# APPENDIX F: NEUTRINO SCATTERING IN FERMI THEORY

Calculation of the cross-section for $\nu_e + n \to p + e^-$ using Fermi theory. The cross-section is given by Fermi's Golden Rule

$$\Gamma = 2\pi \left|M_{fi}\right|^2 \rho\left(E_f\right)$$

where the matrix element, $M_{fi}$, is given by the 4-point interaction with a strength equal to the Fermi constant, $G_F$;

$$\left|M_{fi}\right|^2 \approx G_F^2.$$

There are a total of 4 possible spin states for the spin-$\frac{1}{2}$ $e$ and $\nu$. These correspond to a singlet state $S = 0$ (Fermi transition) and three triplet states $S = 1$ (Gamow-Teller transition). Therefore, the matrix element becomes

$$\left|M_{fi}\right|^2 \approx 4G_F^2.$$

The differential cross-section is then given by,

$$d\sigma = 2\pi 4G_F^2 \frac{E_e^2}{(2\pi)^3} d\Omega$$

where $E_e$ is the energy of the electron in the zero-momentum frame. It follows that

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_e^2}{\pi^2}.$$

The total energy in the zero-momentum frame, $\sqrt{s} = 2E_e$. Hence, the total cross-section can be written as

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{4G_F^2 E_e^2}{\pi} = \underline{\frac{G_F^2 s}{\pi}}.$$

# APPENDIX G: NEUTRINO SCATTERING WITH A MASSIVE W BOSON

From Appendix F, the differential cross-section in Fermi theory is

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_e^2}{\pi^2} \, .$$

The correct theory involves exchange of a massive vector boson of mass $M_W$, which leads to a propagator in the matrix element

$$\frac{1}{q^2 - M_W^2} \, .$$

Fermi theory is equivalent to neglecting the $q^2$ term in the denominator. Hence, treating W-boson exchange correctly, we have

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_e^2}{\pi^2} \left( \frac{M_W^2}{M_W^2 - q^2} \right)^2 \, .$$

Now, for elastic scattering, $q^2 = 0 - |\underline{q}|^2$, where

$$|\underline{q}|^2 = \left( 2E_e \sin\frac{\theta}{2} \right)^2 = \frac{1}{2} s (1 - \cos\theta) \equiv u$$

and so

$$du = \frac{1}{2} s \sin\theta \; d\theta = \frac{s}{4\pi} d\Omega \, .$$

We can thus integrate the differential cross-section in terms of $u$:

$$\begin{aligned}
\sigma &= \int d\Omega \frac{G_F^2 s}{4\pi^2} \left( \frac{M_W^2}{M_W^2 + u} \right)^2 \\
&= \frac{G_F^2 M_W^4}{\pi} \int_0^s du \frac{1}{(M_W^2 + u)^2} \\
&= \frac{G_F^2 M_W^4}{\pi} \left[ \frac{-1}{M_W^2 + u} \right]_0^s \\
&= \frac{G_F^2 M_W^4}{\pi} \left( \frac{1}{M_W^2} - \frac{1}{M_W^2 + s} \right) \\
&= \frac{G_F^2 M_W^2 s}{\pi (M_W^2 + s)}
\end{aligned}$$

At small values of $s$ this reduces to the Fermi theory result, while for $s \gg M_W^2$ the cross-section tends towards the constant value

$$\sigma = \underline{\underline{\frac{G_F^2 M_W^2}{\pi}}}$$

and is no longer divergent.