7. QCD
Particle and Nuclear Physics
In this section...

- The strong vertex
- Colour, gluons and self-interactions
- QCD potential, confinement
- Hadronisation, jets
- Running of $\alpha_s$
- Experimental tests of QCD
Quantum Electrodynamics is the quantum theory of the electromagnetic interaction.

- mediated by massless photons
- photon couples to electric charge
- strength of interaction: \( \langle \psi_f | \hat{H} | \psi_i \rangle \propto \sqrt{\alpha} \)
  \( \alpha = \frac{e^2}{4\pi} = \frac{1}{137} \)

Quantum Chromodynamics is the quantum theory of the strong interaction.

- mediated by massless gluons
- gluon couples to “strong” charge
- only quarks have non-zero “strong” charge, therefore only quarks feel the strong interaction.
- strength of interaction: \( \langle \psi_f | \hat{H} | \psi_i \rangle \propto \sqrt{\alpha_s} \)
  \( \alpha_s = \frac{g_s^2}{4\pi} \sim 1 \)
Basic QCD interaction looks like a stronger version of QED:

\[ \alpha = \frac{e^2}{4\pi} = \frac{1}{137} \]

\[ \alpha_s = \frac{g_s^2}{4\pi} \sim 1 \]

• The coupling of the gluon, \( g_s \), is to the “strong” charge.
• Energy, momentum, angular momentum and charge always conserved.
• QCD vertex never changes quark flavour
• QCD vertex always conserves parity
**Colour**

**QED:**
- Charge of QED is electric charge, a conserved quantum number

**QCD:**
- Charge of QCD is called “colour”
- Colour is a conserved quantum number with 3 values labelled red, green and blue.
  
  Quarks carry colour \( r \ b \ g \)

  Antiquarks carry anti-colour \( \bar{r} \ \bar{b} \ \bar{g} \)

- Colorless particles either have
  - no color at all e.g. leptons, \( \gamma \), \( W \), \( Z \) and do not interact via the strong interaction
  - or equal parts \( r \), \( b \), \( g \) e.g. meson \( q\bar{q} \) with \( \frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g}) \), baryon \( qqq \) with \( rgb \)

- Gluons do not have equal parts \( r \), \( b \), \( g \), so carry color (e.g. \( r\bar{r} \), see later)
QCD as a gauge theory

- Recall QED was invariant under gauge symmetry
  \[ \psi \rightarrow \psi' = e^{iq\alpha(\vec{r}, t)}\psi \]

- The equivalent symmetry for QCD is invariance under
  \[ \psi \rightarrow \psi' = e^{ig\vec{\lambda}.\vec{\Lambda}(\vec{r}, t)}\psi \]

  an “SU(3)” transformation (\(\lambda\) are eight 3x3 matrices).

- Operates on the colour state of the quark field – a “rotation” of the colour state which can be different at each point of space and time.

- Invariance under SU(3) transformations \(\rightarrow\) eight massless gauge bosons, **gluons** (eight in this case). Gluon couplings are well specified.

- Gluons also have self-couplings, i.e. they carry colour themselves...
Gluons are massless spin-1 bosons, which carry the colour quantum number (unlike $\gamma$ in QED which is charge neutral).

Consider a red quark scattering off a blue quark. Colour is exchanged, but always conserved (overall and at each vertex).

Expect 9 gluons (3x3): $r\bar{b} \ r\bar{g} \ g\bar{r} \ g\bar{b} \ b\bar{g} \ b\bar{r} \ r\bar{r} \ b\bar{b} \ g\bar{g}$

However: Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$ is colourless and does not participate in the strong interaction. $\Rightarrow$ 8 coloured gluons

Conventionally chosen to be (all orthogonal):

$$r\bar{b} \ r\bar{g} \ g\bar{r} \ g\bar{b} \ b\bar{g} \ b\bar{r} \ \frac{1}{\sqrt{2}}(r\bar{r} - b\bar{b}) \ \frac{1}{\sqrt{6}}(r\bar{r} + b\bar{b} - 2g\bar{g})$$
Gluon Self-Interactions

QCD looks like a stronger version of QED. However, there is one big difference and that is gluons carry colour charge.

⇒ Gluons can interact with other gluons

Example: Gluon-gluon scattering $gg \rightarrow gg$

Same colour flow in each case: $r\bar{g} + g\bar{b} \rightarrow r\bar{r} + r\bar{b}$
QED Potential:

\[ V_{\text{QED}} = -\frac{\alpha}{r} \]

QCD Potential:

\[ V_{\text{QCD}} = -C\frac{\alpha_s}{r} \]

At short distances, QCD potential looks similar, apart from the “colour factor” \( C \).

For \( q\bar{q} \) in a colourless state in a meson, \( C = 4/3 \)

For \( qq \) in a colourless state in baryon, \( C = 2/3 \)

Note: the colour factor \( C \) arises because more than one gluon can participate in the process \( q \rightarrow qg \). Obtain colour factor from averaging over initial colour states and summing over final/intermediate colour states.
Confinement

Never observe single free quarks or gluons
- Quarks are always confined within hadrons
- This is a consequence of the strong interaction of gluons.

Qualitatively, compare QCD with QED:

![QCD Diagram](image1)

![QED Diagram](image2)

Self interactions of the gluons squeezes the lines of force into a narrow tube or string. The string has a “tension” and as the quarks separate the string stores potential energy.

Energy stored per unit length in field $\sim$ constant $V(r) \propto r$

Energy required to separate two quarks is infinite. Quarks always come in combinations with zero net colour charge $\Rightarrow$ confinement.
QCD potential between quark and antiquark has two components:

- Short range, Coulomb-like term: \(-\frac{4}{3} \alpha_s \frac{1}{r}\)
- Long range, linear term: \(+kr\)

\[ V_{\text{QCD}} = -\frac{4}{3} \alpha_s \frac{1}{r} + kr \]

with \(k \sim 1 \text{ GeV/fm}\)

\[ F = -\frac{dV}{dr} = \frac{4}{3} \frac{\alpha_s}{r^2} + k \]

at large \(r\)

\[ F = k \sim \frac{1.6 \times 10^{-10}}{10^{-15}} \text{ N} = 160,000 \text{ N} \]

Equivalent to weight of \(\sim 150\) people
Jets

Consider the $q\bar{q}$ pair produced in $e^+e^- \rightarrow q\bar{q}$

As the quarks separate, the potential energy in the colour field ("string") starts to increase linearly with separation. When the energy stored exceeds $2m_q$, new $q\bar{q}$ pairs can be created.

As energy decreases, hadrons (mainly mesons) freeze out
As quarks separate, more $q\bar{q}$ pairs are produced. This process is called hadronisation. Start out with quarks and end up with narrowly collimated jets of hadrons.

Typical $e^+e^- \rightarrow q\bar{q}$ event

The hadrons in a quark(antiquark) jet follow the direction of the original quark(antiquark). Consequently, $e^+e^- \rightarrow q\bar{q}$ is observed as a pair of back-to-back jets.
Nucleon-Nucleon Interactions

- Bound \(qqq\) states (e.g. protons and neutrons) are \textit{colourless} (colour singlets).
- They can only emit and absorb another colour singlet state, i.e. not single gluons (conservation of colour charge).
- Interact by exchange of \textit{pions}.

Example: \(pp\) scattering (One possible diagram)

\[
\begin{align*}
V(r) &= -\frac{g^2 e^{-m_{\pi}r}}{4\pi r} \\
\text{Range} &= \frac{1}{m_{\pi}} = (0.140 \text{ GeV})^{-1} = 7 \text{ GeV}^{-1} = 7 \times (\hbar c) \text{ fm} = 1.4 \text{ fm}
\end{align*}
\]
Running of $\alpha_s$

- $\alpha_s$ specifies the strength of the strong interaction.
- **But**, just as in QED, $\alpha_s$ is not a constant. It “runs” (i.e. depends on energy).
- In QED, the bare electron charge is screened by a cloud of virtual electron-positron pairs.
- In QCD, a similar “colour screening” effect occurs.

In QCD, quantum fluctuations lead to a cloud of virtual $q\bar{q}$ pairs.
One of many (an infinite set) of such diagrams analogous to those for QED.

In QCD, the gluon self-interactions **also** lead to a cloud of virtual gluons.
One of many (an infinite set) of such diagrams. No analogy in QED, photons do not carry the charge of the interaction.
**Colour Anti-Screening**

- Due to gluon self-interactions bare colour charge is **screened** by both virtual quarks and gluons.
- The cloud of virtual gluons carries colour charge and the effective colour charge **decreases** at smaller distances (high energy)!
- Hence, at low energies, $\alpha_s$ is large $\rightarrow$ cannot use perturbation theory.
- But at high energies, $\alpha_s$ is small. In this regime, can treat quarks as free particles and use perturbation theory $\rightarrow$ **Asymptotic Freedom**.

\[
\sqrt{s} = 100 \text{ GeV}, \quad \alpha_s \approx 0.12
\]
Scattering in QCD

Example: High energy proton-proton scattering.

Visible jet in direction of $q$

Jet along beam direction

$M \sim \frac{1}{q^2} \sqrt{\alpha_s} \sqrt{\alpha_s}$

$\Rightarrow \frac{d\sigma}{d\Omega} \sim \frac{(\alpha_s)^2}{\sin^4 \theta/2}$

Upper points: Geiger and Marsden data (1911) for the elastic scattering of a particles from gold and silver foils.

Lower points: angular distribution of quark jets observed in $pp$ scattering at $q^2 = 2000$ GeV$^2$.

Both follow the Rutherford formula for elastic scattering.
Scattering in QCD

Example: \( pp \) vs \( \pi^+ p \) scattering

Calculate ratio of \( \sigma(pp)_{\text{total}} \) to \( \sigma(\pi^+ p)_{\text{total}} \)

QCD does not distinguish between quark flavours, only \textit{colour} charge of quarks matters.

At high energy \( (E \gg \text{binding energy of quarks within hadrons}) \), ratio of \( \sigma(pp)_{\text{total}} \) and \( \sigma(\pi^+ p)_{\text{total}} \) depends on number of possible quark-quark combinations.

Predict: \[ \frac{\sigma(\pi p)}{\sigma(pp)} = \frac{2 \times 3}{3 \times 3} = \frac{2}{3} \]

Experiment: \[ \frac{\sigma(\pi p)}{\sigma(pp)} = \frac{24 \text{ mb}}{38 \text{ mb}} \sim \frac{2}{3} \]
QCD in $e^+e^-$ Annihilation

$e^+e^-$ annihilation at high energies provides direct experimental evidence for colour and for gluons.

Start by comparing the cross-sections for $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow q\bar{q}$

$$M \sim \frac{1}{q^2} \sqrt{\alpha} \sqrt{\alpha}$$

$$\Rightarrow \sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

If we neglect the mass of the final state quarks/muons then the only difference is the charge of the final state particles:

$$Q_\mu = -1 \quad Q_q = \pm \frac{2}{3}, \quad -\frac{1}{3}$$
Evidence for Colour

Consider the ratio

\[ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]

For a single quark of a given colour \( R = Q^2_q \)

However, we measure \( \sigma(e^+e^- \rightarrow \text{hadrons}) \) not just \( \sigma(e^+e^- \rightarrow u\bar{u}) \).
A jet from a \( u \)-quark looks just like a jet from a \( d \)-quark etc.
Thus, we need to sum over all available flavours (\( u, d, c, s, t, b \)) and colours (\( r, g, b \)):

\[ R = 3 \sum_i Q^2_i \quad \text{(3 colours)} \]

where the sum is over all quark flavours (\( i \)) that are kinematically accessible at centre-of-mass energy, \( \sqrt{s} \), of the collider.
Evidence for Colour

Expect to see steps in $R$ as energy is increased.

$$R = 3 \sum_i Q_i^2$$

<table>
<thead>
<tr>
<th>Energy</th>
<th>Expected ratio $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s} &gt; 2m_s$, $\sim 1$ GeV</td>
<td>$3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right) = 2$</td>
</tr>
<tr>
<td>$\sqrt{s} &gt; 2m_c$, $\sim 4$ GeV</td>
<td>$3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} \right) = 3\frac{1}{3}$</td>
</tr>
<tr>
<td>$\sqrt{s} &gt; 2m_b$, $\sim 10$ GeV</td>
<td>$3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} \right) = 3\frac{2}{3}$</td>
</tr>
<tr>
<td>$\sqrt{s} &gt; 2m_t$, $\sim 350$ GeV</td>
<td>$3 \left( \frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} \right) = 5$</td>
</tr>
</tbody>
</table>

Dr. Tina Potter
Evidence for Colour

\[ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]

- \( R \) increases in steps with \( \sqrt{s} \)
- Strong evidence for colour

- \( \sqrt{s} < 11 \text{ GeV} \) region observe bound state resonances: charmonium \((c\bar{c})\) and bottomonium \((b\bar{b})\)

- \( \sqrt{s} > 50 \text{ GeV} \) region observe low edge of \( Z \) resonance \( \Gamma \sim 2.5 \text{ GeV} \).
Experimental Evidence for Colour

\[ R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \]

The existence of \( \Omega^- (sss) \)

The \( \Omega^- (sss) \) is a \((L = 0)\) spin-3/2 baryon consisting of three \( s\)-quarks.

The wavefunction: \[ \psi = s \uparrow s \uparrow s \uparrow \]

is symmetric under particle interchange. However, quarks are fermions, therefore require an anti-symmetric wave-function, i.e. need another degree of freedom, namely colour, whose wavefunction must be antisymmetric.

\[ \psi = \left( s \uparrow s \uparrow s \uparrow \right) \psi_{\text{colour}} \]

\[ \psi_{\text{colour}} = \frac{1}{\sqrt{6}} ( rgb + gbr + brg - grb - rgb - bgr ) \]

i.e. need to introduce a new quantum number (colour) to distinguish the three quarks in \( \Omega^- \) – avoids violation of Pauli’s Exclusion Principle.

Drell-Yan process

Need colour to explain cross-section; colours of the annihilating quarks must match to form a virtual photon. Cross-section suppressed by a factor \( N_{\text{colour}}^{-2} \).
Evidence for Gluons

In QED, electrons can radiate photons. In QCD, quarks can radiate gluons.

**Example:** \( e^- e^+ \rightarrow q\bar{q}g \)

\[
\begin{align*}
\gamma & \quad q \\
Qe & \quad \sqrt{\alpha_s} \\
\bar{q} & \quad g
\end{align*}
\]

Given an extra factor of \( \sqrt{\alpha_s} \) in the matrix element, i.e. an extra factor of \( \alpha_s \) in the cross-section.

In QED we can detect the photons. In QCD, we never see free gluons due to confinement.

Experimentally, detect gluons as an additional jet: 3-jet events.

– Angular distribution of gluon jet depends on gluon spin.
Evidence for Gluons

JADE event $\sqrt{s} = 31$ GeV
First direct evidence of gluons (1978)

ALEPH event $\sqrt{s} = 91$ GeV (1990)

Distribution of the angle, $\phi$, between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cm frame). $\phi$ distribution depends on the spin of the gluon.

$\Rightarrow$ Gluon is spin 1
Evidence for Gluon Self-Interactions

Direct evidence for the existence of the gluon self-interactions comes from 4-jet events:

The angular distribution of jets is sensitive to existence of triple gluon vertex (lower left diagram)

- $qqg$ vertex consists of two spin $1/2$ quarks and one spin $1$ gluon
- $ggg$ vertex consists of three spin-1 gluons

$\implies$ Different angular distribution.
Evidence for Gluon Self-Interactions

**Experimental method:**
- Define the two lowest energy jets as the gluons. (Gluon jets are more likely to be lower energy than quark jets).
- Measure angle $\chi$ between the plane containing the “quark” jets and the plane containing the “gluon” jets.

Gluon self-interactions are required to describe the experimental data.
Measurements of $\alpha_s$

$\alpha_s$ can be measured in many ways. The cleanest is from the ratio

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

In practise, measure

$$\gamma e^- e^+ \bar{q} q Q e Q q$$

i.e. don’t distinguish between 2 and 3 jets

When gluon radiation is included:

$$R = 3 \sum Q_q^2 \left(1 + \frac{\alpha_s}{\pi}\right)$$

Therefore,

$$\left(1 + \frac{\alpha_s}{\pi}\right) \sim \frac{3.9}{3.66}$$

$$\alpha_s(q^2 = 25^2) \sim 0.2$$
Measurements of $\alpha_s$

Many other ways to measure $\alpha_s$

Example: 3-jet rate $e^+e^- \rightarrow q\bar{q}g$

$$R_3 = \frac{\sigma(e^+e^- \rightarrow 3 \text{ jets})}{\sigma(e^+e^- \rightarrow 2 \text{ jets})} \propto \alpha_s$$

$\alpha_s$ decreases with energy

$\alpha_s$ runs!

in accordance with QCD
Observed running of $\alpha_s$
Summary

- QCD is a gauge theory, similar to QED, based on SU(3) symmetry
- Gluons are vector gauge bosons, which couple to (three types of) colour charge (r, b, g)
- Gluons themselves carry colour charge – hence they have self-interactions (unlike QED).
- Leads to running of $\alpha_s$, in the opposite sense to QED. Force is weaker at high energies (“asymptotic freedom”) and very strong at low energies.
- Quarks and gluons are confined. Seen as hadrons and jets of hadrons.
- Tests of QCD
  - Evidence for colour
  - Existence of gluons, test of their spin and self-interactions
  - Measurement of $\alpha_s$ and observation that it runs.

Up next...

Section 8: Quark Model of Hadrons