APPENDIX H: GAMOW FACTOR IN ALPHA DECAY

The probability for an $\alpha$ particle to tunnel through the Coulomb barrier can be written as

$$P = \prod_i \exp\{-2G\}$$

where $G$ is the Gamow Factor,

$$G = \int_R^{R'} \frac{[2m(V(r) - E_0)]^{1/2}}{\hbar} \, dr.$$ 

$R$ is the radius of the nucleus of mass number $Z$, $R'$ is the radius at which the $\alpha$ particle escapes, $m$ is the mass of the $\alpha$ particle, $V(r) = 2(Z - 2)e^2/4\pi\epsilon_0r \equiv B/r$ is the Coulomb potential, and $E_0$ is the energy release in the decay.

The $\alpha$ particle escapes the nucleus when $r = R'$. Hence, the potential $V(R') = E_0$ and $R' = B/E_0$. Therefore,

$$G = \left(\frac{2mB}{\hbar^2}\right)^{1/2} \int_R^{R'} \left[ \frac{B}{r} - E_0 \right]^{1/2} \, dr = \left(\frac{2mB}{\hbar^2}\right)^{1/2} \int_R^{R'} \left[ \frac{1}{r} - \frac{1}{R'} \right]^{1/2} \, dr$$

In order to perform the integration, let $r = R'\cos^2\theta$ and $dr = -2R'\cos\theta\sin\theta\,d\theta$. Then

$$\int \left[ \frac{1}{r} - \frac{1}{R'} \right]^{1/2} \, dr = \int \left[ \frac{1}{R'\cos^2\theta} - \frac{1}{R'} \right]^{1/2} (-2R'\cos\theta\sin\theta) \, d\theta$$

$$= \int -2R'^{1/2}\sin^2\theta \, d\theta$$

$$= R'^{1/2} [\sin 2\theta - \theta]$$

Now, using $\cos\theta = (r/R')^{1/2}$, $\sin\theta = (1 - r/R')^{1/2}$ and $\sin 2\theta = 2\sin\theta\cos\theta$, then

$$R'^{1/2} [\sin 2\theta - \theta]|_R^{R'} = R'^{1/2} \left[ 2(1 - r/R')^{1/2}(r/R')^{1/2} - \cos^{-1} (r/R')^{1/2} \right]_R^{R'}$$

$$= R'^{1/2} \left[ \cos^{-1} (R/R')^{1/2} - 2\{(1 - R/R')(R/R')\}^{1/2} \right]$$

Hence, the Gamow factor

$$G = \left(\frac{2mB}{\hbar^2}\right)^{1/2} R'^{1/2} \left[ \cos^{-1} (R/R')^{1/2} - 2\{(1 - R/R')(R/R')\}^{1/2} \right]$$

or

$$G = \left(\frac{2m}{E_0}\right)^{1/2} \frac{B}{\hbar} \left[ \cos^{-1} (R/R')^{1/2} - 2\{(1 - R/R')(R/R')\}^{1/2} \right]$$

For most practical cases, $R \ll R'$, so the term in square brackets is $\approx \pi/2$ and $G$ becomes

$$G \approx \left(\frac{2m}{E_0}\right)^{1/2} \frac{B\pi}{\hbar}$$