10. Electroweak Unification
Particle and Nuclear Physics
In this section...

- GWS model
- Allowed vertices
- Revisit Feynman diagrams
- Experimental tests of Electroweak theory
Electroweak Unification

- Weak CC interactions explained by $W^\pm$ boson exchange
- $W^\pm$ bosons are charged, thus they couple to the $\gamma$

Consider $e^- e^+ \rightarrow W^+ W^-$: 2 diagrams (+interference)

Cross-section diverges at high energy
- Divergence cured by introducing $Z$ boson
- Extra diagram for $e^- e^+ \rightarrow W^+ W^-$
- Idea only works if $\gamma$, $W^\pm$, $Z$ couplings are related
  $\Rightarrow$ Electroweak Unification
Electroweak gauge theory

Postulate invariance under a gauge transformation like:

\[ \psi \rightarrow \psi' = e^{ig\vec{\sigma} \cdot \vec{A}(\vec{r}, t)} \psi \]

an “SU(2)” transformation (\( \sigma \) are 2x2 matrices).

Operates on the state of “weak isospin” – a “rotation” of the isospin state.

Invariance under SU(2) transformations \( \Rightarrow \) three massless gauge bosons \( (W_1, W_2, W_3) \) whose couplings are well specified.

They also have self-couplings.

But this doesn’t quite work...
Predicts \( W \) and \( Z \) have the same couplings – not seen experimentally!
Electroweak gauge theory

The solution...

- Unify QED and the weak force ⇒ electroweak model
- “SU(2)×U(1)” transformation
  - U(1) operates on the “weak hypercharge” \( Y = 2(Q - I_3) \)
  - SU(2) operates on the state of “weak isospin, I”
- Invariance under SU(2)×U(1) transformations ⇒ four massless gauge bosons \( W^+, W^-, W_3, B \)
- The two neutral bosons \( W_3 \) and \( B \) then mix to produce the physical bosons \( Z \) and \( \gamma \)
- Photon properties must be the same as QED ⇒ predictions of the couplings of the \( Z \) in terms of those of the \( W \) and \( \gamma \)
- Still need to account for the masses of the \( W \) and \( Z \). This is the job of the Higgs mechanism (later).
The GWS Model

The Glashow, Weinberg and Salam model treats EM and weak interactions as different manifestations of a single unified electroweak force (Nobel Prize 1979)

Start with 4 massless bosons $W^+$, $W_3$, $W^-$ and $B$. The neutral bosons mix to give physical bosons (the particles we see), i.e. the $W^\pm$, $Z$, and $\gamma$.

\[
\begin{pmatrix}
W^+ \\
W_3 \\
W^-
\end{pmatrix}; \quad B \rightarrow \begin{pmatrix}
W^+ \\
Z \\
W^-
\end{pmatrix}; \quad \gamma
\]


\[
Z = W_3 \cos \theta_W - B \sin \theta_W \\
A = W_3 \sin \theta_W + B \cos \theta_W
\]

$\theta_W$ Weak Mixing Angle

$W^\pm$, $Z$ “acquire” mass via the Higgs mechanism.
The beauty of the GWS model is that it makes exact predictions of the $W^\pm$ and $Z$ masses and of their couplings with only 3 free parameters.

**Couplings given by $\alpha_{EM}$ and $\theta_W$**

\[
\alpha_{EM} = \frac{e^2}{4\pi} \quad g \sim e \quad g_W = \frac{e}{\sin \theta_W} \quad g_Z = \frac{e}{\sin \theta_W \cos \theta_W} = \frac{g_W}{\cos \theta_W}
\]

**Masses also given by $G_F$ and $\theta_W$**

From Fermi theory

\[
\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} = \frac{e^2}{8m_W^2 \sin^2 \theta_W} \quad m_{W^\pm} = \left(\frac{\sqrt{2}e^2}{8G_F \sin^2 \theta_W}\right)^{1/2} \quad m_Z = \frac{m_W}{\cos \theta_W}
\]

If we know $\alpha_{EM}$, $G_F$, $\sin \theta_W$ (from experiment), everything else is defined.
As a result of the mixing, we require that the mass eigenstates should be the $Z$ and $\gamma$, and the mass of the photon be zero.

We then compute the matrix elements of the mass operator:

\[
m_Z^2 = \langle W_3 \cos \theta_W - B \sin \theta_W | \hat{M}^2 | W_3 \cos \theta_W - B \sin \theta_W \rangle
\]
\[
= m_W^2 \cos^2 \theta_W + m_B^2 \sin^2 \theta_W - 2m_{WB}^2 \cos \theta_W \sin \theta_W
\]

\[
m_\gamma^2 = \langle W_3 \sin \theta_W + B \cos \theta_W | \hat{M}^2 | W_3 \sin \theta_W + B \cos \theta_W \rangle
\]
\[
= m_W^2 \sin^2 \theta_W + m_B^2 \cos^2 \theta_W + 2m_{WB}^2 \cos \theta_W \sin \theta_W = 0
\]

\[
m_{Z\gamma}^2 = \langle W_3 \cos \theta_W - B \sin \theta_W | \hat{M}^2 | W_3 \sin \theta_W + B \cos \theta_W \rangle
\]
\[
= (m_W^2 - m_B^2) \sin \theta_W \cos \theta_W + m_{WB}^2 (\cos^2 \theta_W - \sin^2 \theta_W) = 0
\]

Solving these three equations gives

\[
m_Z = \frac{m_W}{\cos \theta_W}
\]
Couplings

- Slightly simplified – see Part III for better treatment. Starting from
  \[ Z = W_3 \cos \theta_W - B \sin \theta_W \]
  \[ A = W_3 \sin \theta_W + B \cos \theta_W \]
- \( W_3 \) couples to \( l_3 \) with strength \( g_W \) and \( B \) couples to \( Y = 2(Q - l_3) \) with \( g' \)
- So, coupling of \( A \) (photon) is
  \[ g_W l_3 \sin \theta_W + g'2(Q - l_3) \cos \theta_W = Qe \quad \text{for all } l_3 \]
  \[ \Rightarrow g' = \frac{g_W \tan \theta_W}{2} \quad \text{and} \quad g' \cos \theta_W = \frac{e}{2} \quad \Rightarrow g_W = \frac{e}{\sin \theta_W} \]
- The couplings of the \( Z \) are therefore
  \[ g_W l_3 \cos \theta_W - g'2(Q - l_3) \sin \theta_W = \frac{e}{\sin \theta_W \cos \theta_W} \left[ l_3 - Q \sin^2 \theta_W \right] \]
  \[ = g_Z \left[ l_3 - Q \sin^2 \theta_W \right] \]
- For right-handed fermions, \( l_3 = 0 \), while for left-handed fermions
  \( l_3 = +1/2(\nu, u, c, t) \) or \( l_3 = -1/2(e^-, \mu^-, \tau^-, d', s', b') \); \( Q \) is charge in units of \( e \)
Discovery of Neutral Currents (1973)

The process $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$ was observed.

Only possible Feynman diagram (no $W^\pm$ diagram).

Indirect evidence for $Z$.

Gargamelle Bubble Chamber at CERN
Evidence for GWS Model

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- **Direct Observation of $W^\pm$ and $Z$ (1983)**
  First direct observation in $p\bar{p}$ collisions at $\sqrt{s} = 540$ GeV via decays into leptons
  
  $p\bar{p} \rightarrow W^\pm + X$
  
  $p\bar{p} \rightarrow Z + X$
  
  $\leftrightarrow e^\pm \nu_e, \mu^\pm \nu_\mu$
  
  $\leftrightarrow e^+ e^-, \mu^+ \mu^-$

  UA1 Experiment at CERN
  Used Super Proton Synchrotron
  (now part of LHC!)
Evidence for GWS Model

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    $p\bar{p} \rightarrow W^\pm + X$
    $p\bar{p} \rightarrow Z + X$
    $\leftrightarrow e^\pm \nu_e, \mu^\pm \nu_\mu$
    $\leftrightarrow e^+ e^-, \mu^+ \mu^-$

  - LEP $e^+ e^-$ collider provided many precision measurements of the Standard Model.

- **Wide variety of different processes consistent with GWS model predictions**
  - and measure same value of
    
    $\sin^2 \theta_W = 0.23113 \pm 0.00015$
    $\theta_W \sim 29^\circ$
All weak neutral current interactions can be described by the $Z$ boson propagator and the weak vertices:

- **Weak NC Lepton Vertex**
  - $e^-, \mu^-, \tau^-$
  - $\nu_e, \nu_\mu, \nu_\tau$

- **Weak NC Quark Vertex**
  - $u, d, s, c, b, t$

- $Z$ never changes type of particle
- $Z$ never changes quark or lepton flavour
- $Z$ couplings are a mixture of EM and weak couplings, and therefore depend on $\sin^2 \theta_W$. 

The Standard Model
Weak NC Lepton Vertex

+ antiparticles

The Standard Model
Weak NC Quark Vertex

+ antiparticles
Examples

$Z \rightarrow e^+ e^-, \mu^+ \mu^-, \tau^+ \tau^-$

$e^-, \mu^-, \tau^-$

$Z \rightarrow \nu_e \bar{\nu}_e, \nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau$

$\nu_e, \nu_\mu, \nu_\tau$

$Z \rightarrow q \bar{q}$

$q$

$e^+ e^- \rightarrow \mu^+ \mu^-$

$e^+, \mu^+, \tau^+$

$e^-, \mu^-, \tau^-$

$\nu_e e^- \rightarrow \nu_e e^-$

$\nu_e$

$\nu_e$

$Z$
Summary of Standard Model (matter) Vertices

**Electromagnetic (QED)**
- $\ell^- \to e$
- $\gamma$

- $\alpha = \frac{e^2}{4\pi}$
- $q = u, d, s, c, b, t$

**Strong (QCD)**
- $q \to g$
- $g_s$

- $\alpha_s = \frac{g_s^2}{4\pi}$

**Weak CC**
- $\ell^- \to W^-$
- $\nu_{\ell}$

- $\alpha_W = \frac{g_W^2}{4\pi}$

**Weak NC**
- $\ell^\pm, \nu_{\ell}$
- $Z$

- $g_Z = \frac{g_W}{\cos \theta_W}$
Feynman Diagrams

1. $\pi^- + p \rightarrow K^0 + \Lambda$

2. $\nu_\tau + e^- \rightarrow \nu_\tau + e^-$

3. $\bar{\nu}_\tau + \tau^- \rightarrow \bar{\nu}_\tau + \tau^-$

4. $D^+ \rightarrow K^- \pi^+ \pi^+$

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Experimental Tests of the Electroweak model at LEP


Precise measurements of the properties of $Z$ and $W^\pm$ bosons provide the most stringent test of our current understanding of particle physics.

- LEP is the highest energy $e^+e^-$ collider ever built $\sqrt{s} = 90 - 209$ GeV
- Large circumference, 27 km
- 4 experiments combined saw $16 \times 10^6$ $Z$ events, $30 \times 10^3$ $W^\pm$ events
OPAL: a LEP detector

OPAL was one of the 4 experiments at LEP. Size: $12 \text{ m} \times 12 \text{ m} \times 15 \text{ m}$. 

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Electromagnetic calorimeters

Muon detectors

Jet chamber

Vertex chamber

μ-Vertex detector

Solenoid and Pressure vessel

Forward detector

SiW luminometer

Muon Chambers

Hadron Calorimeter

Tracking Chambers

Electromagnetic Calorimeter
Typical $e^+e^- \rightarrow Z$ events

$e^+e^- \rightarrow Z \rightarrow e^+e^-$

$e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$
Typical $e^+e^- \rightarrow Z$ events

$$e^+e^- \rightarrow Z \rightarrow \tau^+\tau^-$$

$$e^+e^- \rightarrow Z \rightarrow q\bar{q}$$

Taus decay within the detector (lifetime $\sim 10^{-13}$ s).
Here $\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$, $\tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_{\bar{\tau}}$

3-jet event (gluon emitted by $q/\bar{q}$)
The $Z$ Resonance

Consider the process $e^+ e^- \rightarrow q \bar{q}$

- At small $\sqrt{s} (< 50 \text{ GeV})$, we only considered an intermediate photon
- At higher energies, the $Z$ exchange diagram contributes ($+Z\gamma$ interference)

\[
\sigma(e^+ e^- \rightarrow \gamma \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{3s} \sum 3Q_q^2
\]

- The $Z$ is a decaying intermediate massive state (lifetime $\sim 10^{-25} \text{ s}$)
  \[\Rightarrow\text{ Breit-Wigner resonance}\]
- Around $\sqrt{s} \sim m_Z$, the $Z$ diagram dominates
The Z Resonance

$e^+ e^- \rightarrow \text{hadrons}$

Cross-section (pb)

Centre-of-mass energy (GeV)
The $Z$ Resonance

Breit-Wigner cross-section for $e^+ e^- \rightarrow Z \rightarrow f \bar{f}$ (where $f \bar{f}$ is any fermion-antifermion pair)

Centre-of-mass energy $\sqrt{s} = E_{CM} = E_{e^+} + E_{e^-}$

$$\sigma(e^+ e^- \rightarrow Z \rightarrow f \bar{f}) = \frac{g\pi}{E_e^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(E_{CM} - m_Z)^2 + \frac{\Gamma_Z^2}{4}}$$

with $g = \frac{2J_Z + 1}{(2J_{e^-} + 1)(2J_{e^+} + 1)} = \frac{3}{4}$, $J_Z = 1$; $J_{e^\pm} = \frac{1}{2}$

giving

$$\sigma(e^+ e^- \rightarrow Z \rightarrow f \bar{f}) = \frac{3\pi}{4E_e^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(E_{CM} - m_Z)^2 + \frac{\Gamma_Z^2}{4}} = \frac{3\pi}{s} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(\sqrt{s} - m_Z)^2 + \frac{\Gamma_Z^2}{4}}$$

$\Gamma_Z$ is the total decay width, i.e. the sum over the partial widths for different decay modes

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}$$
The Z Resonance

At the peak of the resonance $\sqrt{s} = m_Z$:

$$\sigma(e^+ e^- \rightarrow Z \rightarrow f \bar{f}) = \frac{12\pi \Gamma_{ee} \Gamma_{f\bar{f}}}{m_Z^2 \Gamma_Z^2}$$

Hence, for all fermion/antifermion pairs in the final state

$$\sigma(e^+ e^- \rightarrow Z \rightarrow \text{anything}) = \frac{12\pi \Gamma_{ee}}{m_Z^2 \Gamma_Z} \quad \Gamma_{f\bar{f}} = \Gamma_Z$$

Compare to the QED cross-section at $\sqrt{s} = m_Z$

$$\sigma_{\text{QED}} = \frac{4\pi \alpha^2}{3s}$$

$$\frac{\sigma(e^+ e^- \rightarrow Z \rightarrow \text{anything})}{\sigma_{\text{QED}}} = \frac{9 \Gamma_{ee}}{\alpha^2 \Gamma_Z} \sim 5700$$

$\Gamma_{ee} = 85 \text{ GeV}, \quad \Gamma_Z = 2.5 \text{ GeV}, \quad \alpha = 1/137$
Measurement of $m_Z$ and $\Gamma_Z$

- Run LEP at various centre-of-mass energies ($\sqrt{s}$) close to the peak of the $Z$ resonance and measure $\sigma(e^+e^- \rightarrow q\bar{q})$
- Determine the parameters of the resonance:
  - Mass of the $Z$, $m_Z$
  - Total decay width, $\Gamma_Z$
  - Peak cross-section, $\sigma^0$

One subtle feature: need to correct measurements for QED effects due to radiation from the $e^+e^-$ beams. This radiation has the effect of reducing the centre-of-mass energy of the $e^+e^-$ collision which smears out the resonance.

![Graph showing measurements, error bars increased by factor 10, QED unfolded, M_Z, and E_cm (GeV) with axes labeled.](image)
Measurement of $m_Z$ and $\Gamma_Z$

$m_Z$ was measured with precision $2$ parts in $10^5$

- Need a detailed understanding of the accelerator and astrophysics.

Tidal distortions of the Earth by the Moon cause the rock surrounding LEP to be distorted – changing the radius by 0.15 mm (total 4.3 km). This is enough to change the centre-of-mass energy.

- Also need a train timetable.

  Leakage currents from the TGV rail via Lake Geneva follow the path of least resistance... using LEP as a conductor.

Accounting for these effects (and many others):

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

$$\sigma_{qq}^0 = 41.450 \pm 0.037 \text{ nb}$$
Number of Generations

- Currently know of three generations of fermions. Masses of quarks and leptons increase with generation. Neutrinos are approximately massless (or are they?)

\[
\begin{pmatrix}
  e^- \\
  \nu_e \\
  \mu^- \\
  \nu_\mu \\
  \tau^- \\
  \nu_\tau \\
  u \\
  d \\
  c \\
  s \\
  t \\
\end{pmatrix}
\]

- Could there be more generations? e.g.

\[
\begin{pmatrix}
  t' \\
  b' \\
\end{pmatrix}
\begin{pmatrix}
  L \\
  \nu_L \\
\end{pmatrix}
\]

- The Z boson couples to all fermions, including neutrinos. Therefore, the total decay width, \(\Gamma_Z\), has contributions from all fermions with \(m_f > m_Z/2\)

\[
\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}
\]

with \(\Gamma_{\nu\bar{\nu}} = \Gamma_{\nu_e\bar{\nu}_e} + \Gamma_{\nu_\mu\bar{\nu}_\mu} + \Gamma_{\nu_\tau\bar{\nu}_\tau}\)

- If there were a fourth generation, it seems likely that the neutrino would be light, and, if so would be produced at LEP

\[
e^+ e^- \rightarrow Z \rightarrow \nu_L \bar{\nu}_L
\]

- The neutrinos would not be observed directly, but could infer their presence from the effect on the Z resonance curve.
Number of Generations

At the peak of the $Z$ resonance, $\sqrt{s} = m_Z$

$$\sigma^0_{f\bar{f}} = \frac{12\pi \Gamma_{ee} \Gamma_{f\bar{f}}}{m_Z^2 \Gamma_Z^2}$$

A fourth generation neutrino would increase the $Z$ decay rate and thus increase $\Gamma_Z$. As a result, a decrease in the measured peak cross-sections for the visible final states would be observed.

Measure the $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ cross-sections for all visible decay models (i.e. all fermions apart from $\nu\bar{\nu}$)

**Examples:**

- $e^+e^- \rightarrow \mu^+\mu^-$
- $e^+e^- \rightarrow \tau^+\tau^-$
Number of Generations

- Have already measured $m_Z$ and $\Gamma_Z$ from the shape of the Breit-Wigner resonance. Therefore, obtain $\Gamma_{f\bar{f}}$ from the peak cross-sections in each decay mode using

$$\sigma_{f\bar{f}}^0 = \frac{12\pi \Gamma_{ee} \Gamma_{f\bar{f}}}{m_Z^2 \Gamma_Z^2}$$

Note, obtain $\Gamma_{ee}$ from

$$\sigma_{ee}^0 = \frac{12\pi \Gamma_{ee}^2}{m_Z^2 \Gamma_Z^2}$$

- Can relate the partial widths to the measured total width (from the resonance curve)

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + N_{\nu} \Gamma_{\nu\nu}$$

where $N_{\nu}$ is the number of neutrino species and $\Gamma_{\nu\nu}$ is the partial width for a single neutrino species.
Number of Generations

The difference between the measured value of $\Gamma_Z$ and the sum of the partial widths for visible final states gives the invisible width $N\nu\Gamma_{\nu\nu}$

<table>
<thead>
<tr>
<th>(\Gamma)</th>
<th>Value (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_Z$</td>
<td>2495.2±2.3</td>
</tr>
<tr>
<td>$\Gamma_{ee}$</td>
<td>83.91±0.12</td>
</tr>
<tr>
<td>$\Gamma_{\mu\mu}$</td>
<td>83.99±0.18</td>
</tr>
<tr>
<td>$\Gamma_{\tau\tau}$</td>
<td>84.08±0.22</td>
</tr>
<tr>
<td>$\Gamma_{qq}$</td>
<td>1744.4±2.0</td>
</tr>
<tr>
<td>$N\nu\Gamma_{\nu\nu}$</td>
<td>499.0±1.5</td>
</tr>
</tbody>
</table>

In the Standard Model, calculate $\Gamma_{\nu\nu} \sim 167$ MeV

Therefore

$$N\nu = \frac{\Gamma_{\text{measured}}^{\nu\nu}}{\Gamma_{\nu\nu}^{\text{SM}}} = 2.984 \pm 0.008$$

$\Rightarrow$ three generations of light neutrinos for $m_\nu < m_Z/2$
Most likely that only 3 generations of quarks and leptons exist.

In addition:

- $\Gamma_{ee}, \Gamma_{\mu\mu}, \Gamma_{\tau\tau}$ are consistent $\Rightarrow$ tests universality of the lepton couplings to the $Z$ boson.

- $\Gamma_{qq}$ is consistent with the expected value which assumes 3 colours – further evidence for colour.
In $e^+e^-$ collisions $W$ bosons are produced in pairs.

Standard Model: 3 possible diagrams:

- $\nu e e^−e^++W^−W^+$
- $\gamma e e^−e^++W^−W^+$
- $Z e e^−e^++W^−W^+$

LEP operated above the threshold for $W^+W^−$ production (1996-2000)

$\sqrt{s} > 2m_W$

Cross-section sensitive to the presence of the Triple Gauge Boson vertex
In the Standard Model $W\ell\nu$ and $Wq\bar{q}$ couplings are $\sim$ equal.

$W^-$ \quad \text{to} \quad \bar{e}, \bar{\mu}, \bar{\tau}$

$W^-$ \quad \text{to} \quad \bar{u}, \bar{c}$

$m_W < m_t \times 3 \text{ for colour}$

**Expect** (assuming 3 colours)

\[
B(W^\pm \rightarrow q\bar{q}) = \frac{6}{9} = \frac{2}{3}
\]

\[
B(W^\pm \rightarrow \ell\nu) = \frac{3}{9} = \frac{1}{3}
\]

QCD corrections $\sim (1 + \frac{\alpha_s}{\pi})$

\[
\Rightarrow B(W^\pm \rightarrow q\bar{q}) = 0.675
\]

**Measured BR**

- $W^+W^- \rightarrow \ell\nu\ell\nu \quad 10.5\%$
- $W^+W^- \rightarrow q\bar{q}\ell\nu \quad 43.9\%$
- $W^+W^- \rightarrow q\bar{q}q\bar{q} \quad 45.6\%$

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$W^+ W^- \rightarrow e\nu\mu\nu$

$W^+ W^- \rightarrow q\bar{q}e\nu$

$W^+ W^- \rightarrow q\bar{q}q\bar{q}$
Unlike $e^+e^- \rightarrow Z$, $W$ boson production at LEP was not a resonant process. $m_W$ was measured by measuring the invariant mass in each event.

$$m_W = \frac{1}{2} (m_{q\bar{q}} + m_{\ell\nu})$$

$m_W = 80.423 \pm 0.038$ GeV

$\Gamma_W = 2.12 \pm 0.11$ GeV
In the Standard Model, the $W$ boson decay width is given by

$$\Gamma(W^\rightarrow e^- \bar{\nu}_e) = \frac{g_W^2 m_W}{48\pi} = \frac{G_F m_W^3}{6\sqrt{2}\pi}$$

$\mu$-decay: $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$  

LEP: $m_W = 80.423 \pm 0.038$ GeV

$$\Rightarrow \Gamma(W^\rightarrow e^- \bar{\nu}_e) = 227 \text{ MeV}$$

Total width is the sum over all partial widths:

$W^\rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau,$

$W^\rightarrow d'\bar{u}, s'\bar{c}, \times 3$ for colour

If the $W$ coupling to leptons and quarks is equal and there are 3 colours:

$$\Gamma = \sum_i \Gamma_i = (3 + 2 \times 3)\Gamma(W^\rightarrow e^- \bar{\nu}_e) \sim 2.1 \text{ GeV}$$

Compare with measured value from LEP: $\Gamma_W = 2.12 \pm 0.11$ GeV

- Universal coupling constant
- Yet more evidence for colour!
Summary of Electroweak Tests

Now have 5 precise measurements of fundamental parameters of the Standard Model

\[ \alpha_{EM} = \frac{1}{(137.03599976 \pm 0.00000050)} \]  
\[ G_F = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2} \]  
\[ m_W = 80.385 \pm 0.015 \text{ GeV} \]  
\[ m_Z = 91.1875 \pm 0.0021 \text{ GeV} \]  
\[ \sin^2 \theta_W = 0.23143 \pm 0.00015 \] 

In the Standard Model, only 3 are independent.

The measurements are consistent, which is an incredibly powerful test of the Standard Model of Electroweak Interactions.
Summary

- Weak interaction with $W^\pm$ fails at high energy.
- Introduction of unified theory involving and relating $Z$ and $\gamma$ can resolve the divergences.
- One new parameter, $\theta_W$, allows predictions of $Z$ couplings and mass relations.
- Extensively and successfully tested at LEP.

Up next...
Section 11: The Top Quark and the Higgs Mechanism