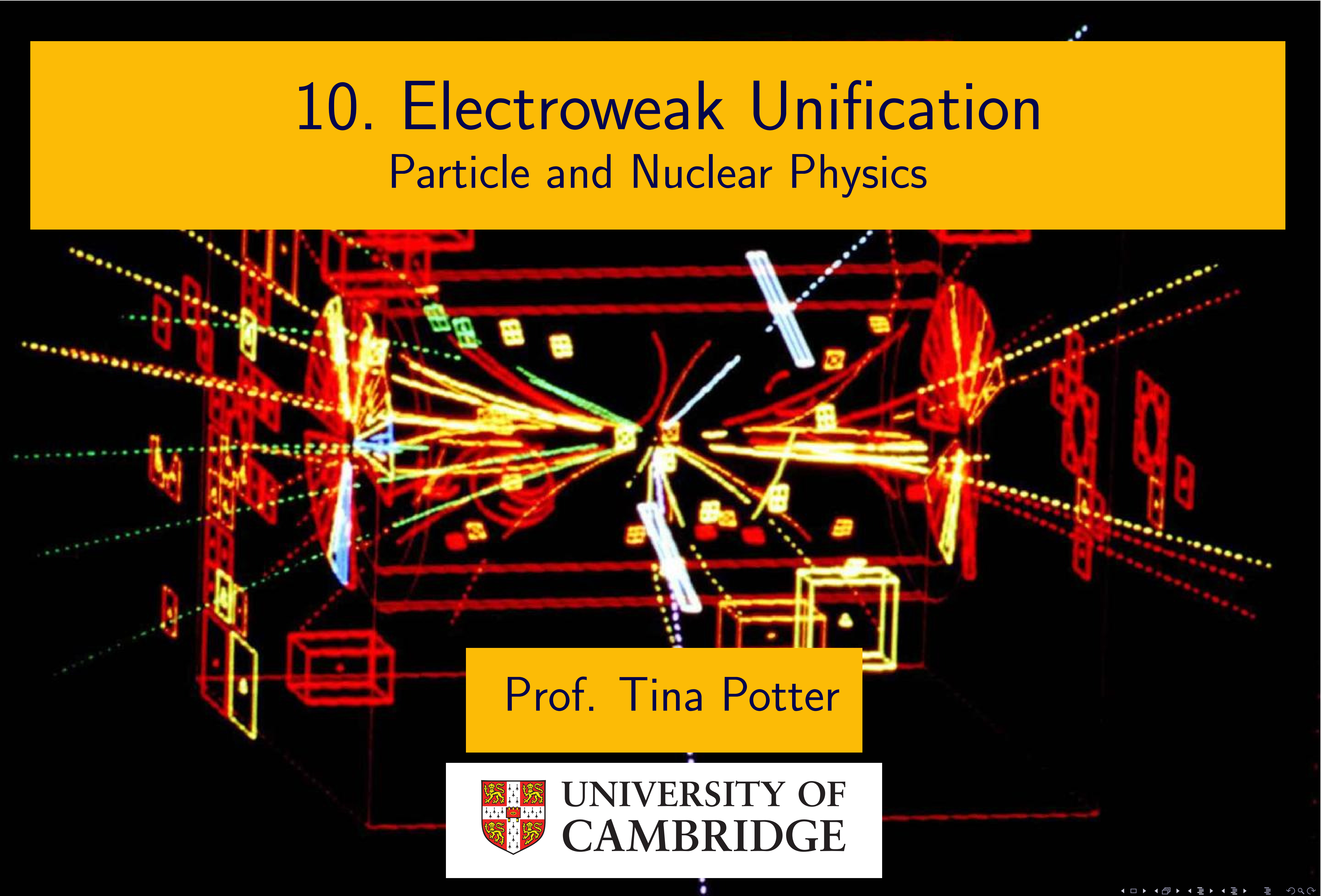
10. Electroweak Unification

Particle and Nuclear Physics



Prof. Tina Potter

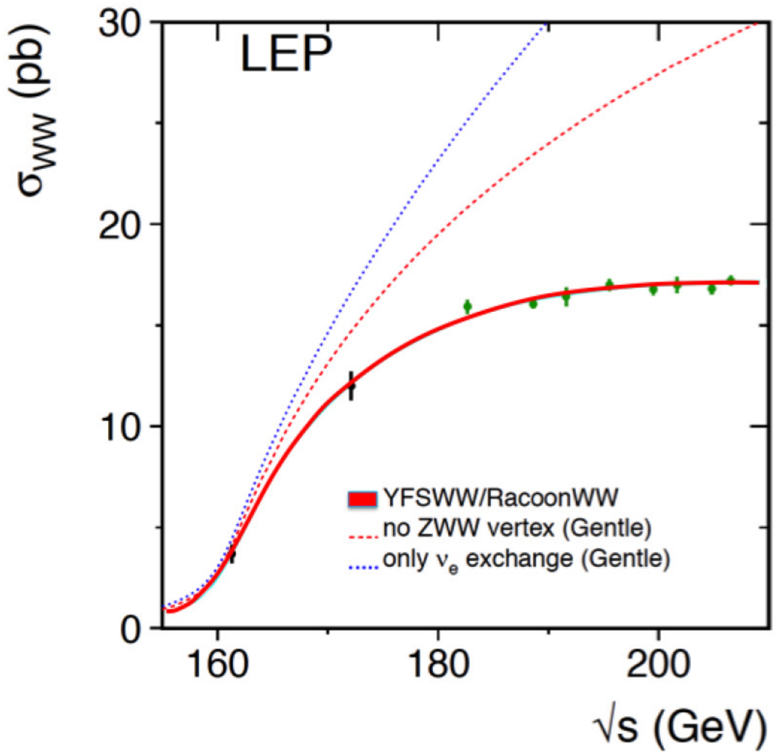
UNIVERSITY OF CAMBRIDGE

In this section...

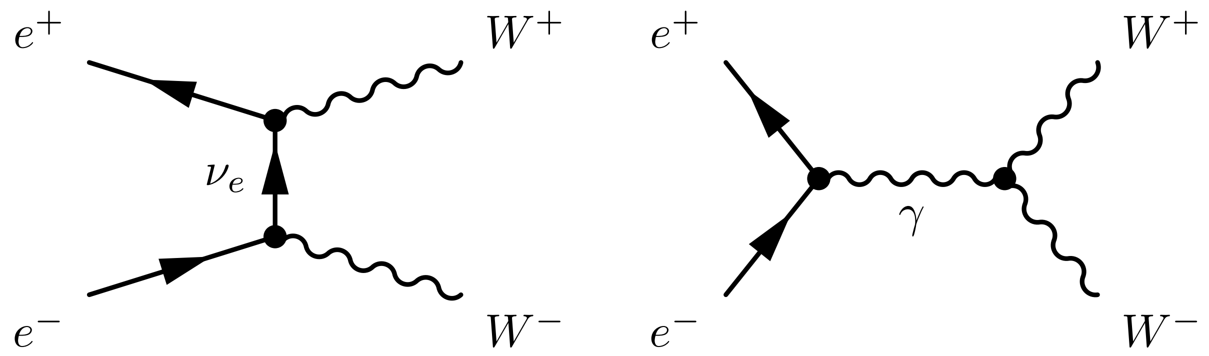
- GWS model
- Allowed vertices
- Revisit Feynman diagrams
- Experimental tests of Electroweak theory

Electroweak Unification

- Weak CC interactions explained by $W^\pm$ boson exchange
- $W^\pm$ bosons are charged, thus they couple to the γ

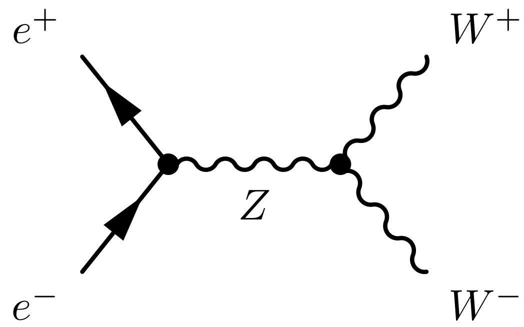


Consider $e^-e^+ \to W^+W^-$: 2 diagrams (+interference)



- Cross-section diverges at high energy
- Divergence cured by introducing Z boson
- Extra diagram for $e^-e^+ \to W^+W^-$
- Idea only works if γ, $W^\pm$, Z couplings are related

$\Rightarrow$ **Electroweak Unification**



Electroweak gauge theory *(non-examinable)*

- Postulate invariance under a gauge transformation like:

$$\psi \rightarrow \psi' = \mathrm{e}^{\mathrm{i}g\vec{\sigma}.\vec{\Lambda}(\vec{r},t)}\psi$$

  an "SU(2)" transformation (σ are 2x2 matrices).
- Operates on the state of "weak isospin" – a "rotation" of the isospin state.
- Invariance under SU(2) transformations $\Rightarrow$ three massless gauge bosons (W_1, W_2, W_3) whose couplings are well specified.
- They also have self-couplings.

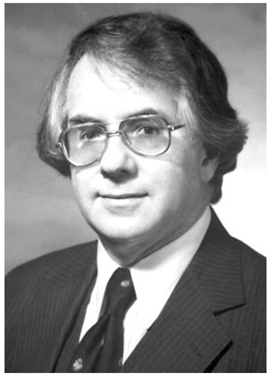
But this doesn't quite work...
Predicts W and Z have the same couplings – not seen experimentally!

Electroweak gauge theory

The solution...

- Unify QED and the weak force $\Rightarrow$ electroweak model
- "SU(2)xU(1)" transformation
  U(1) operates on the "weak hypercharge" $Y = 2(Q - I_3)$
  SU(2) operates on the state of "weak isospin, I"
- Invariance under SU(2)xU(1) transformations $\Rightarrow$ four massless gauge bosons W^+, W^-, W_3, B
- The two neutral bosons W_3 and B then **mix** to produce the physical bosons Z and γ
- Photon properties must be the same as QED $\Rightarrow$ predictions of the couplings of the Z in terms of those of the W and γ
- Still need to account for the masses of the W and Z. This is the job of the Higgs mechanism (later).

The GWS Model



The Glashow, Weinberg and Salam model treats EM and weak interactions as different manifestations of a single unified electroweak force (Nobel Prize 1979)

Start with 4 massless bosons W^+, W_3, W^- and B. The neutral bosons mix to give physical bosons (the particles we see), i.e. the $W^\pm$, Z, and γ.

$$\begin{pmatrix} W^+ \\ W_3 \\ W^- \end{pmatrix} ; \quad B \quad \rightarrow \quad \begin{pmatrix} W^+ \\ Z \\ W^- \end{pmatrix} ; \quad \gamma$$

Physical fields: W^+, Z, W^- and A (photon).

$$Z = W_3 \cos\theta_W - B \sin\theta_W$$

$$A = W_3 \sin\theta_W + B \cos\theta_W$$

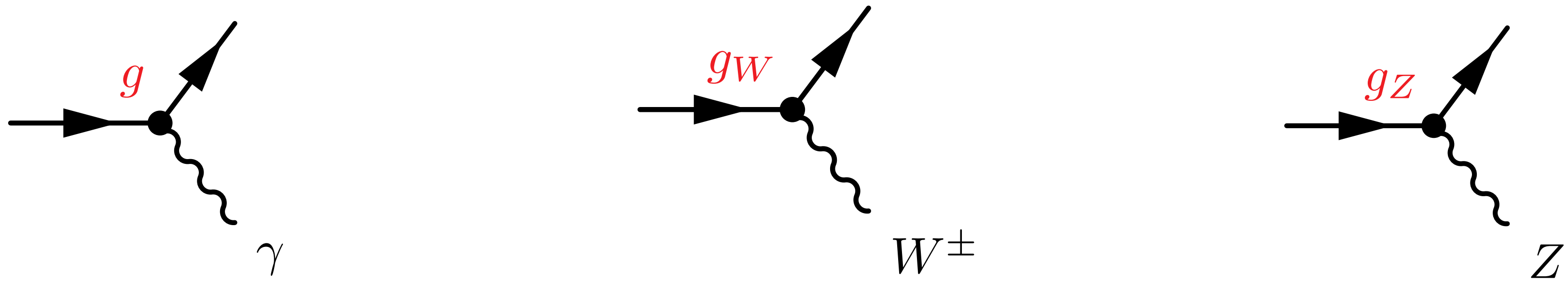
θ_W Weak Mixing Angle

$W^\pm$, Z "acquire" mass via the Higgs mechanism.

The GWS Model

The beauty of the GWS model is that it makes exact predictions of the $W^{\pm}$ and Z masses and of their couplings with only 3 free parameters.

Couplings given by α_{EM} and θ_W



$$\alpha_{EM} = \frac{e^2}{4\pi} \quad g \sim e \qquad g_W = \frac{e}{\sin\theta_W} \qquad g_Z = \frac{e}{\sin\theta_W \cos\theta_W} = \frac{g_W}{\cos\theta_W}$$

Masses also given by G_F and θ_W

From Fermi theory

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} = \frac{e^2}{8m_W^2 \sin^2\theta_W} \qquad m_{W^{\pm}} = \left(\frac{\sqrt{2}e^2}{8\,G_F \sin^2\theta_W}\right)^{1/2} \qquad m_Z = \frac{m_W}{\cos\theta_W}$$

If we know α_{EM}, G_F, $\sin\theta_W$ (from experiment), everything else is defined.

Example — mass relation (non-examinable)

- As a result of the mixing, we require that the mass eigenstates should be the Z and γ, and the mass of the photon be zero.
- We then compute the matrix elements of the mass operator:

$$m_Z^2 = \langle W_3 \cos\theta_W - B \sin\theta_W | \hat{M}^2 | W_3 \cos\theta_W - B \sin\theta_W \rangle$$

$$= m_W^2 \cos^2\theta_W + m_B^2 \sin^2\theta_W - 2m_{WB}^2 \cos\theta_W \sin\theta_W$$

$$m_\gamma^2 = \langle W_3 \sin\theta_W + B \cos\theta_W | \hat{M}^2 | W_3 \sin\theta_W + B \cos\theta_W \rangle$$

$$= m_W^2 \sin^2\theta_W + m_B^2 \cos^2\theta_W + 2m_{WB}^2 \cos\theta_W \sin\theta_W \quad = 0$$

$$m_{Z\gamma}^2 = \langle W_3 \cos\theta_W - B \sin\theta_W | \hat{M}^2 | W_3 \sin\theta_W + B \cos\theta_W \rangle$$

$$= (m_W^2 - m_B^2) \sin\theta_W \cos\theta_W + m_{WB}^2 (\cos^2\theta_W - \sin^2\theta_W) \quad = 0$$

- Solving these three equations gives

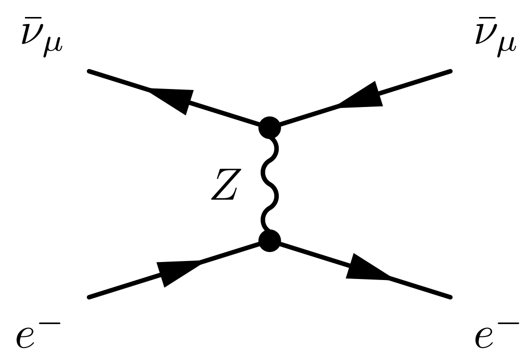
$$m_Z = \frac{m_W}{\cos\theta_W}$$

Couplings

- Slightly simplified – see Part III for better treatment. Starting from
$$Z = W_3 \cos\theta_W - B \sin\theta_W$$
$$A = W_3 \sin\theta_W + B \cos\theta_W$$
- W_3 couples to I_3 with strength g_W and B couples to $Y = 2(Q - I_3)$ with g'
- So, coupling of A (photon) is
$$g_W I_3 \sin\theta_W + g' 2(Q - I_3)\cos\theta_W = Qe \qquad \text{for all } I_3$$
$$\Rightarrow g' = \frac{g_W \tan\theta_W}{2} \quad \text{and} \quad g' \cos\theta_W = \frac{e}{2} \qquad \Rightarrow g_W = \frac{e}{\sin\theta_W}$$
- The couplings of the Z are therefore
$$g_W I_3 \cos\theta_W - g' 2(Q - I_3)\sin\theta_W = \frac{e}{\sin\theta_W \cos\theta_W}\left[I_3 - Q\sin^2\theta_W\right]$$
$$= g_Z\left[I_3 - Q\sin^2\theta_W\right]$$
- For right-handed fermions, $I_3 = 0$, while for left-handed fermions $I_3 = +1/2(\nu, u, c, t)$ or $I_3 = -1/2(e^-, \mu^-, \tau^-, d', s', b')$; Q is charge in units of e

Evidence for GWS Model

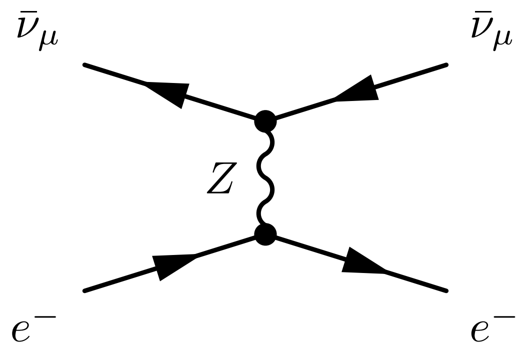
- **Discovery of Neutral Currents (1973)**
  The process $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$ was observed.
  Only possible Feynman diagram (no $W^\pm$ diagram).
  Indirect evidence for Z.



Gargamelle Bubble Chamber at CERN

Evidence for GWS Model

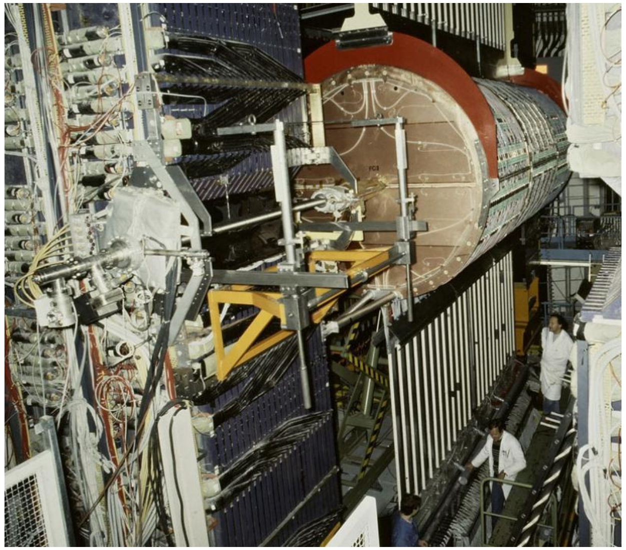
- **Discovery of Neutral Currents (1973)**
  The process $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ was observed.
  Only possible Feynman diagram (no $W^\pm$ diagram).
  Indirect evidence for Z.



- **Direct Observation of $W^\pm$ and Z (1983)**
  First direct observation in $p\bar{p}$ collisions at $\sqrt{s} = 540\ \mathrm{GeV}$ via decays into leptons

$$p\bar{p} \to W^\pm + X \qquad\qquad p\bar{p} \to Z + X$$
$$\hookrightarrow e^\pm \nu_e, \mu^\pm \nu_\mu \qquad\qquad \hookrightarrow e^+e^-, \mu^+\mu^-$$

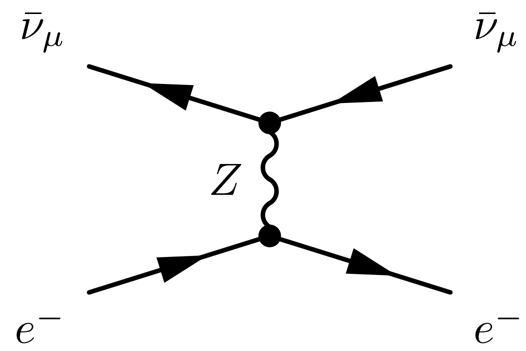
UA1 Experiment at CERN
Used Super Proton Synchrotron
(now part of LHC!)



Evidence for GWS Model

- **Discovery of Neutral Currents (1973)**

  The process $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$ was observed.
  Only possible Feynman diagram (no $W^\pm$ diagram).
  Indirect evidence for Z.



- **Direct Observation of $W^\pm$ and Z (1983)**

  First direct observation in $p\bar{p}$ collisions at $\sqrt{s} = 540\ \mathrm{GeV}$ via decays into leptons

$$p\bar{p} \to W^\pm + X \qquad \hookrightarrow e^\pm \nu_e, \mu^\pm \nu_\mu$$

$$p\bar{p} \to Z + X \qquad \hookrightarrow e^+ e^-, \mu^+ \mu^-$$

- **Precision Measurements of the Standard Model (1989-2000)**

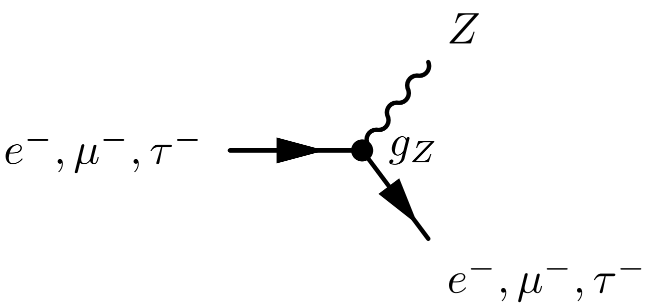
  LEP e^+e^- collider provided many precision measurements of the Standard Model.

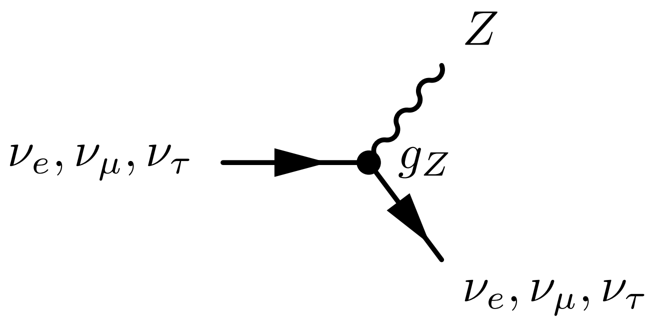
- Wide variety of different processes consistent with GWS model predictions and measure same value of

$$\sin^2\theta_W = 0.23113 \pm 0.00015 \qquad \theta_W \sim 29^\circ$$

The Weak NC Vertex

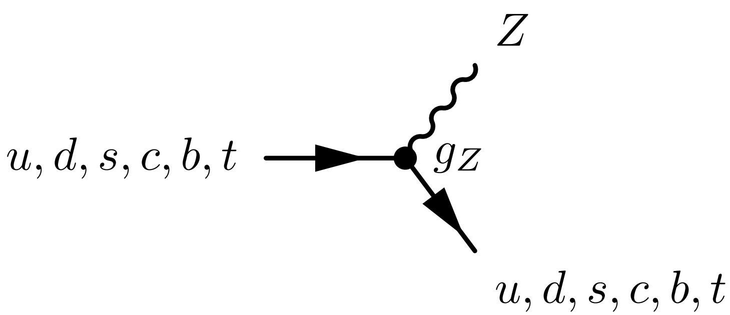
All weak neutral current interactions can be described by the Z boson propagator and the weak vertices:





The Standard Model Weak NC Lepton Vertex

+ antiparticles



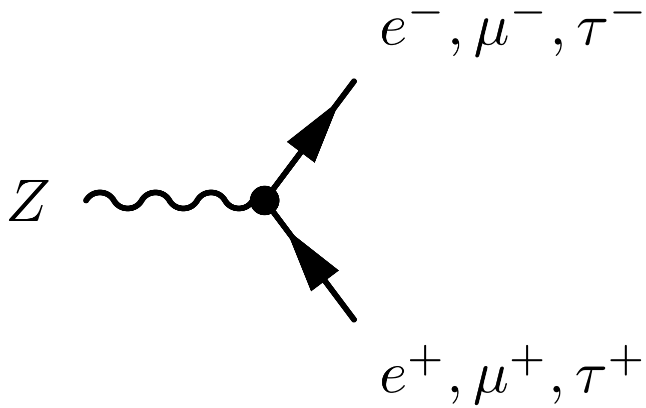
The Standard Model Weak NC Quark Vertex

+ antiparticles

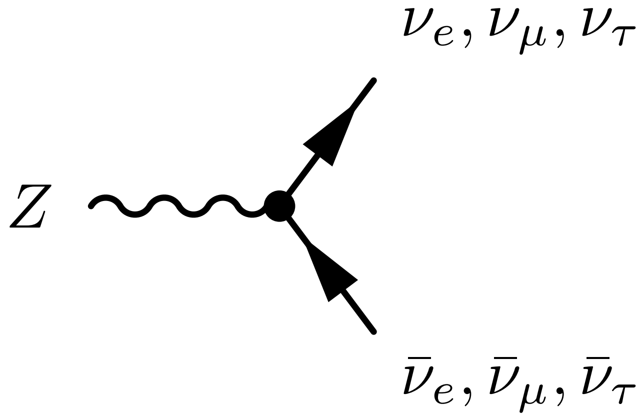
- Z never changes type of particle
- Z never changes quark or lepton flavour
- Z couplings are a mixture of EM and weak couplings, and therefore depend on $\sin^2\theta_W$.

Examples

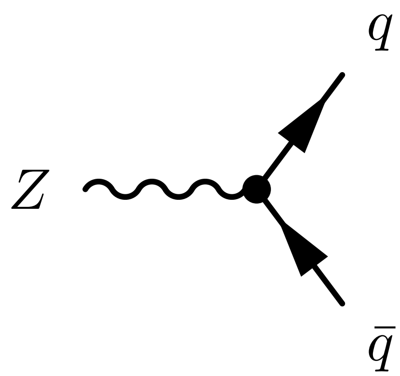
$Z \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$



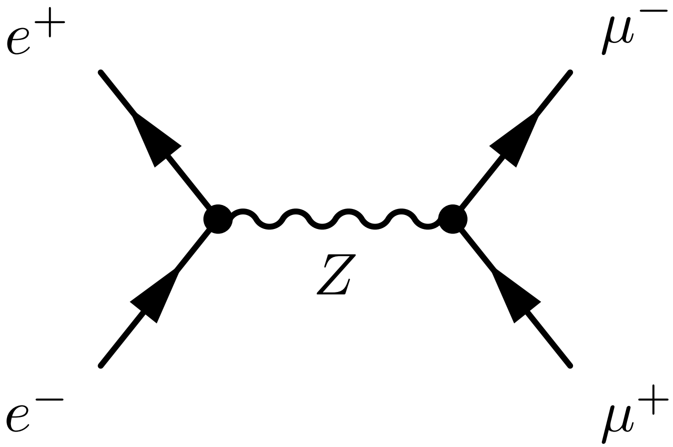
$Z \rightarrow \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$



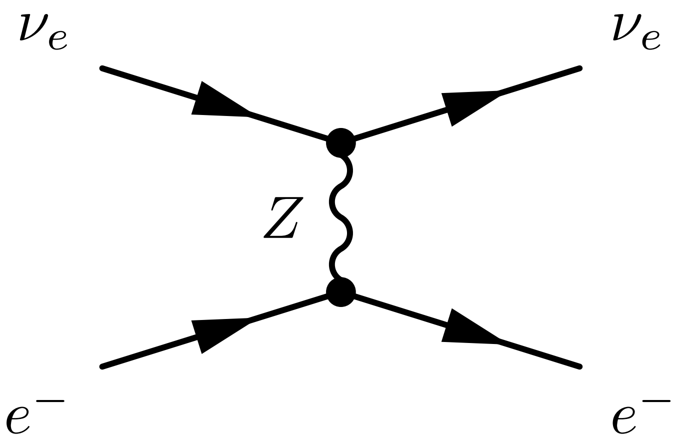
$Z \rightarrow q\bar{q}$



$e^+e^- \rightarrow \mu^+\mu^-$

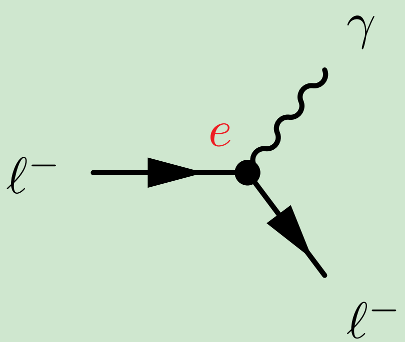


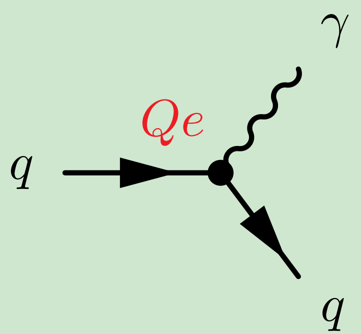
$\nu_e e^- \rightarrow \nu_e e^-$



Summary of Standard Model (matter) Vertices

Electromagnetic (QED)



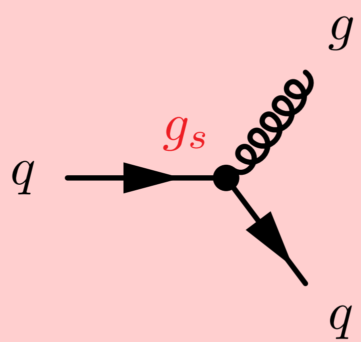


$$\alpha = \frac{e^2}{4\pi}$$

$q = u, d, s, c, b, t$

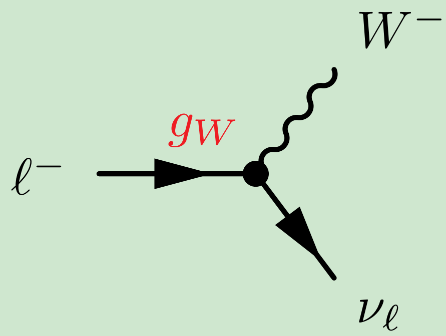
+ antiparticles

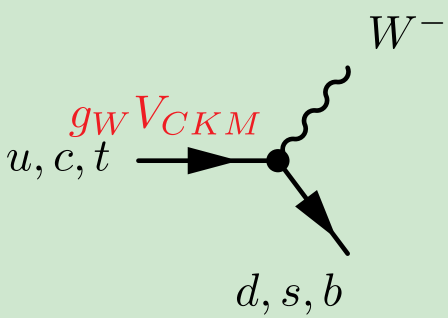
Strong (QCD)



$$\alpha_s = \frac{g_s^2}{4\pi}$$

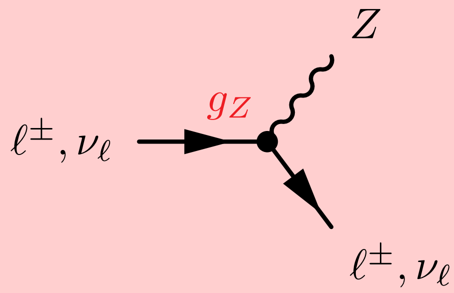
Weak CC

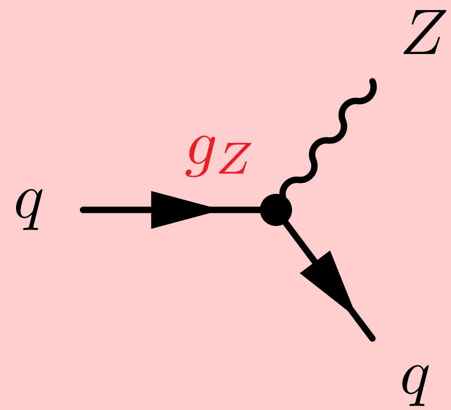




$$\alpha_W = \frac{g_W^2}{4\pi}$$

Weak NC





$$g_Z = \frac{g_W}{\cos\theta_W}$$

Feynman Diagrams *a reminder*

1. $\pi^- + p \rightarrow K^0 + \Lambda$

2. $\nu_\tau + e^- \rightarrow \nu_\tau + e^-$

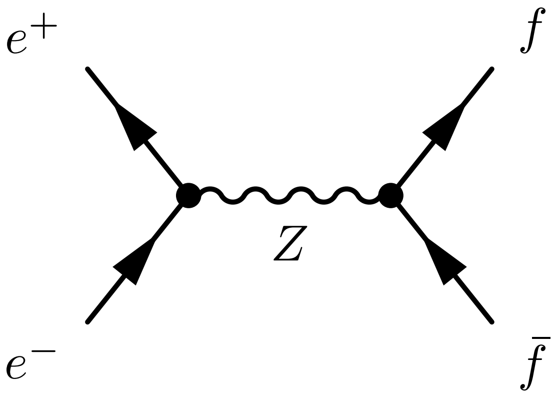
3. $\bar{\nu}_\tau + \tau^- \rightarrow \bar{\nu}_\tau + \tau^-$

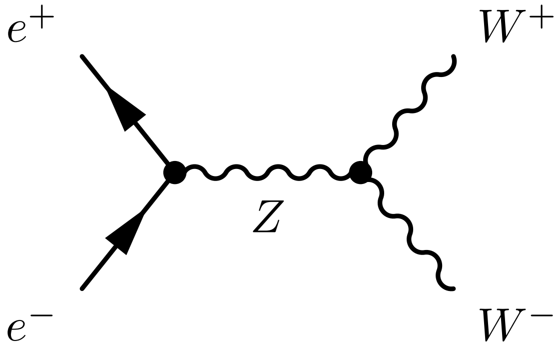
4. $D^+ \rightarrow K^-\pi^+\pi^+$

Experimental Tests of the Electroweak model at LEP

The Large Electron Positron (LEP) collider at CERN provided high precision measurements of the Standard Model (1989-2000).

Designed as a Z and $W^{\pm}$ boson factory



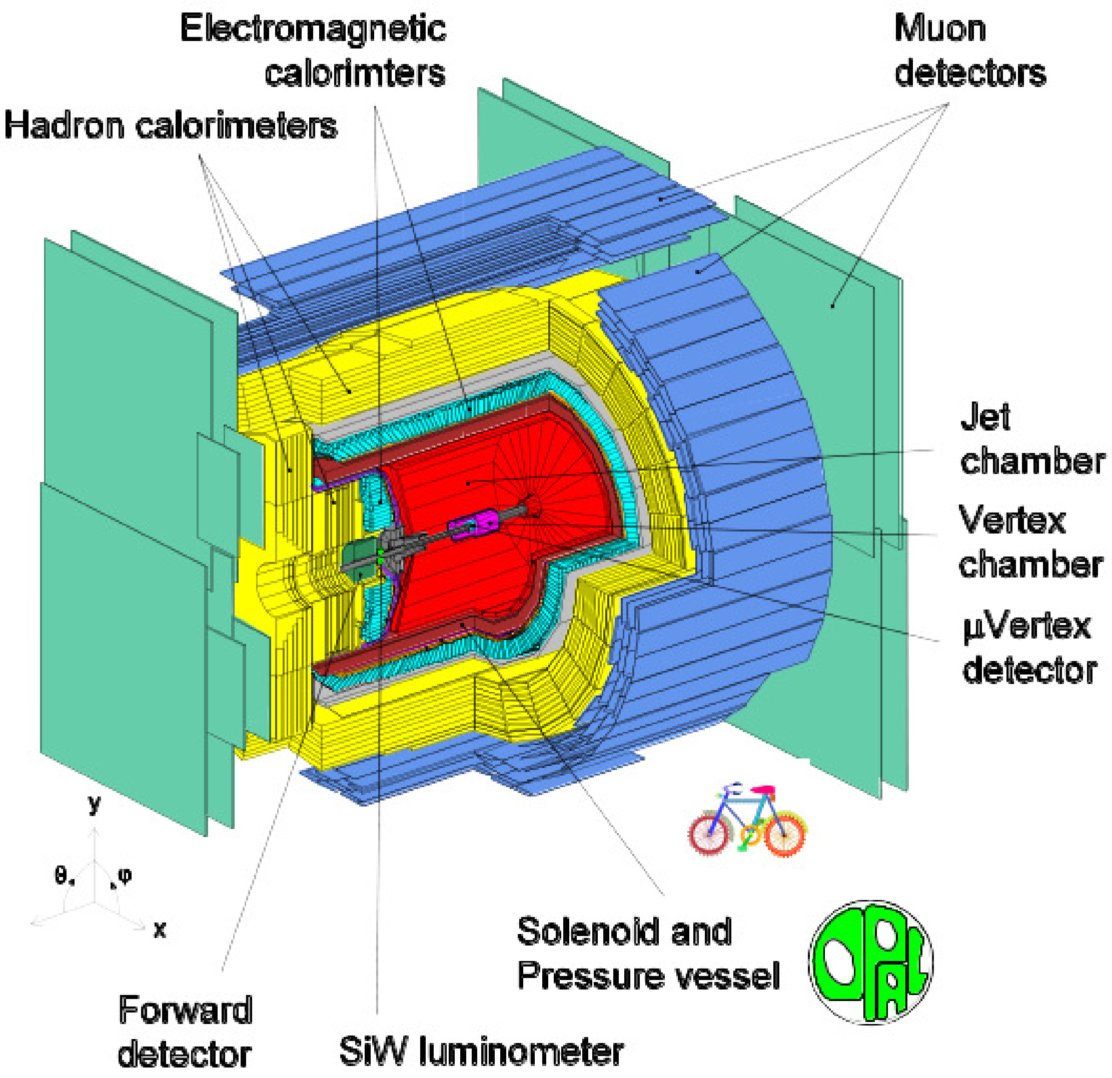


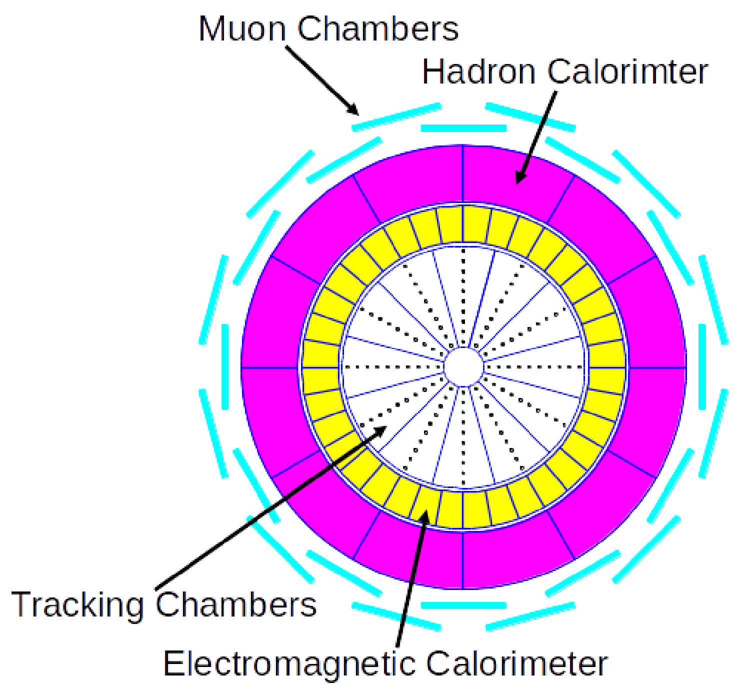
Precise measurements of the properties of Z and $W^{\pm}$ bosons provide the most stringent test of our current understanding of particle physics.

- LEP is the highest energy e^+e^- collider ever built $\sqrt{s} = 90 - 209$ GeV
- Large circumference, 27 km
- 4 experiments combined saw 16×10^6 Z events, 30×10^3 $W^{\pm}$ events

OPAL: a LEP detector

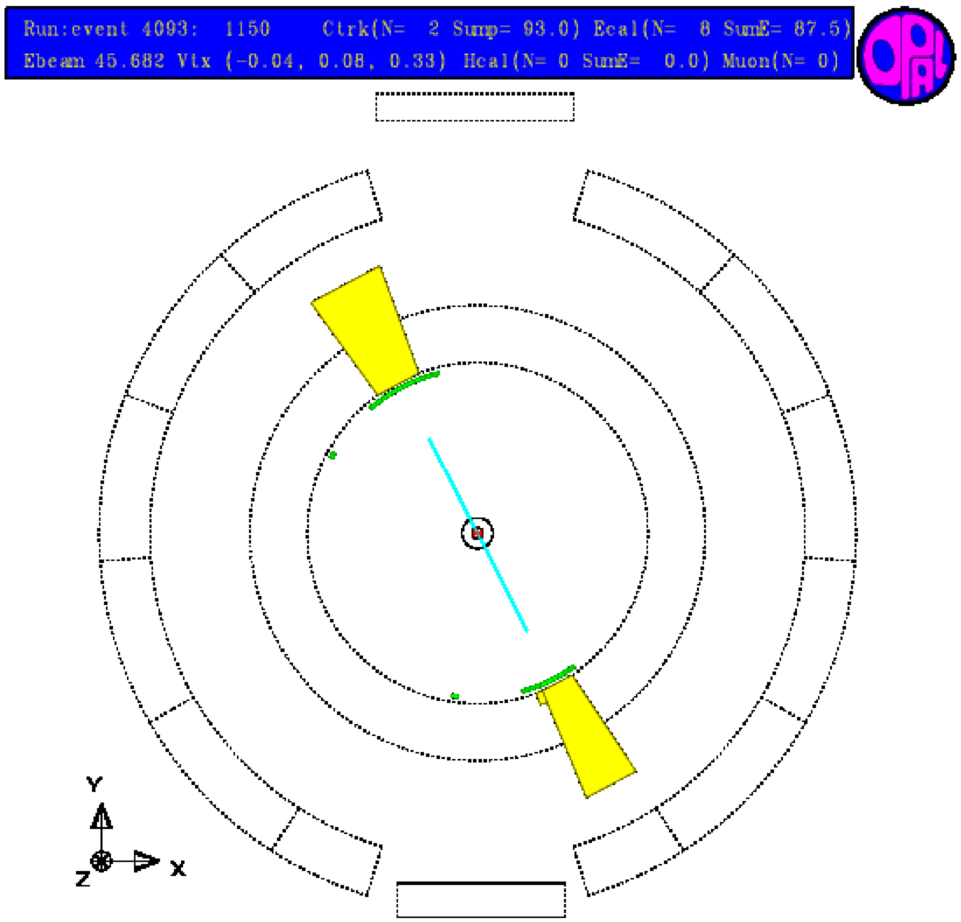
OPAL was one of the 4 experiments at LEP. Size: $12\,\mathrm{m} \times 12\,\mathrm{m} \times 15\,\mathrm{m}$.



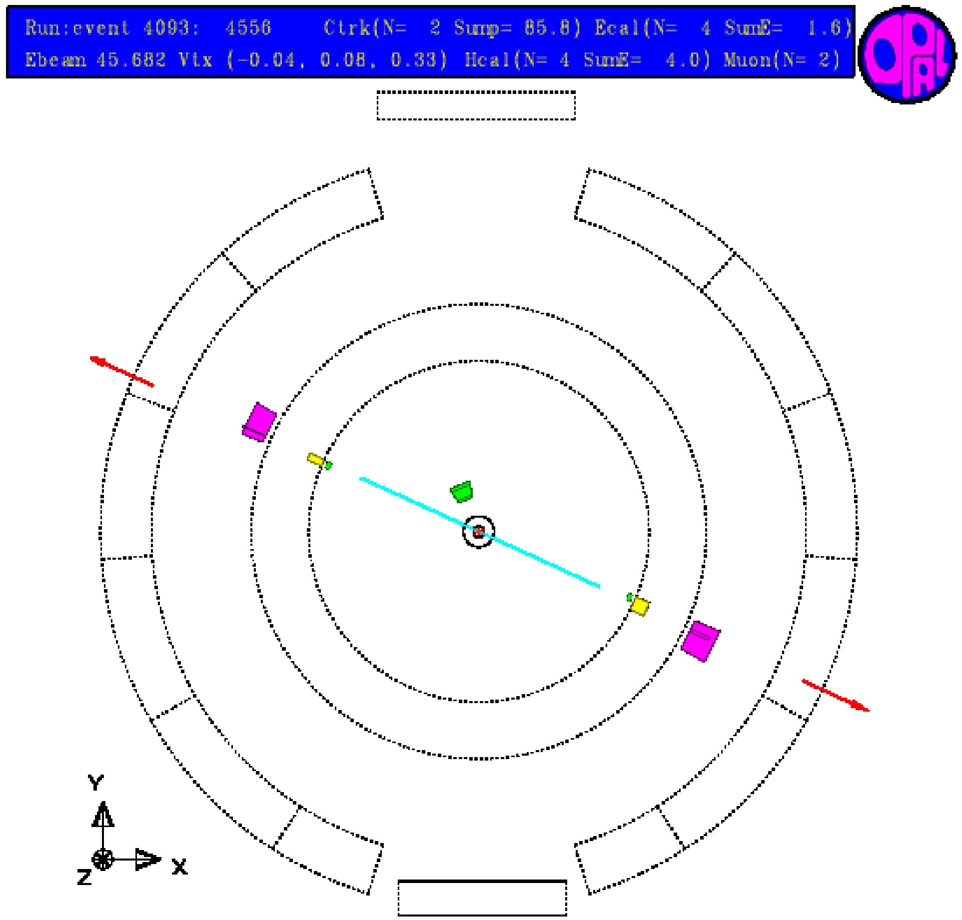


Typical $e^+e^- \to Z$ events

$e^+e^- \to Z \to e^+e^-$

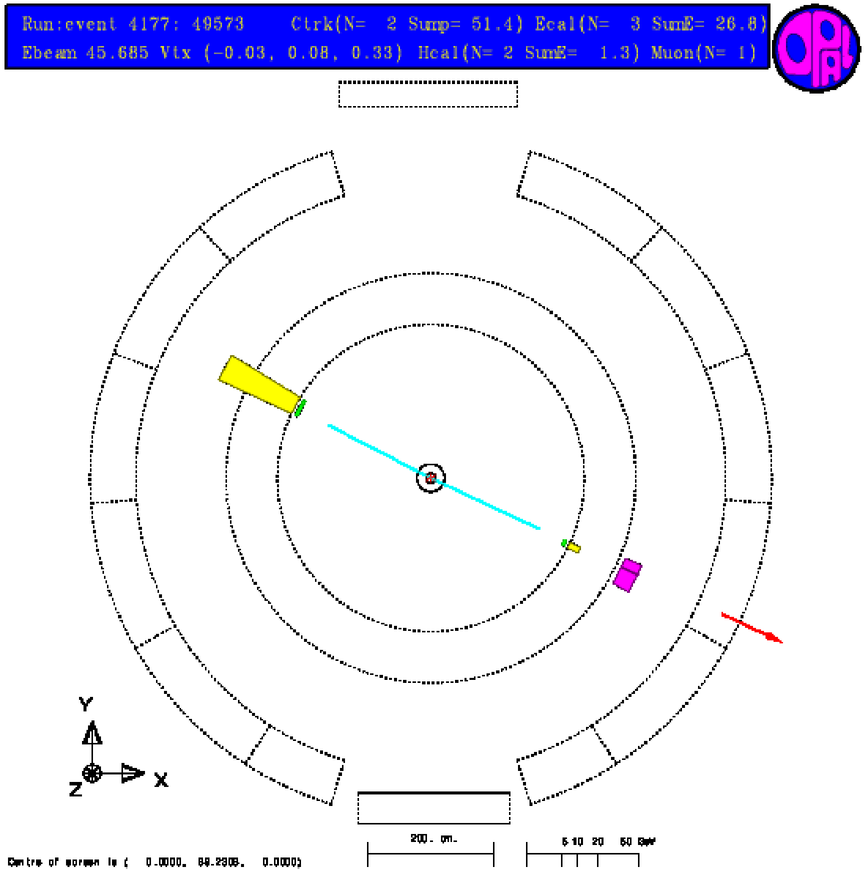


$e^+e^- \to Z \to \mu^+\mu^-$



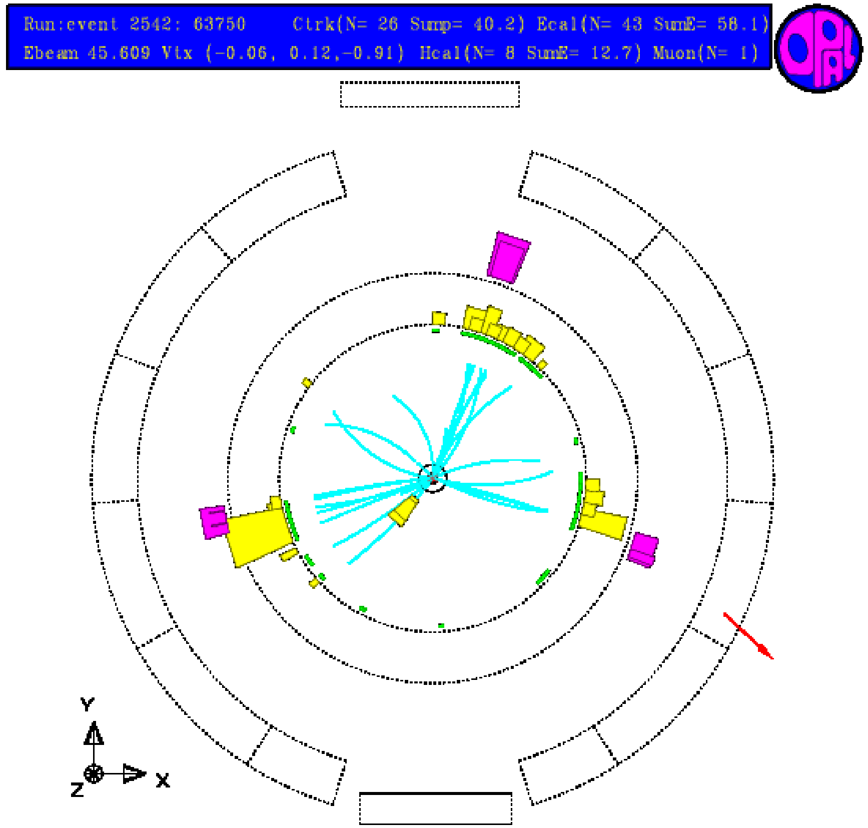
Typical $e^+e^- \to Z$ events

$e^+e^- \to Z \to \tau^+\tau^-$



Taus decay within the detector (lifetime $\sim 10^{-13}$ s).
Here $\tau^- \to e^-\bar{\nu}_e\nu_\tau$, $\tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau$

$e^+e^- \to Z \to q\bar{q}$

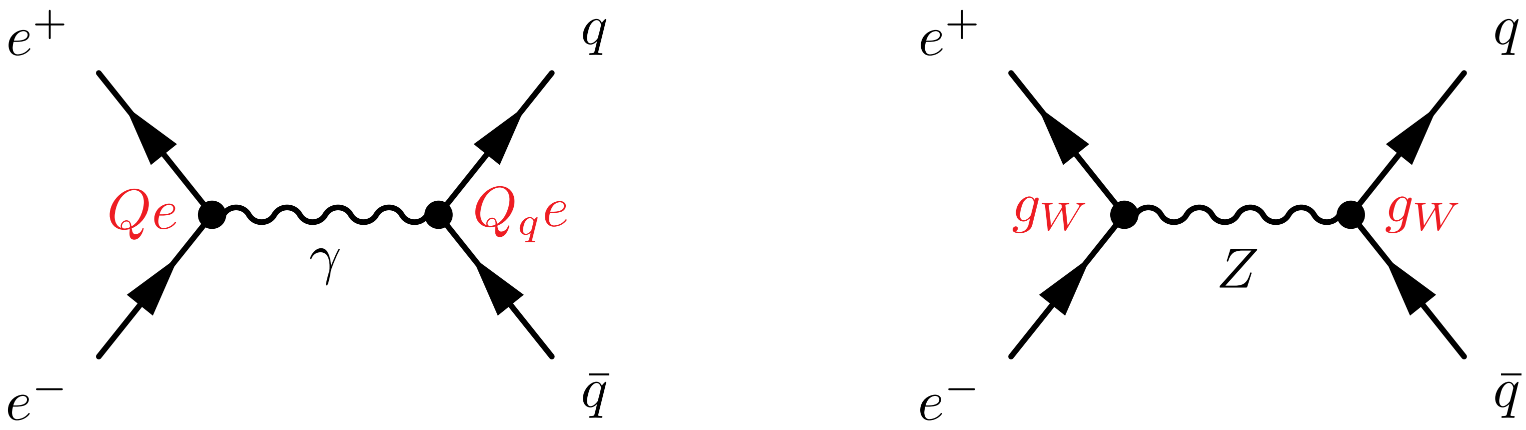


3-jet event (gluon emitted by $q/\bar{q}$)

The Z Resonance

Consider the process $e^+e^- \to q\bar{q}$

- At small $\sqrt{s}(< 50\ \mathrm{GeV})$, we only considered an intermediate photon
- At higher energies, the Z exchange diagram contributes ($+Z\gamma$ interference)

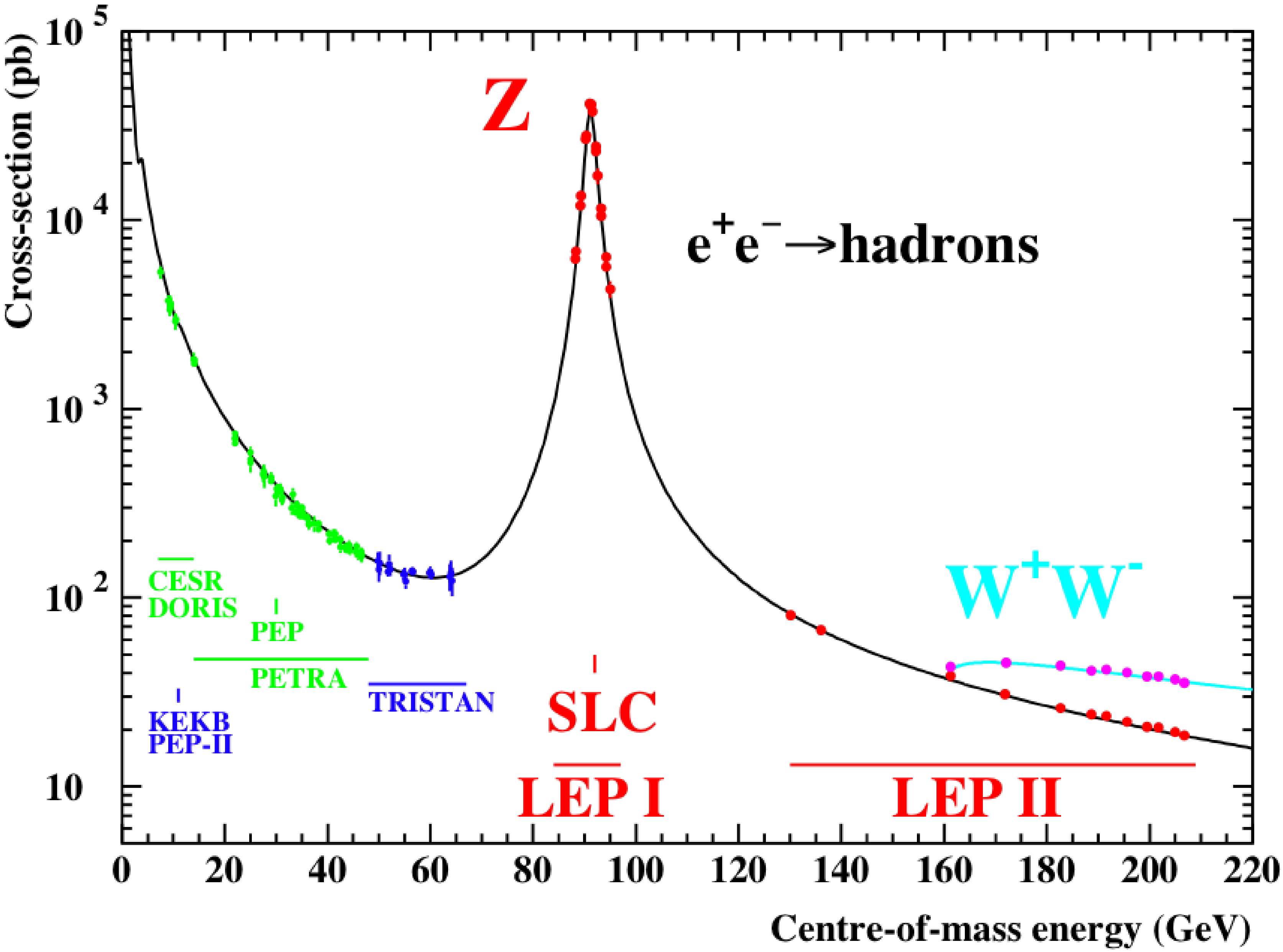


$$\sigma(e^+e^- \to \gamma \to q\bar{q}) = \frac{4\pi\alpha^2}{3s}\sum 3Q_q^2$$

- The Z is a decaying intermediate massive state (lifetime $\sim 10^{-25}$ s)

  $\Rightarrow$ Breit-Wigner resonance
- Around $\sqrt{s} \sim m_Z$, the Z diagram dominates

The Z Resonance



The Z Resonance

Breit-Wigner cross-section for $e^+e^- \to Z \to f\bar{f}$ (where $f\bar{f}$ is any fermion-antifermion pair)

Centre-of-mass energy $\sqrt{s} = E_{CM} = E_{e^+} + E_{e^-}$

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{g\pi}{E_e^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(E_{CM} - m_Z)^2 + \frac{\Gamma_Z^2}{4}}$$

with $$g = \frac{2J_Z + 1}{(2J_{e^-} + 1)(2J_{e^+} + 1)} = \frac{3}{4} \qquad J_Z = 1;\; J_{e^\pm} = \frac{1}{2}$$

giving

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{3\pi}{4E_e^2} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(E_{CM} - m_Z)^2 + \frac{\Gamma_Z^2}{4}} = \frac{3\pi}{s} \frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{(\sqrt{s} - m_Z)^2 + \frac{\Gamma_Z^2}{4}}$$

Γ_Z is the total decay width, i.e. the sum over the partial widths for different decay modes

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}$$

The Z Resonance

At the peak of the resonance $\sqrt{s} = m_Z$:

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

Hence, for all fermion/antifermion pairs in the final state

$$\sigma(e^+e^- \to Z \to \text{anything}) = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}}{\Gamma_Z} \qquad\qquad \Gamma_{f\bar{f}} = \Gamma_Z$$

Compare to the QED cross-section at $\sqrt{s} = m_Z$

$$\sigma_{\text{QED}} = \frac{4\pi\alpha^2}{3s}$$

$$\frac{\sigma(e^+e^- \to Z \to \text{anything})}{\sigma_{\text{QED}}} = \frac{9}{\alpha^2}\frac{\Gamma_{ee}}{\Gamma_Z} \sim 5700$$

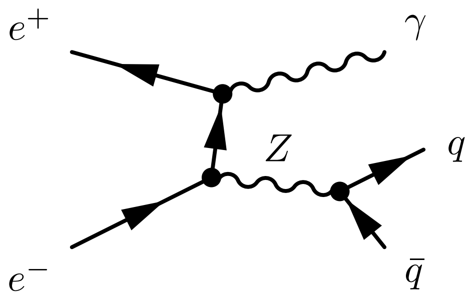
$$\Gamma_{ee} = 85\ \text{MeV}, \quad \Gamma_Z = 2.5\ \text{GeV}, \quad \alpha = 1/137$$

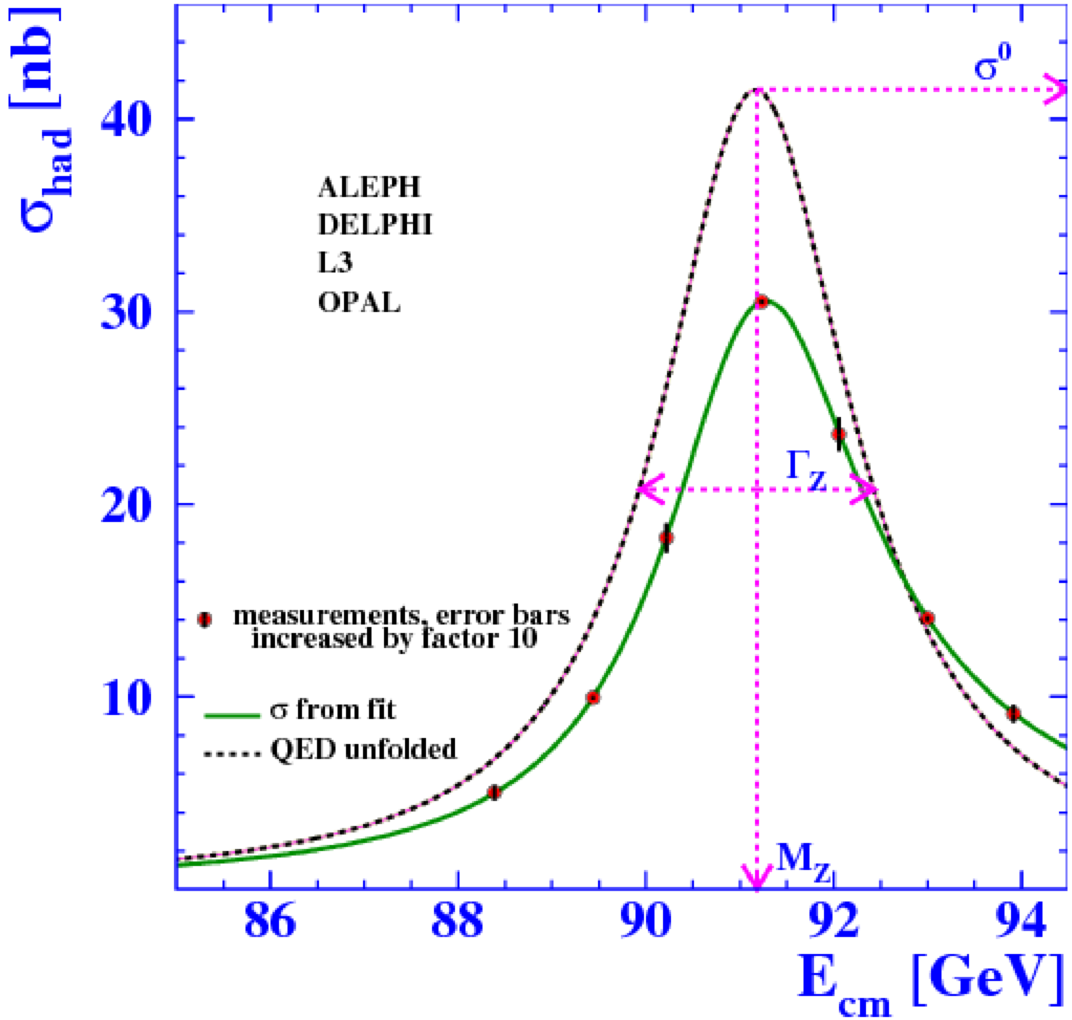
Measurement of m_Z and Γ_Z

- Run LEP at various centre-of-mass energies ($\sqrt{s}$) close to the peak of the Z resonance and measure $\sigma(e^+e^- \to q\bar{q})$
- Determine the parameters of the resonance:

  Mass of the Z, m_Z
  Total decay width, Γ_Z
  Peak cross-section, σ^0

One subtle feature: need to correct measurements for QED effects due to radiation from the e^+e^- beams. This radiation has the effect of reducing the centre-of-mass energy of the e^+e^- collision which smears out the resonance.



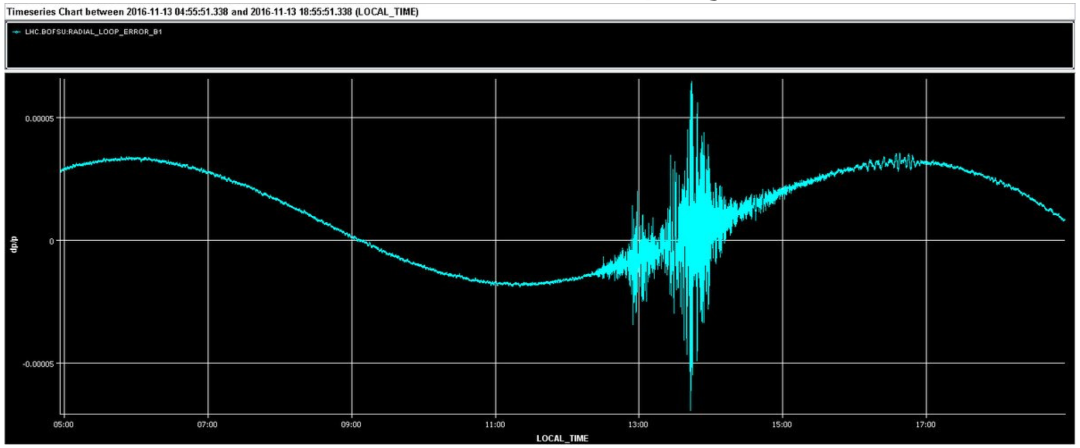


Measurement of m_Z and Γ_Z

m_Z was measured with precision 2 parts in 10^5

- Need a detailed understanding of the accelerator and astrophysics.

Tidal distortions of the Earth by the Moon cause the rock surrounding LEP to be distorted – changing the radius by 0.15 mm (total 4.3 km). This is enough to change the centre-of-mass energy.



LHC ring is stretched by 0.1mm by the 7.5 magnitude earthquake in New Zealand, Nov 2016. Tidal forces can also be seen.

- Also need a train timetable.

  Leakage currents from the TGV rail via Lake Geneva follow the path of least resistance... using LEP as a conductor.

Accounting for these effects (and many others):

$$m_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

$$\sigma^0_{q\bar{q}} = 41.450 \pm 0.037 \text{ nb}$$

Number of Generations

- Currently know of three generations of fermions. Masses of quarks and leptons increase with generation. Neutrinos are approximately massless (or are they?)

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}\begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}\begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix} \qquad \begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} t \\ b \end{pmatrix}$$

- Could there be more generations? e.g. $\begin{pmatrix} t' \\ b' \end{pmatrix} \quad \begin{pmatrix} L \\ \nu_L \end{pmatrix}$

- The Z boson couples to all fermions, including neutrinos. Therefore, the total decay width, Γ_Z, has contributions from all fermions with $m_f < m_Z/2$

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + \Gamma_{\nu\bar{\nu}}$$

$$\text{with} \qquad \Gamma_{\nu\bar{\nu}} = \Gamma_{\nu_e\bar{\nu}_e} + \Gamma_{\nu_\mu\bar{\nu}_\mu} + \Gamma_{\nu_\tau\bar{\nu}_\tau}$$

- If there were a fourth generation, it seems likely that the neutrino would be light, and, if so would be produced at LEP $\quad e^+e^- \to Z \to \nu_L\bar{\nu}_L$

- The neutrinos would not be observed directly, but could infer their presence from the effect on the Z resonance curve.

Number of Generations

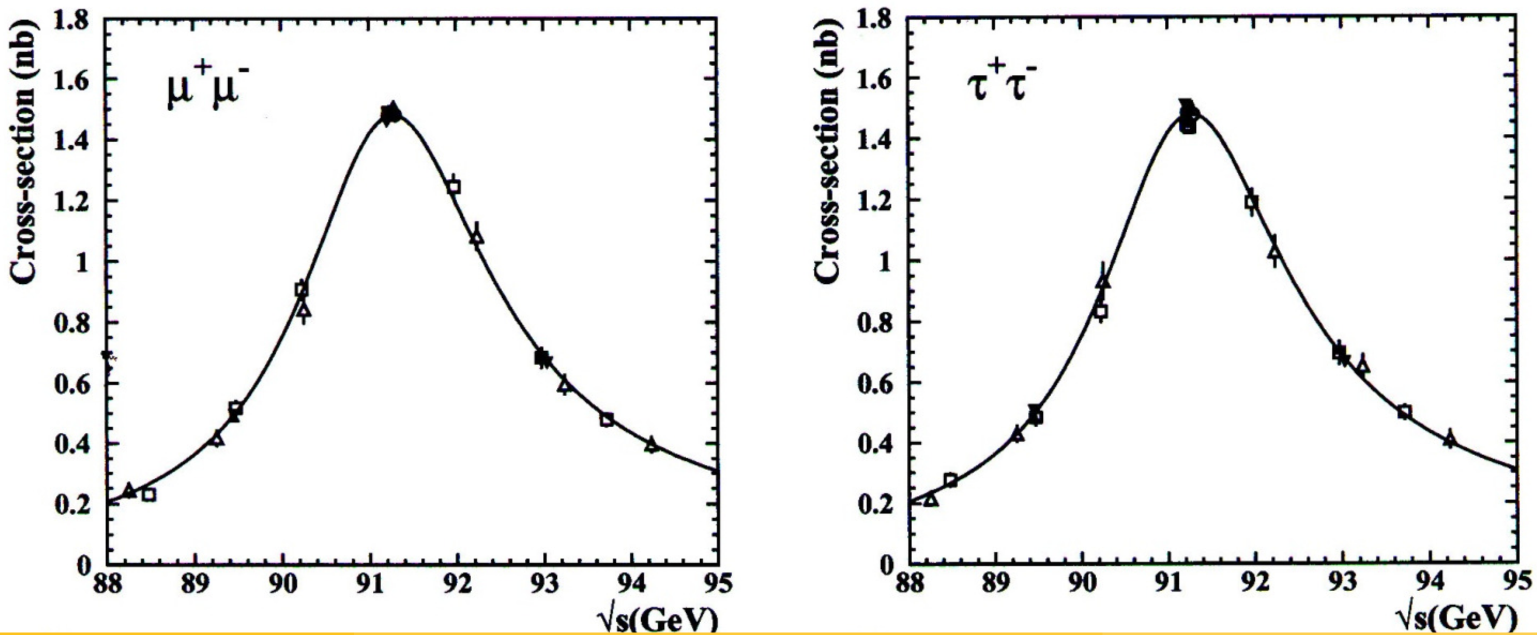
At the peak of the Z resonance, $\sqrt{s} = m_Z$

$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$

A fourth generation neutrino would increase the Z decay rate and thus increase Γ_Z. As a result, a decrease in the measured peak cross-sections for the visible final states would be observed.

Measure the $e^+e^- \to Z \to f\bar{f}$ cross-sections for all visible decay models (i.e. all fermions apart from $\nu\bar{\nu}$)

Examples: $e^+e^- \to \mu^+\mu^-$ $e^+e^- \to \tau^+\tau^-$



Number of Generations

- Have already measured m_Z and Γ_Z from the shape of the Breit-Wigner resonance. Therefore, obtain $\Gamma_{f\bar{f}}$ from the peak cross-sections in each decay mode using
$$\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}$$
Note, obtain Γ_{ee} from $\quad \sigma^0_{ee} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}^2}{\Gamma_Z^2}$

- Can relate the partial widths to the measured total width (from the resonance curve)
$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{q\bar{q}} + N_\nu \Gamma_{\nu\nu}$$
where N_ν is the number of neutrino species and $\Gamma_{\nu\nu}$ is the partial width for a single neutrino species.

Number of Generations

The difference between the measured value of Γ_Z and the sum of the partial widths for visible final states gives the invisible width $N_\nu \Gamma_{\nu\nu}$

Γ_Z	**2495.2±2.3 MeV**
Γ_{ee}	83.91±0.12 MeV
$\Gamma_{\mu\mu}$	83.99±0.18 MeV
$\Gamma_{\tau\tau}$	84.08±0.22 MeV
Γ_{qq}	1744.4±2.0 MeV
$N_\nu \Gamma_{\nu\nu}$	499.0±1.5 MeV

In the Standard Model, calculate $\Gamma_{\nu\nu} \sim 167\ \mathrm{MeV}$

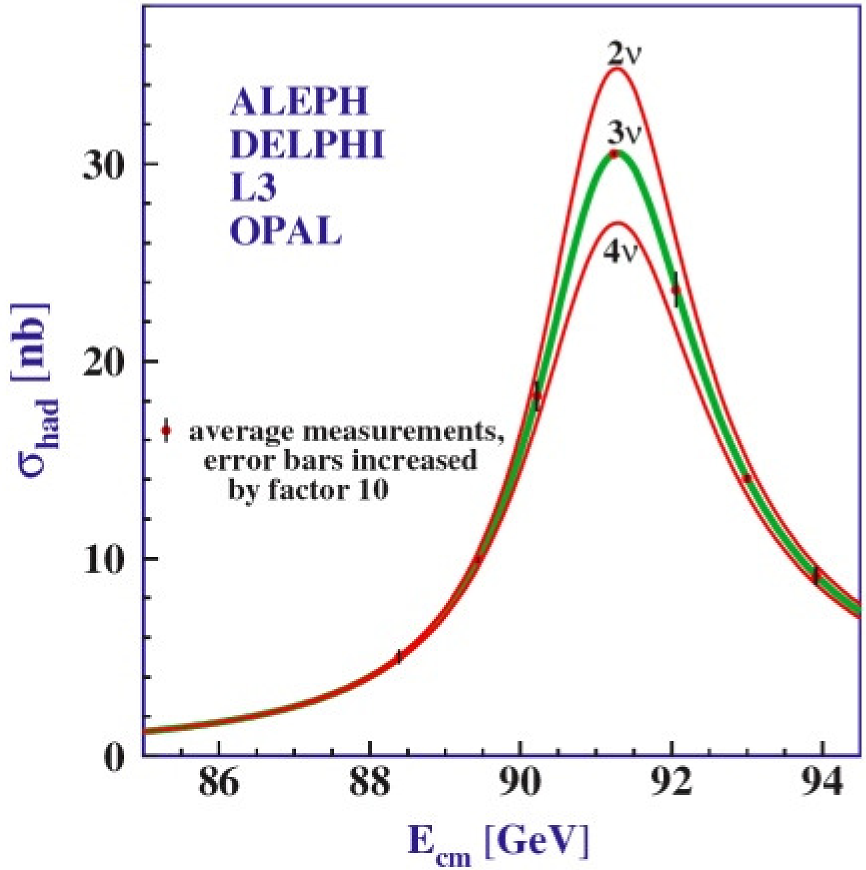
Therefore

$$N_\nu = \frac{\Gamma_{\nu\nu}^{\mathrm{measured}}}{\Gamma_{\nu\nu}^{\mathrm{SM}}} = 2.984 \pm 0.008$$

$\Rightarrow$ three generations of light neutrinos for $m_\nu < m_Z/2$

Number of Generations

Most likely that only 3 generations of quarks and leptons exist

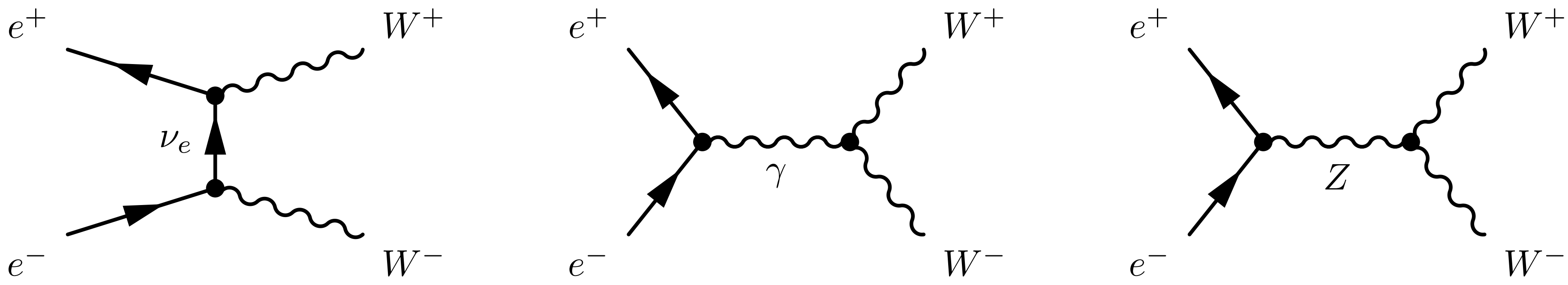


In addition

- Γ_{ee}, $\Gamma_{\mu\mu}$, $\Gamma_{\tau\tau}$ are consistent $\Rightarrow$ tests universality of the lepton couplings to the Z boson.
- Γ_{qq} is consistent with the expected value which assumes 3 colours – further evidence for colour

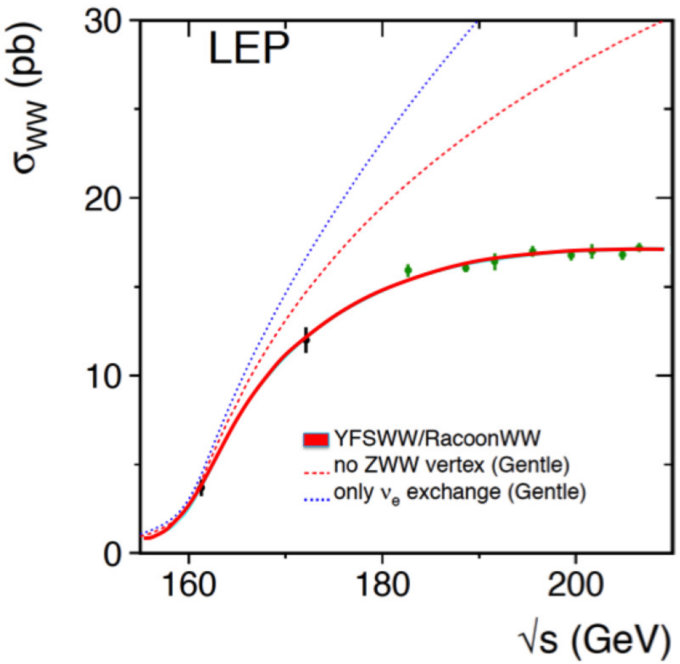
W^+W^- at LEP

- In e^+e^- collisions W bosons are produced in pairs.
- Standard Model: 3 possible diagrams:



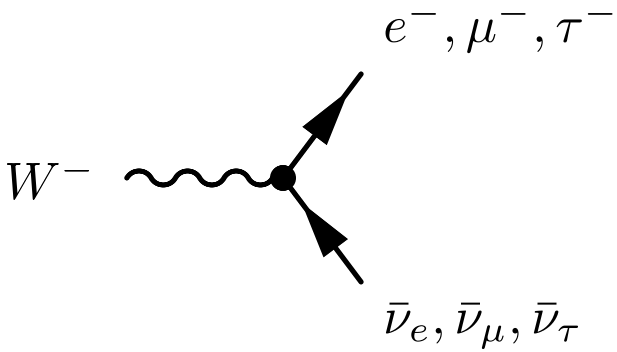
- LEP operated above the threshold for W^+W^- production (1996-2000) $\sqrt{s} > 2m_W$

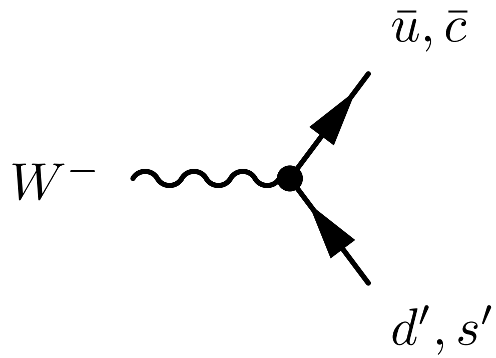
- Cross-section sensitive to the presence of the Triple Gauge Boson vertex



W^+W^- at LEP

In the Standard Model $W\ell\nu$ and $Wq\bar{q}$ couplings are $\sim$ equal.





$m_W < m_t$

$\times 3$ for colour

Expect (assuming 3 colours)

$$B(W^\pm \to q\bar{q}) = \frac{6}{9} = \frac{2}{3}$$

$$B(W^\pm \to \ell\nu) = \frac{3}{9} = \frac{1}{3}$$

QCD corrections $\sim \left(1 + \frac{\alpha_s}{\pi}\right)$

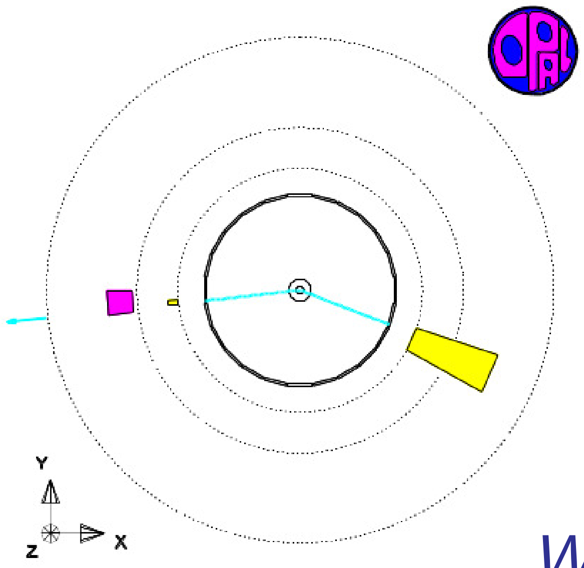
$$\Rightarrow B(W^\pm \to q\bar{q}) = 0.675$$

Measured BR

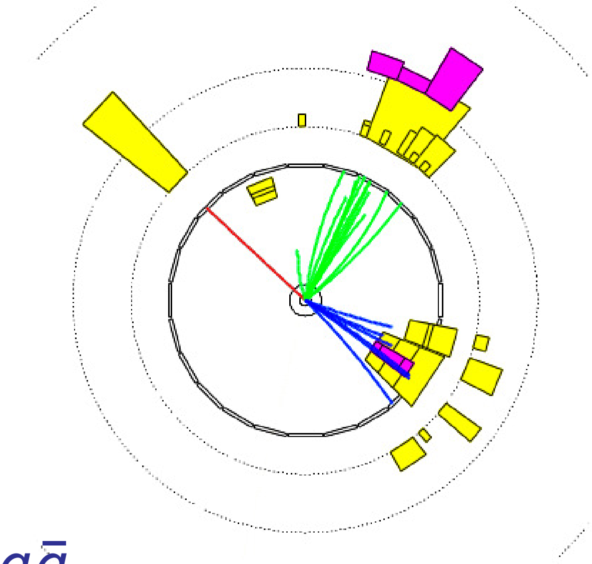
$W^+W^- \to \ell\nu\ell\nu$	10.5%
$W^+W^- \to q\bar{q}\ell\nu$	43.9%
$W^+W^- \to q\bar{q}q\bar{q}$	45.6%

W^+W^- events in OPAL

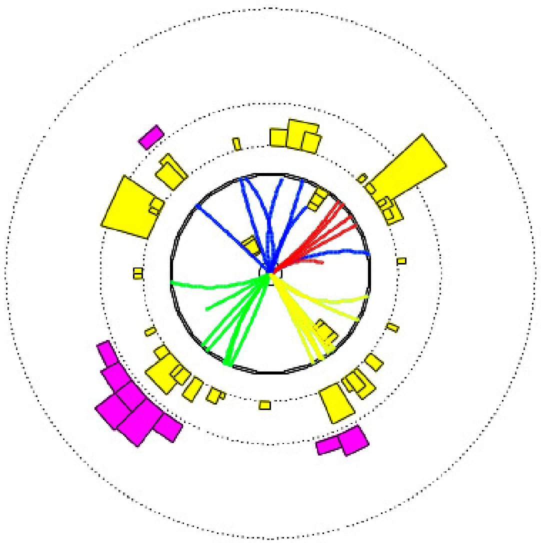
$W^+W^- \to e\nu\mu\nu$



$W^+W^- \to q\bar{q}e\nu$



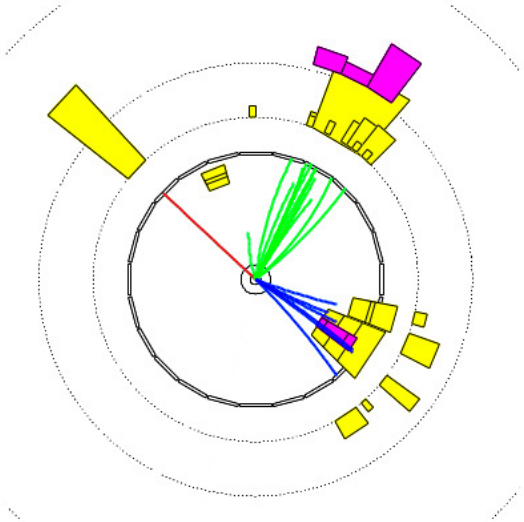
$W^+W^- \to q\bar{q}q\bar{q}$

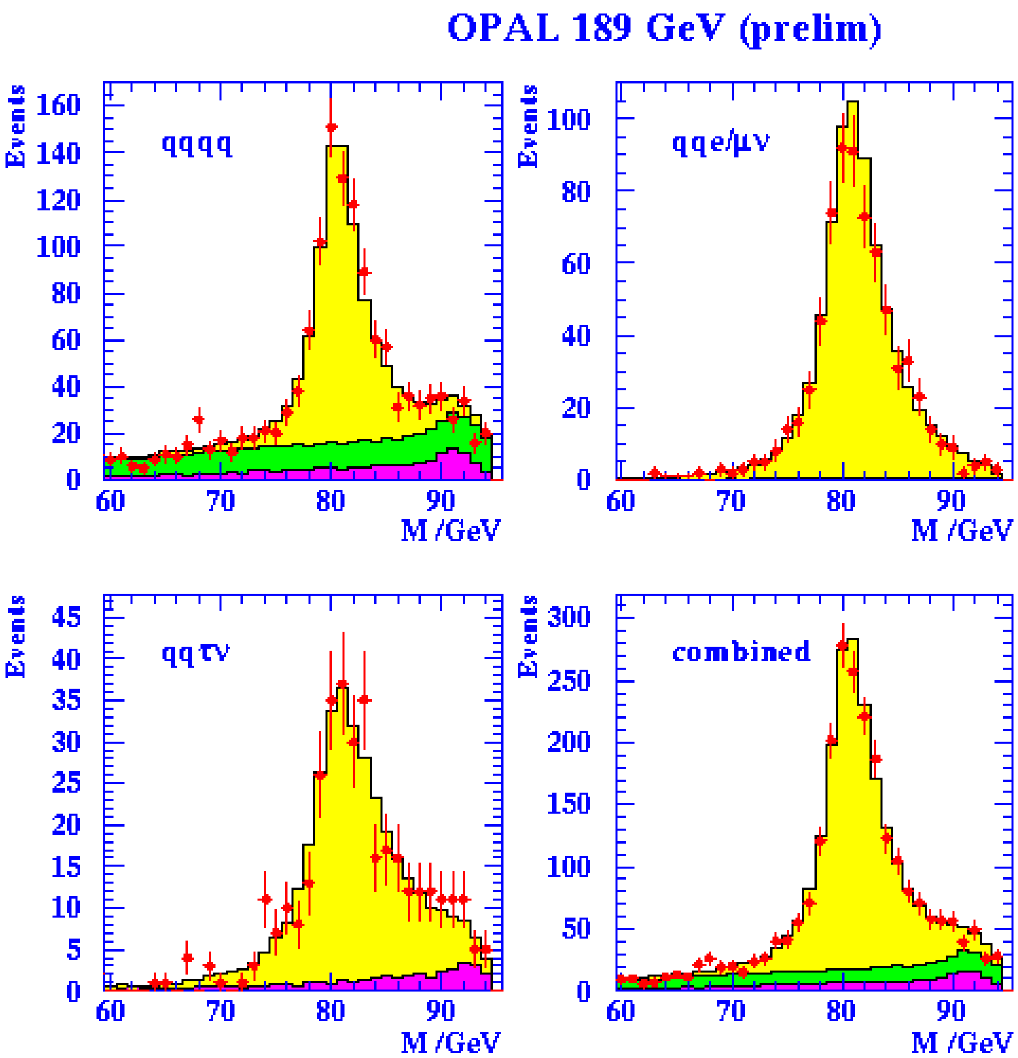


Measurement of m_W and Γ_W

Unlike $e^+e^- \rightarrow Z$, W boson production at LEP was not a resonant process.

m_W was measured by measuring the invariant mass in each event





4-momenta $p_{q1}, p_{q2}, p_e, p_\nu$

$$m_W = \frac{1}{2}\left(m_{q\bar{q}} + m_{\ell\nu}\right)$$

$$m_W = 80.423 \pm 0.038\ \mathrm{GeV}$$

$$\Gamma_W = 2.12 \pm 0.11\ \mathrm{GeV}$$

W Boson Decay Width

In the Standard Model, the W boson decay width is given by

$$\Gamma(W^- \to e^- \bar{\nu}_e) = \frac{g_W^2 m_W}{48\pi} = \frac{G_F m_W^3}{6\sqrt{2}\pi}$$

μ-decay: $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ LEP: $m_W = 80.423 \pm 0.038\ \mathrm{GeV}$

$$\Rightarrow \Gamma(W^- \to e^- \bar{\nu}_e) = 227\ \mathrm{MeV}$$

Total width is the sum over all partial widths:

$$W^- \to e^- \bar{\nu}_e,\ \mu^- \bar{\nu}_\mu,\ \tau^- \bar{\nu}_\tau,$$
$$W^- \to d' \bar{u},\ s' \bar{c}, \qquad \times 3 \text{ for colour}$$

If the W coupling to leptons and quarks is equal and there are 3 colours:

$$\Gamma = \sum_i \Gamma_i = (3 + 2 \times 3)\Gamma(W^- \to e^- \bar{\nu}_e) \sim 2.1\ \mathrm{GeV}$$

Compare with measured value from LEP: $\Gamma_W = 2.12 \pm 0.11\ \mathrm{GeV}$

- Universal coupling constant
- Yet more evidence for colour!

Summary of Electroweak Tests

Now have 5 precise measurements of fundamental parameters of the Standard Model

$$\alpha_{EM} = 1/(137.03599976 \pm 0.00000050) \qquad \text{(at } q^2 = 0 \text{)}$$

$$G_F = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$$m_W = 80.385 \pm 0.015\ \mathrm{GeV}$$

$$m_Z = 91.1875 \pm 0.0021\ \mathrm{GeV}$$

$$\sin^2\theta_W = 0.23143 \pm 0.00015$$

In the Standard Model, only 3 are independent.

The measurements are consistent, which is an incredibly powerful test of the Standard Model of Electroweak Interactions.

Summary

- Weak interaction with $W^{\pm}$ fails at high energy.
- Introduction of unified theory involving and relating Z and γ can resolve the divergences.
- One new parameter, θ_W, allows predictions of Z couplings and mass relations.
- Extensively and successfully tested at LEP.

Problem Sheet: q.26-27

Up next...
Section 11: The Top Quark and the Higgs Mechanism