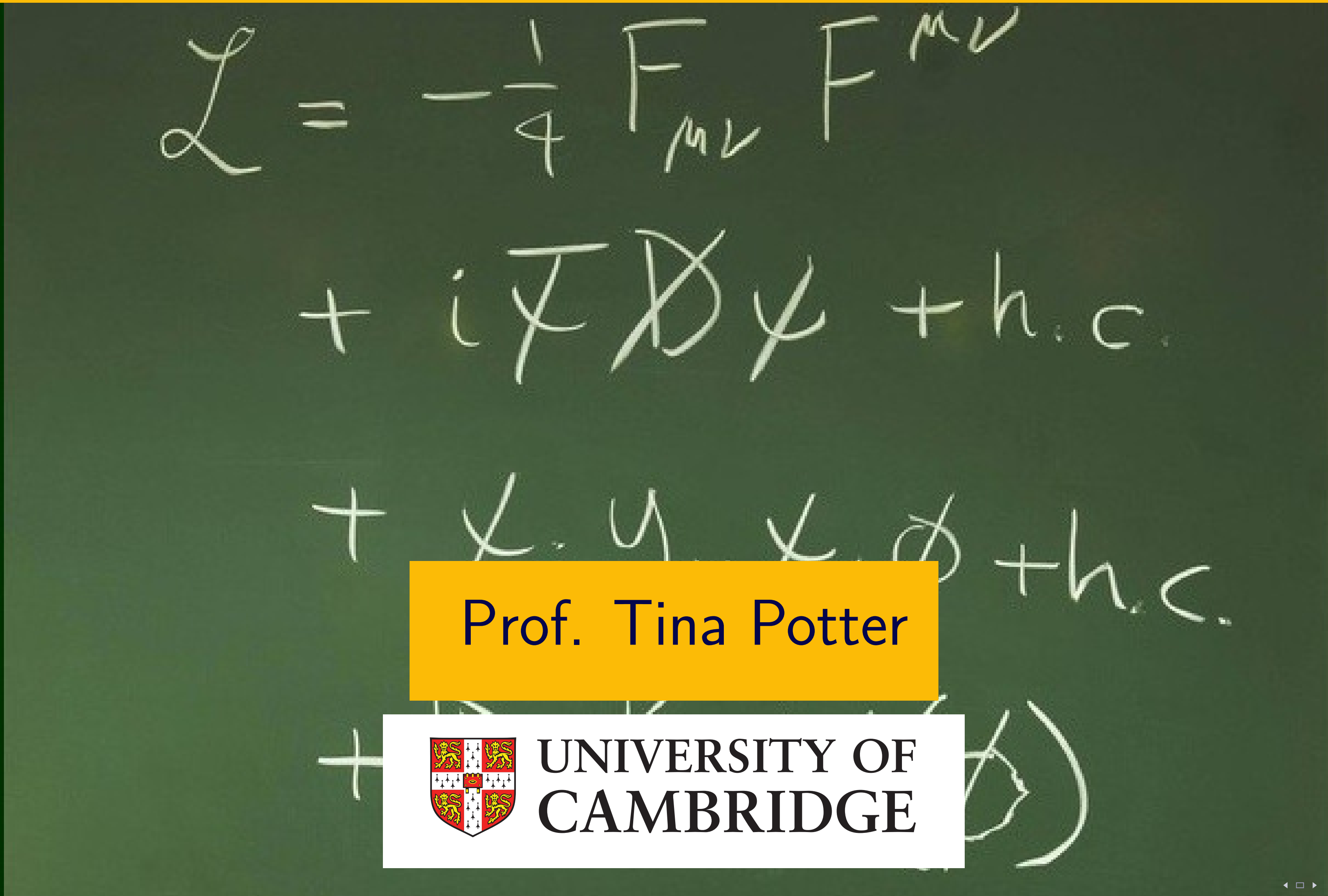
4. The Standard Model

Particle and Nuclear Physics



Prof. Tina Potter

UNIVERSITY OF CAMBRIDGE

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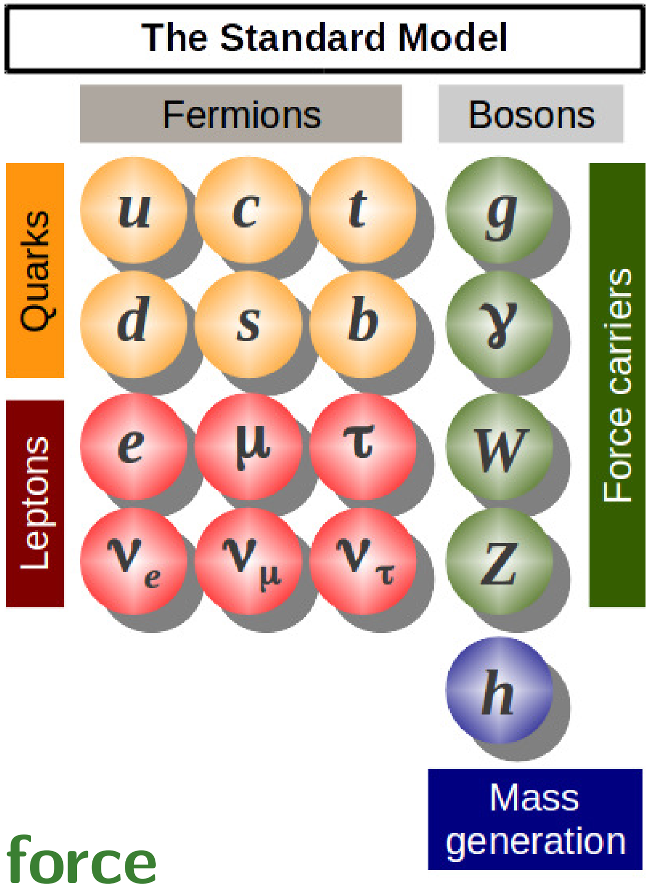
- Standard Model particle content
- Klein-Gordon equation
- Antimatter
- Interaction via particle exchange
- Virtual particles

The Standard Model



Spin-1/2 fermions

		Charge (units of e)
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$	$+\frac{2}{3}$ $-\frac{1}{3}$
Leptons	$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$	-1 0

Plus antileptons and antiquarks

Spin-1 bosons

		Mass (GeV/c^2)	
Gluon	g	0	**Strong force**
Photon	γ	0	**EM force**
W and Z bosons	$W^\pm, Z$	91.2, 80.3	**Weak force**

Spin-0 bosons

Higgs	h	125	**Mass generation**

Theoretical Framework

	Macroscopic	Microscopic
Slow	Classical Mechanics	Quantum Mechanics
Fast	Special Relativity	Quantum Field Theory

The Standard Model is a collection of related Gauge Theories which are Quantum Field Theories that satisfy Local Gauge Invariance.

Electromagnetism: **Quantum Electrodynamics (QED)**

1948 Feynman, Schwinger, Tomonaga (1965 Nobel Prize)

Electromagnetism + Weak: **Electroweak Unification**

1968 Glashow, Weinberg, Salam (1979 Nobel Prize)

Strong: **Quantum Chromodynamics (QCD)**

1974 Politzer, Wilczek, Gross (2004 Nobel Prize)

The Schrödinger Equation

To describe the fundamental interactions of particles we need a theory of
Relativistic Quantum Mechanics

Schrödinger Equation for a free particle $$\hat{E}\psi = \frac{\hat{p}^2}{2m}\psi$$

with energy and momentum operators $$\hat{E} = i\frac{\partial}{\partial t}, \quad \hat{p} = -i\boldsymbol{\nabla}$$

($\hbar = 1$ natural units)

giving $$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\boldsymbol{\nabla}^2\psi$$

which has plane wave solutions: $$\psi(\vec{r}, t) = Ne^{-i(Et-\vec{p}.\vec{r})}$$

- 1^{st} order in time derivative
- 2^{nd} order in space derivatives

Not Lorentz Invariant!

Schrödinger equation cannot be used to describe the physics of relativistic particles.

Klein-Gordon Equation

Use the KG equation to describe the physics of relativistic particles.

From Special Relativity: $E^2 = p^2 + m^2$

use energy and momentum operators $\hat{E} = \mathrm{i}\frac{\partial}{\partial t}, \quad \hat{p} = -\mathrm{i}\boldsymbol{\nabla}$

giving $-\frac{\partial^2 \psi}{\partial t^2} = -\boldsymbol{\nabla}^2\psi + m^2\psi$ $\qquad \frac{\partial^2 \psi}{\partial t^2} = (\boldsymbol{\nabla}^2 - m^2)\psi$ Klein-Gordon Equation

Second order in both space and time derivatives $\Rightarrow$ Lorentz invariant.

Plane wave solutions $\psi(\vec{r}, t) = N\mathrm{e}^{-\mathrm{i}(Et - \vec{p}.\vec{r})}$

but this time requiring $E^2 = \vec{p}^2 + m^2$, allowing $E = \pm\sqrt{|\vec{p}|^2 + m^2}$

Negative energy solutions required to form complete set of eigenstates.

$\Rightarrow$ **Antimatter**

Antimatter *and the Dirac Equation*

In the hope of avoiding negative energy solutions, Dirac sought a linear relativistic wave equation:

$$\mathrm{i}\frac{\partial \psi}{\partial t} = (-\mathrm{i}\vec{\alpha}.\vec{\boldsymbol{\nabla}} + \beta m)\psi$$

$\vec{\alpha}$ and β are appropriate 4x4 matrices.
ψ is a column vector "spinor" of four wavefunctions.

Two of the wavefunctions describe the states of a fermion, but the other two still have negative energy.

Dirac suggested the vacuum had all negative energy states filled. A hole in the negative energy "sea" could be created by exciting an electron to a positive energy state. The hole would behave like a positive energy positive charged "positron". Subsequently detected.

However, this only works for fermions...

We now interpret negative energy states differently...

Antimatter *and the Feynman-Stückelberg Interpretation*

Consider the negative energy solution in which a negative energy particle travels backwards in time.

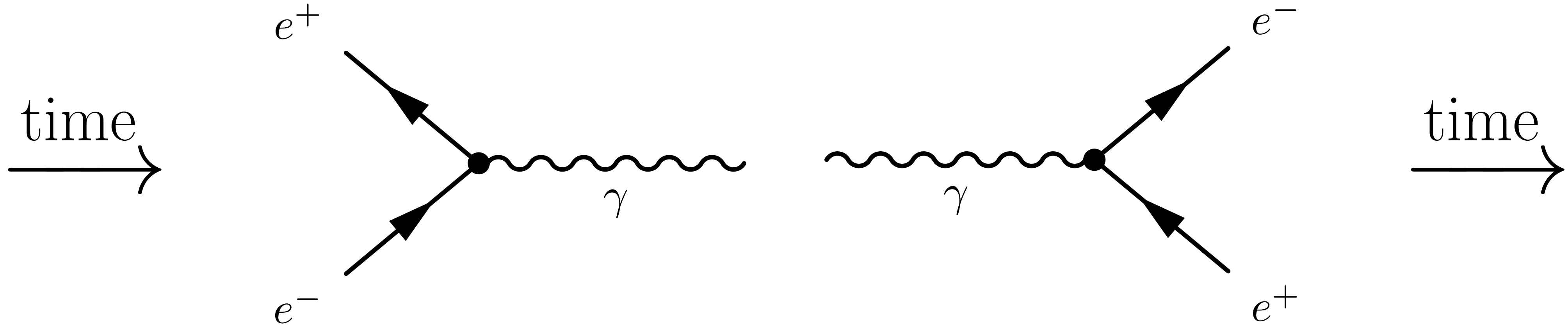
$$e^{-iEt} \equiv e^{-i(-E)(-t)}$$

Interpret as a **positive** energy **anti**particle travelling **forwards** in time.

Then all solutions can be used to describe physical states with positive energy, going forward in time.

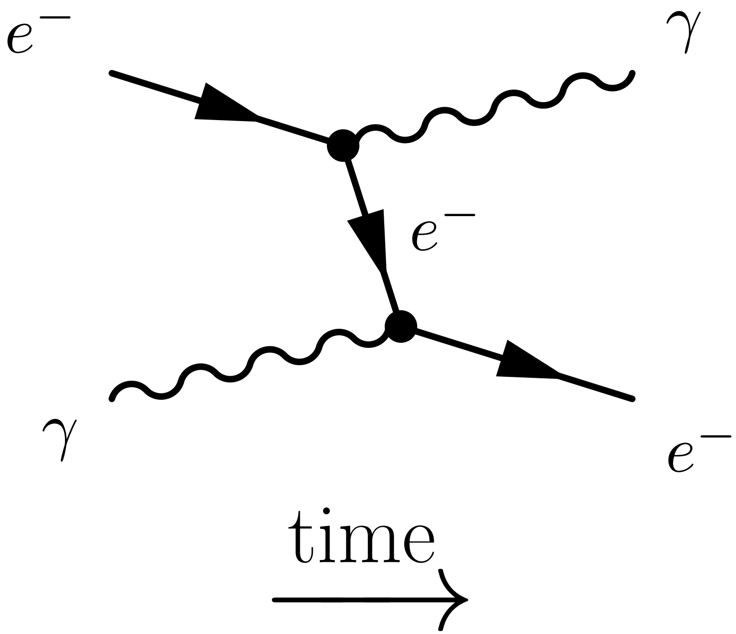
e.g.

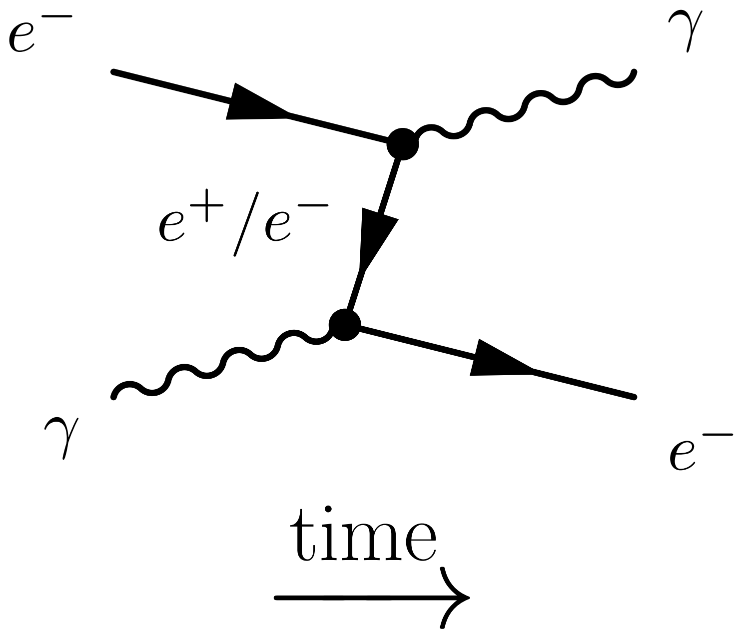
e^+e^- annihilation   pair production



All quantum numbers carried into a vertex by the e^+ are the same as if it is regarded as an outgoing e^-, or vice versa.

Antimatter *and the Feynman-Stückelberg Interpretation*



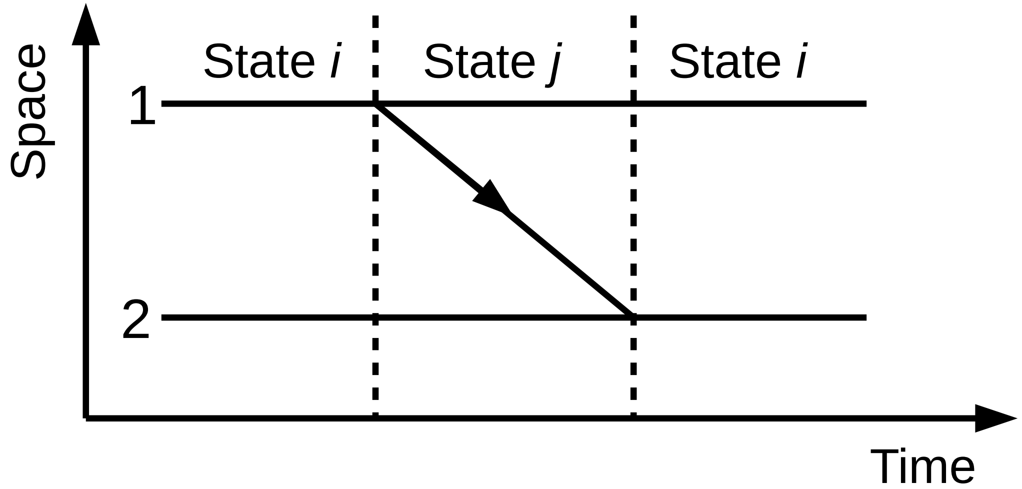


The interpretation here is easy. The first photon emitted has less energy than the electron it was emitted from. No need for "anti-particles" or negative energy states.

The emitted photon has more energy than the electron that emitted it. Either view the top vertex as "emission of a negative energy electron travelling backwards in time" or "absorption of a positive energy positron travelling forwards in time".

Interaction via Particle Exchange

Consider two particles, fixed at $\vec{r}_1$ and $\vec{r}_2$, which exchange a particle of mass m.



$$p^\mu = (E, \vec{p})$$

$$E = E_j - E_i$$

Calculate the shift in energy of state i due to this exchange (using second order perturbation theory):

$$\Delta E_i = \sum_{j \neq i} \frac{\langle i|H|j\rangle \langle j|H|i\rangle}{E_i - E_j}$$

Sum over all possible states j with different momenta

where $\langle j|H|i\rangle$ is the transition from i to j at $\vec{r}_1$
where $\langle i|H|j\rangle$ is the transition from j to i at $\vec{r}_2$

Interaction via Particle Exchange

Consider $\langle j|H|i\rangle$ (transition from $i \to j$ by emission of m at $\vec{r}_1$)

$$\psi_i = \psi_1\psi_2 \qquad \text{Original 2 particles}$$

$$\psi_j = \psi_1\psi_2\psi_3 \qquad \psi_3 = N\,\mathrm{e}^{-\mathrm{i}(Et-\vec{p}.\vec{r})}$$

ψ_3 represents a free particle with $p^\mu = (E, \vec{p})$

normalise $\psi_1^*\psi_1 = \psi_2^*\psi_2 = \psi_3^*\psi_3 = 1$

Let g be the probability of emitting m at r_1

$g/\sqrt{2E}$ is required on dimensional grounds, c.f. AQP vector potential of a photon.

$$\langle j|H|i\rangle = \int \mathrm{d}^3\vec{r}\,\psi_1^*\psi_2^*\psi_3^*\frac{g}{\sqrt{2E}}\psi_1\psi_2\,\delta^3(\vec{r}-\vec{r}_1)$$

$$= \frac{g}{\sqrt{2E}}N\mathrm{e}^{\mathrm{i}(Et-\vec{p}.\vec{r}_1)}$$

Dirac δ function

$$\int \mathrm{d}^3\vec{r}\delta^3(\vec{r}-\vec{r}_1) = 1 \quad \text{for } \vec{r} = \vec{r}_1$$

$$= 0 \quad \text{for } \vec{r} \neq \vec{r}_1$$

Similarly $\langle i|H|j\rangle$ is the transition from j to i at $\vec{r}_2$

$$\langle i|H|j\rangle = \frac{g}{\sqrt{2E}}N\mathrm{e}^{-\mathrm{i}(Et-\vec{p}.\vec{r}_2)}$$

Shift in energy state $\Delta E_i^{1\to 2} = \sum_{j\neq i}\frac{g^2}{2E}\frac{N^2\mathrm{e}^{\mathrm{i}\vec{p}.(\vec{r}_2-\vec{r}_1)}}{E_i - E_j} = \sum_{j\neq i}\frac{g^2N^2\mathrm{e}^{\mathrm{i}\vec{p}.(\vec{r}_2-\vec{r}_1)}}{-2E^2}$

$(E = E_j - E_i)$

Interaction via Particle Exchange

Putting the pieces together

Different states j have different momenta $\vec{p}$ for the exchanged particle.

Therefore sum is actually an integral over all momenta:

$$\Delta E_i^{1\to 2} = \int \frac{g^2 N^2 e^{i\vec{p}.(\vec{r}_2-\vec{r}_1)}}{-2E^2}\rho(p)\,\mathrm{d}p \quad = \int \frac{g^2 e^{i\vec{p}.(\vec{r}_2-\vec{r}_1)}}{-2E^2}\frac{1}{L^3}\left(\frac{L}{2\pi}\right)^3 p^2\,\mathrm{d}p\,\mathrm{d}\Omega$$

$$= -g^2\left(\frac{1}{2\pi}\right)^3 \int \frac{e^{i\vec{p}.(\vec{r}_2-\vec{r}_1)}}{2E^2} p^2\,\mathrm{d}p\,\mathrm{d}\Omega$$

$$N = \sqrt{\frac{1}{L^3}}, \quad \rho(p) = \left(\frac{L}{2\pi}\right)^3 p^2\,\mathrm{d}\Omega$$

$$E^2 = p^2 + m^2$$

The integral can be done by taking the z-axis along $\vec{r} = \vec{r}_2 - \vec{r}_1$

Then $\vec{p}.\vec{r} = pr\cos\theta$ and $\mathrm{d}\Omega = 2\pi\,\mathrm{d}(\cos\theta)$

$$\Delta E_i^{1\to 2} = -\frac{g^2}{2(2\pi)^2}\int_0^\infty \frac{p^2}{p^2+m^2}\frac{e^{ipr} - e^{-ipr}}{ipr}\,\mathrm{d}p \qquad \text{(see Appendix D)}$$

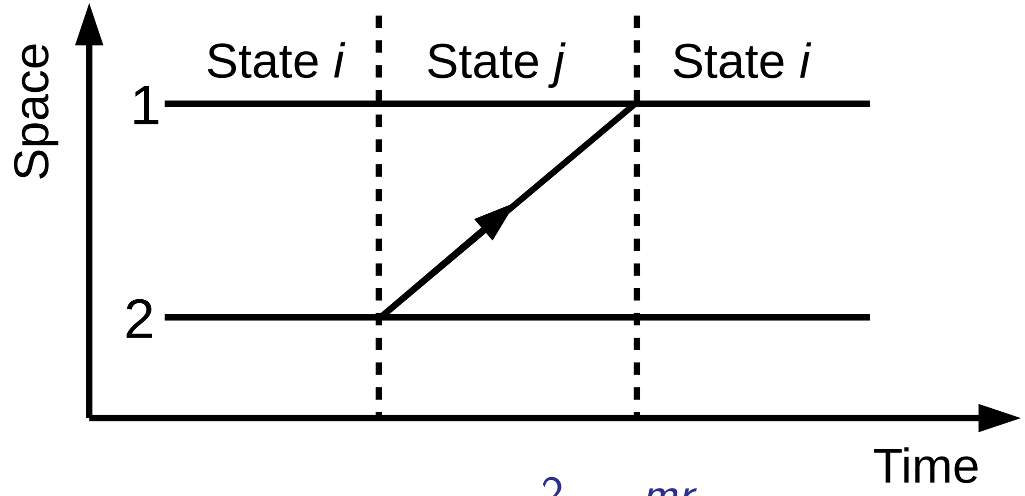
Write this integral as one half of the integral from $-\infty$ to $+\infty$, which can be done by residues giving

$$\Delta E_i^{1\to 2} = -\frac{g^2}{8\pi}\frac{e^{-mr}}{r}$$

Interaction via Particle Exchange

Final stage

Can also exchange particle from 2 to 1:



Get the same result: $$\Delta E_i^{2\to 1} = -\frac{g^2}{8\pi}\frac{e^{-mr}}{r}$$

Total shift in energy due to particle exchange is

$$\Delta E_i = -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$ Yukawa Potential

Attractive force between two particles, decreasing exponentially with range r.

Yukawa Potential

Hideki Yukawa
1949 Nobel Prize

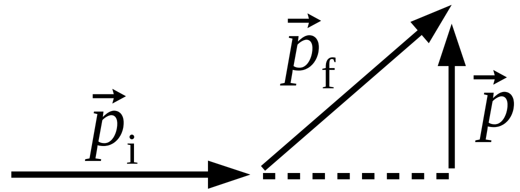
$$V(r) = -\frac{g^2}{4\pi}\frac{\mathrm{e}^{-mr}}{r}$$ Yukawa Potential

- Characteristic range $= 1/m$
  (Compton wavelength of exchanged particle)
- For $m \to 0$, $V(r) = -\frac{g^2}{4\pi r}$ infinite range (Coulomb-like)

Yukawa potential with $m = 139\ \mathrm{MeV}/c^2$ gives a good description of long range part of the interaction between two nucleons and was the basis for the prediction of the existence of the pion.

Scattering from the Yukawa Potential

Consider elastic scattering (no energy transfer)



Born Approximation $\quad M_\mathrm{fi} = \int \mathrm{e}^{\mathrm{i}\vec{p}.\vec{r}} V(r)\, \mathrm{d}^3\vec{r}$

Yukawa Potential $\quad V(r) = -\dfrac{g^2}{4\pi}\dfrac{\mathrm{e}^{-mr}}{r}$

$$M_\mathrm{fi} = -\frac{g^2}{4\pi}\int \frac{\mathrm{e}^{-mr}}{r}\mathrm{e}^{\mathrm{i}\vec{p}.\vec{r}}\,\mathrm{d}^3\vec{r} = -\frac{g^2}{|\vec{p}|^2 + m^2}$$

$q^\mu = (E, \vec{p})$

$q^2 = E^2 - |\vec{p}|^2$

q^2 is invariant

"Virtual Mass"

The integral can be done by choosing the z-axis along $\vec{r}$, then $\vec{p}.\vec{r} = pr\cos\theta$ and $\mathrm{d}^3\vec{r} = 2\pi r^2\,\mathrm{d}r\,\mathrm{d}(\cos\theta)$

For elastic scattering, $q^\mu = (0, \vec{p})$, $q^2 = -|p|^2$ and exchanged massive particle is highly "virtual"

$$M_\mathrm{fi} = \frac{g^2}{q^2 - m^2}$$

Virtual Particles

Forces arise due to the exchange of unobservable virtual particles.

- The effective mass of the virtual particle, q^2, is given by

$$q^2 = E^2 - |\vec{p}|^2$$

  and is not equal to the physical mass m, i.e. it is off-shell mass.

- The mass of a virtual particle can be +ve, -ve or imaginary.

- A virtual particle which is off-mass shell by amount Δm can only exist for time and range

$$t \sim \frac{\hbar}{\Delta m c^2} = \frac{1}{\Delta m}, \qquad \text{range} = \frac{\hbar}{\Delta m c} = \frac{1}{\Delta m} \qquad \hbar = c = 1$$

- If $q^2 = m^2$, the the particle is real and can be observed.

Virtual Particles

For virtual particle exchange, expect a contribution to the matrix element of

$$M_{\mathrm{fi}} = \frac{g^2}{q^2 - m^2}$$

where

where	g	Coupling constant
	g^2	Strength of interaction
	m^2	Physical (on-shell) mass
	q^2	Virtual (off-shell) mass
	$\frac{1}{q^2-m^2}$	Propagator

Qualitatively: the propagator is inversely proportional to how far the particle is off-shell. The further off-shell, the smaller the probability of producing such a virtual state.

- For $m \to 0$; e.g. single γ exchange, $M_{\mathrm{fi}} = g^2/q^2$
- For $q^2 \to 0$, very low momentum transfer EM scattering (small angle)

Virtual Particles *Example*

Summary

- SM particles: 12 fermions, 5 spin-1 bosons, 1 spin-0 boson.
- Need relativistic wave equations to describe particle interactions.
  Klein-Gordon equation (bosons), Dirac equation (fermions).
- Negative energy solutions describe antiparticles.
- The exchange of a massive particle generates an attractive force between two particles.
- Yukawa potential $$V(r) = -\frac{g^2}{4\pi}\frac{\mathrm{e}^{-mr}}{r}$$
- Exchanged particles may be virtual.

Problem Sheet: q.10

Up next...
Section 5: Feynman Diagrams