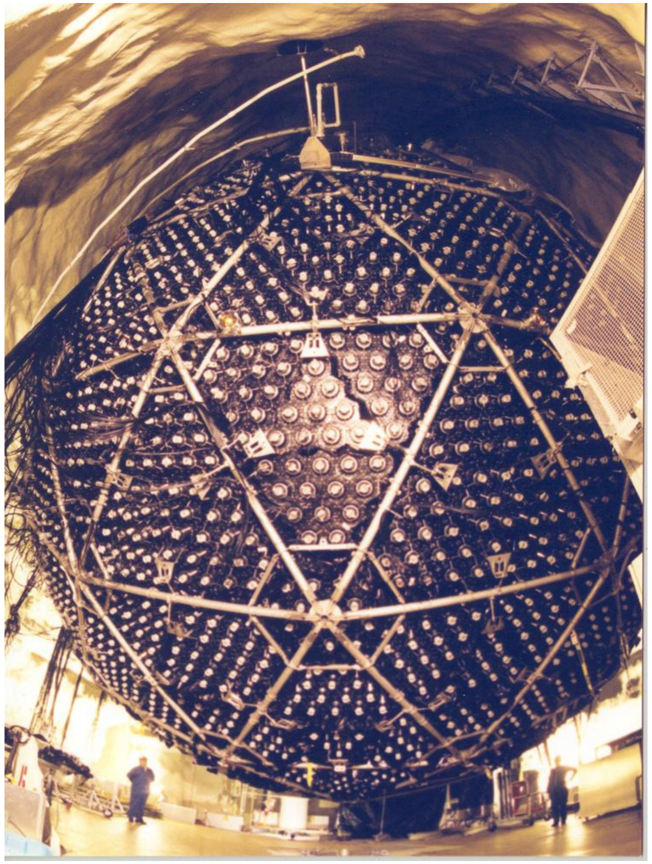
Observed ν flux $\sim$ 1/3 expected ν flux "Solar ν problem"

Solar Neutrinos

The Solar ν problem has recently been resolved by the Sudbury Neutrino Observatory (SNO) collaboration. They have reported evidence for a non-ν_e neutrino component in the solar ν flux

$\rightarrow$ Neutrino Oscillations

SNO (1000 tons D_2O in spherical vessel) measures the ^8B solar ν flux using three reactions:



Measure ν_e flux

$$\nu_e + d \rightarrow e^- + p + p$$

Measure total flux for all ν species

$$\nu_X + d \rightarrow \nu_X + p + n$$
$$\nu_X + e^- \rightarrow \nu_X + e^-$$

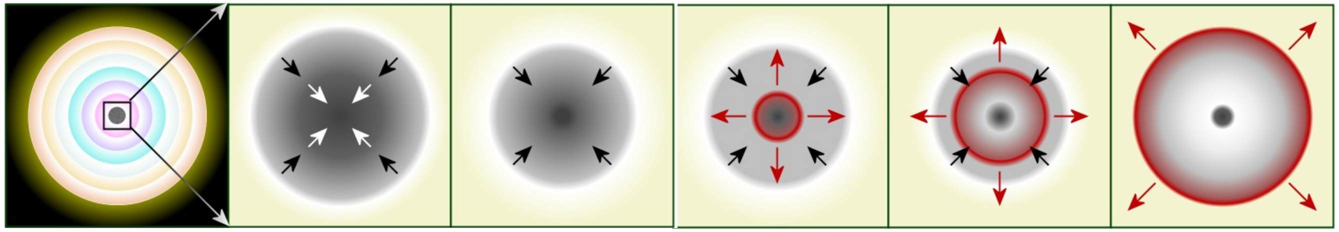
Observe a depletion in the ν_e flux, while the flux summed over all neutrino flavours agrees with expected solar flux.

Evidence for $\nu_e \Leftrightarrow \nu_X$ at 5σ

Further nuclear processes in astrophysics

Creating the heavy elements

Once the hydrogen is exhausted in a star, further gravitational collapse occurs and the temperature rises.



Eventually, it is hot enough to "burn" ^{4}He via fusion:

$$^4\text{He} + {}^4\text{He} \rightarrow {}^8\text{Be} + \gamma$$
$$^4\text{He} + {}^8\text{Be} \rightarrow {}^{12}\text{C} + \gamma$$
$$^4\text{He} + {}^{12}\text{C} \rightarrow {}^{16}\text{O} + \gamma$$

When the ^{4}He is exhausted, star undergoes further collapse

$\rightarrow$ further fusion reactions (and repeat)

Until we have the most tightly bound nuclei ^{56}Fe, ^{56}Co, ^{56}Ni.

Heavier elements are formed in supernova explosions:

$$n + {}^{56}\text{Fe} \rightarrow {}^{56}\text{Fe} + \gamma$$
$$n + {}^{57}\text{Fe} \rightarrow {}^{58}\text{Fe} + \gamma$$
$$n + {}^{58}\text{Fe} \rightarrow {}^{59}\text{Fe} + \gamma$$
$$^{59}\text{Fe} \rightarrow {}^{59}\text{Co} + e^- + \bar{\nu}_e$$

etc etc

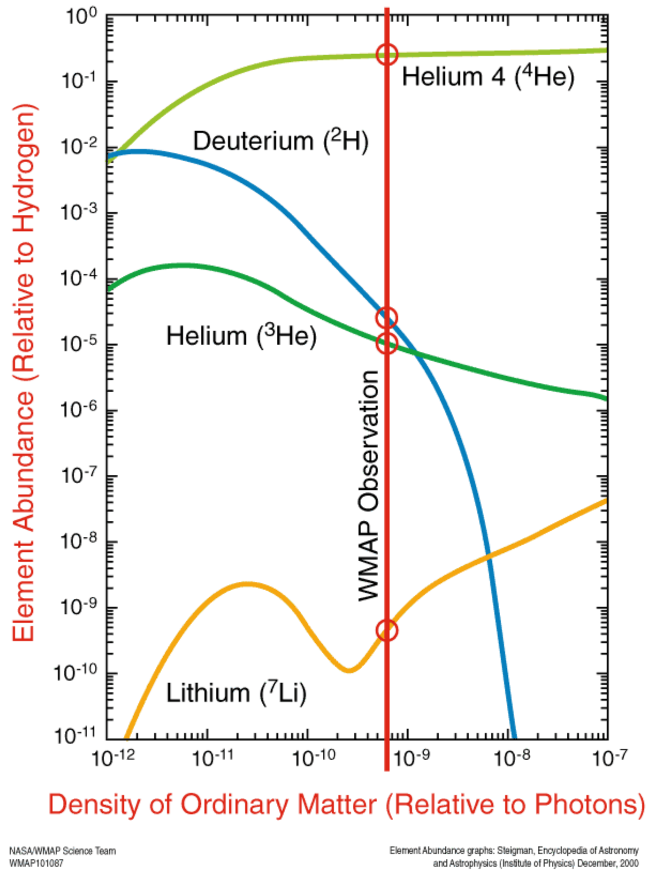
Further nuclear processes in astrophysics

Big bang nucleosynthesis

Fusion processes are also important in the Big Bang.
Both p and n present, at $T \gg 10^9$K.
Typical reactions:

$$n + p \rightarrow d + \gamma \qquad d + p \rightarrow {}^3\text{He} + \gamma$$
$$d + n \rightarrow {}^3\text{H} + \gamma \qquad d + d \rightarrow {}^3\text{H} + p$$
$${}^3\text{H} + p \rightarrow {}^4\text{He} + \gamma \qquad d + d \rightarrow {}^3\text{He} + n$$
$${}^3\text{He} + n \rightarrow {}^4\text{He} + \gamma$$

Observed abundances of these light elements provide a sensitive test of the Big Bang model.



In particular, they depend on aspects of particle physics which determine the n/p ratio, which depends on the temperature at which the reactions

$$p + \bar{\nu}_e \rightarrow n + e^+ \qquad n + \nu_e \rightarrow p + e^-$$

"freeze out", which in turn depends on the number of neutrino species.

Fusion in the lab

Fusion in the laboratory was first demonstrated in 1932, here at the Cavendish (Oliphant).

For fusion we need sufficiently high temperatures and controlled conditions.
The challenge now is to generate more power than expended.

Possible fusion reactions:

$$d + d \rightarrow {}^3\text{He} + n \qquad Q = 3.3\ \text{MeV}$$
$$d + d \rightarrow {}^3\text{H} + p \qquad Q = 4.0\ \text{MeV}$$
$$d + {}^3\text{H} \rightarrow {}^4\text{He} + n \qquad Q = 17.6\ \text{MeV}$$

The $d+{}^3\text{H}$ (aka DT) reaction is especially attractive

✓ largest energy release (α particle very stable)

✓ lowest Coulomb barrier

✗ 80% of the energy is released in the neutron – less easy to use, and doesn't help to heat the plasma.

✗ ${}^3\text{H}$ (tritium) unstable ($\tau_{1/2} \sim 12$ yr); need to produce it via $n+{}^6\text{Li} \rightarrow {}^4\text{He}+{}^3\text{H}$ or $n+{}^7\text{Li} \rightarrow {}^4\text{He}+{}^3\text{H}+n$ using some of the neutrons formed in the fusion reaction.

A recipe for controlled fusion

- Need $T \sim 10^8\,\mathrm{K}$ i.e. $E \sim 10\ \mathrm{keV} \gg$ ionisation energy $\Rightarrow$ **plasma**
  reminder: plasmas are electrically conductive and can be controlled with magnetic fields.
- Heat plasma by applying r.f. energy.
  Declare **Ignition** when the process is self-sustaining: the heating from $3.5\ \mathrm{MeV}$ α-particles produced in fusion exceeds the losses (due to bremsstrahlung, for example).
- **Break even** achieved when there is more power out (incl. losses) than in.

Fusion rate $= n_D n_T \langle \sigma v \rangle = \frac{1}{4} n^2 \langle \sigma v \rangle$ (assumes $n_D = n_T = \frac{1}{2} n$, where n is the electron density).
Rate of generation of energy $= \frac{1}{4} n^2 \langle \sigma v \rangle Q$
Rate of energy loss $= W/\tau$ where $W = 3nkT$ is the energy density in the plasma ($3kT/2$ for electrons and the same for the ions) and τ is the lifetime of the plasma due to losses.
Break even if $\frac{1}{4} n^2 \langle \sigma v \rangle Q \; > \; 3nkT/\tau$, i.e.

$$\textbf{Lawson criterion} \qquad n\tau \; > \; \frac{12kT}{Q\langle \sigma v \rangle}$$

For DT, this is $n\tau > 10^{20}\,\mathrm{m}^{-3}\mathrm{s}$ at $kT \gg 10\ \mathrm{keV}$.

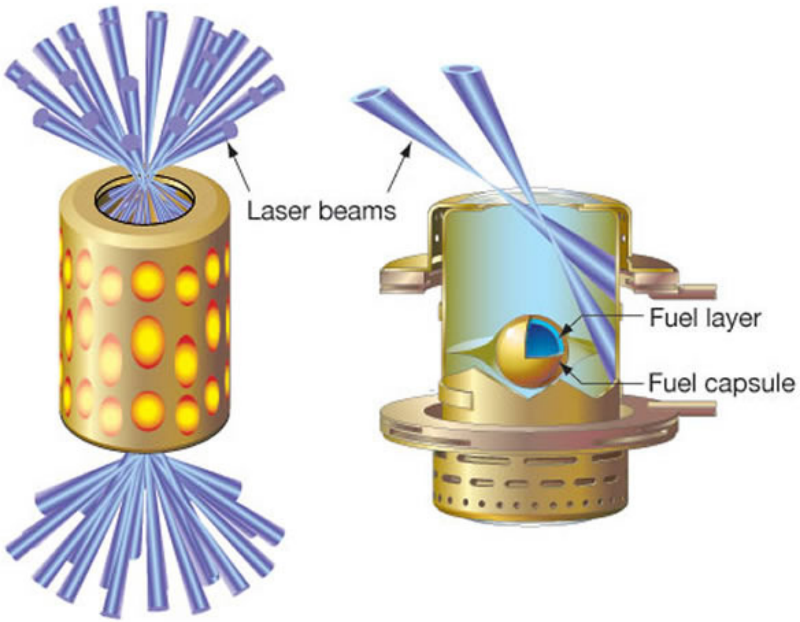
People commonly look at the "triple product" $n\tau T$ for fusion processes.

Controlled fusion – confinement

Need $T \sim 10^8\,\mathrm{K}$ i.e. $E \sim 10\ \mathrm{keV} \gg$ ionisation energy $\Rightarrow$ need to **control** a plasma

Inertial confinement

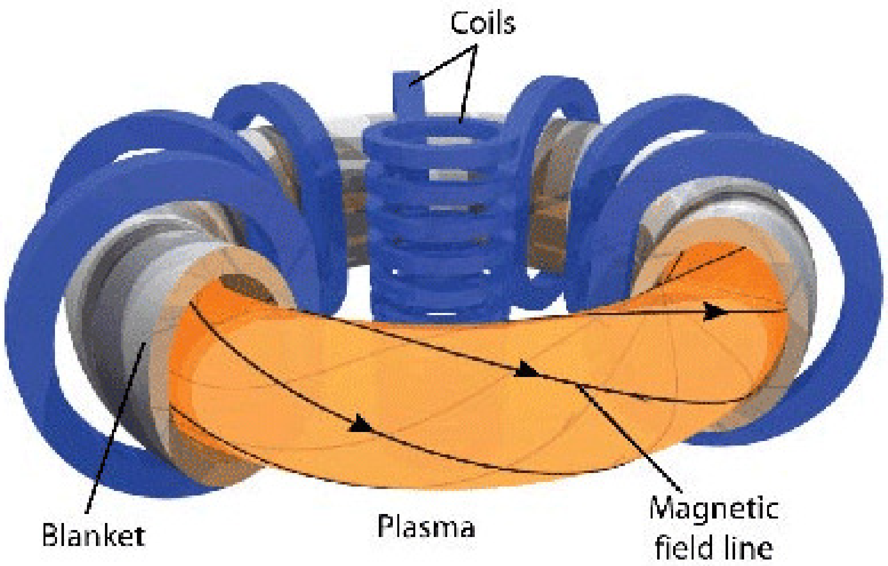
Use a pellet containing $d+{}^3\mathrm{H}$ zapped from all sides with lasers or particle beams to heat it. Need very high power lasers + repeated feeding of fuel.



e.g. National Ignition Facility, LBNL, US

Magnetic confinement

Use a configuration of magnetic fields to control the plasma (Tokamak) and keep it away from walls.



e.g. International Thermonuclear Experimental Reactor, France

Controlled fusion – the status today

JET (Joint European Torus),
TFTR (Tokamak Fusion Test Reactor), both achieved appropriate values of plasma density (n) and lifetime (τ), but not simultaneously $\Rightarrow$ yet to break even.

NIF (National Ignition Facility) closing in on ignition.

ITER (International Thermonuclear Experimental Reactor) should break even. Build time ∼10 years; then ∼20 years of experimentation starting 2025.

Commercial fusion power can't realistically be expected before 2050.

Recent progress

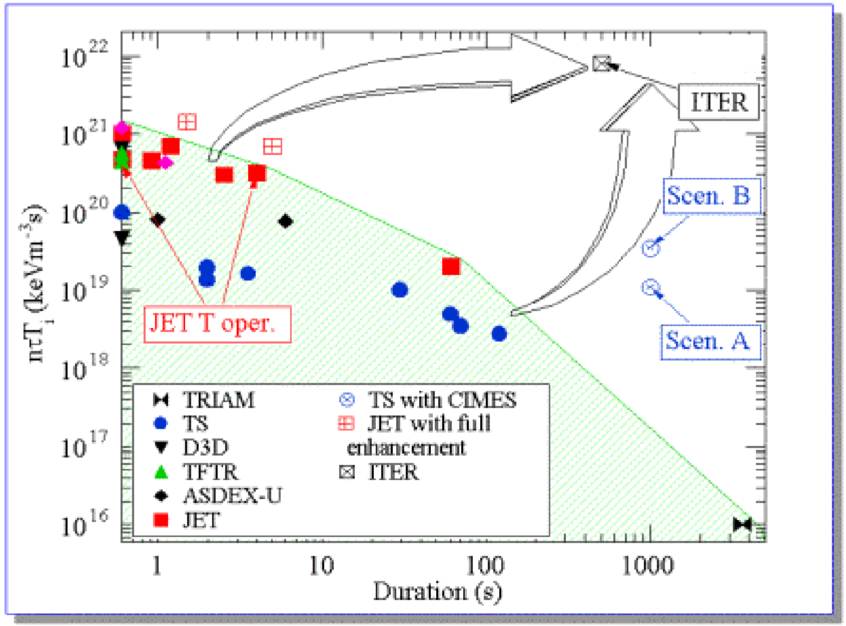
Aug 2021: NIF produced 1.3 MJ – 70% of delivered laser energy.

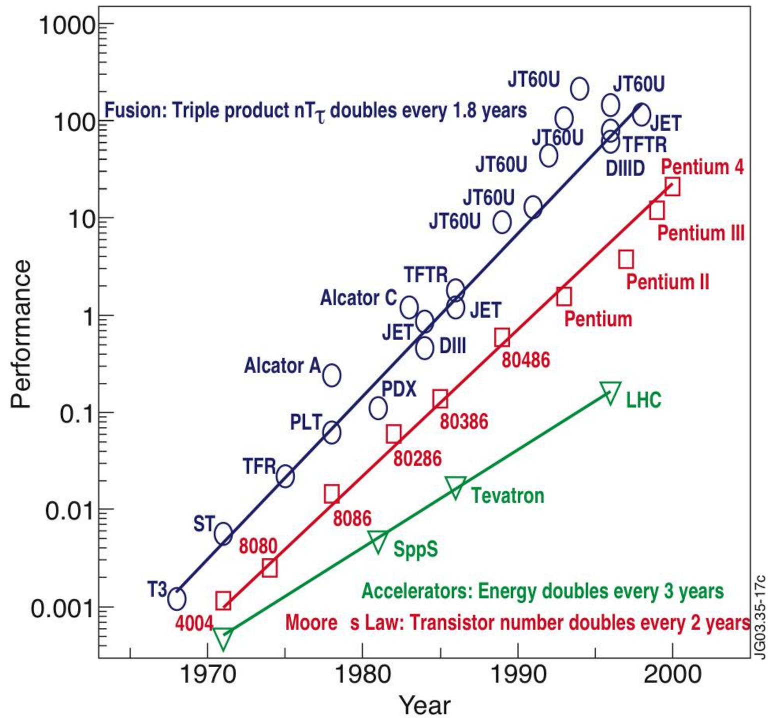
Dec 2022: NIF produced 3.15 MJ – 150% of delivered laser energy. *Breakeven!*

Feb 2022: JET broke 23 year old energy record – 59 MJ over 5 s

Feb 2024: JET – 69 MJ over 5 s

This could be *your* work!





Summary

- Spontaneous fission – energetically possible for many nuclei, but tunnelling needed – rate only competitive for a few heavy elements.
- Neutron induced fission – neutron absorption into a fissile excited state. Practical importance in power generation and bombs.
- Asymmetric fission; neutrons liberated
- Chain reaction. Use of delayed neutron component for control.
- Fusion – again a tunnelling problem. Needs very high temperatures for useful rates.
- Fusion processes in the sun (solar neutrinos).
- Nucleosynthesis in the big bang.
- Controlled fusion.

Problem Sheet: q.42-44

Thank you for being a great class! Farewell!