Lectures on

QCD
(in the LHC precision era)

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Contents

NOTE: the lectures are for experimental and theoretical students. They are light on proofs and derivations and try not to be overly technical. The accent is on "getting the gist of it" and on helping the students develop understanding about how SM works at Colliders.

- I will discuss e^+e^- colliders in order to gain insight into the nature of parton-hadron transitions
- I will then move to hadron colliders where we will use QCD in all it generality and glory.
- No discussion of DIS.
- DIS was important to *establish* QCD; to *understand* QCD we will stick with hadron colliders.

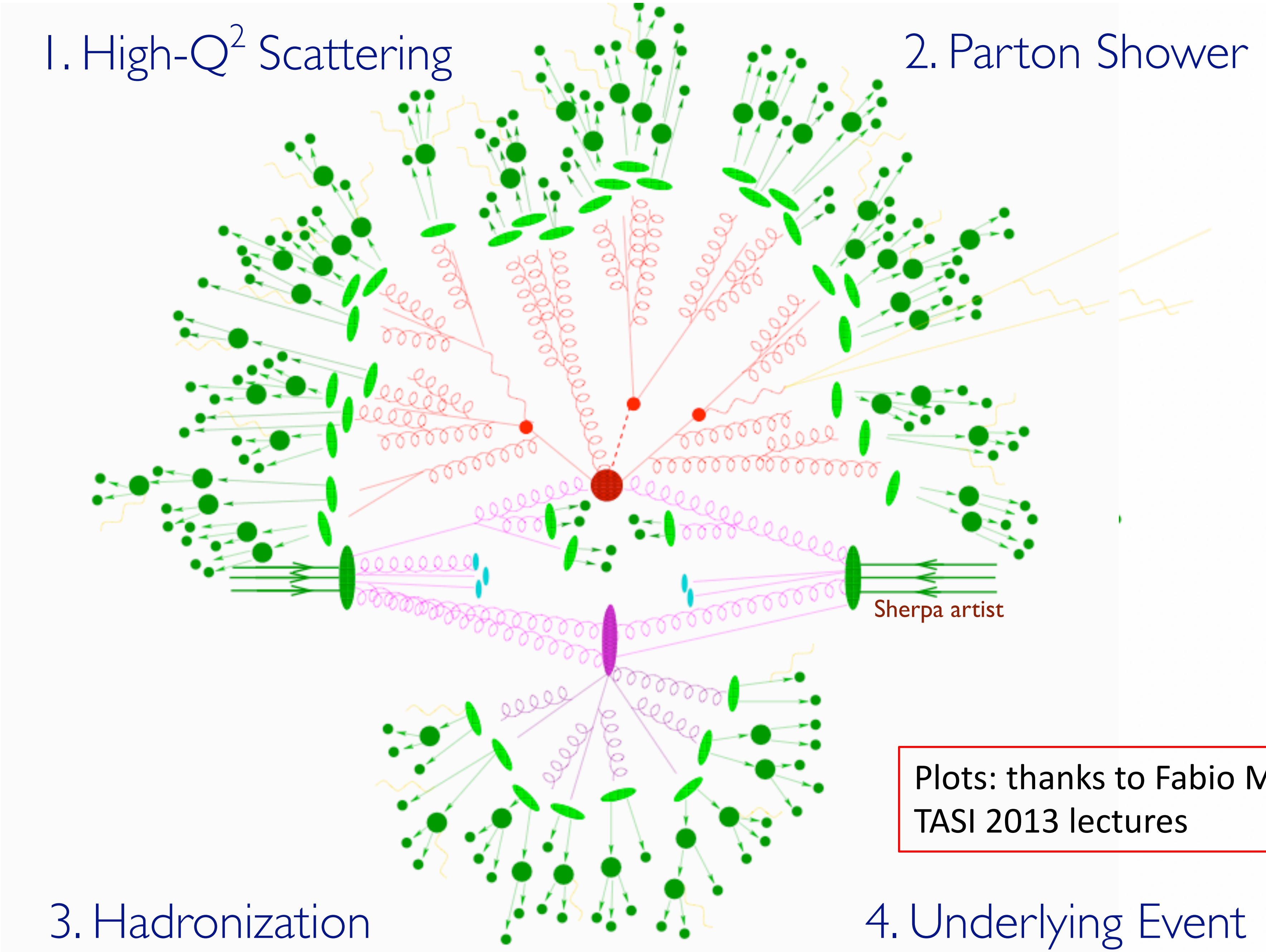
Introduction: why care about strong interactions?

- Because we mostly use hadron colliders. They collide hadrons = strongly interacting particles
- Because most of the particles produced and observed at colliders are hadrons
- Because ***we can manage*** perturbation theory and **it really works**:
  - there are 3 constants in the Standard Model:
    - the fine structure constant (it is small) $\boxed{\alpha(\mu = 0) \approx \frac{1}{137}}$
    - The Fermi constant (it is even smaller)
    - The strong coupling constant (large) $\boxed{\alpha_S(\mu = m_Z) \approx 0.1}$

✓ The effects due to strong interactions are by far the largest and most important ones.

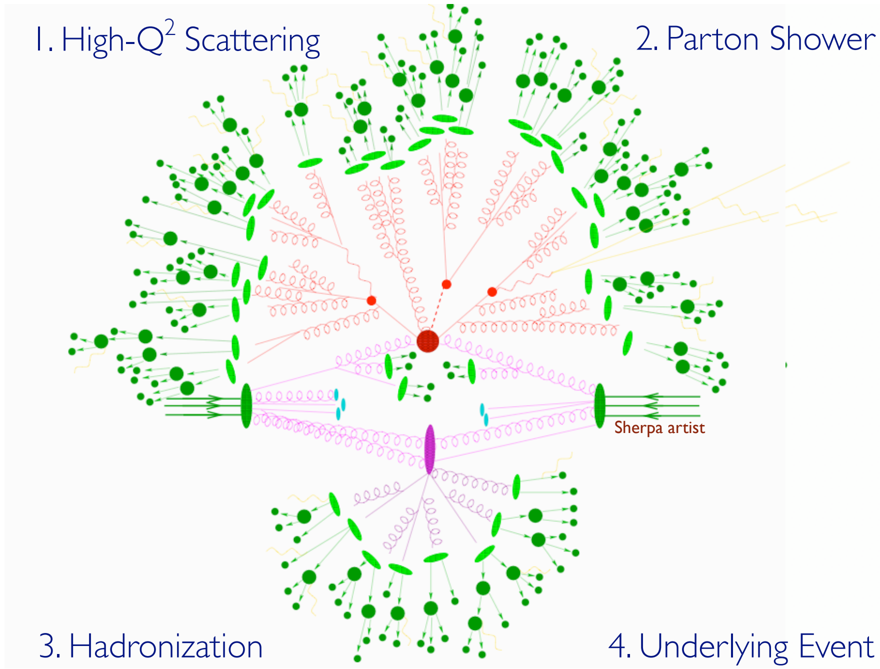
✓ We need to have a handle on them for any meaningful collider phenomenology.

Schematic view of a typical high energy event and its main evolution stages



Plots: thanks to Fabio Maltoni
TASI 2013 lectures

Schematic view of a typical high energy event and its main evolution stages



- How realistic is this picture?
  - As it turns out, it is overly simplistic, not overly complicated.
  - The above picture is inherently classical; no proper quantum effects are included yet
  - Inclusion of proper quantum effects is a dramatic complication, that is not yet fully achieved.

Overly simplistic dictionary: classic = LO; quantum = NLO or NNLO, etc

QCD: the formal bit

Extra material

- QCD is a SU(3) gauge theory. The strong charge is called color.
- The QCD Lagrangian reads:

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a}(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m_q \delta_{ab})\psi_{q,b} - \frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu}$$

$$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu$$

- $\psi_{q,a}$ - quark fields. They carry two indices:
    - Flavor: q=u,d,s,c,b,t (6 flavors)
    - Color: a=1…3

QCD is symmetric in the 6 quark flavors. They are distinguished only by their masses!

- $\mathcal{A}^C_\mu$ - gauge fields (gluons). They also carry two indices:
    - Lorentz index
    - Color index: C=1…8 (dimension of the color algebra)

- t^C_{ab} - eight SU(3) generators (in the fundamental (3x3) representation).

- f_{ABC} - the structure constants of the algebra

- g_s - the ONLY gauge coupling of the theory: $\alpha_s = \frac{g_s^2}{4\pi}$

QCD: color

Extra material

- Color is not observed (everyone knows that!). So how do we then choose the color matrices?
- A particular representation is given by the Gell-Mann matrices.
- All that is important are their commutation relations and traces.

$$
\begin{aligned}
[T^a, T^b] &= i f^{abc} T^c \,, \\
\{T^a, T^b\} &= \frac{1}{N}\delta^{ab} + d^{abc} T^c \,, \\
\mathrm{Tr}(T^a T^b) &= T_R \delta^{ab} \ , \ \text{where} \ \ T_R = \frac{1}{2} \,, \\
\mathrm{Tr}(T^a) &= 0 \,.
\end{aligned}
$$

$$
\begin{aligned}
\sum_{a,j} T^a_{ij} T^a_{jk} &\equiv (T\cdot T)_{ik} = C_F \delta_{ik} \ , \ \ C_F = \frac{N^2-1}{2N} \quad (= 4/3) \\
\sum_{a,b} f^{abc} f^{abd} &= C_A \delta_{cd} \ , \ \ C_A = N \quad (=3) \,, \\
\sum_{abc} d^{abc} d^{abd} &= \frac{N^2-4}{N}\delta_{cd} \ , \ \ d^{aac} = 0 \,,
\end{aligned}
$$

- C_F and C_A : the first Casimir of SU(3) in the fundamental/adjoint representation
- QCD has also a second Casimir (which rarely appears). A prominent place is the Tevatron A_{FB}
- Some useful relations:

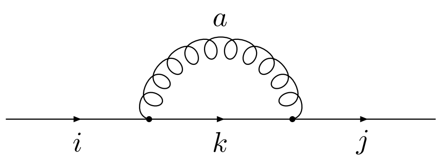
$$
\begin{aligned}
T^a_{ij} T^a_{kl} &= \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N}\delta_{ij}\delta_{kl}\right) \,, \\
\mathrm{Tr}\left(T^a T^b T^c\right) &= \frac{1}{4}(d^{abc} + i f^{abc}) \,, \\
\mathrm{Tr}\left(T^a T^b T^a T^c\right) &= -\frac{1}{4N}\delta_{bc} \,,
\end{aligned}
$$

- For the theory behind computation of colour factors see [hep-ph/9802376].
- For any serious computation use the program FORM (by Jos Vermaseren).

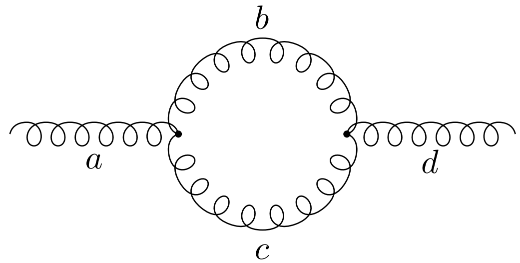
QCD: color

Extra material

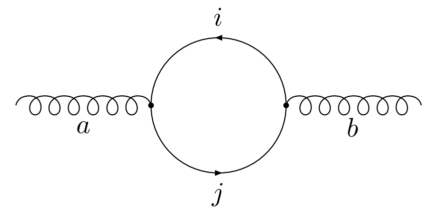
- Some examples of color factors for Feynman diagrams:



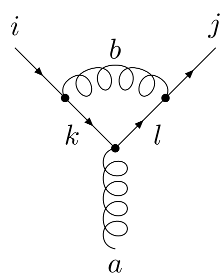
$$\begin{aligned} T^a_{ki}T^a_{jk} &\overset{2.10}{=} \frac{1}{2}\left(\delta_{kk}\delta_{ij} - \frac{1}{N}\delta_{ki}\delta_{jk}\right) \\ &= \frac{1}{2}\left(N\delta_{ij} - \frac{1}{N}\delta_{ij}\right) \\ &= \frac{1}{2}\left(N - \frac{1}{N}\right)\delta_{ij} = C_F\delta_{ij} \end{aligned}$$



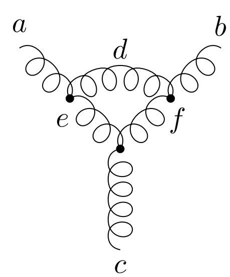
$$f^{abc}f^{dbc} \overset{2.16}{=} N\delta_{ab} = C_A\delta_{ab}$$



$$\begin{aligned} T^a_{ji}T^b_{ij} &= \mathrm{Tr}(T^aT^b) \\ &\overset{2.8}{=} \frac{1}{2}\delta_{ab} = T_F\delta_{ab} \end{aligned}$$



$$\begin{aligned} T^b_{ki}T^b_{jl}T^a_{lk} &\overset{2.10}{=} \frac{1}{2}\left(\delta_{kl}\delta_{ij} - \frac{1}{N}\delta_{ki}\delta_{jl}\right)T^a_{lk} \\ &= \frac{1}{2}\delta_{ij}T^a_{kk} - \frac{1}{2N}T^a_{ji} \\ &\overset{2.7}{=} -\frac{1}{2N}T^a_{ji} \end{aligned}$$



$$f^{ade}f^{efc}f^{dbf} = f^{ade}f^{cef}f^{bfd} \overset{2.18}{=} -\frac{N}{2}f^{abc}$$

More on QCD

Extra material

- QCD allows another peculiar term: $\theta \frac{\alpha_s}{8\pi} F^A_{\mu\nu} \tilde{F}^{A\,\mu\nu}$
- This term involves CP violation and is typically set to zero. We know it is small: $|\theta| \lesssim 10^{-10}$
- This terms can be rotated away for massless quarks. But quarks have non-zero masses …
- Quantum numbers of quarks:

	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	$+1$	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	$+1$

- Electric charge: $\mathsf{Q} = \mathsf{I}_z + \frac{\mathcal{B} + \mathsf{S} + \mathsf{C} + \mathsf{B} + \mathsf{T}}{2}$
- Hypercharge: $\mathsf{Y} = \mathcal{B} + \mathsf{S} - \frac{\mathsf{C} - \mathsf{B} + \mathsf{T}}{3}$

The Standard Model: masses

- The gluons are massless. *Exactly* massless due to the gauge symmetry.
- Quark masses are free parameters in QCD. In the SM they are set through the EW sector via the Yukawa coupling between quarks and Higgs (more later).

- Quark masses are strange concept in QCD.
- QCD, *per se*, does not need masses. QCD dynamics generates most of the hadronic masses.
- At perturbative level the Lagrangian mass appear as “usual” mass (like the electron mass).
- This point is subtle, however. Quark masses are not observables (unlike m_e).
- Quark masses for light quarks are defined through the meson masses. Usually running masses are used (MSbar, 1S, etc).
- For top quark the pole mass is suitable because the top decays before hadronization.

We will keep revisiting the issue of masses.

- The EW couplings of quarks read schematically:

$$
\begin{aligned}
\mathscr{L}_F = & \sum_i \overline{\psi}_i \left(i \, \partial\!\!\!/ - m_i - \frac{m_i H}{v} \right) \psi_i \\
& - \frac{g}{2\sqrt{2}} \sum_i \overline{\Psi}_i \, \gamma^\mu \, (1-\gamma^5)(T^+ \, W_\mu^+ + T^- \, W_\mu^-) \, \Psi_i \\
& - e \sum_i Q_i \, \overline{\psi}_i \, \gamma^\mu \, \psi_i \, A_\mu \\
& - \frac{g}{2\cos\theta_W} \sum_i \overline{\psi}_i \, \gamma^\mu (g_V^i - g_A^i \gamma^5) \, \psi_i \, Z_\mu \ .
\end{aligned}
$$

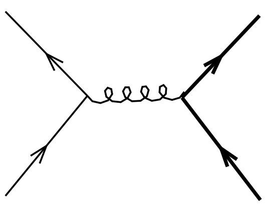
QCD: how we compute things

- … using Feynman diagrams and rules (or by using one of the many programs on the market)

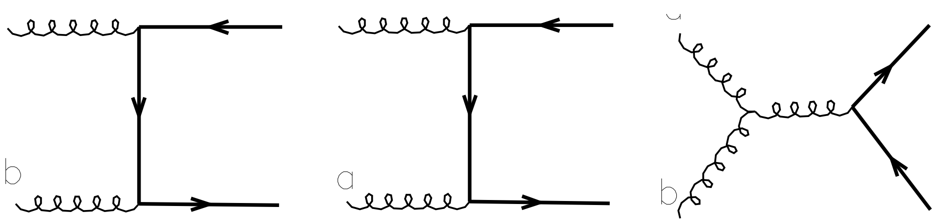
- Given the process we want to compute and the level of approximation (LO, NLO, etc) we:

  - construct the full set of Feynman diagrams [hint: use existing software]
  - square them : (…) x (…)*
  - integrate over loops and phase-space, as appropriate.

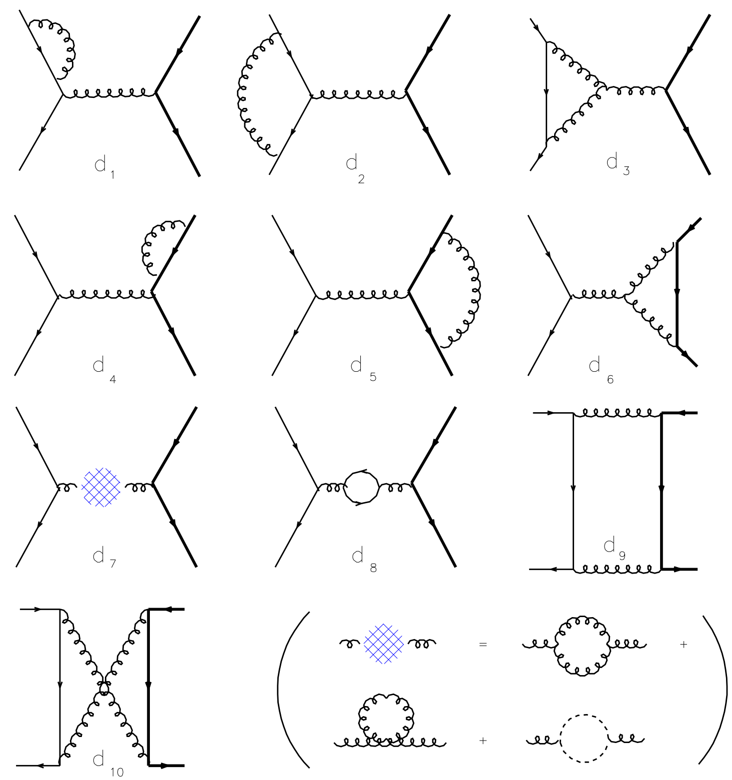
✓ Example: top quark pair production at hadron colliders



LO: qq->tt



LO gg->tt



NLO: qq->tt

QCD versus QED: twins that can be very far apart

- Perturbatively QCD and QED are very similar. In fact they are almost identical once color has been handled:

  The transition QCD → QED: $t^A \rightarrow 1$; $f^{ABC} \rightarrow 0$; $C_A \rightarrow 0$; $C_F \rightarrow 1$.

- In calculations it is often useful to compare the two, or to think of the abelian limit of QCD.

- The true differences first appear due to coupling running (not obvious in the Lagrangian):

  - In QED: coupling decreases with distance
  - In QCD: coupling increases with distance

- At large distances (or small energies) QCD becomes confining, i.e. the constituent particles, <u>the quarks, cannot be separated</u>.

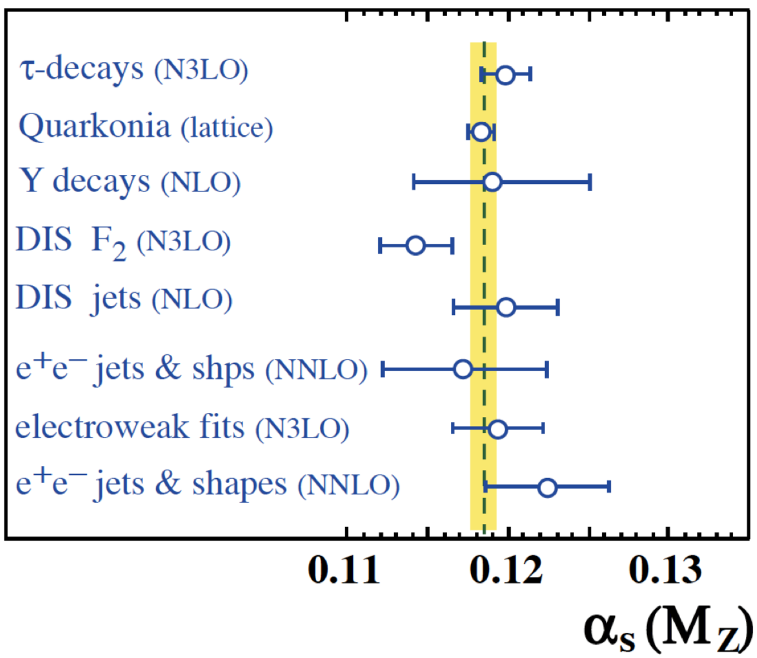
- In observables quarks always form bound states: the hadrons.

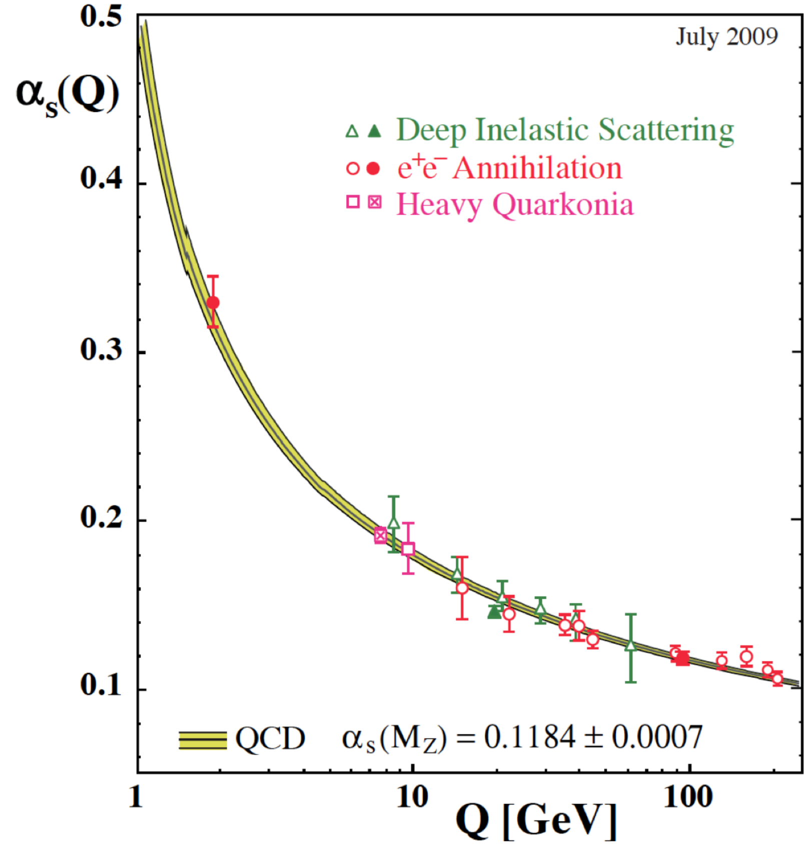
- No quark (or gluon) can be observed alone.

- Formalize:
  - only colorless states can be observed (i.e. hadrons are always colorless).

QCD versus QED: twins that can be very far apart

- The true differences first appear due to coupling running:
  - QED: coupling decreases with distance
  - QCD: coupling increases with distance (decreases with energy)

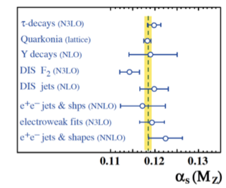


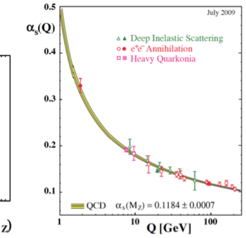


- A firm prediction of the theory: the coupling must be process independent.
- Its running too. It is known to 4 loops!

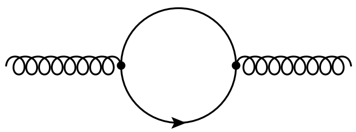
By now plenty of <u>precision</u> data that confirms this. A triumph for QCD!

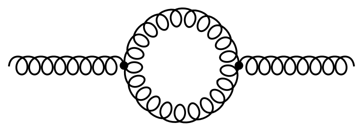
Why do we care about the running of the coupling?





- Perturbative computations at low scales are meaningless!!!
- Running of the coupling is computed from diagrams like:

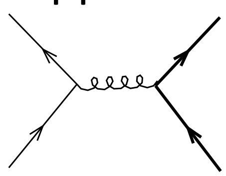




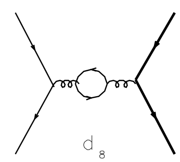
$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)}$$

$$\boxed{\begin{array}{l} \Lambda \cong 200 \text{ MeV} \\ \beta_0 \cong 11N_C - 2N_F \end{array}}$$

- The point is that we cannot – at least not yet – calculate cross-section ***exactly***.
- What we do is an approximated calculation: LO, NLO, etc.



- Imagine we perform a LO calculation; effectively a classical result, that knows nothing about running of the coupling.
- But if we replace the coupling constant with the running coupling, we effectively add some higher order ***quantum effects*** that improve the predictions



- If we do a higher order calculation, say at NLO, then some of the quantum effects are already directly included so the extra running-coupling effects become less needed.

QCD: quarks or hadrons?

- We defined the Standard Model but we didn't see anywhere strongly interacting particles like
  - Proton
  - Neutron
  - Mesons

- These are not less important; after all, the LHC collides protons!

- This brings us to the main problem: namely, how to describe bound states.

- We imagine that hadrons (like the proton) are bound states of quarks and gluons.
- We cannot describe this from first principles (due to the confining nature of QCD) (lattice gauge theory aims at solving QCD *numerically* and *non-perturbatively*)

  - Some understanding exists: we know how to describe bound states in
    - Non-relativistic QM
    - QED (hydrogen atom, positronium) [Bethe, Salpeter '51]

- A lot of ***scientific*** modeling is involved in the description of the production and decay of strongly interacting bound-states at Colliders and this would be a major subject for the remainder of these lectures.

Scientific modeling means we preserve the predictivity of the theory, not just model what we can't calculate!

QCD: quarks or hadrons?

✓ Let's build some intuition first

✓ The process of forming a bound state:
- ✓ Imagine a cloud of protons, just sitting in space.
- ✓ An electron flies in.
- ✓ What can happen?
  - An electron goes out, likely deflected from its original path
  - A bound state is formed.

$\boxed{\text{Detailed calculation required in order to predict the details.}}$

✓ On the other hand, we can answer less detailed questions without additional effort.

- Here is a good example: what is the amount of bound states to be produced?
- Answer: the same as the <u>net</u> amount of electrons ($N_{in} - N_{out}$). Thus no need to calculate bound states!

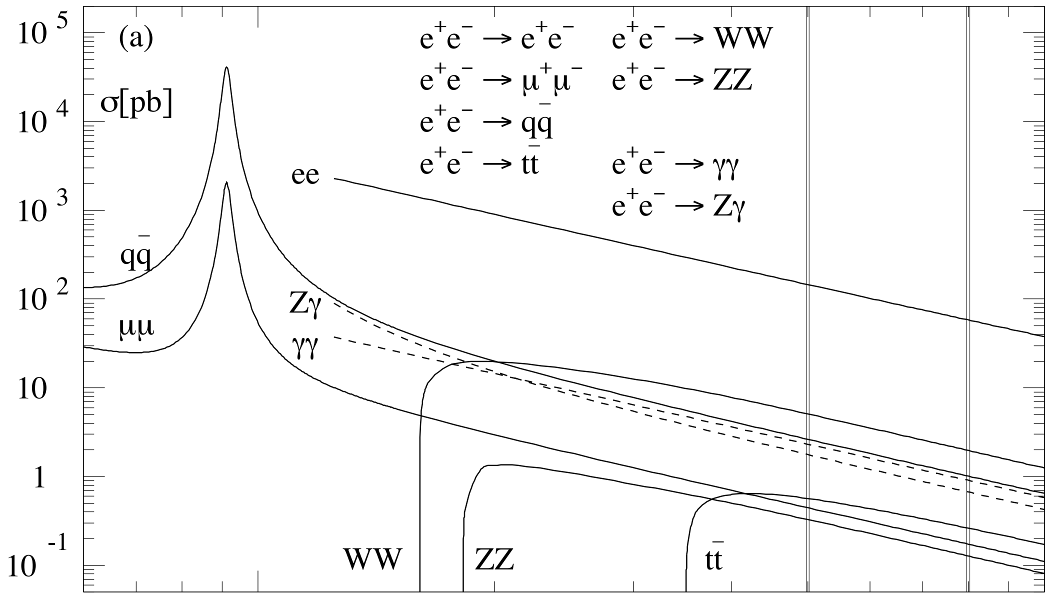
✓ This is an example of inclusive observable (i.e. we are not interested in the details…)
✓ Inclusive observables are encountered often. They are very useful. Get to know them!

First QCD calculation:

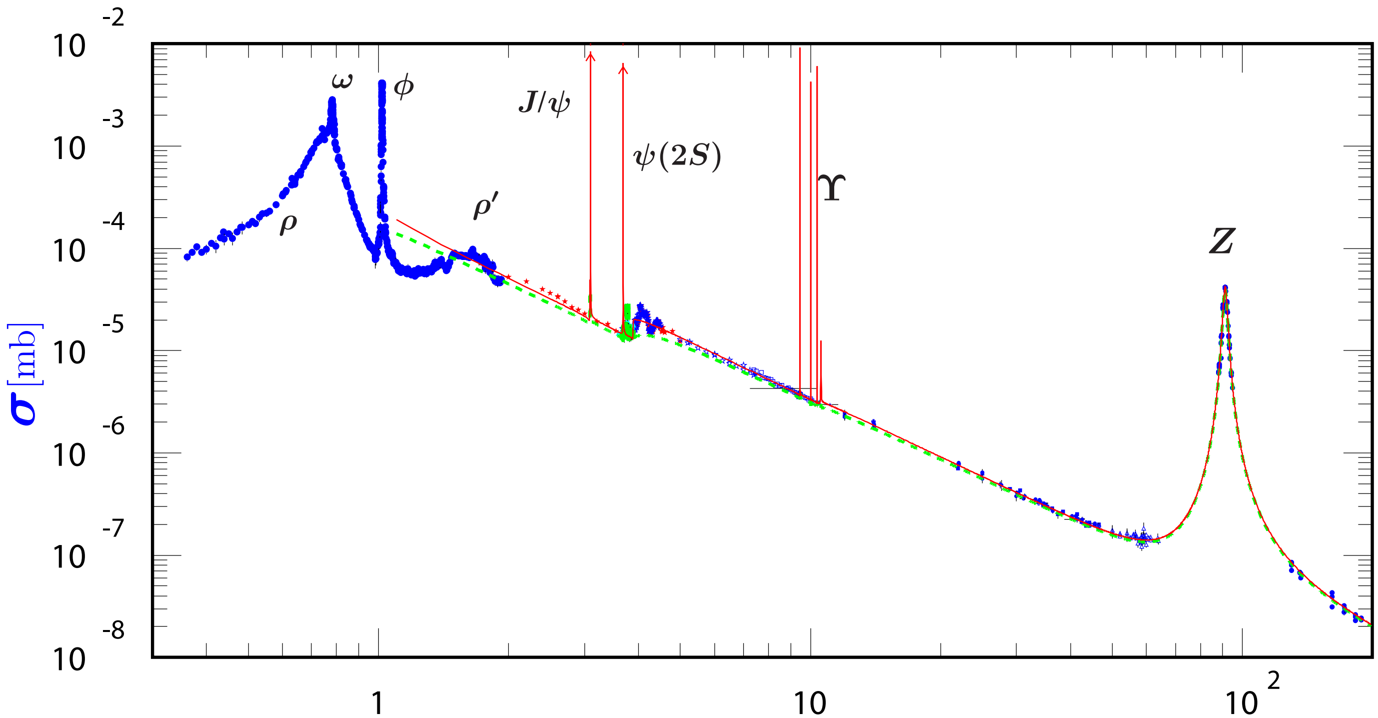
$e^+e^- \rightarrow$ hadrons

$e^+ e^-$ colliders

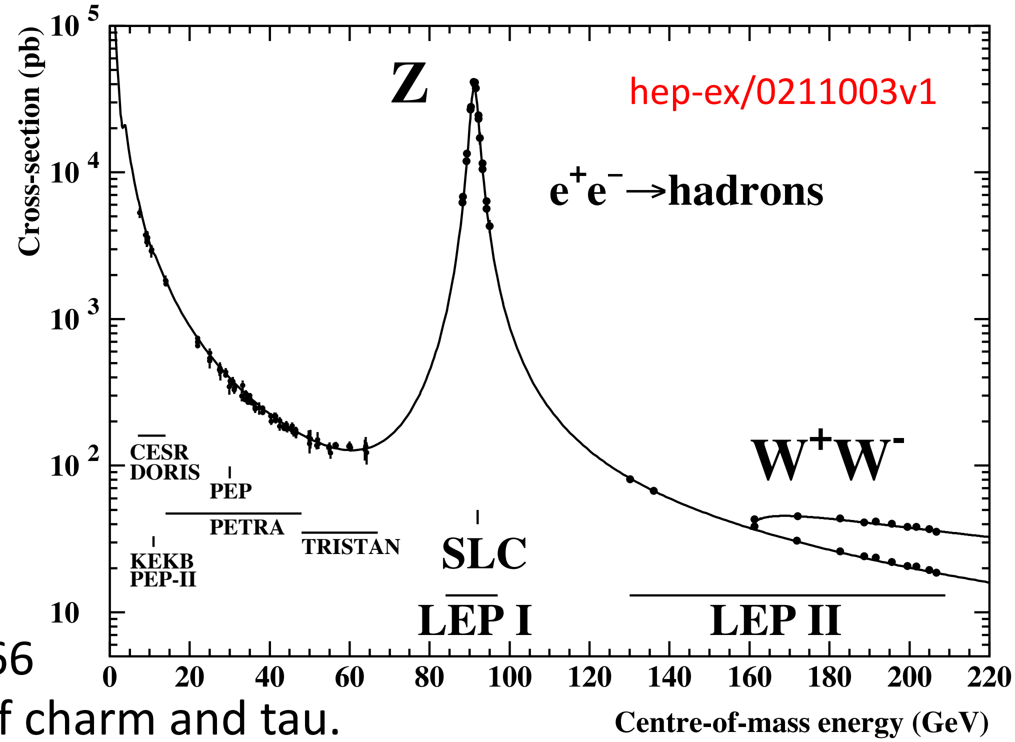
- The cross-sections for various SM process at e^+e^- colliders as a function of the collider energy



Note the resonance structures!



e^+e^- colliders come in two shapes: circular and linear. Some past/present/future experiments



- LEP
  - CERN (1989 to 2000)
  - $\sqrt{s}$ = 89 to 206 GeV
  - 4 points: Aleph, Delphi, L3, Opal.
  - 27 km tunel now houses the LHC
  - Legacy: validated the SM
- Stanford Linear Collider (SLC)
  - 1990's (same as LEP)
  - SLD detector
  - 2 miles linear accelerator exists since 1966
  - Legacies: Studies of Z-boson; discovery of charm and tau.
- B-physics
  - BaBar (SLAC linac)
  - Belle (KEK, Japan)
  - Operational till late 2000's.
  - Asymmetric e^+e^- colliders in the 10 GeV range. Mostly: e^+e^- -> $\Upsilon(4S)$ -> B^+B^- or B^0B^0
  - Legacy: CP violation in B-sector; unprecedented precision in B-physics.
  - Future upgrade Belle II (to be operational sometime during LHC Run 2)
- ILC,CLIC
  - Proposed future colliders with c.m. energy 250-1000 GeV
  - Precision Higgs/top studies. BSM searches and identification of possible TeV BSM physics.

QCD: time to get our hands dirty!

✓ We are ready to do an inclusive calculation in QCD

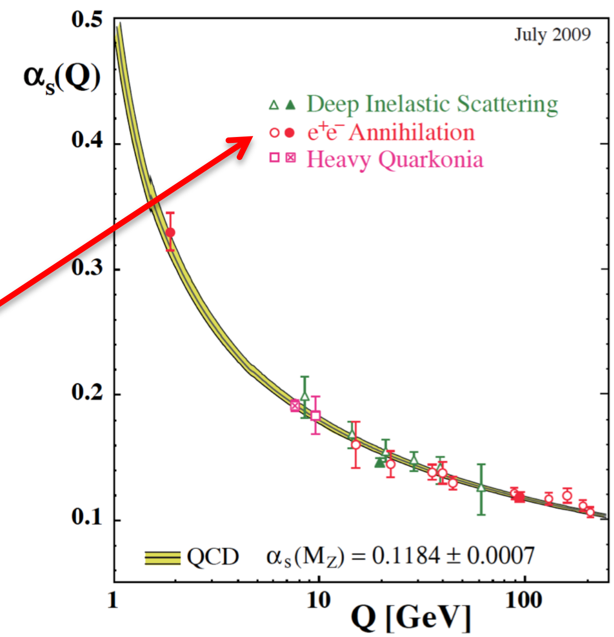
Make sure you understand this well!

$e^+ e^- \rightarrow$ hadrons

(i.e. find the probability that by scattering $e^+ e^-$ pairs, we produce ***any*** hadron)

✓ This is an example of inclusive observable. We <u>do not</u> ask:

- what kind of hadrons are produced?
- what is their distribution
- what is their multiplicity
- etc.



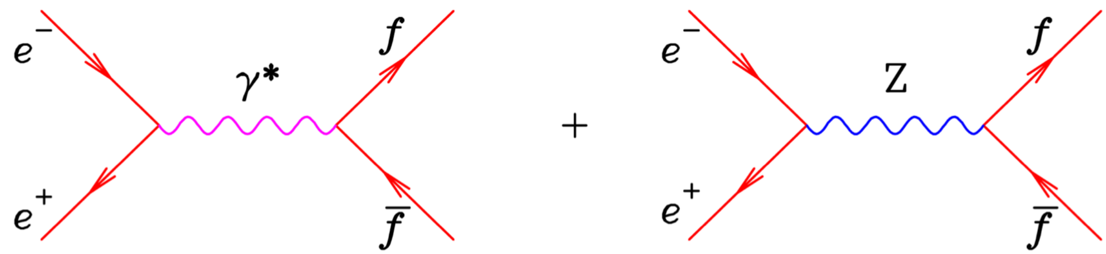
✓ Essentially, this is just a counting experiment.
And an important one!

Quark – hadron duality: in <u>very</u> inclusive observables, quarks and hadrons are the same thing

Note this keyword

QCD @ $e^+ e^- \rightarrow$ hadrons: main features

✓ The lowest order Feynman diagrams (Born process):



- e^+e^- annihilation proceeds through the exchange of:
  - (at low energy): a virtual photon
  - (at higher energy): also a Z-boson

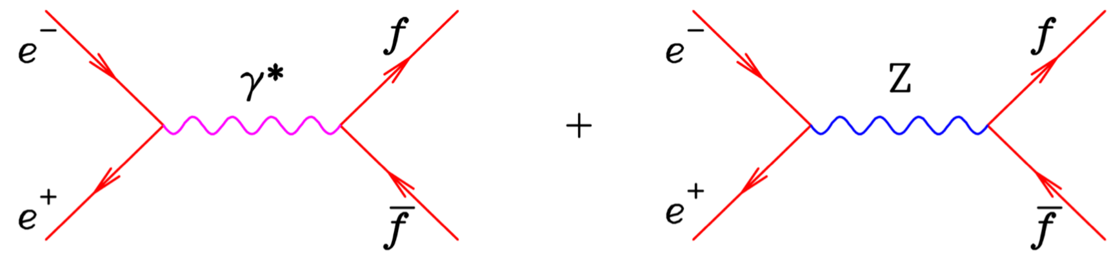
- Main feature of this reaction is that (typically*) the center of mass frame for the hadronic final state coincides with the lab frame (i.e. the detector).
- Since we can easily modify the energy of the reaction, we can achieve a great deal of control over the produced final state.
- The produced hadronic final state originating form the "decaying" gauge boson is in a color singlet state. This is very important: it implies that the hadronic system has no strong interactions with the "outside world".

➢ The above points are the MAIN difference w/r to hadron colliders!

* Unless a real photon is emitted prior to the gauge boson producing the hadronic state.
* Then we speak of Initial State Radiation (ISR).
* This highly antiquated terminology still in use today at hadron colliders.

QCD: $e^+ e^- \rightarrow$ hadrons

✓ The lowest order Feynman diagrams (Born process):



✓ The R-ratio (very well measured observable; insensitive to the details of hadronization).

QCD

QED

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \frac{\sum_q \sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3\sum_q Q_q^2$$

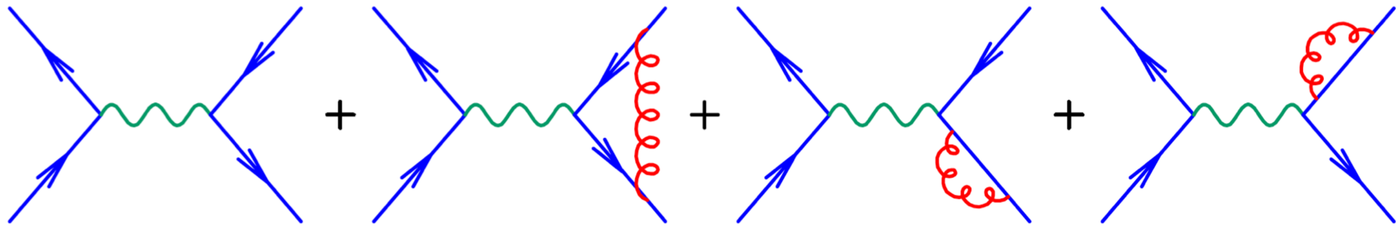
From here we can experimentally constrain:

- the electric charge of quarks
- the number of colors
- the number of quark generations

- We will not dwell on the precise expressions for the EW couplings. They are not relevant for our subsequent discussion. If interested, huge textbook literature available on this.

QCD: $e^+ e^- \rightarrow$ hadrons

✓ The one-loop quantum corrections:



✓ In dimensional regularization, i.e. in d=4-2ε dimensions, and after UV renormalization:

$$\sigma^{q\bar{q}} = 3\sigma_0 \left\{1 + \frac{2\alpha_S}{3\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right]\right\}$$

✓Oooops! The result is divergent (when ε→ 0) and so is meaningless !!!

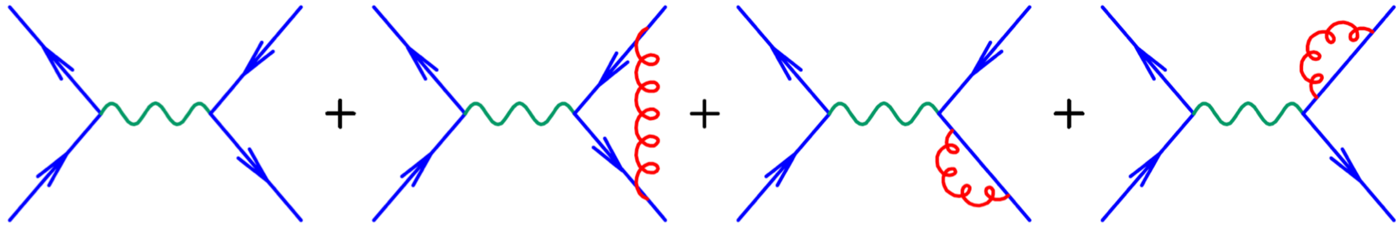
✓ This is not a mistake but an indication of a serious deficiency! We must be doing something very wrong.

✓ Let's re-analyze the whole setup.

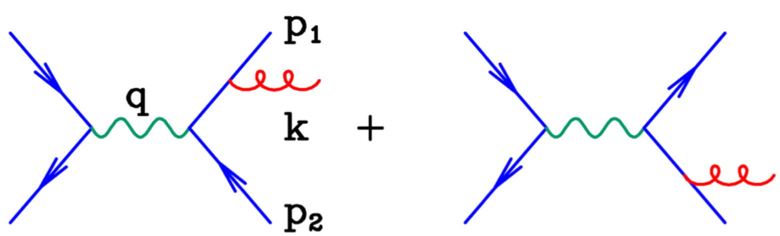
✓ Any ideas? Or questions up to here?

QCD: back to $e^+ e^- \rightarrow$ hadrons

✓ The one-loop quantum corrections



✓ need to be supplemented by real-radiation ones:



Q: But why should this work?

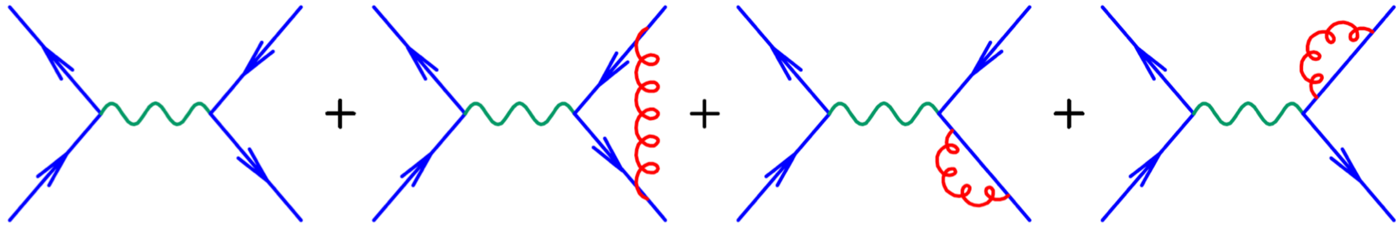
- One way to see why this works, is to recall the definition of R:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_q \sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q Q_q^2$$

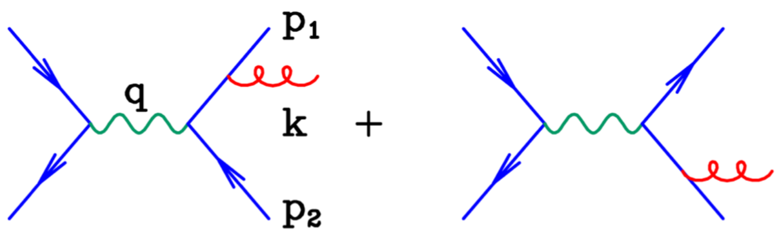
- True only at Born Level. At higher orders we have to allow for all possible combinations, not just qq!

QCD: back to $e^+ e^- \rightarrow$ hadrons

✓ The one-loop quantum corrections



✓ need to be supplemented by real-radiation ones:



$$\sigma^{q\bar{q}} = 3\sigma_0 \left\{ 1 + \frac{2\alpha_S}{3\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right] \right\}$$

$$\sigma^{q\bar{q}g} = 2\sigma_0 \frac{\alpha_S}{\pi} H(\epsilon) \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right]$$

Nice! The sum of the two is now finite ☺

$$\boxed{R = 3 \sum_q Q_q^2 \left\{ 1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2) \right\}}$$

- The key concept here is the notion if inclusiveness: two final states might be formally different but if we cannot distinguish them experimentally, then they are effectively the same!

QCD: back to $e^+ e^- \rightarrow$ hadrons

✓ The following equation is remarkable also because it demonstrates the existence of 3 colors!

$$R = 3 \sum_q Q_q^2 \left\{ 1 + \frac{\alpha_S}{\pi} + \mathcal{O}(\alpha_S^2) \right\}$$

N_C = number of quark colors (N_C=3)

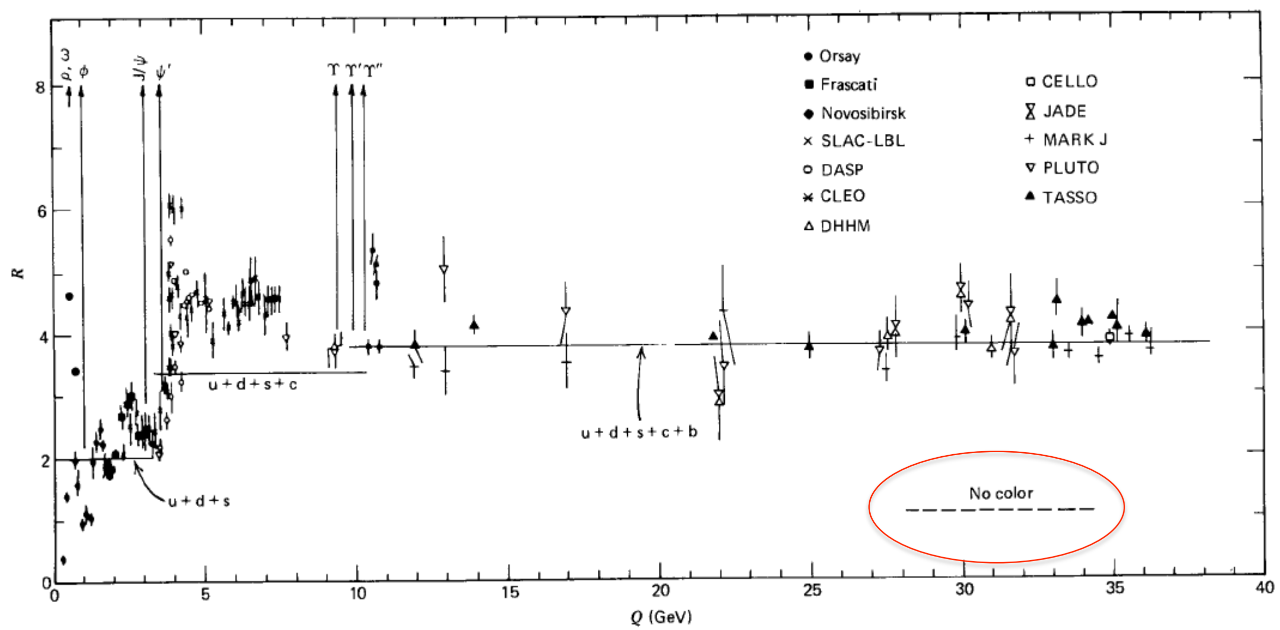


Fig. 11.3 Ratio R of (11.6) as a function of the total $e^- e^+$ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

M. Botje, "Lectures on QCD"

✓ Data consistent with N_C=3; excludes N_C=1

More on the IR singularities

✓ There are two types of IR singularities:

- Soft
- Collinear

✓ They are very significant physically; appear everywhere. Let's understand them well.

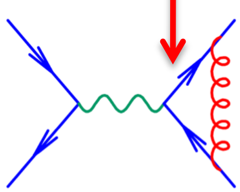
- Soft singularity: due to emission (real or virtual) of a soft massless gauge boson (photon or gluon) with vanishing energy.

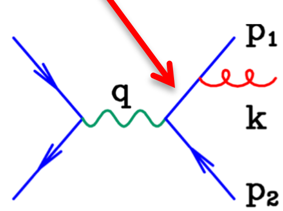
- Collinear singularity: due to emission (real or virtual) of a massless gauge boson (photon or gluon) with a momentum parallel to the emitting massless quark.

✓ Technically, these singularities are due to vanishing propagators.

✓ The singularities are regulated, say dimensionally, and then when you integrate over them (in the loop or over the phase space) explicit poles are generated.

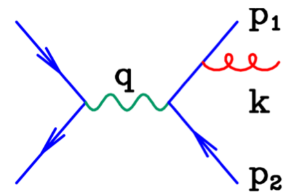
Diverging propagators





Go into the deep: augment the formal theory with physics intuition
Infra Red (IR) singularities

✓ Where do these singularities come from?



$$\left.\frac{1}{(p-k)^2}\right|_{p^2=k^2=0} = \frac{1}{2p\cdot k} = \frac{1}{p^0k^0(1-\cos(\theta))}$$

$$\frac{1}{(p-k)^2} \text{ diverges when } = \begin{cases} k^0 \to 0 & \text{soft singularity ,} \\ \cos(\theta) \to 1 & \text{collinear singularity .} \end{cases}$$

✓ What does their presence indicate? (i.e. what are they trying to tell us?)

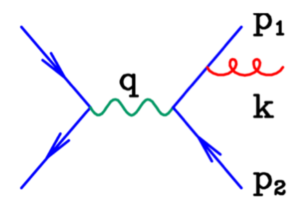
- A state with a soft gluon is <u>indistinguishable from</u> a state without it.
  - Note: this is conceptual limitation! Detectors have finite resolution.
  - A profound consequence: if we allow one soft gluon, we should allow arbitrary many.
  - This leads to resummation (more later)
- A state with a gluon collinear to the emitting particle is <u>indistinguishable from</u> a state without it. We can have many collinear gluons.

✓ So, a state is not a simple concept after all. It can contain many “basic” states. We always have to sum over all allowed states. The biggest complication in collider phenomenology!

✓ Infrared safety: only infrared safe observables are meaningful.

Go into the deep: augment the formal theory with physics intuition
Infra Red (IR) singularities

$$\left.\frac{1}{(p-k)^2}\right|_{p^2=k^2=0} = \frac{1}{2p\cdot k} = \frac{1}{p^0 k^0 (1-\cos(\theta))}$$



$$\frac{1}{(p-k)^2} \text{ diverges when } = \begin{cases} k^0 \to 0 & \text{soft singularity}\,, \\ \cos(\theta) \to 1 & \text{collinear singularity}\,. \end{cases}$$

- Question:
  - OK … this is all fine… but do we need to care about these emissions?
  - After all we cannot see them or detect them even if there is a large number of them!

- The answer is yes, we need to care about them *very much*.
- The reason is that although the individual emissions are very soft/collinear the probability for their emission becomes very large (formally infinite) as can be seen from the divergent propagator above.

- Another reason specific to QCD: soft gluons may carry zero energy but they carry a unit of color charge! Therefore soft gluon emission has dramatic effect on the color flow in the whole process!

- The implications of soft/collinear emissions are, therefore, global and very significant.

Definition of a final state

We noted that:

✓ So, a state is not a simple concept after all. It can contain many "basic" states. We always have to sum over all allowed states: the biggest complication in collider phenomenology!

1. Definition of observable must be independent of the perturbative order
2. Definition of a final state does depend on the perturbative order

Example: Higgs boson production

- Inclusive Higgs production (i.e. H+*anything*)
  Anything = (0,H,HH,g,ggggg,qqgg, etc.)

  - Born level: *anything* = 0 Leading Order (LO)
  - First correction: *anything* = g   Next-to-Leading Order (NLO)
  - Second correction: *anything* = (gg,qq) Next-to-Next-to-Leading Order (NNLO)
  - …

Note:

1. The emitted particles are not only soft or collinear; they can be anything (e.g. hard).
2. When we integrate over them, there is always a region where they become soft/collinear.
3. This leads to divergences. They cancel when all states are included.
4. IR safety implies we always need to include at least soft/collinear radiation.

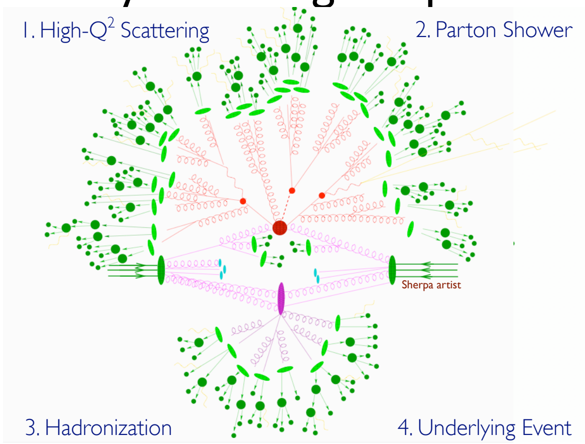
Differential observables
(i.e. identified particles in the final state)

Collinear factorization. Partons and hadrons.

- ✓ Up to here we didn't pay any attention to the distinction quarks/gluons/hadrons.
- ✓ The reason was we ***chose*** to work only with ***very inclusive*** observables.
- ✓ Most interesting observables are not fully inclusive. They are ***differential*** *observables*.
- ✓ For differential observables we need to distinguish between quarks/gluons and hadrons.

Some terminology:

- ✓ Parton: a quark or a gluon, i.e. an object that we can treat perturbatively
- ✓ Hadronization: the process of forming a hadron. It is initiated by an energetic parton.
- ✓ Fragmentation ≈ Hadronization



Why things change when we go to differential observables?

Idea: at the differential level we start to ask questions
about the nature and structure of the final state!

Example: what is the momentum of the parton?

- But that supposes we can distinguish the partons from each other.
- We now know this is not IR safe.
- So we expect new divergences to appear

Collinear factorization. Partons and hadrons.

- We saw that once real and virtual corrections *to an inclusive observable* are added, the IR divergences cancel.
- But what happens when we look at *differential* distributions?

If you compute the NLO corrections to the energy distribution of a quark (x=normalized energy) it is still divergent (but now only a single power of ε). The result reads:

$$\frac{d\sigma}{dx}(e^+e^- \to q + X) = \frac{1}{\epsilon}\sigma_{\rm born}\, P_{qq}^{(0)}(x) + \text{finite terms}$$

$$P_{qq}^{(0)}(x) = \frac{4}{3}\left(\frac{1+x^2}{1-x}\right)_+ \qquad \int_0^1 dx\, P_{qq}^{(0)}(x) \;=\; 0 \qquad \int_0^1 [f(x)]_+ g(x)dx = \int_0^1 f(x)\,(g(x)-g(1))\,dx$$

- P: the Altarelli-Parisi splitting function (at 1 loop; known to 3 loops)
- Notice – their integral is zero (these are not functions but distributions)
- there are also functions for any splitting (q→q, q→g, g→q,g→g)

Check:

$$\int_0^1 dx\, \frac{d\sigma}{dx}(e^+e^- \to q + X) = \sigma(e^+e^- \to X)$$

Good! We reproduced what we already know from the inclusive calculation (that the differential distribution is divergent, but the total inclusive one is finite)

Collinear factorization. Fragmentation functions.

- What to do with the remaining singularity (since it doesn't just cancel)?
- The issue stems from the fact that what we are calculating does not correspond to what we are measuring!

  - We calculate partons
  - We measure hadrons

$$\boxed{\frac{d\sigma}{dx}(e^+e^- \to q + X)} \quad \text{versus} \quad \boxed{\frac{d\sigma}{dx}(e^+e^- \to H + X)}$$

- So, this must be it! We just have incomplete calculation and QCD reminds us we are not done!

  → OK: how do we describe the hadrons then?

The only good way we know is based on factorization

- Hadrons are non-perturbative objects
- This means we cannot describe them in perturbation theory
- Non-perturbative phenomena are described by QCD - we believe - but:
- We cannot solve QCD non-perturbatively (a big open question; lattice calculations may help)

- Therefore we have to model the parton -> hadron transition

$$\frac{d\sigma}{dx}(e^+e^- \to H + X) = \frac{d\hat{\sigma}}{dx} \otimes D_{q\to H}(x)$$

Perturbative, finite (singularities removed by hand).

Non-perturbative. Extract from experiment.

Collinear factorization. Fragmentation functions.

The way to think about the collinear divergences is that they are not real divergences;
They are just artifacts of our idealized picture that the process of producing a hadron goes through an on-shell massless parton.

✓ How do we model the fragmentation functions $D_{q\text{->}H}(x)$?

- They are non-perturbative
- process independent i.e. universal
- Therefore, can be extracted from experiment
- They only depend on q and H.

$$\frac{d\sigma}{dx}(e^+e^- \to H + X) = \frac{d\hat{\sigma}}{dx} \otimes D_{q\to H}(x)$$

✓ Few more things to note:

- fragmentation functions are not observables
- they are not unique: depend on the definition of the counterterms
- usually in the MS-bar scheme.

- Fragmentation of massive quarks (charm, bottom) requires additional perturbative component for resummation of ln(m) terms.
- This was the resolution of the b-production puzzle at Tevatron ~20 years ago.

The bottom line on fragmentation functions

- Fragmentation functions are relevant at any collider. They are best measured at e^+e^-
- The proper definition is (at e^+e^- colliders):

$$\frac{1}{\sigma_0}\frac{d\sigma^h}{dx} = F^h(x,s) = \sum_i \int_x^1 \frac{dz}{z} C_i(z,\alpha_s(\mu),\frac{s}{\mu^2}) D_i^h(\frac{x}{z},\mu^2) + \mathcal{O}(\frac{1}{\sqrt{s}})$$

- C_i: coefficient functions. They are process dependent but purely perturbative. Can be computed at LO, NLO, NNLO (all 3 known) , … It is standard to define them in Msbar scheme.

$$C_{a,i}(z,\alpha_s) = (1-\delta_{aL})\,\delta_{iq} + \frac{\alpha_s}{2\pi}\,c_{a,i}^{(1)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 c_{a,i}^{(2)}(z) + \ldots .$$

- The FF D_i^h is process independent. However its definitions in NOT unambiguous!
- The way D_i^h is extracted is:
  - Take some experimental data (i.e. LHS of the above equation)
  - Compute perturbatively C_i
  - Extract D_i^h. Usually one assumes some functional form for D_i^h and then fits its parameters. A typical example is:

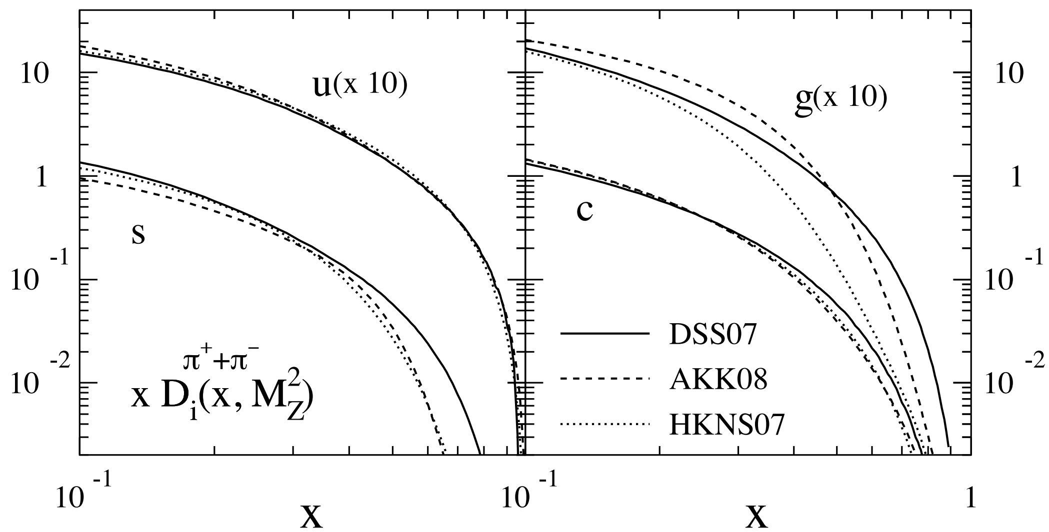
$$D_i^h(x,\mu_0^2) = N x^\alpha (1-x)^\beta \left(1+\gamma(1-x)^\delta\right)$$

- A number of groups extract FF's:

D. de Florian *et al.*, Phys. Rev. **D76**, 074033 (2007);
D. de Florian *et al.*, Phys. Rev. **D75**, 114010 (2007).
S. Albino *et al.*, Nucl. Phys. **B803**, 42 (2008).
S. Kretzer *et al.*, Eur. Phys. J. **C22**, 269 (2001).
S. Kretzer, Phys. Rev. **D62**, 054001 (2000).
L. Bourhis *et al.*, Eur. Phys. J. **C19**, 89 (2001).
M. Hirai *et al.*, Phys. Rev. **D75**, 094009 (2007).
C. Aidala *et al.*, Phys. Rev. **D83**, 034002 (2011).

More on fragmentation functions

Some examples:



Note: for light flavor fragmentation the FF's are peaked at low-x

- Fragmentation functions satisfy sum rules (the momentum of the incoming parton is conserved when summed over all final state hadrons)

$$\sum_h \int_0^1 dz\, z\, D_i^h(z, \mu^2) \;=\; 1$$

- Fragmentation functions depend on the partonic fraction "z" but also on an energy scale. This scale dependence can be predicted within pQCD (time-like evolution):

$$\frac{\partial}{\partial \ln \mu^2} D_i(x, \mu^2) \;=\; \sum_j \int_x^1 \frac{dz}{z} P_{ji}(z, \alpha_s(\mu^2)) D_j(\frac{x}{z}, \mu^2)$$

(Timelike) DGLAP equation

where: $$P_{ji}(z, \alpha_s) \;=\; \frac{\alpha_s}{2\pi} P_{ji}^{(0)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ji}^{(1)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^3 P_{ji}^{(2)}(z) + \ldots$$

Altarelli-Parisi splitting functions

More on fragmentation functions

Extra material

- Heavy quark fragmentation functions are peculiar. We suppose that the non-perturbative transition b->B is initiated only by the corresponding heavy flavor “b”. Same for charm…

- Then the perturbative part can be computed perturbatively and it reads:

$$\frac{d\sigma_{\mathcal{Q}}}{dz}(z,Q,m) = \sum_a \int_z^1 \frac{dx}{x}\frac{d\hat{\sigma}_a}{dx}(x,Q,\mu) D_{a/\mathcal{Q}}\left(\frac{z}{x},\frac{\mu}{m}\right) + \mathcal{O}\left(\frac{m}{Q}\right)^p$$

where Q is a heavy quark (on-shell) with mass m.

- The FF $D_{a/Q}$ satisfies DGLAP evolution equation. Its boundary condition can be evaluated perturbatively at some low scale $\mu_0 \approx m$:

$$\boxed{D^{\mathrm{ini}}_{a/\mathcal{Q}}\left(z,\frac{\mu_0}{m}\right) = \sum_{n=0}\left(\frac{\alpha_s(\mu_0)}{2\pi}\right)^n d_a^{(n)}\left(z,\frac{\mu_0}{m}\right)}$$

$$d_a^{(0)}(z) = \delta_{a\mathcal{Q}}\delta(1-z),$$

$$d^{(1)}_{a=\mathcal{Q}}\left(z,\frac{\mu_0}{m}\right) = C_F\left[\frac{1+z^2}{1-z}\left(\ln\left(\frac{\mu_0^2}{m^2(1-z)^2}\right)-1\right)\right]_+$$

$$d^{(1)}_{a=g}\left(z,\frac{\mu_0}{m}\right) = T_R\left(z^2+(1-z)^2\right)\ln\left(\frac{\mu_0^2}{m^2}\right)$$

$$d^{(1)}_{a\neq\mathcal{Q},g}\left(z,\frac{\mu_0}{m}\right) = 0,$$

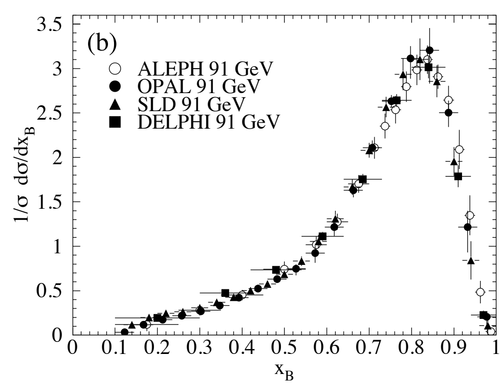
The NNLO term $d^{(2)}$ also known.

NNLO application exists:
https://arxiv.org/abs/2102.08267

The non-perturbative part is fitted from data

$$\text{Peterson } et\ al.\ [233]: \quad D_{\mathrm{np}}(z) \propto \frac{1}{z}\left(1-\frac{1}{z}-\frac{\epsilon}{1-z}\right)^{-2}$$

$$\text{Colangelo\&Nason } [236]: \quad D_{\mathrm{np}}(z) \propto (1-z)^\alpha z^\beta$$



Peaked at large-x

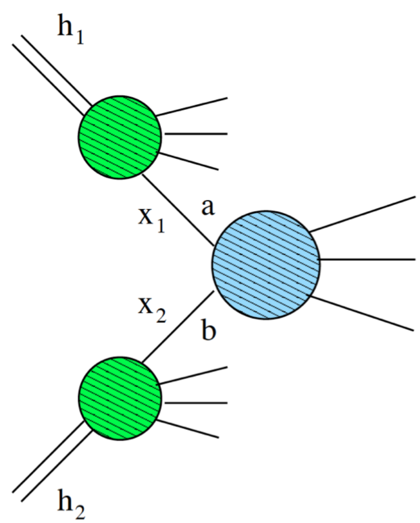
Physics at hadron colliders

Collinear factorization. Parton distribution functions.

✓ Recall the factorized form of the cross-section in e^+e^- collisions:

$$\frac{d\sigma}{dx}(e^+e^- \to H + X) = \frac{d\hat{\sigma}}{dx} \otimes D_{q\to H}(x)$$

✓ We encounter conceptually the same story when we consider hadrons in the initial state



(🡐 Drell-Yan-type process)

✓ Even for fully inclusive observables collinear singularities remain.
They are associated with the Initial hadron → parton transition.

✓ This non-perturbative transition is described by parton distribution functions.

✓ They are very similar to fragmentation functions but are not the same!

✓ Extracted from experiment

✓ Universal (i.e. process independent)

✓ Scheme dependent (typically MS-bar)

Hadron colliders: the basics

Extra material

The natural kinematical variables at hadron colliders are dictated by a) the cylindrical geometry of the beam-detector system and b) the fact that the initial state momentum (along the z-direction) is unknown.

$$\begin{aligned} p^\mu &= (E, p_x, p_y, p_z) \\ &= (m_T \cosh(y), p_T \sin(\phi), p_T \cos(\phi), m_T \sinh(y)) \end{aligned}$$

where we have introduced:

$$\begin{aligned} p_T^2 &= p_x^2 + p_y^2 \quad \text{(transverse momentum)}\,, \\ m_T &= \sqrt{p_T^2 + m^2} \quad \text{(transverse mass)}\,, \\ y &= \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) \quad \text{(rapidity)}\,. \end{aligned}$$

An easier to measure variable is the pseudorapidity:

$$\eta = -\ln \tan(\theta/2) \quad \text{with} \quad \eta = y\big|_{m=0}\,.$$

In general we have:

$$y = \ln \frac{\sqrt{m^2 + p_T^2 \cosh^2 \eta} + p_T \sinh \eta}{\sqrt{m^2 + p_T^2}}\,, \quad \text{and :} \quad \eta = y\big|_{m=0}\,.$$

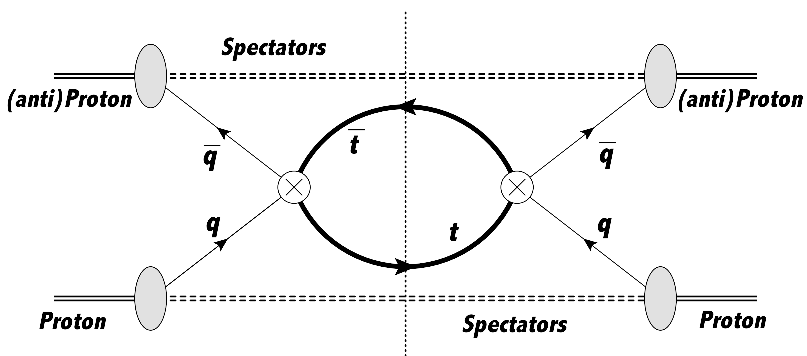
- The pseudorapidity is easy to measure directly, in terms of the angle θ.
- Rapidity differences are boost invariant.

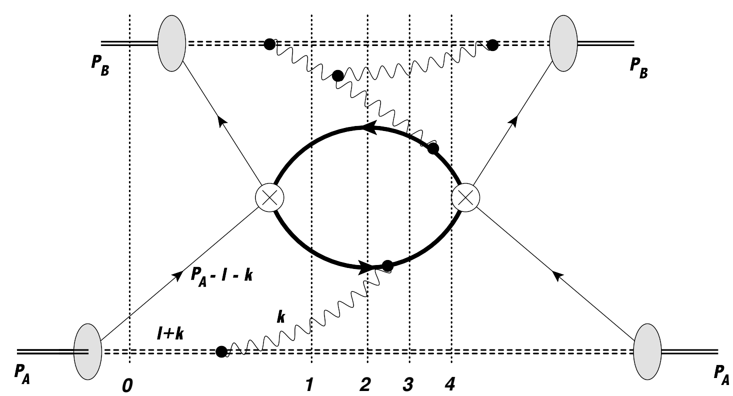
Hadron colliders: differences w/r to e^+e^-

- Partonic fraction "z" versus "x":
  - In fragmentation we have q -> H (i.e. H carries fraction $0 < z < 1$ of the momentum of q)
  - In PDF's we have H -> q (i.e. q carries fraction $0 < x < 1$ of the momentum of H)
  - Thus z and x are "related" as z <---> 1/x (careful, this is kinematically forbidden!)

- The initial *partonic* state momentum is not known in hadronic collisions. This is easy to see: Let the two protons have 3-momenta P and –P. The initial state partons have 3-momenta x_1P and $-x_2P$. Since the partonic fractions x_1 and x_2 can be as low as zero, then the momentum of the initial state can be very large or very small, symmetric or very asymmetric i.e. boosted along the beam direction.

- The final state is not evolving on its own. In hadron-hadron collisions there are spectators. The final state resulting from the hard collision (i.e. what we care about) can interact with the beam remnants.



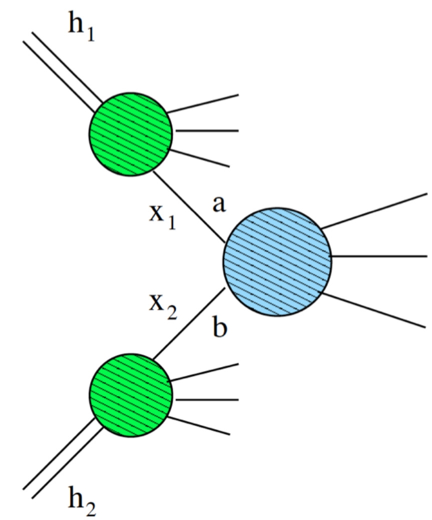


These are mostly small but can be significant sometimes.

Collinear factorization in all its glory

Some hadron

$$\frac{d\sigma}{dp_T}(pp \to H+X) = \sum_{i,j,k=q,\bar{q},g} f_i \otimes f_j \otimes \frac{d\hat{\sigma}}{dp_T}(ij \to k+X) \otimes D_{k\to H}(x) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)$$



- ✓ Two pdf's for LHC collisions (1 for DIS)
- ✓ One fragmentation function for each observed final state hadron
  No need for it if we have gauge bosons, Higgs, jets.
- ✓ We sum over all possible partons in the initial/final state
- ✓The factorization formula does not automatically apply to every process
  - ✓ For some never been proven,
  - ✓For other may not apply (or remainders changes)
- ✓ It is not exact. It misses terms that are small
- ✓ Q is a "typical" scale. A hard scale which is large. That's why we can neglect the remainder.
- ✓ There is, of course, scheme dependence (how collinear singularities are factorized). Msbar …
- ✓ The scheme dependence means that what is meaningful is the LHS and
  not any one term on the RHS

Factorization and factorization scales

- We saw that factorization is useful because it offers the right way to interpret IR singularities in hard scattering cross-sections with initial/final state ***identified*** partons.

? Question: why at hadron colliders – even for fully integrated over final state – we still have leftover collinear singularities?

- It turns out there is much more to factorization than the above!

Every factorization leads to the appearance of a factorization scale

- This actually makes sense: "factorization" means we separate long-distance from short-distance physics. But where is the formal separation boundary? For this we need to introduce a scale – called factorization scale μ_F – which tells us what is long- and what short-distance. The theory does not tell us what the value of this scale is.

- And how could it? Such scale is unphysical – it is a formality, an artifact – of our formal separation (recall: when we factorize we neglect terms that are "small", i.e. factorized expressions are not exact but approximately exact).

- If the factorization scale is unphysical then observables should not depend on it! This is important requirement that has profound implications!

DGLAP evolution equation

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi: 1970's

- The space-like evolution equation (for hadron colliders) reads:

$$\frac{\partial}{\partial \ln \mu^2} f_i(x,\mu^2) \;=\; P_{ij}(x,\mu^2) \otimes f_j(x,\mu^2)$$

- The splitting functions have perturbative expansion (now known to 3 loops) and satisfy some all-order relations:

$$P_{ij}^{\mathrm{N^mLO}}(x,\mu^2) \;=\; \sum_{k=0}^{m} a_{\mathrm{s}}^{k+1}(\mu^2)\, P_{ij}^{(k)}(x)$$

$$\begin{aligned} P_{\mathrm{q}_i\mathrm{q}_k} &= P_{\bar{\mathrm{q}}_i\bar{\mathrm{q}}_k} = \delta_{ik} P_{\mathrm{qq}}^{\mathrm{v}} + P_{\mathrm{qq}}^{\mathrm{s}} \\ P_{\mathrm{q}_i\bar{\mathrm{q}}_k} &= P_{\bar{\mathrm{q}}_i\mathrm{q}_k} = \delta_{ik} P_{\mathrm{q}\bar{\mathrm{q}}}^{\mathrm{v}} + P_{\mathrm{q}\bar{\mathrm{q}}}^{\mathrm{s}} \\ P_{\mathrm{gq}} &\equiv P_{\mathrm{gq}_i} = P_{\mathrm{g}\bar{\mathrm{q}}_i} \\ P_{\mathrm{qg}} &\equiv n_f\, P_{\mathrm{q}_i\mathrm{g}} = n_f\, P_{\bar{\mathrm{q}}_i\mathrm{g}} \end{aligned}$$

- This is a matrix equation which mixes all quark flavors, all antiquarks and the gluon. We need to quasi-diagonalize to solve it. It is useful to think in terms of the flavor symmetry of all quarks (they are all massless!). There is one combination which transforms as the gluon (flavor singlet) and it cannot be decoupled from the gluon:

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} q_{\mathrm{s}} \\ g \end{pmatrix} = \begin{pmatrix} P_{\mathrm{qq}} & P_{\mathrm{qg}} \\ P_{\mathrm{gq}} & P_{\mathrm{gg}} \end{pmatrix} \otimes \begin{pmatrix} q_{\mathrm{s}} \\ g \end{pmatrix} \quad \text{for} \quad q_{\mathrm{s}} \;=\; \sum_{r=1}^{n_f} (q_r + \bar{q}_r)$$

The non-singlets can be defined as

$$q_{\mathrm{ns},ik}^{\pm} \;=\; q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k)\;, \qquad q_{\mathrm{ns}}^{\mathrm{v}} \;=\; \sum_{r=1}^{n_f} (q_r - \bar{q}_r)$$

And they evolve (1-dim equations) with

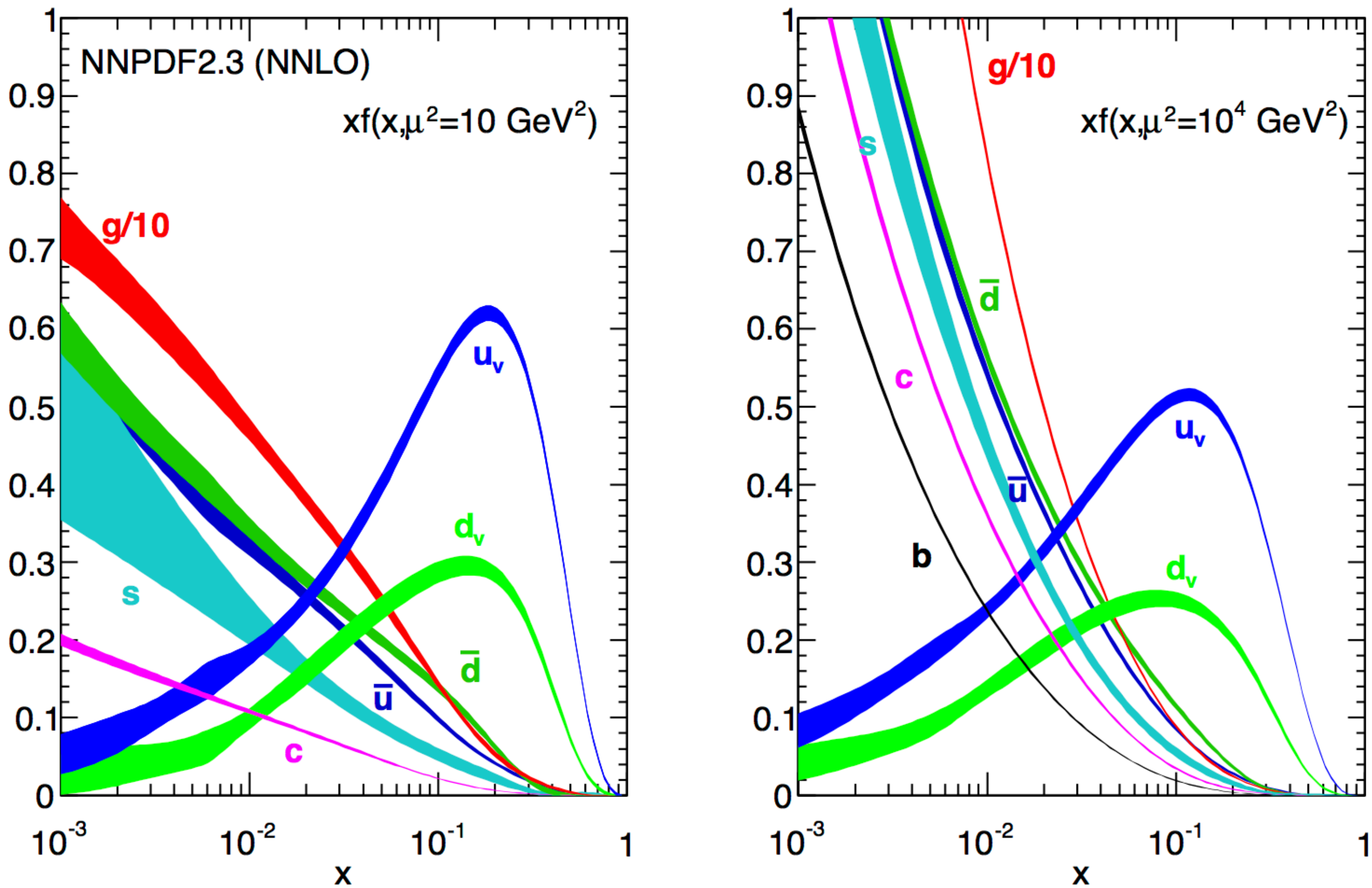
$$\begin{aligned} P_{\mathrm{ns}}^{\pm} &= P_{\mathrm{qq}}^{\mathrm{v}} \pm P_{\mathrm{q}\bar{\mathrm{q}}}^{\mathrm{v}}\;, \\ P_{\mathrm{ns}}^{\mathrm{v}} &= P_{\mathrm{qq}}^{\mathrm{v}} - P_{\mathrm{q}\bar{\mathrm{q}}}^{\mathrm{v}} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} - P_{\mathrm{q}\bar{\mathrm{q}}}^{\mathrm{s}}) \equiv P_{\mathrm{ns}}^{-} + P_{\mathrm{ns}}^{\mathrm{s}} \end{aligned}$$

The quark singlet evolves with: $P_{\mathrm{qq}} \;=\; P_{\mathrm{ns}}^{+} + n_f (P_{\mathrm{qq}}^{\mathrm{s}} + P_{\bar{\mathrm{q}}\mathrm{q}}^{\mathrm{s}})$

See http://arxiv.org/pdf/hep-ph/0408244v1.pdf for more details

DGLAP: initial conditions

✓ An example: the NNPDF2.3 pdf set (one of several major sets of pdfs)



- Left: scale Q=3GeV
- Right: scale Q=100 GeV

✓ The width of the curves indicates the uncertainty

More on the factorization and renormalization scales

- The most controversial topic. Ever.
- (as we discussed already) Factorization scale separates long- from short-distance physics
- The renormalization scale is unrelated; this is simply the scale at which the running coupling is evaluated. Usually the two are taken to be equal. But they need not be.
- An observable is formally independent of these scales. However this is only true if we can compute the full perturbative series. In such case, the sum of all terms (and each term is scale dependent) becomes scale-independent.
- In reality, we only compute the perturbative expansion to some fixed order (LO or NLO or NNLO, etc). This means that due to the missing higher order terms the cancellation of the scale independence is spoiled and the final predicted result does depend on these scales. Here is an example (for factorization and renormalization scales being equal):

$$\begin{aligned}\sigma_{ij,\mathbf{I}}(\beta,\mu,m) &= \sigma^{(0)}_{ij,\mathbf{I}}\Bigg\{1+\frac{\alpha_s(\mu^2)}{4\pi}\left[\sigma^{(1,0)}_{ij,\mathbf{I}}+\sigma^{(1,1)}_{ij,\mathbf{I}}\ln\left(\frac{\mu^2}{m^2}\right)\right] \\ &\quad+\left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2\left[\sigma^{(2,0)}_{ij,\mathbf{I}}+\sigma^{(2,1)}_{ij,\mathbf{I}}\ln\left(\frac{\mu^2}{m^2}\right)+\sigma^{(2,2)}_{ij,\mathbf{I}}\ln^2\left(\frac{\mu^2}{m^2}\right)\right]+\mathcal{O}(\alpha_s^3)\Bigg\}\end{aligned}$$

- Clearly, the more terms we add in the perturbative expansion, the smaller the dependence on these scales.
- This is verified in all known cases where higher-order perturbative calculations have been made.
- Scale variation is the standard procedure for establishing the error due to missing higher order terms. It is not based on hard science but can be verified with NNLO calculations.
- One systematic approach is BLM. [Brodsky, Lepage, Mackenzie ’83]

Resummation

Resummation: why large logs?

- While we managed to dispose of all IR singularities present in fixed order calculations, as it turns out, we haven't removed all traces of them!
- Upon cancellation of soft singularities, or upon the factorization of the collinear ones, finite but numerically large remainders remain in the perturbative results.
- These terms are of logarithmic nature, i.e. they look like Log[s], and s<<1. Such logs are thus large.
- The problem is not just in the fact that these terms are large; after all, all kinds of numbers appear in perturbative calculations; some are big and some are small (e.g. Pi^2~10 which is large) without deep underlying reason.
- The problem is that such terms ~Log[s] appear systematically to all orders in the perturbative expansion.

Resummation: why large logs?

Extra material

Let s be some small parameter, $s << 1$, while $s > 0$. Denote $L = \log(s)$. Let O be some observable with the following perturbative expansion:

$$O = O^{(0)} + \alpha_S\,(a_1 L + \dots) + \alpha_S^2\left(a_2 L^2 + b_1 L + \dots\right) + \mathcal{O}(\alpha_S^3)$$

Basically, at each order of α_S we get higher and higher accompanying powers of L.

The maximum power of L is different for soft and collinear logs. For the example above, for each power of α_S^n we get terms like:

- $\alpha_S^n L^n$ (Leading Logs, or LL),
- $\alpha_S^n L^{n-1}$ (Next-to-Leading Logs or NLL),
- $\alpha_S^n L^{n-2}$ (Next-to-Next-to-Leading Logs or NNLL), etc.

Therefore, what we call leading log is not just one term, but a series of terms to *all orders in* α_S: $\alpha_S L + (\alpha_S L)^2 + (\alpha_S L)^3 + \dots$.

This is significant, because (if we restrict ourselves to LL for now) the perturbative expansion of the observable O is actually in two parameters:

$$O(\alpha_S, \alpha_S L) = \sum_{k,n}^{\infty} o_{k,n}\alpha_S^k(\alpha_S L)^n$$

Recall that $\alpha_S \sim 0.1 \ll 1$ and $\alpha_S L = \mathcal{O}(1)$. Thus since $\alpha_S L \gg \alpha_S$ we effectively have:

$$O(\alpha_S, \alpha_S L) = \sum_{n}^{\infty} o_n(\alpha_S L)^n$$

This is the problem: we are trying to compute O in an expansion which is not convergent at all! This implies that while we can compute, at least in principle, any fixed order expansion of $O(\alpha_S)$ this makes no sense at all because the expansion is not convergent. Thus perturbation theory fails.

The only way to restore the predictivity of the theory is to *resum* all terms $\sim (\alpha_S L)^n$, to **all** orders in α_S.

Once this is done at the LL level, we have remaining the next subleasing expansion (which now becomes the dominant one), i.e. the NLL one in $(\alpha_S L)^n/L$. The story here is the same. These have to be resummed, too. And so on and so forth.

How factorization leads to resummation

Extra material

Let's consider the following *very simplified* expression for some observable in e^+e^-:

$$O(x) = C(x, \mu_F) \times D(x, \mu_F) + \text{small terms}\,,$$

and since the observable O is independent of μ_F we have:

$$\mu_F \frac{d}{d\mu_F}\left(O(x)\right) = 0\,.$$

This implies that:

$$\mu_F \frac{d}{d\mu_F} C(x, \mu_F) = \gamma = -\mu_F \frac{d}{d\mu_F} D(x, \mu_F)\,.$$

Above γ is an integration constant, known as "anomalous dimension". We can easily integrate the above to make the scale dependence of D explicit:

$$D(x, \mu_F) \sim \exp\left[\gamma \int^{\mu_F} d\ln\mu_F\right] \sim (\mu_F)^\gamma \times D(x, 1)\,.$$

The above equation indicates that the scale dependence of D can be predicted - even if D is non-perturbative!

- The above argument captures most relevant features; in reality there are complications.

How factorization leads to resummation

Extra material

- <u>Problem</u>: the product "X" is usually a convolution, not multiplication. This fails the above argument.
- <u>Solution</u>: go from <u>momentum</u> "x" space to Mellin <u>moment</u> "N" space.

$$f(N) = \int_0^1 x^{N-1} f(x) dx$$

In this space a convolution is turned into simple product:

$$f \otimes g(x) \longrightarrow f(N)g(N)$$

- The μ_F dependence also enters through the running coupling …

- The dependence on μ_F is through dimensionless ratios μ_F/Q for some kinematical variable Q (or some mass).

- The main point is:

$$D(x, \mu_F) \sim \exp\left[\gamma \int^{\mu_F} d\ln\mu_F\right] \sim (\mu_F)^\gamma \times D(x, 1)$$

expresses the major wisdom***: factorization leads to evolution***!

How factorization leads to resummation

Extra material

- Let's go back and collect what we found so far (and be more careful about the arguments):

$$O\left(N, \frac{Q^2}{m^2}, \alpha_S\right) = C\left(N, \frac{Q^2}{\mu_F^2}, \alpha_S\right) \cdot D\left(N, \frac{\mu_F^2}{m^2}, \alpha_S\right)$$

- Q is some large scale
- m is some small (but perturbative) scale
- N is some kinematic variable (rather its Mellin conjugate)

- The factorization scale dependence enters exactly as above; this follows from the differential equations we solved.

- How is the evolution useful? Rewrite it as:

$$O\left(N, \frac{Q^2}{m^2}, \alpha_S\right) = C\left(N, \frac{Q^2}{\mu_F^2}, \alpha_S\right) \cdot E\left(N, \frac{\mu_F^2}{\mu_0^2}, \alpha_S\right) \cdot D\left(N, \frac{\mu_0^2}{m^2}, \alpha_S\right)$$

- Above μ_F and μ_0 are two distinct factorization scales.
- Choose: $\mu_F \approx Q$ and $\mu_0 \approx m$
- Then the functions C and D have no large ratios of scales: C(N,1) and D(N,1).
- C(N,1) can be computed perturbatively (in this case perturbation theory works at its best)
- D(N,1) does not depend on any scales – just on one kinematic variable and can be extracted from data.
- The only remaining piece is E. It is perturbative. It is process-independent. It only depends on the anomalous dimension and the strong coupling. It depends on the ratio of two very different scales.

How factorization leads to resummation

Extra material

$$O\left(N,\frac{Q^2}{m^2},\alpha_S\right)=C\left(N,\frac{Q^2}{\mu_F^2},\alpha_S\right)\cdot E\left(N,\frac{\mu_F^2}{\mu_0^2},\alpha_S\right)\cdot D\left(N,\frac{\mu_0^2}{m^2},\alpha_S\right)$$

- Let’s go back and collect what we found so far (and be more careful about the arguments):

$$\begin{aligned}E\left(N,\frac{\mu_F^2}{\mu_0^2},\alpha_S\right)&=\exp\left[\int_{m^2}^{Q^2}\gamma(\alpha_S(\mu))\frac{d\mu}{\mu}\right]\,E\left(N,1,\alpha_S\right)\\&=\exp\left[\int_{m^2}^{Q^2}\frac{\gamma(\alpha_S)}{\beta(\alpha_S)}d\alpha_S\right]\,E\left(N,1,\alpha_S\right)\end{aligned}$$

and E(N,1) can be computed perturbatively.

- The above equation is very general

- In particular, if the anomalous dimension depends on the kinematics (i.e. N) we get the DGLAP evolution equation; the anomalous dimension are the AP splitting functions!

- There is more: rewriting the above result the way we did, allows us to remove any large log from our results (will explain later):

$$O\left(N,\frac{Q^2}{m^2},\alpha_S\right)=C\left(N,1,\alpha_S\right)\cdot E\left(N,1,\alpha_S\right)\cdot D\left(N,1,\alpha_S\right)\cdot\exp\left[\int_{m^2}^{Q^2}\gamma(\alpha_S(\mu))\frac{d\mu}{\mu}\right]$$

We close the circle: Factorization ➔ Evolution ➔ Resummation

Resummation: collinear logs

- The logs are two types:
  - Collinear
  - Soft

- Collinear logs are simpler; they originate from small masses. Imagine we have a small mass m. By small we mean not the absolute size of m but relative to other hard scales in the problem. Let Q be one such scale and m<<Q. Let also assume that all other kinematical scales (if present) are also large and of the order of Q.
- Thus: s=m^2/Q^2<<1 and L=Log[m^2/Q^2]>>1 .

- How do we resum collinear logs? With the DGLAP equation. Recall what we just derived:

$$O\left(N, \frac{Q^2}{m^2}, \alpha_S\right) = C\left(N, 1, \alpha_S\right) \cdot E\left(N, 1, \alpha_S\right) \cdot D\left(N, 1, \alpha_S\right) \cdot \exp\left[\int_{m^2}^{Q^2} \gamma(\alpha_S(\mu)) \frac{d\mu}{\mu}\right]$$

- If γ is the LO splitting function then we have $\gamma=\gamma_0\alpha_S$ and the exp term reads:

Recall: $L = \ln\left(\frac{m^2}{Q^2}\right)$ Therefore: $e^{\gamma_0 \alpha_S L} = \left(\frac{m^2}{Q^2}\right)^{\gamma_0 \alpha_S}$

Well-behaving function!

- We now can expand the exponent and will get: 1 + const α_S L + const $(\alpha_S L)^2$ + … which is indeed the LL tower we wanted to resum!

Resummation: soft logs

- The soft logs are trickier.

  - We get up to two powers of logs per power of α_S.
  - Soft logs are more entangled. They “exponentiate” not just as simple numbers (like the collinear logs) but rather as color matrices.

>> Exponentiation is synonymous to resummation <<

  - For soft logs the small parameter “s” is not a fixed parameter (like the mass was) but is a kinematical variable!

  - Therefore, s can be small in some kinematical configurations but large in others!

  - To understand soft logs one first have to identify the kinematical configurations where they can emerge. How do we do that?

  - This is done on a case by case basis, separately for each process and observable.

  - These are configurations that are close to the edge of phase space. In such configurations the emission of hard radiation is impossible since in this kinematical corner there is simply no energy available for radiating anything but a soft gauge boson.

Resummation: soft logs

- Examples
  - Drell-Yan $z = Q^2/s$ (s-partonic c.m. energy; Q mass of the lepton pair)
  - Inclusive Higgs production $z = m_H^2/s$
  - Inclusive top pair production $z = 4m^2/s$, m- mass of top quark
  - Top quark pair invariant mass $z = M_{t\bar{t}}^2/s$
- In all cases the "soft limit" is when z->1. The large logs are $\mathrm{Log}^n[1-z]/(1-z)$. In Mellin space z<->N this corresponds to $\mathrm{Log}^n(N)$ (for N->∞).
- What this implies is that in all cases, when z->1, all the energy available to the system is barely enough to produce the required final state, and very little energy left for extra radiation.
- The total energy available for radiation is $E_{rad}=(1-z)\sqrt{s} \to 0$.

Resummation: how to resum soft logs?

Extra material

- The way to describe soft gluon radiation (i.e. the system in the "soft limit") is to work in the eikonal approximation.

- In this approximation we take the so called Born configuration and treat it semi-classically.

- We think the basic Born-level particles follow their classical trajectories as if no radiation takes place, and on top of this we add radiation which does not exert any back-reaction on the hard emitting particles (called eikonals).

- In effect we need to compute the S-matrix in the eikonal approximation. This is much easier.

- Think about it this way: the theory is rather complicated in general. But once we approach a singular limit then everything simplifies. Why? Because close to a singularity the whole dynamics is dominated by the leading power in the singular variable and everything else is strongly suppressed.

- In other words, close to the z->1 limit (or N->∞) the cross-section is dominated by its most singular term and it naturally can be furthered factorized. An example follows:

Resummation: how to resum soft logs?

Extra material

- Resummation in top pair production (the most complex case you may consider)

The fully differential cross-section:

$$M^4 \frac{d\sigma_{h_1 h_2 \to Q\bar{Q}}}{dM^2 dy d\hat{\eta}} = \sum_f \int_\tau^1 dz \int \frac{dx_a}{x_a}\frac{dx_b}{x_b}\, \phi_{f/h_1}(x_a,\mu^2)\,\phi_{\bar{f}/h_2}(x_b,\mu^2)$$
$$\times\, \delta\left(z - \frac{\tau}{x_a x_b}\right)\delta\left(y - \frac{1}{2}\ln\frac{x_a}{x_b}\right)$$
$$\times\, \omega_{f\bar{f}\to Q\bar{Q}}\left(z, \hat{\eta}, \frac{M^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s(\mu^2)\right),$$

- Define $z = \frac{\tau}{x_a x_b} = \frac{M^2}{x_a x_b S}$ and go to Mellin space:

$$\sigma(N) = \int_0^1 dz z^{N-1}\sigma(z)$$
$$= \int_0^1 dz e^{-(N-1)(1-z)}\sigma(z) + \mathcal{O}(1/N)$$

- The cross-section factorizes as:

$$\omega_P\left(N, \hat{\eta}, \frac{M^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s(\mu^2)\right) = J_1(N, \alpha_s(\mu^2))\ldots J_k(N, M/\mu, m/\mu, \alpha_s(\mu^2))$$
$$\times\, \mathrm{Tr}\left[\mathbf{H}^P\left(\frac{M^2}{\mu^2}, \frac{m^2}{\mu^2}, \hat{\eta}, \alpha_s(\mu^2)\right)\mathbf{S}^P\left(\frac{N^2\mu^2}{M^2}, \frac{M^2}{m^2}, \hat{\eta}, \alpha_s(\mu^2)\right)\right] + \mathcal{O}(1/N)$$

- The most important ingredient is the so-called "soft function" which we derive by computing the S-matrix in the eikonal approximation.

$$\mathbf{S}\left(\frac{N^2\mu^2}{M^2}, \beta_i\cdot\beta_j, \alpha_s(\mu^2)\right)\Bigg|_{\mu=M} = \overline{\mathcal{P}}\exp\left\{-\int_{M/\bar{N}}^{M}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_S^\dagger\left(\beta_i\cdot\beta_j, \alpha_s\left(\mu'^2\right)\right)\right\}$$
$$\times\mathbf{S}\left(1, \beta_i\cdot\beta_j, \alpha_s\left(M^2/\bar{N}^2\right)\right)$$
$$\times\mathcal{P}\exp\left\{-\int_{M/\bar{N}}^{M}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_S\left(\beta_i\cdot\beta_j, \alpha_s\left(\mu'^2\right)\right)\right\}$$

- Do you see the exponents? They produce the LL, NLL, etc

Resummation: the Sudakov formfactor

Extra material

- Jet functions read

$$\ln J^P_{\rm in}(N,Q) = \frac{1}{2}\int_0^1 dx \frac{x^{N-1}-1}{1-x}\left\{\int_{\mu_F^2}^{(1-x)^2Q^2}\frac{dq^2}{q^2}\, 2\, A_P\left(\alpha_s\left[q^2\right]\right) + D_P\left(\alpha_s\left[(1-x)^2Q^2\right]\right)\right\}.$$

Real radiation

Virtual corrections (i.e. no-emission probability)

- Jet functions are process independent and can be extracted, for example, from Drell-Yan and/or Higgs production and hadron colliders.

- The jet function describes the independent evolution of an initial hard, well-separated, parton (quark or gluon).

- It contains both real emissions and pure virtual corrections.

- Note the virtual piece. It can be interpreted as no-emission probability. It is a fundamental ingredient in parton showers and many higher-order calculations!
- In reality, the R and V pieces are separately divergent (when z→1, i.e. soft emissions). The proper definition of no-emission probability requires virtual + unresolved soft emissions, which is finite. We will return to it later.

Resummation: final comments

- What is the moral of the story? There is hardly any other subject in our field which stirs more emotions and controversies than the application of soft-gluon resummation.

- Everyone agrees that in principle it is needed. But there is no clear guidance when it is useful.
- In particular, since it is orders of magnitude easier to do an NNLL resummation that a full NNLO calculation, often we are tempted to expand the NNLL exponent up to the NNLO terms (in α_S). Such expansion produces some of the terms that would be found in a full NNLO calculation. This way we have <u>some</u> approximation to the full NNLO.
- This approach is now simply known as *approximate NNLO*.

- Is this approach useful?
- My personal impression is that it *rarely* is.
    1. The formulation of the approximation is by itself ambiguous,
    2. In a number of important cases this approximation fails to be close to the full result.

- In general this approximation may only be expected to be good if the soft terms are dominant in the full NNLO. This is rarely, if ever, the case at hadron colliders.

- And a word about quasi-collinear logs Log[m/Q]: LEP studies of b-production have shown the resummation of these logs is important at Z-pole energies, i.e. Log[m_b/m_Z] is large enough and needs to be resummed.

Soft gluon Resummation: examples

- The best example I know of that shows how important soft-gluon resummation is the reduction in the size of the relative theory error of the tt total cross-section as a function of the collider energy
- As the collider energy gets smaller there is less and less energy for radiation
- In this region the difference between fixed order and resummed calculations becomes apparent.

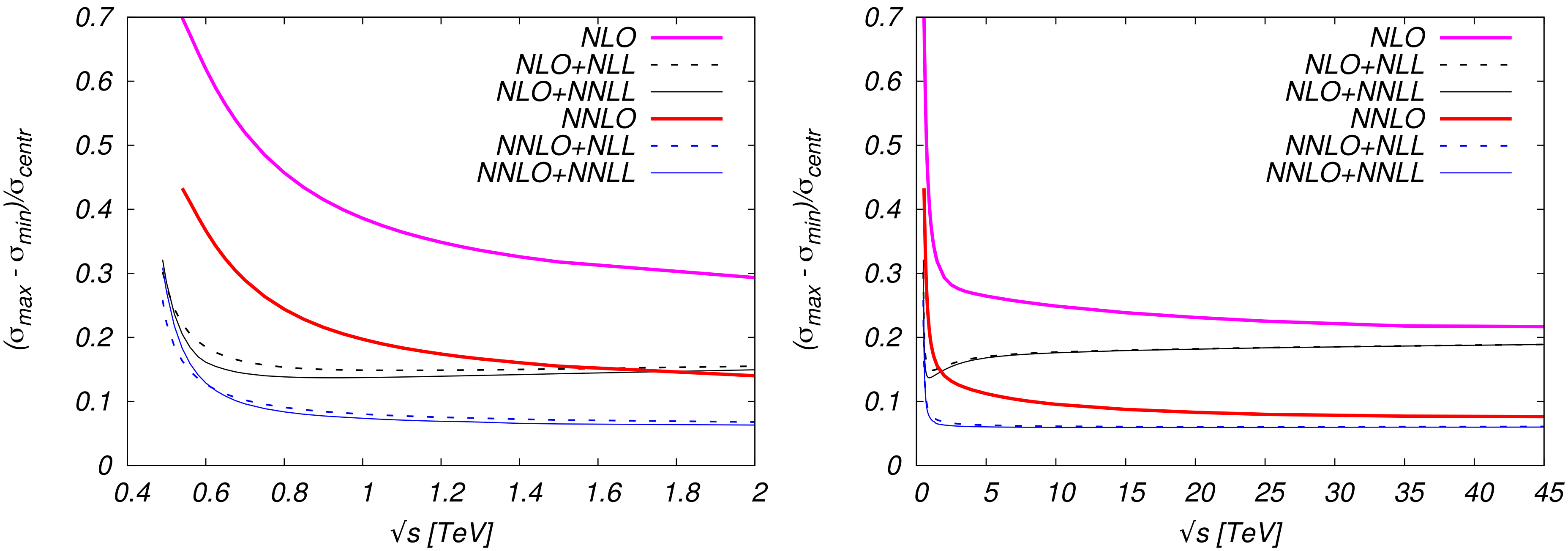


Fig. 3. – The relative scale uncertainty of the $t\bar{t}$ cross-section, computed as a function of the LHC collider energy at fixed order (NLO and NNLO) and including with soft-gluon resummation (NLL and NNLL).

See http://arxiv.org/pdf/1305.3892.pdf for details

Soft gluon Resummation: examples

- Thrust distribution in e^+e^- -> 3 jets at NNLO:

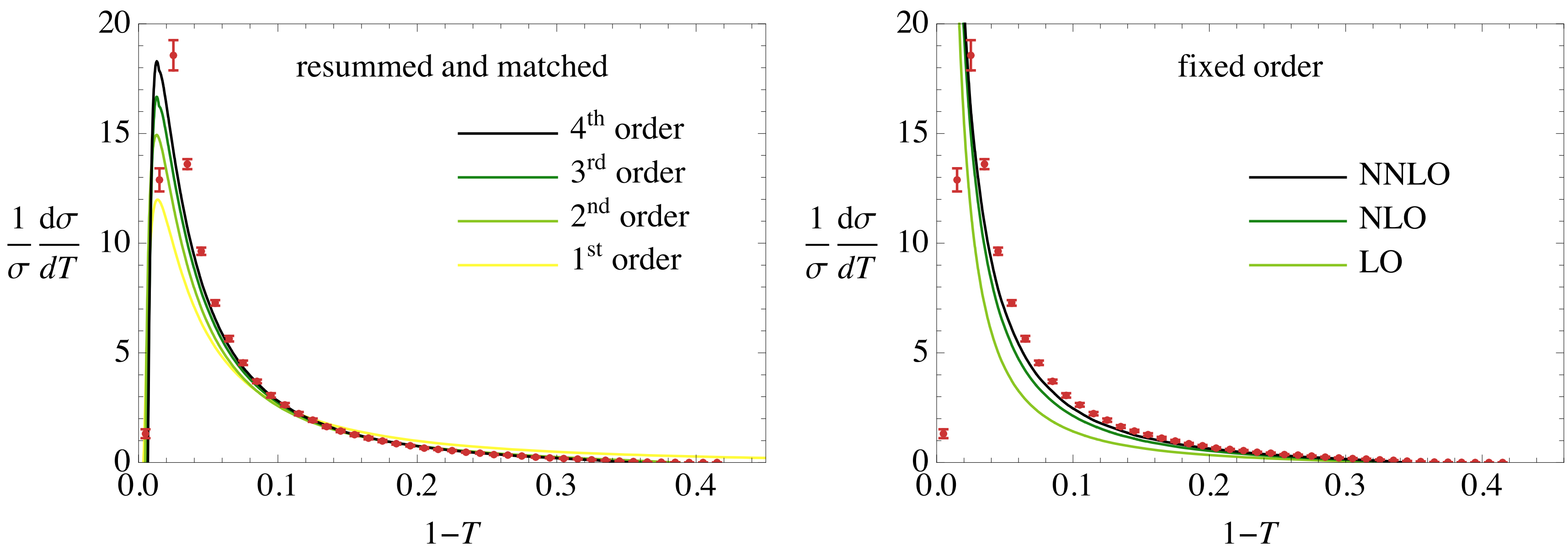
See http://arxiv.org/pdf/0803.0342.pdf

$$T = \max_{\mathbf{n}} \frac{\sum_i |\mathbf{p}_i \cdot \mathbf{n}|}{\sum_i |\mathbf{p}_i|}, \tag{1}$$

where the sum is over all momentum 3-vectors $\mathbf{p}_i$ in the event, and the maximum is over all unit 3-vectors $\mathbf{n}$. In the endpoint region, $T \to 1$ or $\tau = (1-T) \to 0$, no fixed-order calculation could possibly describe the full distribution due to the appearance of large logarithms. For example, at leading order in perturbation theory the thrust distribution has the form

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \delta(\tau) + \frac{2\alpha_s}{3\pi}\left[\frac{-4\ln\tau - 3}{\tau} + \dots\right], \tag{2}$$

$$R(\tau) = \int_0^\tau \mathrm{d}\tau' \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau'} = 1 + \frac{2\alpha_s}{3\pi}\left[-2\ln^2\tau - 3\ln\tau + \dots\right]$$



Final states at hadron colliders (particles and jets)

Final states at hadron colliders (particles and jets)

- The particles observed at colliders are:

  - Strongly interacting ones (mesons, baryons)
  - Weakly interacting ones
    - The gauge bosons (γ,Z,W)
    - Higgs (which decays of course)
    - Leptons

- The problem is that there are many of them

- Even at high PT the number of identifiable tracks is in the dozens and hundreds.

- It would be very very, hard to describe such large multiplicities down to the individual particle.

- Luckily, they tend to clump together along the direction of some hard parton (which we do not see directly) that initiated them.

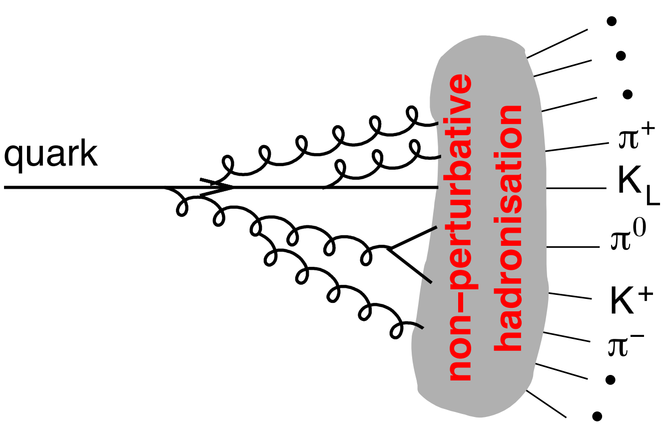
- Such clumps of particles are called jets.

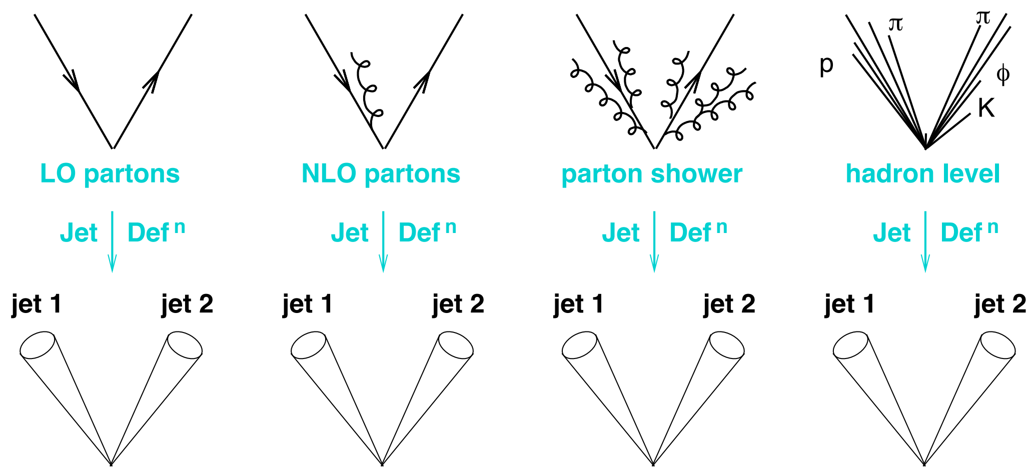
Jets at hadron colliders: an alternative way of thinking about hadron production

- ✓ Jets are not "Physical" objects: they are merely clusters of hadrons
- ✓ No two jets are the same!
- ✓ But Jets are natural at hadron colliders:
  - ✓ Ex: describing the water molecules in Jet d'eau is hard
  - ✓ The water jet itself depends on the dynamics among the constituent
  - ✓ It is the natural thing to study when the detector is close to the water source.
- ✓ Have been measured at colliders since the late 1970-ies.

Photo credits [http://en.wikipedia.org/wiki/Jet_d'Eau]

The process of jet formation:





More details: Gavin Salam, TASI lectures on Jets (2013)

Jets and infrared safety

- Jets are defined through some algorithm (or jet function) which tells us how the measured particles are grouped into jets.
- Many such definitions exist. A very popular one nowadays is the anti-KT algorithm of Cacciari, Salam and Soyez '08.
- If you are interested in this subject, you want to read this paper http://arxiv.org/pdf/0802.1189v2.pdf

Infrared safety means that we work with observables that are not singular when soft/collinear emissions are made (Note: fragmentation functions are the opposite extreme since they *are* collinearly unsafe; there we collect all collinear singularities and put them into the hadron).

$$\frac{d\sigma}{dO} = \sum_n \int |M^{(n)}|^2 \delta\left(O - F_O^{(n)}(p_1, \ldots, p_n)\right) d\Phi^{(n)}$$

The "Jet" observation function has to have the following property:

$$F_O^{(n+1)}(p_1, \ldots, p_n, p_{n+1}) = F_O^{(n)}(p_1, \ldots, p_n), \quad \text{if} \quad p_{n+1} \to 0$$
$$F_O^{(n+1)}(p_1, \ldots, p_n, p_{n+1}) = F_O^{(n)}(p_1, \ldots, p_n + p_{n+1}), \quad \text{if} \quad p_n || p_{n+1}$$

Jets: definition

- In a realistic LHC event one has to cluster around 10^3-10^4 particles.
- Few particles are hard, most are soft (soft: i.e. momentum->0)
- For this we need:
  - A jet algorithm (IR safe one!) that will cluster all these partons into jets
  - Speed (i.e. fast algorithm)

- Some popular jet algorithms:
  - K_T
  - Cambridge-Aachen
  - Anti-K_T

- How does the clustering work?

- Define a distance function d_{ij} between any two particles or proto-jets (proto-jet: a collection of particles that may not yet be a jet), as well as a distance d_{iB} between each particle and the beam

K_T algorithm:

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\, \Delta R_{ij}^2 / R^2$$
$$d_{iB} = p_{ti}^2$$

Cambridge/Aachen

$$d_{ij} = \Delta R_{ij}^2 / R^2$$
$$d_{iB} = 1\,.$$

anti-K_T algorithm:

$$d_{ij} = \min(1/p_{ti}^2, 1/p_{tj}^2)\, \Delta R_{ij}^2 / R^2$$
$$d_{iB} = 1/p_{ti}^2\,.$$

And: $\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$ R: a parameter (cone size); R=0.4-1

Jets: definition

- The distance ΔR_{ij} is boost invariant
- The jet size R:
  - If R – large, then the concept of jet looses its meaning because it becomes equivalent to the total cross-section.
  - If R-very small, we have a problem: the jet is formally IR finite the the space where IR cancellation takes place becomes very tight and the IR cancellation becomes imperfect. As a result, in the limit R->0 we get terms like Log[R].
  - Indeed, in the limit R->0 we must run into trouble since we conceptually go to the fragmentation function case we described previously (which was IR unsafe).

- Implementation:
  - Construct the set of all measured momenta p_i
  - Compute all distances d_{iB} and d_{ij} (defined on previous page)
  - If d_{iB} is the smallest on the list: call "i" a jet and remove from the list
  - If d_{ij} is the smallest: then add p_i and p_j and replace them in the list with their sum.
  - Continue until all particle (proto-jets) momenta in the list are exhausted.
  - The resulting objects are our jets.
  - At the end, only jets above certain p_T cut are used in the final analysis.

- Speed: for N particles, the number of comparisons is $O(N^3)$. This is large.
- A library called ***FastJet*** exists, which reduces the time to N log[N]. Moreover it provides common implementation and interface to many jet algorithms.

The shapes of Jets; addition of soft radiation

- Some jets are better at handling additional soft radiation (i.e. are less sensitive to it)

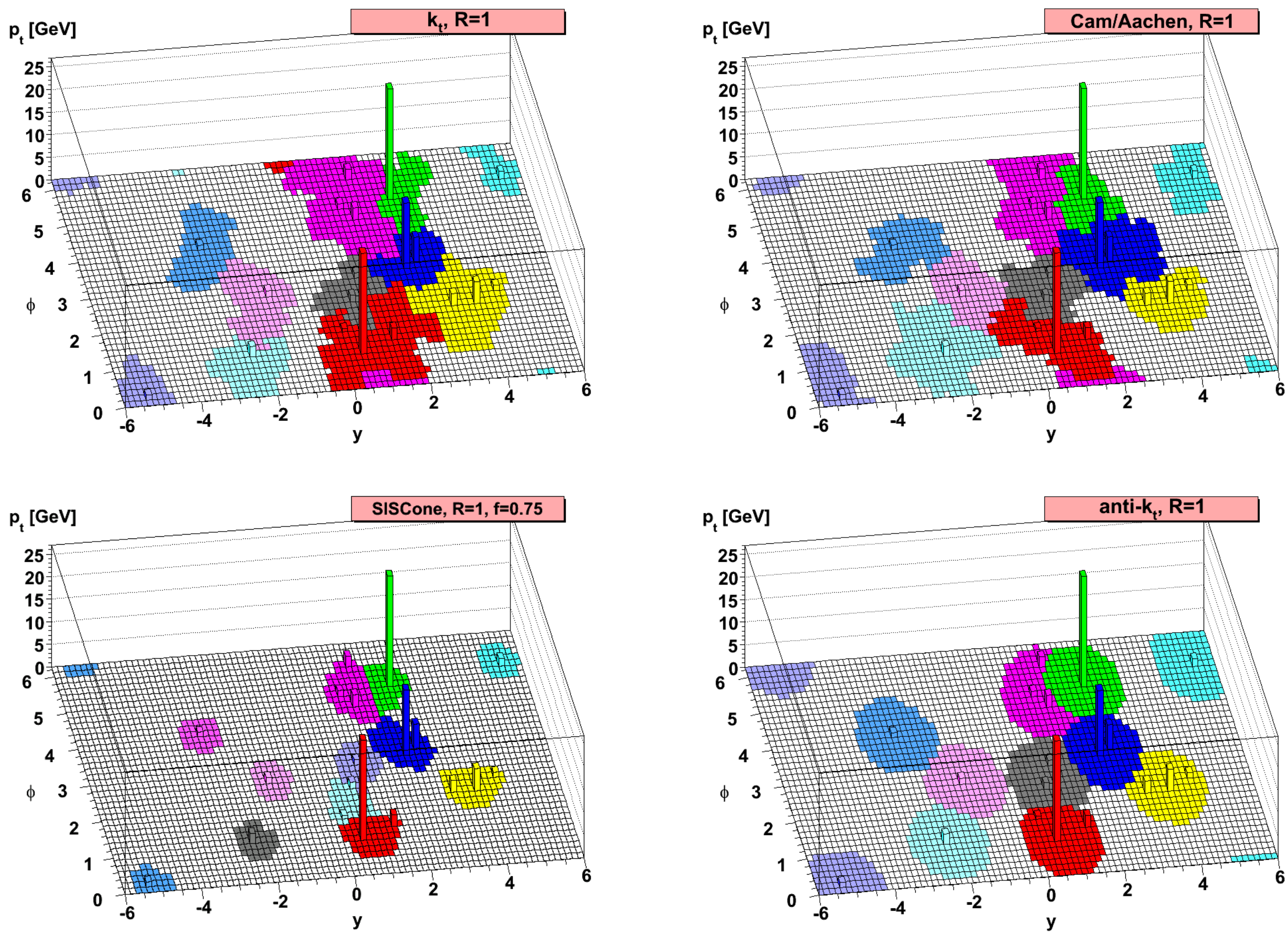


Figure 1: A sample parton-level event (generated with Herwig [8]), together with many random soft "ghosts", clustered with four different jets algorithms, illustrating the "active" catchment areas of the resulting hard jets. For k_t and Cam/Aachen the detailed shapes are in part determined by the specific set of ghosts used, and change when the ghosts are modified.

Latest developments in Jets

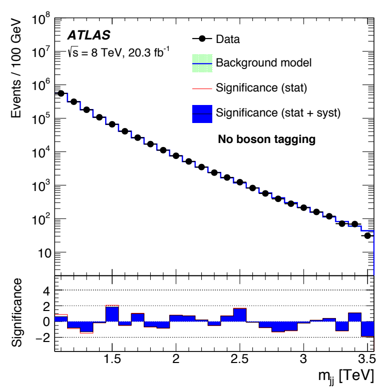
For more info see the proceedings of the annual BOOST conference http://boost2015.uchicago.edu

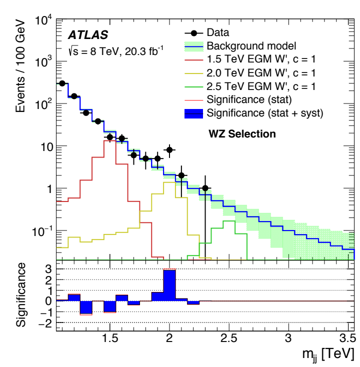
- Boosted objects as jets.
  - Imagine W decaying to jets (W→ qq). In cases the W itself is very energetic its decay products will appear as a single jet. Same for decaying boosted tops.
  - The real motivation for considering such cases is searches for new physics:
    - Imagine a heavy resonance decaying to pair of tops (typical bSM possibility)
    - Each one of the tops will be highly boosted
    - The top decay products will be collimated.
- Jet substructure
  - A way of distinguishing normal QCD jets (they are not supposed to have any characteristic internal structure) from highly boosted decays is to try to identify the presence of sub-jets in a highly energetic and massive jets.
  - Explosion of interest and literature on this topic in the last few years. Many techniques developed: *N-subjettiness, etc; Jet trimming, Jet filtering, Jet pruning*.

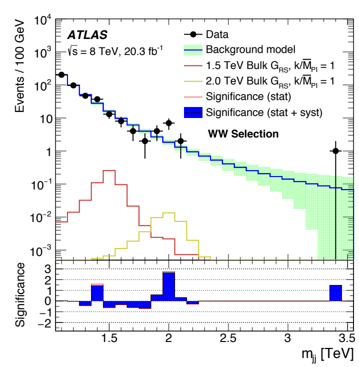
For more info see http://arxiv.org/abs/1307.0007 , http://arxiv.org/abs/1311.2708

- Fat jets and recent diboson 8TeV excess (ATLAS, CMS) (a search which is optimized towards heavy objects decaying to gauge bosons) See http://arxiv.org/abs/1506.00962

Backgrounds seem OK





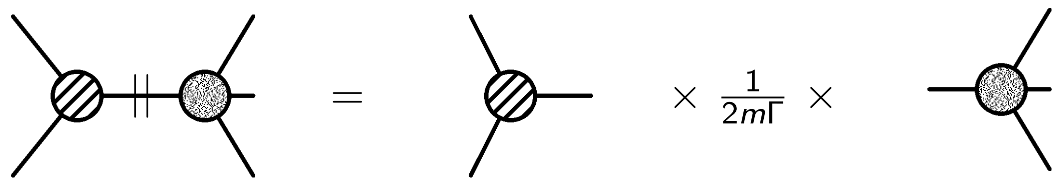


Local excess after additional W,Z selection

Not seen in 13 TeV data!

Decay of unstable particles: narrow width approximation

For $\Gamma \ll m$



The propagator of an unstable particle (Breit-Wigner resonance) of momentum q, mass m and width Γ is:

$$P(q, m, \Gamma) = \frac{1}{(q^2 - m^2)^2 + m^2\Gamma^2}$$

(this is just the modulus square of the usual propagator for a particle of width Γ)

In the NWA we take the formal limit:

$$\frac{1}{(q^2 - m^2)^2 + m^2\Gamma^2} \longrightarrow \frac{\pi}{m\Gamma}\delta(q^2 - m^2)$$

i.e. the decaying particle is placed on-shell. This way the Phase space for the complete n-body process factories into the product of the phase space of all particles but the decay products, times the decay of the unstable particle.

The NWA leads to drastic simplification:

$$\sigma = \sigma_{\text{prod}} \times BR\,,$$
$$BR = \frac{\Gamma_{\text{partial}}}{\Gamma_{\text{tot}}}\,.$$

where Γ_{tot} is the same as Γ above. If there is only one decay mode then $BR = 1$.

- The error is O(Γ/m). Works well in SM (top, W,Z). Application more subtle in bSM context. See: http://arxiv.org/abs/0807.4112 for further details.

Monte Carlo integration methods

Monte Carlo integration methods

- So far we discussed only analytical integration in our discussion of cross-sections.
- This is, of course fine, but as it turns out it is very restrictive given the realities of experimental analyses at colliders.

So, what's the problem?

- The problem is that analytical integration is, by its very nature, inclusive.

- Let's look at an example:
  - f(x) is some probability density (we imagine it corresponds to some differential distribution)
  - Within the analytical integration approach a question we can ask is: what is the value:

$$F(a,b) = \int_a^b f(x)dx$$

  - In effect this is a bin.
  - Clearly this is well defined only if a=/= b
  - Therefore we cannot ask, or predict, what will be the measured value of F in a single point F(a,a). In other words, within this approach, we cannot predict single events.

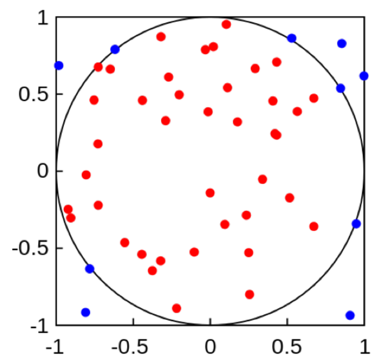
- Yet single events happen all the time at colliders.

- MC comes to the rescue!

- Monte Carlo techniques are super useful for two very important reasons:

  - Formally, they are an integration technique, i.e. we can use them to do integration numerically. Compare, for example with Gaussian integration.

Example: calculating the value of π



- Generate N points randomly over the (x,y) square
- Count the points inside the circle (n)
- Derive: $\pi/4 \approx n/N$
- A simple counting experiment. Error: $\sim \dfrac{1}{\sqrt{N}}$

    - MC integration has smaller numerical precision in 1D,
    - In higher dimensions MC has no competition.

  - The way the integration is done is by summing up discrete points in the continuous variable(s) being integrated. We interpret such discrete points as collider events.

> Therefore, Monte Carlo integration offers the possibility to compute formally continuous distributions by summing up individual, discrete events, while being proper integration technique at the same time!

- A word of caution: although the interpretation of such MC events as the real-life collider events is absolutely tantalizing, one should be careful: this is only an integration technique which carries the inherent uncertainties of the underlying theoretical approximation.

Monte Carlo integration methods

- Note about probability density interpretation:

Going beyond LO many of the contributions are not positive anymore.

- This is OK, since:

  - The observable is positive definite
  - It is a sum over partonic reactions, each of which is unphysical (their separation is scheme dependent)
  - The LO usually are positive but higher orders can individually be negative.

Fixed order calculations with MC techniques: LO

- A fully differential observable at LO is defined as:

$$\frac{d\sigma}{dO} = \sum_{ij=g,u,\bar{u},\ldots} \int \frac{f_i(x_1) f_j(x_2)}{2s} \left| M_{ij}(p_1, p_2 \to q_1, \ldots, q_n) \right|^2 \delta\Big(O - F_O(q_1, \ldots, q_n)\Big) \underbrace{d\Phi(q_1, \ldots, q_n) dx_1 dx_2}_{\text{3(n-1)+2 variables; (some redundant)}}$$

  - $p_1 = x_1 P_1 \quad ; \quad p_2 = x_2 P_2$
  - $d\Phi(q_1, \ldots, q_n) = (2\pi)^4 \delta^4(p_1 + p_2 - q_1 \cdots - q_n) \frac{d^3 q_1}{(2\pi)^3 2E_1} \cdots \frac{d^3 q_{n-1}}{(2\pi)^3 2E_{n-1}}$
  - O labels some observable (say P_T) and $F_O(\ldots)$ is its analytic representation through the final state momenta

- To integrate analytically, we first separate the pdf's and the integrations over $x_{1,2}$
- Then perform analytically the integrations over the independent 3-momenta keeping $x_{1,2}$ fixed

$$\frac{d\sigma}{dO} = \sum_{ij=g,u,\bar{u},\ldots} \int \frac{f_i(x_1) f_j(x_2)}{2s} \underbrace{\left| M_{ij}(p_1, p_2 \to q_1, \ldots, q_n) \right|^2 \delta\Big(O - F_O(q_1, \ldots, q_n)\Big) d\Phi(q_1, \ldots, q_n)}_{\text{partonic cross-section}} dx_1 dx_2$$

- Finally perform the remaining $x_{1,2}$ integrations numerically.
- If the partonic x-section is simple enough we can get analytic expression and therefore compute the O-dependence as a smooth curve; no bins needed.

Fixed order calculations with MC techniques: LO

$$\frac{d\sigma}{dO} = \sum_{ij=g,u,\bar{u},\ldots} \int \frac{f_i(x_1)f_j(x_2)}{2s} \Big| M_{ij}(p_1,p_2 \to q_1,\ldots,q_n) \Big|^2 \delta\Big(O - F_O(q_1,\ldots,q_n)\Big) d\Phi(q_1,\ldots,q_n) dx_1 dx_2$$

- Performing such integrations analytically is often not practical. In complicated cases it is hardly possible.

- MC integration is much simpler! Consider the above integral as a simultaneous integral over all final and initial states variables (i.e. consider $x_{1,2}$ on equal footing with the final state ones)

- Parameterize the final state momenta through independent variables $z_1 \ldots z_{3(n-1)}$ such that they take values on the unit hypercube $0 \le z_i \le 1$:

$$d\Phi(q_1,\ldots,q_n) = \frac{d^3q_1}{(2\pi)^3 2E_1} \cdots \frac{d^3q_{n-1}}{(2\pi)^3 2E_{n-1}} = J(z_1,\ldots,z_{3(n-1)}) dz_1 \ldots dz_{3(n-1)}$$

- Note: the z_i's are simply normalized energies and cos(angles)
- The matrix element depends on scalar products: $(p_{1,2}.q_i)$ and $(q_i.q_j)$. Rewrite them through z_i. The x-section now reads:

$$\frac{d\sigma}{dO} = \int w(x_1,x_2,z_1,\ldots,z_{3(n-1)}) \delta\Big(O - F_O(z_1,\ldots,z_{3(n-1)})\Big) dx_1 dx_2 dz_1 \ldots dz_{3(n-1)}$$

- Here is how we actually implement the MC integration:

Fixed order calculations with MC techniques: LO

$$\frac{d\sigma}{dO} = \int w(x_1, x_2, z_1, \ldots, z_{3(n-1)}) \delta\Big(O - F_O(z_1, \ldots, z_{3(n-1)})\Big) dx_1 dx_2 dz_1 \ldots dz_{3(n-1)}$$

- Attempting the above integral with a MC, as it is, is a bad idea:
  - Fix the value of O
  - Take a random point $(x_{1,2}, z_i)$
  - F_O at this point will not be equal to the chosen O

- For MC integration we need to bin "events". Therefore we need to replace the delta-function with a binning function:

$$d\sigma_O = \int w(x_1, x_2, z_1, \ldots, z_{3(n-1)}) B\Big(F_O(z_1, \ldots, z_{3(n-1)})\Big) dx_1 dx_2 dz_1 \ldots dz_{3(n-1)}$$

- The binning function is a set of theta functions; basically it takes values 1 or 0.

- Binning could be done simultaneously in several variables.
- Or even fully exclusively:
  1. Decide binning for each variable of interest
  2. Generate a point $(x_{1,2}, z_i)$. We call it "event". Beware it is not exactly a physical event!
  3. Compute the value of B(….) at this point. It is non-zero for only one bin.
  4. Compute the value of the weight w(…) at this point. Add it to the bin determined in 3.
  5. Continue the process until sufficiently large number of "events" generated in each bin.
  6. Divide by the number of "events" in each bin (i.e. obtain the average w in each bin)

Fixed order calculations with MC techniques: LO

$$d\sigma_O = \int w(x_1, x_2, z_1, \ldots, z_{3(n-1)}) B\Big(F_O(z_1, \ldots, z_{3(n-1)})\Big) dx_1 dx_2 dz_1 \ldots dz_{3(n-1)}$$

- Binned distributions are very easy to manipulate.
- One could even compute the "events" without regard of any binning!
  - Generate events
  - Save the event information: ($x_{1,2}$,z_i) and weight w(…) for each event.
  - At a later point analyze and bin the events.

- Such approach allows unprecedented flexibility

- Without having to re-compute the matrix elements |M(…)|^2, one could *a posteriori*, after events are computed, change:

  - Value of the renormalization scale (recall |M()|^2 ~ $\alpha_S^k(\mu_R)$; so divide by this an multiply by $\alpha_S^k(\mu_R')$ evaluated a some different scale μ_R' =/= μ_R.
  - Value of factorization scale*
  - The pdf set*

- In practice computing at LO is fast enough; the above approach is very handy at NLO, and beyond.

* To do this, one has to save separately the contributions from all contributing partonic reactions, not just their sum, as implied by the above equation!

Fixed order calculations with MC techniques: NLO

- NLO x-sections can be computed following the LO MC methods described above.
- However, there are dramatic complications that arise at NLO, and which we describe next.

- At NLO we need to sum over all cuts (i.e. all different partonic final states) that contribute to the observable at hand.

- We expect this from our previous discussions of IR safety.

- Dijets as an example:

  1. If we want to have exactly 2 jets
     - then extra radiation has to be only Unresolved
  2. If we want at least 2 jets (i.e. could be 2,3 or more)
     - Then extra radiation could be anything (i.e. Unresolved or Resolved)

- Putting it all together we get:

Fixed order calculations with MC techniques: NLO

- The so-called "Virtual" 2→n contribution:

$$\frac{d\sigma(2\to n;\varepsilon)}{dO}=\sum_{ij}\int\frac{f_i(x_1)f_j(x_2)}{2s}\left|M_{ij}^{(1Loop)}(2\to n;\varepsilon)\right|^2\delta\Big(O-F_O(1,\ldots,n)\Big)d\Phi(q_1,\ldots,q_n)dx_1dx_2$$

  - The 1 Loop contribution is divergent; it contains explicit poles in epsilon.

$$V\equiv\left|M_{ij}(2\to n)\right|^2=\left|M_{ij}^{(Born)}(2\to n)\right|^2+2\mathrm{Re}\left(M_{ij}^{(Born)}(2\to n)\times M_{ij}^{(1Loop)}(2\to n;\varepsilon)\right)+\text{NNLO terms}$$

  - Phase space in d-dimensions. Note the phase space integration of this piece is regular but normally has to be performed in d-dim since terms ~ eps can multiply poles from V which results in finite contributions to the x-section.

- The so-called "Real" 2→n+1 contribution:

$$\frac{d\sigma(2\to n+1;\varepsilon)}{dO}=\sum_{ij}\int\frac{f_i(x_1)f_j(x_2)}{2s}\left|M_{ij}^{(Born)}(2\to n+1)\right|^2\delta\Big(O-F_O(1,\ldots,n+1)\Big)d\Phi(q_1,\ldots,q_{n+1})dx_1dx_2$$

  - The amplitude is the Born one, so no poles in eps (still sub-leading terms in eps might have to be retained)

  - The phase space is done in d-dimensions:
    - The integrand is finite
    - Upon integration over phase space eps poles are generated

Fixed order calculations with MC techniques: NLO

Extra material

- There is another source of divergences: collinear divergences.
- They are simpler (of complexity of one-loop less, i.e. Born, but still have to be accounted for)
- Here is what happens, schematically, through NNLO:
  - After Real and Virtual corrections are added together the x-section is still divergent. Through NNLO we have (ρ stands for the relevant kinematic variable):

$$\tilde{\sigma}(\varepsilon,\rho) = \tilde{\sigma}^{(0)}(\varepsilon,\rho) + \alpha_S \tilde{\sigma}^{(1)}(\varepsilon,\rho) + \alpha_S^2 \tilde{\sigma}^{(2)}(\varepsilon,\rho) + \dots$$

  - Subtract collinear singularities for hadron colliders (i.e. factor them into the hadrons) as:

Divergent

$$\frac{\tilde{\sigma}_{ij}(\epsilon,\rho)}{\rho} = \sum_{k,l} \left[\frac{\hat{\sigma}_{kl}(x)}{x} \otimes \Gamma_{ki} \otimes \Gamma_{lj} \right](\rho)$$

Finite

  - From the above we derive the finite x-section
  - The process-independent collinear counter-terms are:

$$\begin{aligned}
\Gamma_{ij}(\epsilon,x) &= \delta_{ij}\delta(1-x) + \alpha_S \Gamma_{ij}^{(1)}(\epsilon,x) + \alpha_S^2 \Gamma_{ij}^{(2)}(\epsilon,x)\,, \\
\Gamma_{ij}^{(1)}(\epsilon,x) &= -\frac{1}{2\pi}\,\frac{P_{ij}^{(0)}(x)}{\epsilon}\,, \\
\Gamma_{ij}^{(2)}(\epsilon,x) &= \left(\frac{1}{2\pi}\right)^2 \left\{ \frac{1}{2\epsilon^2}\left[P_{ik}^{(0)} \otimes P_{kj}^{(0)}(x) + \beta_0 P_{ij}^{(0)}(x)\right] - \frac{1}{2\epsilon} P_{ij}^{(1)}(x) \right\}
\end{aligned}$$

Fixed order calculations with MC techniques: NLO

- Adding Real and Virtual corrections is, unfortunately, highly non-trivial.

$$\frac{d\sigma(2\to n;\varepsilon)}{dO} = \sum_{ij}\int \frac{f_i(x_1)f_j(x_2)}{2s}\left|M_{ij}^{(1Loop)}(2\to n;\varepsilon)\right|^2 \delta\Big(O - F_O(1,\dots,n)\Big)\, d\Phi(q_1,\dots,q_n)dx_1dx_2$$

$$\frac{d\sigma(2\to n+1;\varepsilon)}{dO} = \sum_{ij}\int \frac{f_i(x_1)f_j(x_2)}{2s}\left|M_{ij}^{(Born)}(2\to n+1)\right|^2 \delta\Big(O - F_O(1,\dots,n+1)\Big)\, d\Phi(q_1,\dots,q_{n+1})dx_1dx_2$$

- Here are the problems:
  - The integration over the Real phase-space has to be done in d-dim.
  - This generates explicit poles and we have to control them analytically
  - This is against the spirit of MC integrations where everything is done numerically.

  - The next complication:
    - We want to have fully differential calculations
    - This means Real and Virtual poles must cancel in every point (local cancellation)
    - And for any measurement function (recall that the measurement functions for the Real and Virtual corrections are different).
    - Therefore we have to ensure that poles cancel even before the observation functions have been specified!
  - It turns out it is sufficient to know their limiting behavior in soft/collinear limit (recall our discussion of jets)

$$F_O^{(n+1)}(p_1,\dots,p_n,p_{n+1}) = F_O^{(n)}(p_1,\dots,p_n), \quad \text{if} \quad p_{n+1}\to 0$$

$$F_O^{(n+1)}(p_1,\dots,p_n,p_{n+1}) = F_O^{(n)}(p_1,\dots,p_n+p_{n+1}), \quad \text{if} \quad p_n || p_{n+1}$$

Fixed order calculations with MC techniques: NLO

- Two methods are available on the market.
  - Slicing method (older; not exact; being developed even to NNLO)
  - Subtraction method (newer; exact; has been extended to NNLO)

- To get a feeling for how the methods work we will simplify them;
  - Ignore the presence of collinear singularities (as the previous slide)
  - Hide the presence of pdf's etc.
  - will consider only one kinematical variable (called x); think of it as the energy of the additional emitted gluon

$$d\sigma_O = \int_0^1 \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \left(\frac{V^{\mathrm{pole}}}{\varepsilon} + V^{\mathrm{fin}}\right) F_O(n)$$

  - Recall that

$$F_O(n+1;x=0) = F_O(n) \quad ; \quad R(0) = V^{\mathrm{pole}}$$

  - … and the function R(x) is finite for x=0 (follows from the factorization of amplitudes)

- In the phase-space slicing method we split the integration over x in two:

$$\begin{aligned} d\sigma_O &= \int_0^1 \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \left(\frac{V^{\mathrm{pole}}}{\varepsilon} + V^{\mathrm{fin}}\right) F_O(n) \\ &= \int_0^\delta \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \int_\delta^1 \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \left(\frac{V^{\mathrm{pole}}}{\varepsilon} + V^{\mathrm{fin}}\right) F_O(n) \end{aligned}$$

- Take δ very small, δ<<1. In the first term we can approximate R(x)≈R(0)
- Set eps=0 in the second term (integration is now finite). Integrate it numerically

Fixed order calculations with MC techniques: NLO

- The slicing method:

$$
\begin{aligned}
d\sigma_O &= \int_0^1 \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \left(\frac{V^{\rm pole}}{\varepsilon} + V^{\rm fin}\right) F_O(n) \\
&= \int_0^\delta \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \int_\delta^1 \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \left(\frac{V^{\rm pole}}{\varepsilon} + V^{\rm fin}\right) F_O(n) \\
&\approx \left(-\frac{1}{\varepsilon} + \log(\delta)\right) V^{\rm pole} F_O(n) + \int_\delta^1 \frac{dx}{x} R(x) F_O(n+1;x) + \left(\frac{V^{\rm pole}}{\varepsilon} + V^{\rm fin}\right) F_O(n) \\
&\approx \int_\delta^1 \frac{dx}{x} R(x) F_O(n+1;x) + \left(\log(\delta) V^{\rm pole} + V^{\rm fin}\right) F_O(n) + \mathcal{O}(\delta)
\end{aligned}
$$

- When δ→ 0 the approach becomes exact however the numerical integration becomes unstable.
- One has to show that the error due to finite δ is small.

- The behavior for x→0 (i.e. In the singular limit) can be predicted with resummation

  - Q_T resummation technique (Catani, Grazzini)

    - Used for computing all 2-to-2 reactions at NNLO where the Born final state is color singlet (WW,ZZ,γγ). Now is being developed also for colorful final states like top-pair.
  - N-subjettiness. New technique developed in the last year (Boughezal, Focke, Liu, Petriello)
    - NNLO corrections to (W+jet; Higgs+jet)

Fixed order calculations with MC techniques: NLO

- The subtraction method. Use the mathematical identity:

$$x^{-1+\varepsilon} = \frac{1}{\varepsilon}\delta(x) + \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!}\left[\frac{\ln^n(x)}{x}\right]_+$$

- Then rewrite (everything is exact)

$$\begin{aligned}
d\sigma_O &= \int_0^1 \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \left(\frac{V^{\text{pole}}}{\varepsilon} + V^{\text{fin}}\right) F_O(n) \\
&= \int_0^1 \frac{dx}{x}\left(R(x)F_O(n+1;x) - R(0)F_O(n+1;0)\right) + V^{\text{fin}} F_O(n) \\
&= \int_0^1 \frac{dx}{x} \underbrace{\Big(R(x)F_O(n+1;x) - V^{\text{pole}} F_O(n)\Big)}_{\text{Vanishes for } x \to 0} + V^{\text{fin}} F_O(n)
\end{aligned}$$

- The above integral is now finite
  - In any kinematical point
  - For any observable
  - Therefore, we can easily construct an MC (partonic MC)

- Method has been developed at NLO (basis for the MC@NLO); also at NNLO (Czakon). Used for top pair; Higgs +jet, top decay).

Fixed order calculations with MC techniques: NLO

$$
\begin{aligned}
d\sigma_O &= \int_0^1 \frac{dx}{x^{1+\varepsilon}} R(x) F_O(n+1;x) + \left(\frac{V^{\text{pole}}}{\varepsilon} + V^{\text{fin}}\right) F_O(n) \\
&= \int_0^1 \frac{dx}{x} \left(R(x)F_O(n+1;x) - R(0)F_O(n+1;0)\right) + V^{\text{fin}} F_O(n) \\
&= \int_0^1 \frac{dx}{x} \Big(R(x)F_O(n+1;x) - V^{\text{pole}} F_O(n)\Big) + V^{\text{fin}} F_O(n)
\end{aligned}
$$

Event (n+1 kinematics)

Counter-event (n kinematics)

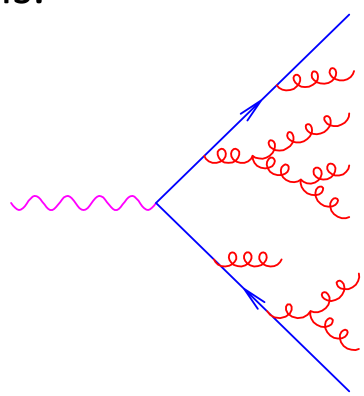
- Although the method is exact and implementable in MC, it brings addt'l complications at NLO
  - Finiteness is achieved through combination of events and counter-events; events have positive weight; counter-events – negative weight.
  - Separately, they can be arbitrary large but added together they are finite.
  - Events and counter-events are strongly correlated.
    - Generate event and its counter-event at the same time
  - Two separate calculations are now needed (they are individually finite):
    1. Subtracted real contributions
    2. Finite Virtual term

Parton showers and event generators (Leading Order)

What is a shower?

- Hard emissions (at some scale Q) are described well in fixed order perturbation theory.
- The probability for such emissions is suppressed by powers of $\alpha_S(Q) << 1$.
- Soft emissions (at some soft scale S) are ubiquitous because $\alpha_S(S) \approx 1$.
- In particular there can be many such emissions.

- Example: a typical event for Z → hadrons; down to ~ GeV scales in average around 7 gluons are emitted.

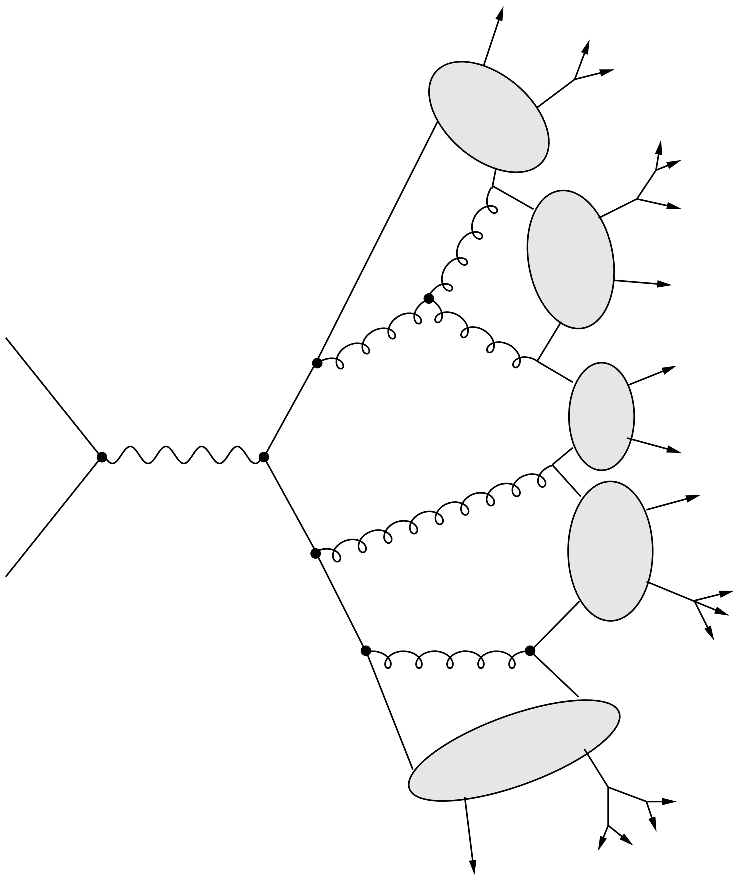


Credit: Bryan Webber

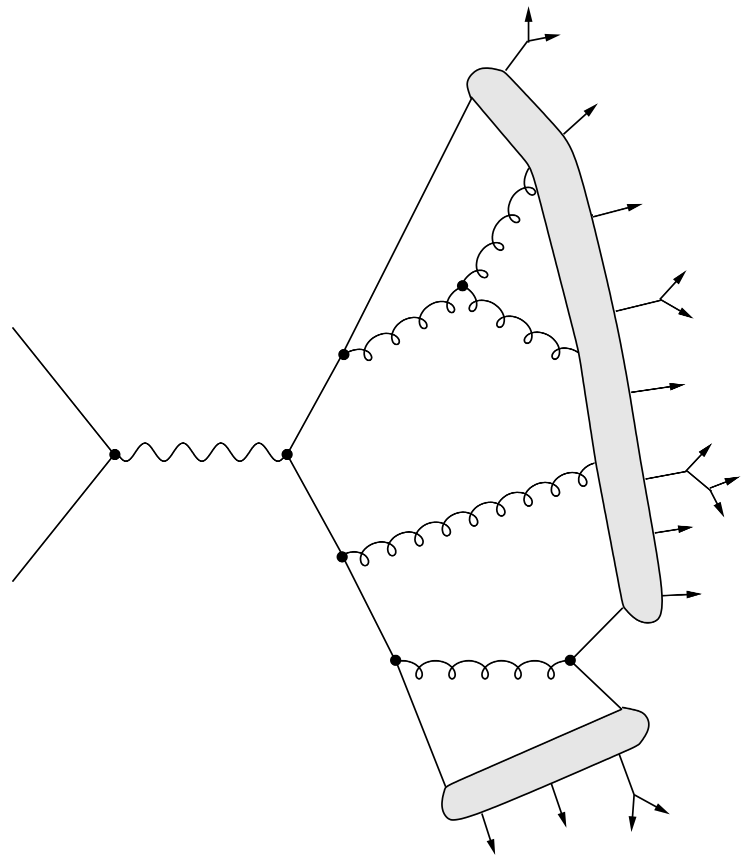
- Here is how a typical hard collision event is developing:
  - After the collision hard radiation is possible; few very hard particles are produced. This part is well described by fixed order perturbation theory.
  - The produced partons are off-shell and can still radiate. Typically these are soft emissions (real and/or virtual).
  - As discussed previously such emissions are cheap and can be copious.
  - This stage is described by a “parton shower”, i.e. a calculator that simulates soft and/or collinear emissions. Due to their universality and factorizability, such calculations are much easier than full FO calculations.
  - Once the system is at very low scales O(GeV), perturbation theory completely breaks down. We enter the hadronization stage. Hadronization can be modeled “exclusively”.
  - Programs that do all steps above are called event generators (like HERWIG, PYTHIA).

Main hadronization models

Credit: Ellis, Stirling, Webber



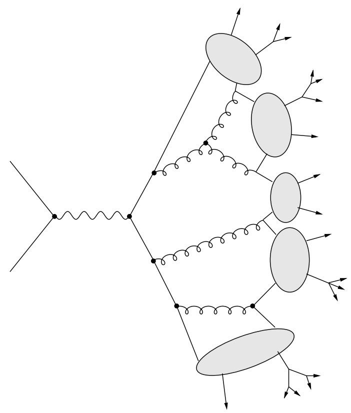
Cluster



String (like string theory)

Main hadronization models

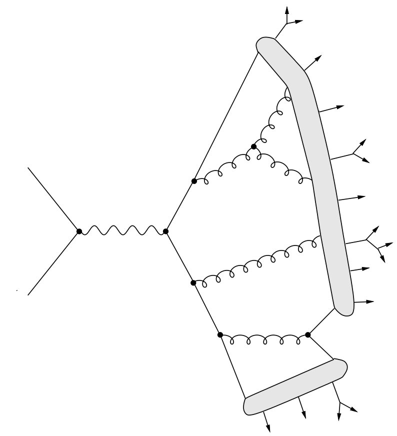
Cluster hadronization model



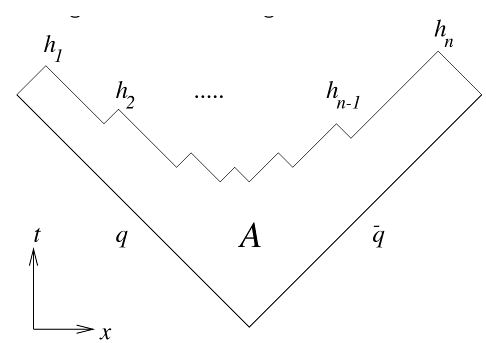
- Used in HERWIG
- Assumed that color singlet cluster are formed from neighboring q-qbar pairs
- These color cluster then decay into hadrons
- The mass of the clusters is few GeV
- How they decay to hadrons is model dependent. But a simple phase space-based model already works well.

- The model does not work very well with very massive cluster
- Problems with Baryons and heavy quarks.

String hadronization model



- Used in PYTHIA
- The color string formed from a quark pair breaks down into hadrons
- String is consistent with linear confining potential



$$|M(q\bar{q} \to h_1 \cdots h_n)|^2 \propto e^{-bA}$$

- A – area
- Kinks – from gluon emissions

Extra material

Parton showers and event generators:

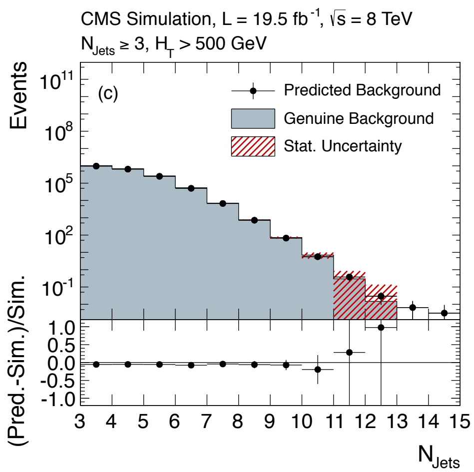
Matching to LO calculations

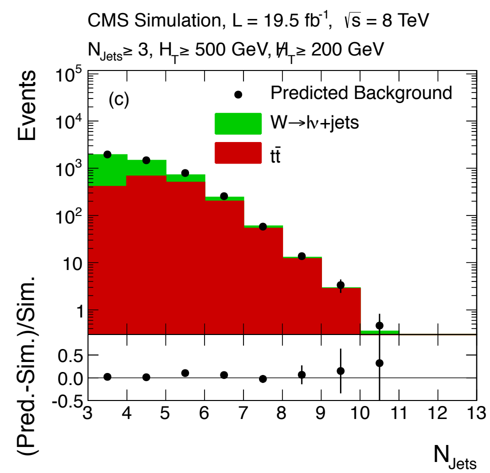
Why we need LO+PS matching?

Extra material

- Multijet events are omnipresent at the LHC. QCD produces many of those; bSM too. To find bSM we need good understanding of the genuine QCD backgrounds.

CMS ’14 http://arxiv.org/pdf/1402.4770v2.pdf





Background simulations

Selection N_{Jets}	H_{T} [GeV]	$\not{H}_{\mathrm{T}}$ [GeV]	$Z \to \nu\bar{\nu}$	$t\bar{t}/W \to e,\mu$+X	$t\bar{t}/W \to \tau_h$+X	QCD	Total background	Data
3–5	500–800	200–300	1820±390	2210±450	1750±210	310±220	6090±670	6159
3–5	500–800	300–450	990±220	660±130	590±70	40±20	2280±270	2305
3–5	500–800	450–600	273±63	77±17	66.3±9.5	$1.3^{+1.5}_{-1.3}$	418±66	454
3–5	500–800	>600	42±10	9.5±4.0	5.7±1.3	$0.1^{+0.3}_{-0.1}$	57.4±11.2	62
…								
≥8	500–800	>200	$0.0^{+0.8}_{-0.0}$	1.9±1.5	2.8±1.4	$0.1^{+0.4}_{-0.1}$	$4.8^{+2.3}_{-2.1}$	8
≥8	800–1000	>200	0.6±0.6	4.8±2.9	2.3±1.2	$0.5^{+0.9}_{-0.5}$	$8.3^{+3.4}_{-3.3}$	9
≥8	1000–1250	>200	0.6±0.5	$1.4^{+1.5}_{-1.4}$	2.9±1.3	$0.7^{+1.0}_{-0.7}$	$5.6^{+2.3}_{-2.1}$	8
≥8	1250–1500	>200	$0.0^{+0.9}_{-0.0}$	5.1±3.5	1.4±0.9	$0.5^{+0.9}_{-0.5}$	$7.1^{+3.8}_{-3.6}$	5
≥8	>1500	>200	$0.0^{+0.7}_{-0.0}$	$0.0^{+4.2}_{-0.0}$	2.4±1.4	$0.9^{+1.3}_{-0.9}$	$3.3^{+4.7}_{-1.7}$	2

Data vs. background

- Notice the limit of the simulation
- Notice the large number of jets that are actually measured. And this is for 8 TeV. The LHC now operates at 13 TeV!

Why we need LO+PS matching?

Extra material

- The genuine shower programs cannot predict such events with any reasonable accuracy.

  - Pythia, for example, has only 2 → 1 and 2 → 2processes genuinely built in. To generate many hard jets with a shower, one has to use the soft and collinear radiation of the shower well outside its intended "comfort" zone.
  - A warning – this can be achieved by playing with the scales – but would this be correct? And a more general warning: programs can produce any number. It is up to the user to make sense of produced results. The logic of "an imperfect number is better than no number" could be very useful but also very dangerous. One has to be very careful there!

- Large number of hard emissions are naturally described in fixed order perturbation theory. But these are single, colorful (typically massless) on-shell partons that look nothing like the jets we measure.

- Clearly, we need a combination of:
  - hard emissions generated by a complete fixed order calculation (these will give the proto-jets)
  - parton shower (builds the highly complex internal structure of the jets).

- Combining fixed order calculations with parton showers is a non-trivial task which is by now solved in many ways at LO. Doing this at NLO is still a very advanced problem. At NNLO this hasn't even been seriously contemplated (not yet – but may not be far into the future!)

Before we match LO with PS: what's a merging scheme?

Extra material

- First, recall the distinction between inclusive and exclusive observable
  - Example: inclusive jets: typically, within QCD, this means two or more jets.
  - At LO however, we only include 2-to-2 diagrams. This way no final states with 3 or more particles (jets). At the same time a state with, e.g. 8 jets, also belongs to this inclusive observable.
  - Question is: how to account for such multi-jet events?
  - Exclusive event: one with a fixed number of final state particles (jets). For example:
    - Exactly 2
    - Exactly 3
    - ...
  - Note: at LO (and only at LO) the various final states are mutually exclusive, i.e. an inclusive sample is just a sum of exclusive ones. This absolutely doesn't work at NLO and beyond!

- Thus, we arrive at the basic idea of merging samples at LO:
  - Introduce a separation measure between final states with n and n+1 partons.
  - Generate samples for all processes with n final states, $n<N_{max}$. N_{max} ~O(10) – see previous slide.
  - Add the samples. They are non-overlapping by construction, i.e. any double counting is avoided.
  - A question: this seems an easy thing to do. But then why do we need NLO calculations?

The difference between a merged LO sample and an NLO (or N^{k}LO) calculation

Extra material

- Lets take as an example Higgs production:
  - Inclusive Higgs at LO: pp→H
  - Inclusive Higgs at NLO: pp→ h; h+j
  - Inclusive Higgs at NNLO: pp->h;h+j;h+jj

- A merged LO sample with N_{max}=2 would cover all of the above final states.
- But not in the full kinematic range!

- For example, in the merged LO sample we are not allowed to take any two final state partons too close to each other. In fact, the result would diverge if we attempted to do that!

- Thus, the LO merged sample depends on the parameter that separates the different multiplicities.

- In contrast, in an NLO calculation one can take the extra emitted final state parton and make it as close as desired to any other parton. The divergence is compensated by the divergence in the loop virtual corrections that are absent in the merged sample!

- Similarly at NNLO: there up to two partons can become very close to any other parton. The divergences are much worse than at NLO but this is again compensated by the (now even more complicated) loop corrections.

From merged LO samples to matching with PS

Extra material

A note of caution: the terms merging and matching are not always assigned the same meaning in the literature. Keep an open mind and all should eventually be clear from the context.

- There is no one “best” or unique way of doing this: the final result always contains ambiguities and dependence on unphysical scales as long as we work to finite orders in perturbation theory
- The main requirements for a good matching scheme are:
  - Avoid double counting (all emissions look the same: be they hard, or from the shower)
  - Avoid dead regions (i.e. kinematical regions unpopulated by radiation while they should be)
  - One scheme is better than another one if it is a better approximation (in the sense that both LO and NLO are imperfect, but NLO is clearly better than LO).

Common strategies for PS matching procedures

Extra material

Follow the comparative study http://arxiv.org/pdf/0706.2569v2.pdf

1. A jet measure is defined and all relevant cross sections including jets are calculated for the process under consideration. I.e. for the production of a final state X in pp-collisions, the cross sections for the processes pp → X + n jets with n = 0, 1, ..., $N = N_{max}$ are evaluated.
2. Hard parton samples are produced with a probability proportional to the respective total cross section, in a corresponding kinematic configuration following the matrix element.
3. The individual configurations are accepted or rejected with a dynamical, kinematics-dependent probability that includes both effects of running coupling constants and of Sudakov form factors. In case the event is rejected, step 2 is repeated, i.e. a new parton sample is selected, possibly with a new number of jets.
4. The parton shower is invoked with suitable initial conditions for each of the legs. In some cases, like, e.g. in the MLM procedure, this step is performed together with the step before, i.e. the acceptance/rejection of the jet configuration. In all cases the parton shower is constrained not to produce any extra jet; stated in other words: configurations that would fall into the realm of matrix elements with a higher jet multiplicity are vetoed in the parton shower step.

The matching procedures discussed below differ mainly in:

- the jet definition used in the matrix elements;
- how acceptance/rejection of jet configurations from the matrix element is performed;
- Details of, and the jet vetoing inside, the parton showering.

Restate the problem

Extra material

- Let's make it even more evident where's the problem
  - We need a separation parameter at parton level R_{part}, i.e. any two partons must have $R>R_{part}$. It is needed, because if $R_{part} \rightarrow 0$ then the partonic x-section is IR divergent.
  - We also need a jet-level separation parameter (connected to jet definition, etc) R_{jet} which separates jets from each other.
- Clearly, only R_{jet} is physical because it is related to the measurement; not R_{part}.
- Yet, it is easy to see that an unmatched sample has strong dependence on the value of R_{part}: by taking smaller and smaller values for R_{part} the x-section grows unbounded.
- Basically our prediction strongly depends on an unphysical parameter. This is a problem.
- R_{part} should be smaller than R_{jet} (because otherwise we will have unpopulated regions – or dead zones) which is undesirable.
- The goal of the FO+PS matching procedure is to minimize the dependence on this parton level cut

  <u>Ideally it should be independent of it, but this is never the case.</u>
- How to achieve this is not obvious. There are 3 main proposals.

Main LO + PS matching procedures

Extra material

- Main algorithms:
  - MLM (Mangano '02)
  - CKKW (Catani, Kraus, Kuhn, Webber '01)
  - Dipole (Lonnblad'02)

- Their approaches are: if two partons are very close (can happen when R_{part} is small) we somehow suppress or outright veto such event (a veto is a form of suppression).

- In MLM the parton level generation and shower are done without any intermediate checks.
  - Only the final jets are checked for:
    - mutual separation
    - If each jet can be associated with one hard parton
    - all jets and partons can be paired
  - Any event where the above are not satisfied is vetoed.
  - The R_{part} sensitivity is reduced because if two partons are very close they will produce jets that are close to each other and this is vetoed.

- In CKKW there are both parton-level and jet-level checks:
  - Associate a Sudakov factor at each vertex. This is an exponential which dampens parton-level events with small separation.
  - PS emissions which are hard (off-jet) are vetoed.
  - Important: this gives a prescription for how to choose the value of the renormalization scale at each vertex (i.e. for each emission)!

CKKW merging procedure

Extra material

- The separation of the matrix-element and parton-shower domains for different multi-jet processes is achieved through a $k_\perp$ measure which controls the internal separation cut, also called the merging scale;

- The acceptance/rejection of jet configurations proceeds through a reweighting of the matrix elements with analytical Sudakov form factors and factors due to different scales in α_s;

- A vetoed parton-shower algorithm is used to guarantee that no unwanted hard jets are produced during jet evolution.

- The starting scale for the parton shower evolution of each parton is given by the scale where it appeared first:

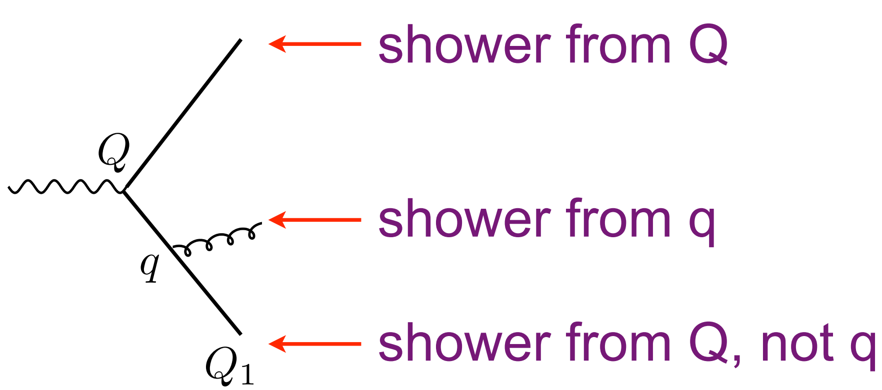


Fig. courtesy of B. Webber

The Sudakov Formfactor

Extra material

- In our discussion of soft-gluon resummation (lecture 2) we encountered the Sudakov formfactor
- It was resumming soft emission from an independently evolving hard parton
- It included both virtual (loop) and real emission corrections.
- Separately, Real and Virtual corrections were divergent, but together they were finite.
- We interpreted it as a probability for no resolved emission.

- In the context of parton showers it reads:

A-P splitting function with azimuthal correlations

$$\Delta_{i\to j}(q_1,q_2) = \exp\left[-\int_{q_2^2}^{q_1^2}\frac{dq^2}{q^2}\frac{\alpha_S(q^2)}{2\pi}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz\int_0^{2\pi} d\phi P_{ij}(z,\phi)\right]$$

  - Evolution of parton i from scale q_1 down to scale q_2 without resolved radiation
  - Q_0 is a scale 1GeV, at which the shower terminates.

- How does the shower work?
  - Start with a hard parton i;
  - It is on-shell; we shift it off-shell and assign some virtuality Q (the initial scale) to the parton (all momenta need to be reshuffled for this!).
  - Solve the equation $\Delta(Q,q_1) = R$ for q_1, where R is a uniform random number.
    - If $q_1 < Q_0$: terminate the shower (no resolved emission was made)
    - If $q_1 > Q_0$: then splitting $i \rightarrow j$ occurred.
  - Repeat the above for the secondary parton j starting from a scale q_1.

The MC@NLO approach

Extra material

Frixione, Webber hep-ph/0204244

- Combine NLO matrix element with a shower

  - The NLO matrix element is formulated within the subtractions method we discussed yesterday. The matrix elements either have an emission or not (virtual)
  - The shower is based upon the Sudakov formfactor and describes emissions (0,1,2…) that are independent of the matrix element emissions

  - The goal is to ensure:

    - No double counting (after all, emissions from matrix elements and Shower look the same…)
    - Simple interpretation: the definition of the observable enters through the shower; the matrix elements only modify the weight of the shower (compared to the LO case).
    - Improved numerical convergence compared to a fixed order calculation

The MC@NLO approach

Extra material

- Assume the following form:

$$d\sigma_{\mathrm{MC@NLO}}(O) = W_0 S(O, x_M) + \int_{x_0}^{x_M} dy W_i(y) S(O, y) + \dots$$

- A combination of sets of (0,1,…)-emission events with:
  - Weight W_0,W_1, to be determined by matching to a fixed order calculation
  - Each emission is interfaced to a shower S(). The kinematics of the shower is dependent on the real emission (i.e. the shower can emit only the energy left after the hard emission is made)
  - x_0 and x_M are the minimum/maximum energies available to radiate.
  - Denote the energy of the real emission, if any, as y: $x_0 < y < x_M$.

- The Sudakov formfactor is:

$$\Delta(x_M, x_0) = \exp\left[-a \int_{x_0}^{x_M} \frac{dz}{z} P(z)\right] = 1 - a \int_{x_0}^{x_M} \frac{dz}{z} P(z) + \mathcal{O}(a^2)$$

- It drives the shower (i.e. shower contains all possible emissions with probability derived from the Sudakov):

$$\begin{aligned} S(O, x(y)) = & \left[1 - a \int_{x_0}^{x} \frac{dz}{z} P(z) + \mathcal{O}(a^2)\right] \delta(O - O(y)) \\ & + \left[a \int_{x_0}^{x} \frac{dx_1}{x_1} P(x_1) + \mathcal{O}(a^2)\right] \delta(O - O(y; x_1)) \\ & + \mathcal{O}(a^2) \end{aligned}$$

← No emission from shower

← 1 emission from shower

← 2 emissions from shower

The MC@NLO approach

Extra material

$$d\sigma_{\mathrm{MC@NLO}}(O) = W_0 S(O, x_M) + \int_{x_0}^{x_M} dy W_i(y) S(O,y) + \dots$$

- To determine the weights W0,W1 one expands MC@NLO and requires that to NLO it agrees with the NLO result (derived within the subtraction method)

$$\begin{aligned} d\sigma_{\mathrm{NLO}} &= B + a\left(\frac{V^{\mathrm{pole}}}{\varepsilon} + V^{\mathrm{fin}}\right) + \mathcal{O}(a^2) \\ &\quad + a\int_0^1 \frac{dx}{x^{1+\varepsilon}} R(x) + \mathcal{O}(a^2) \\ &= B + a\left(V^{\mathrm{fin}} + \int_0^1 \frac{dx}{x}\left(R(x) - R(0)\right)\right) + \mathcal{O}(a^2) \end{aligned}$$

- The MC@NLO weights read:

$$\begin{aligned} W_0 &= B + a\left[V^{\mathrm{fin}} + \overbrace{\int_0^1 \frac{dx}{x}\left(BP(x) - R(0)\right)}^{\text{Just a number ~ Born}}\right] \\ W_1 &= a\frac{R(x) - BP(x)}{x} \end{aligned}$$

- Notice the “miracle”:
  - Weights are similar to the NLO ones but the real emission counter-term got replaced by the shower: R(0)==B → BP(x)
  - This implies that the subtraction kinematics is the same as the one for the event. Improved convergence; less negative weight events compared to fixed order calculation

The state of the art for FO + PS

- NNLO + PS results have been derived for processes like Higgs, DY, etc.
  - Recently also for colored final states like ttbat

  - Within MINLO approach (2013): Hamilton, Nason, Re, Zanderighi
  - GENEVA collaboration (2015): Alioli, Bauer, Berggren, Tackmann, Walsh

- The MINLO approach is based on fully differential NLO calculation which has NNLO normalization through reweighting.

- The GENEVA result is based on the slicing method with N-jettiness variable.

- Extensions beyond these (2→ 1) processes is in progress.

- Expect lots of activity and hopefully new results.

PDF evolution and number of active flavors in the proton

- How to treat the heavy flavors (c,b,t) in the proton?
- It depends on the scale at which we measure the pdfs:

$$
f_i(x,\mu) \quad \begin{cases} i = u,d,s \quad \text{if} \quad \mu < m_c \\ i = u,d,s,c \quad \text{if} \quad m_c < \mu < m_b \\ i = u,d,s,c,b \quad \text{if} \quad m_b < \mu < m_t \\ i = u,d,s,c,b,t \quad \text{if} \quad \mu > m_t \end{cases}
$$

- Unlikely to need top quark pdf's at the LHC but should be absolutely essential at a future 100 TeV hadron collider

PDF (and fragmentation functions) at different orders (LO NLO,…)

$$O = \sum_{ij} f_i f_j \left(d\sigma_{ij}^{(0)} \right)$$

$$O = \sum_{ij} f_i f_j \left(d\sigma_{ij}^{(0)} + d\sigma_{ij}^{(1)} \right)$$

$$O = \sum_{ij} f_i f_j \left(d\sigma_{ij}^{(0)} + d\sigma_{ij}^{(1)} + d\sigma_{ij}^{(2)} \right)$$

- Using the same data, we can extract pdf's at LO, NLO, NNLO,…
- Clearly, the change in perturbative cross-section gets "absorbed" by a change in the pdf.
- Therefore, pdf's at LO, NLO,… are different.
- They should be used consistently in subsequent computations.

NNLO calculations

NNLO approaches (1)

- First were Smith, van Neerven and co. [Drell-Yan, e+e-] through mid-'90's
- Early modern work was analytic (elegant but couldn't cope with less-inclusive observables)

[Higgs, Drell-Yan] Anastasiou, Dixon, Melnikov Petriello '01-04

- Early numeric work based on sector decomposition (lead to tremendous progress; implementation is process dependent)

Binoth and Heinrich '04
[Higgs, Drell-Yan] Anastasiou, Melnikov, Petriello '03

- Antenna subtraction (ongoing progress)

[e+e- → 3 jets] Weinzierl '08-09
Gerhmann-De Ridder, Gehrmann, Glover, Heinrich '07
[dijets] Currie, Gehrmann-De Ridder, Gerhmann, Glover, Pires, Wells '13-15
[H+j] Chen, Gehrmann, Glover, Jacquier '14
[Z+j] Gehrmann-De Ridder, Gerhmann, Glover, Huss, Morgan '15
[tt (quarks)] Abelof, Gehrmann-De Ridder '14

-
- Colorful subtraction (promising development)

del Duca, Somogyi, Trocsanyi '05
[Higgs → bb] del Duca, Duhr, Somogyi, Tramontano, Trocsanyi '15
[e+e- → 3j] del Duca, Duhr, Kardos, Somogyi, Trocsanyi '16

NNLO approaches (2)

- q_T-subtraction
  - elegant and effortless for colorless final states:

    Catani, Grazzini '07
    [γγ] Catani, Cieri, de Florian, Ferrara, Grazzini '11
    [Wγ, Zγ] Grazzini, Kallweit, Rathlev,Torre '13-15
    [ZH] Ferrara, Grazzini, Tramontano '14
    [WH] Ferrara, Grazzini, Tramontano '11-13
    [WW] Gehrmann, Grazzini, Kallweit, Maierhoeffer,von Manteuffel,Pozzorini, Rathlev, Tancredi '14
    [ZZ] Cascioli, Gehrmann, Grazzini, Kallweit, Maierhoeffer, von Manteuffel,Pozzorini, Rathlev, Tancredi, Weihs '14

  - being developed for general final states:

    Zhu, Li, Li, Shao, Yang '12
    Catani, Grazzini, Torre '14
    [tt−offdiagonal] Bonciani, Catani, Grazzini, Sargsyan, Torre '15

- N-jettiness (new and very promising development)

  Gaunt, Stahlhofen, Tackmann, Walsh, '15
  [Vj] Boughezal, Focke, Liu, Petriello '15-16
  [Zj] Boughezal, Focke, Giele, Liu, Petriello '15
  [Hj] Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello '15
  [γγ] Campbell, Ellis, Li, Williams '16

NNLO approaches (3)

- Sector-improved residue subtraction

Czakon ‘10-11
Czakon, Heymes ‘14
Boughezal, Melnikov, Petriello, '11
[tt] Barnreuther, Czakon, Fiedler, Heymes, Mitov ’12-16
[Hj] Boughezal, Caola, Melnikov, Petriello, Schulze, '13-15
[B-decay] Caola, Czernecki, Liang, Melnikov, Szafron, '14
[t-decay] Brucherseifer, Caola, Melnikov, '13

- Future developments within this approach:
  - Independent implementation in a new code *STRIPPER*

    Czakon, Heymes, van Hameren

  - Process-independent (currently used for top production; adding top decay)
  - Important stability and numerics-related improvements being implemented
  - Linked to fastNLO: could output tables for any process

    Britzger, Rabbertz, Sieber, Stober, Wobisch

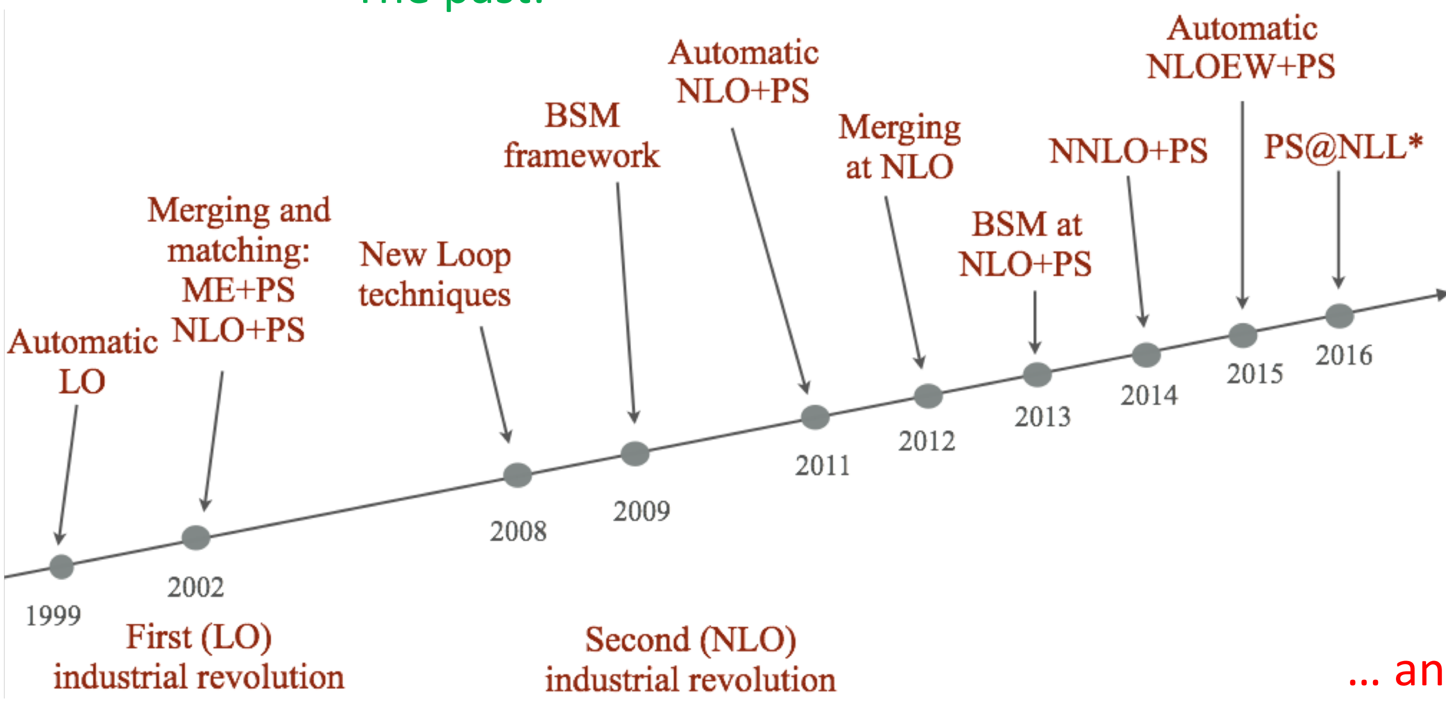
    - ✓ Very useful for pdf studies

NNLO: future needs and directions

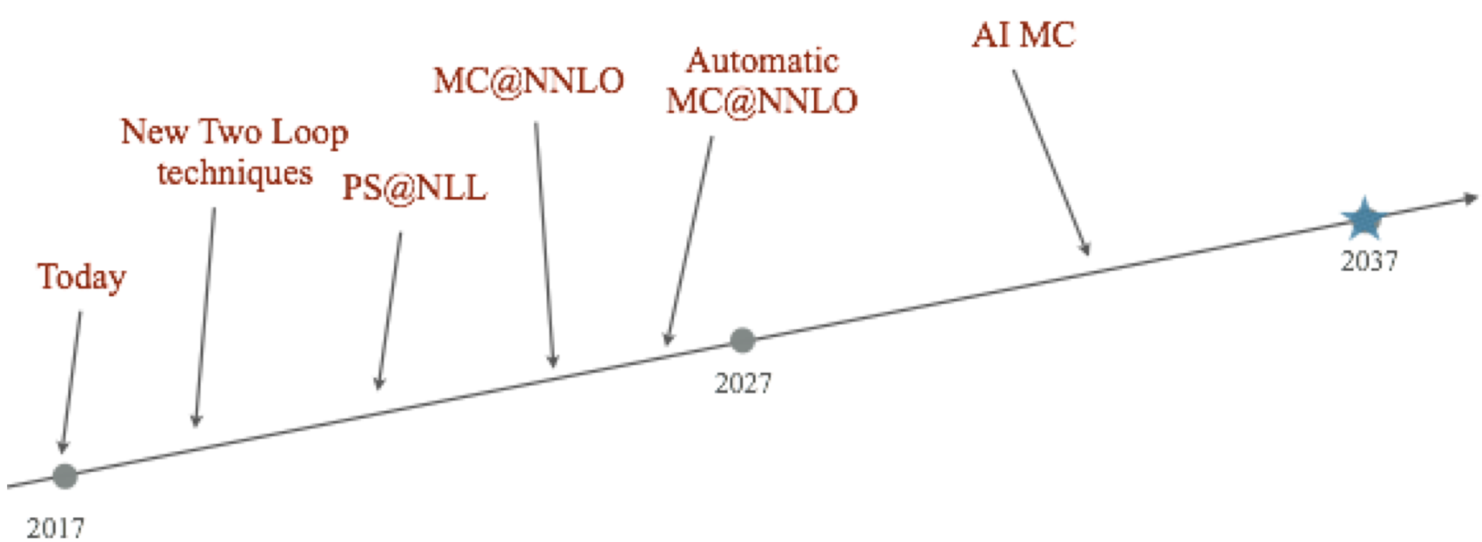
- Establishing a connection between various processes:
  - Access all (many) processes under the same roof
- NLO is a good lead (although should not be followed verbatim due to computational cost at NNLO):
  - MCFM
  - MC@NLO
  - Powheg
  - Sherpa
  - …
- Matching NNLO to showers and description of realistic final states
- Exploring new frontiers (beyond 2-to-2)
  - (Some of) the NNLO methods can in principle cope with any-multiplicity processes.
  - Numerics is however another issue…
  - However, no 2-loop amplitude is known beyond 2-to-2

And finally: the big picture (thanks to Fabio Maltoni)

The past:



... and the future



LHC applications

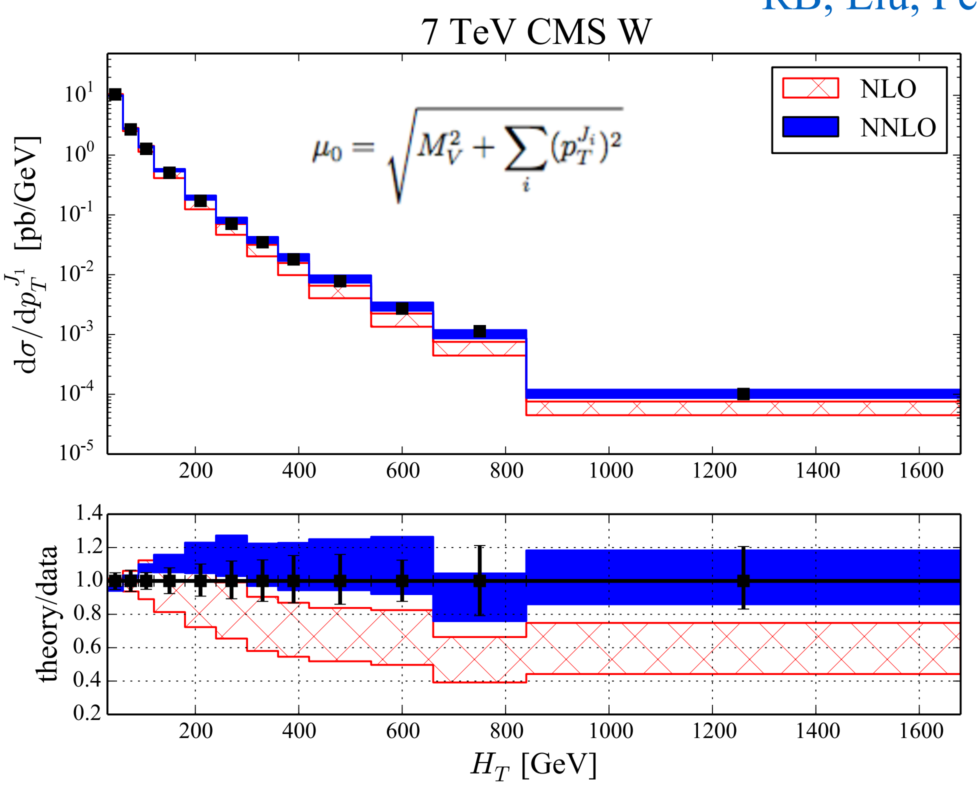
Drell-Yan, W and Z

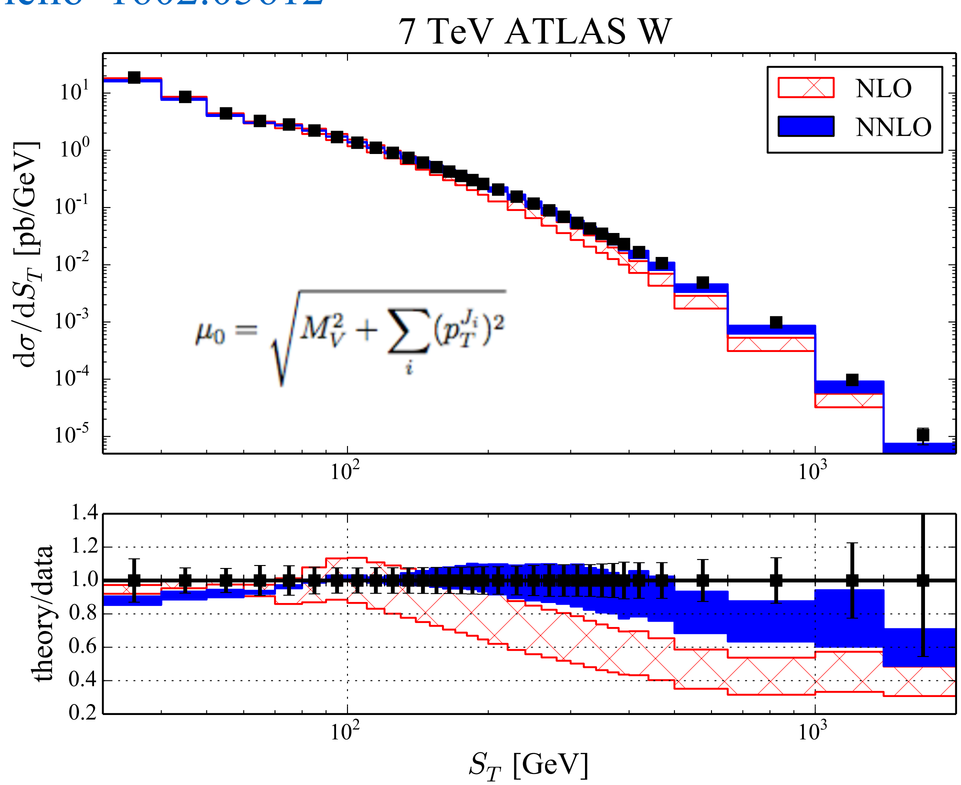
- These processes are the best known ones at hadron colliders
- See program FEWZ   Latest version of FEWZ http://arxiv.org/pdf/1208.5967.pdf
- Known fully differentially through NNLO in QCD for all processes
- For Drell-Yan also NLO EW included.
- Relevance of these processes:
  - Standard calibration tool for detectors.
  - Were proposed for LHC luminosity measurement due to the very good theory control.
  - Searches for Z’ and related bSM processes
  - The W+ and W- asymmetries allow direct access to the flavor asymmetries of the proton pdf’s.
  - Uncertainties at percent level.
- W/Z + jet now also known at NNLO (several groups)
- DY/W/Z also merged with PS’s

W and Z + jet @ NNLO

- V+jet @ NNLO is needed to describe the V P_T at NNLO
- H_T distribution for W+jet:

RB, Liu, Petriello 1602.05612





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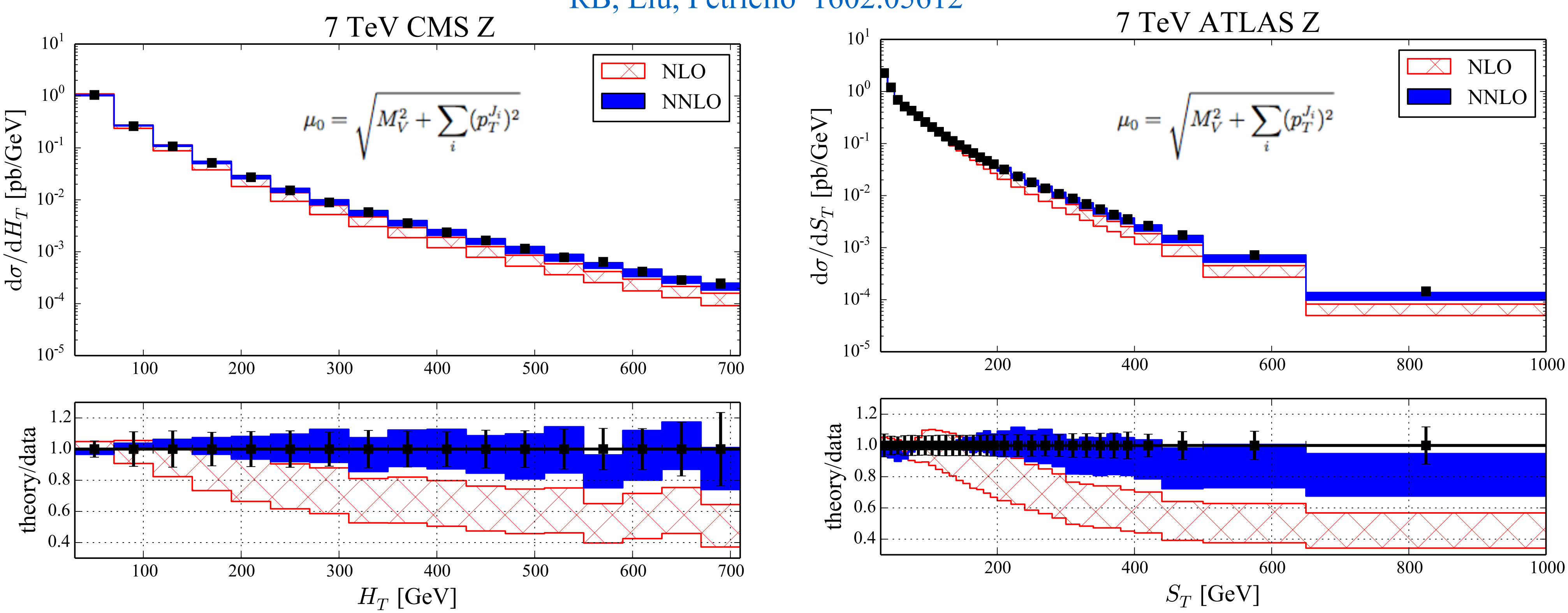
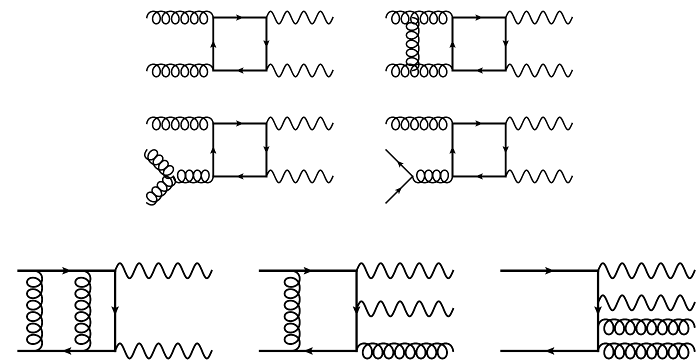
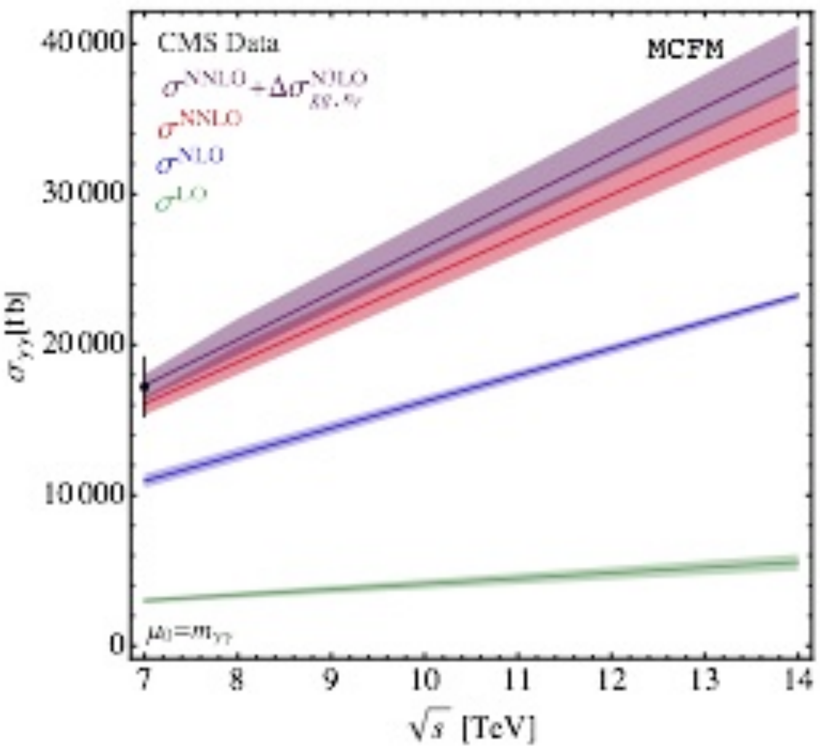


Di-photon @ NNLO

MCFM: http://arxiv.org/pdf/1603.02663v2.pdf

- The main background to Higgs → γγ
- NNLO (and beyond) makes a dramatic difference





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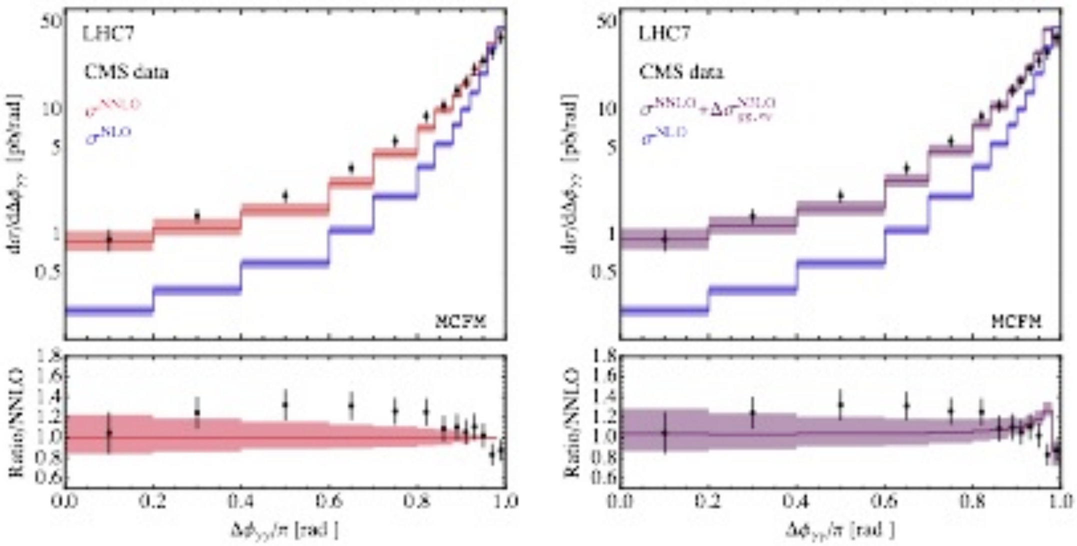
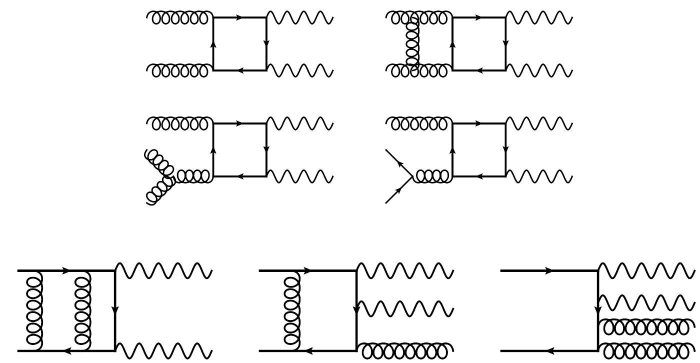
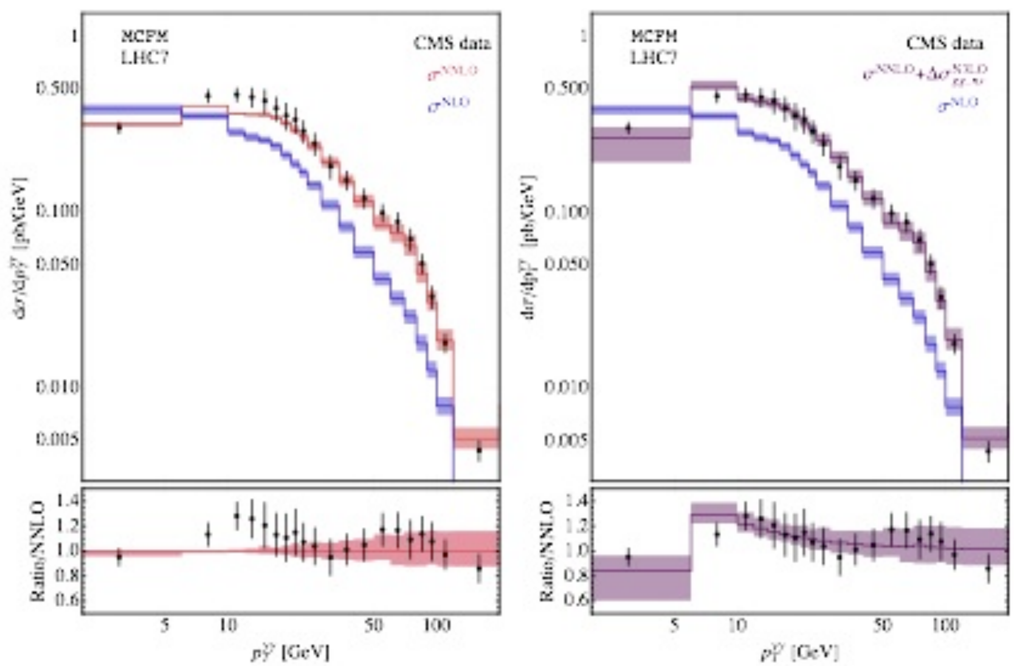




I expect this to be known at N^{3}LO soon



Higgs production

- The slow convergence of the perturbative expansion in Higgs production prompted work beyond NNLO QCD.

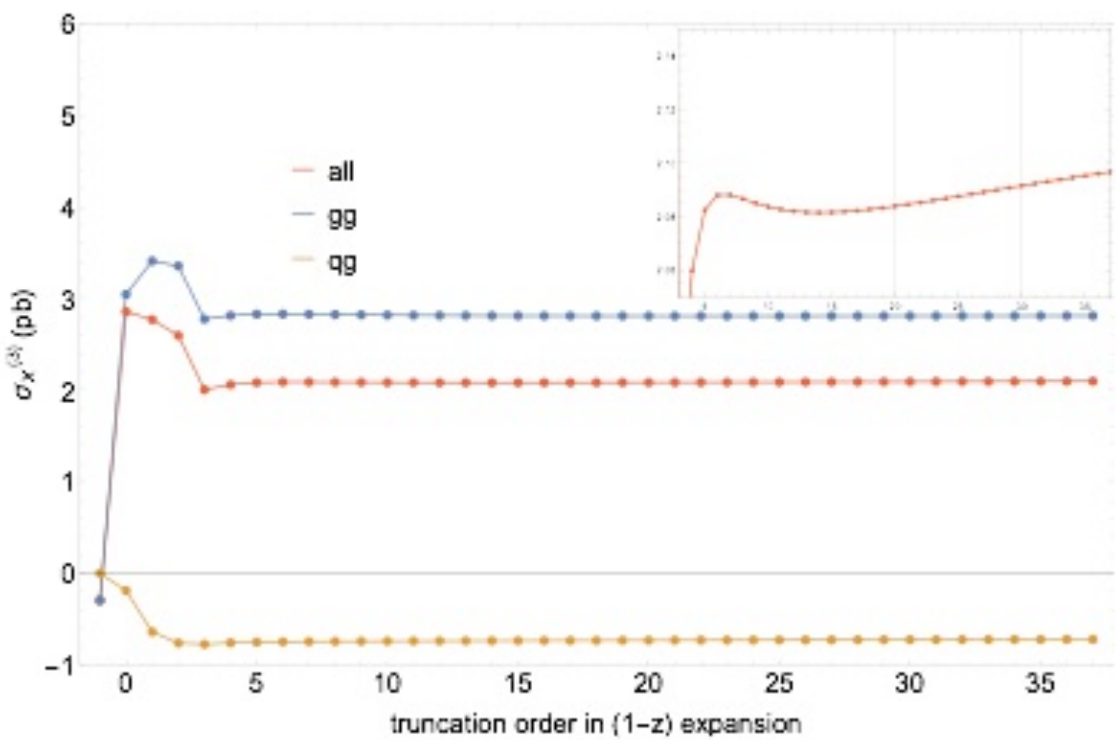
$$\sigma = \tau \sum_{ij} \left(f_i \otimes f_j \otimes \frac{\hat{\sigma}_{ij}(z)}{z} \right)(\tau) \qquad \tau = \frac{m_H^2}{S} \;\text{ and }\; z = \frac{m_H^2}{s} \qquad \frac{\hat{\sigma}_{ij}(z)}{z} = \frac{\pi\, C^2}{8V} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k \eta_{ij}^{(k)}(z)$$

- Expansion around the soft limit (normally n=1):

$$\frac{\hat{\sigma}_{ij}(z)}{z^{1+n}} \simeq \hat{\sigma}_{ij}(z)|_{(1-z)^{-1}} + \hat{\sigma}_{ij}(z)|_{(1-z)^0} + n(1-z)\,\hat{\sigma}_{ij}(z)|_{(1-z)^{-1}} + \mathcal{O}(1-z)^1$$

- A deep expansion around the soft limit at N^{3}LO is now known

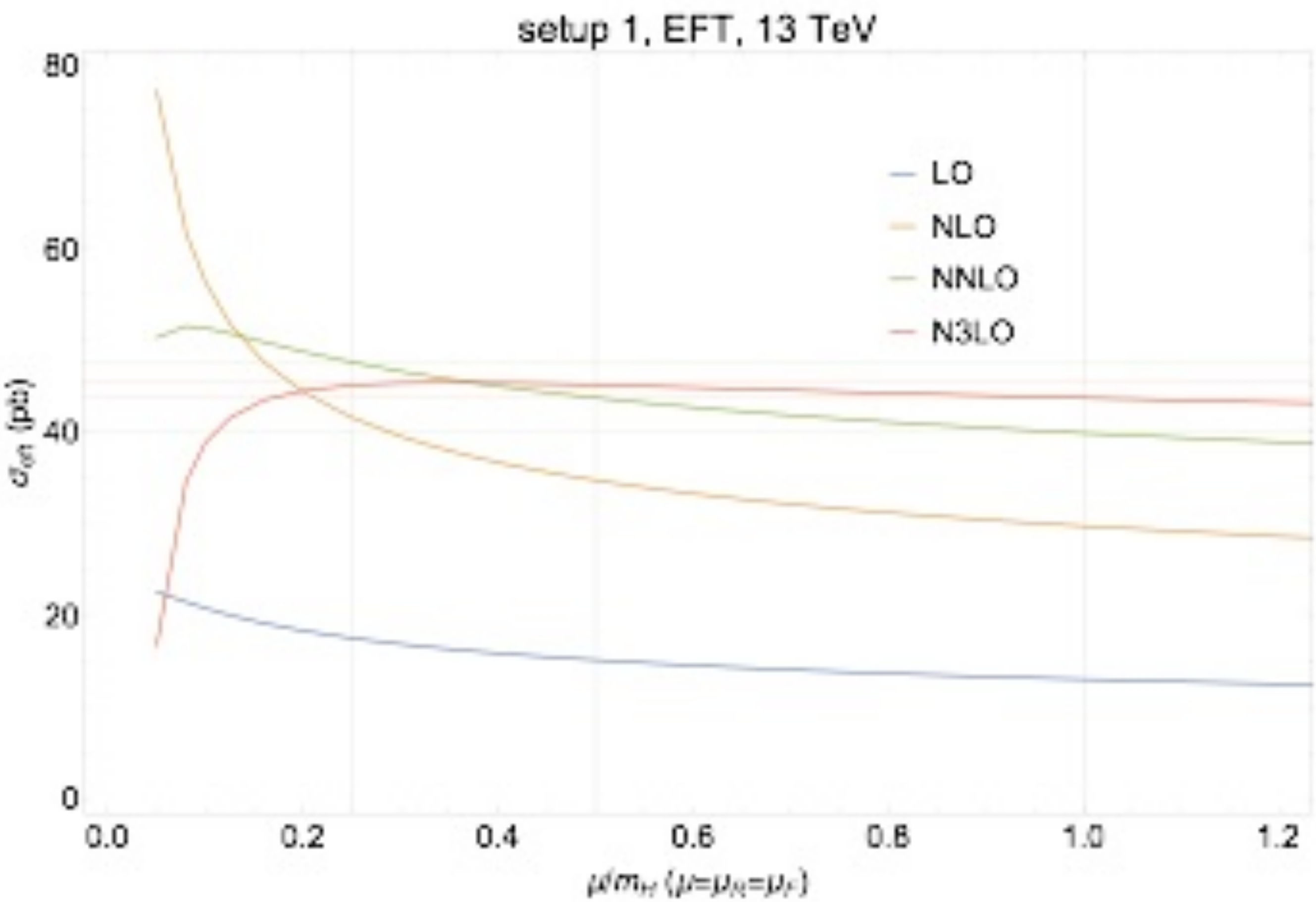
See https://arxiv.org/abs/1602.00695



Higgs production

- The (essentially) full N^{3}LO result is a reason for joy!

See https://arxiv.org/abs/1602.00695



		$\Delta^{\text{scale}}_{EFT,k}$
LO	$(k=0)$	$\pm 14.8\%$
NLO	$(k=1)$	$\pm 16.6\%$
NNLO	$(k=2)$	$\pm 8.8\%$
N^{3}LO	$(k=3)$	$\pm 1.9\%$

- So far the choice of central scale was always tenuous.
- It seems at N^{3}LO it doesn't mater as much (which is a very good news)

Higgs + jet production

- Now also H+j is known at NNLO
  - Needed to describe the Higgs P_T at NNLO

See http://arxiv.org/pdf/1505.03893.pdf

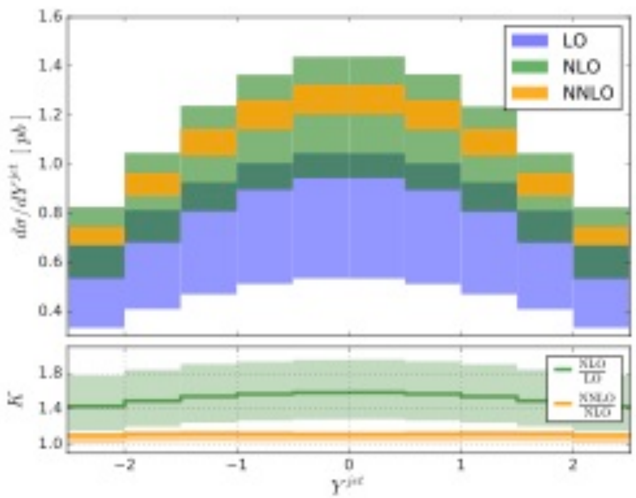


Figure 2: The rapidity of the leading jet at LO, NLO, and NNLO in the strong coupling constant. The lower inset shows the ratios of NLO over LO cross sections, and NNLO over NLO cross sections. Both shaded regions in the upper panel and the lower inset indicate the scale-variation errors.

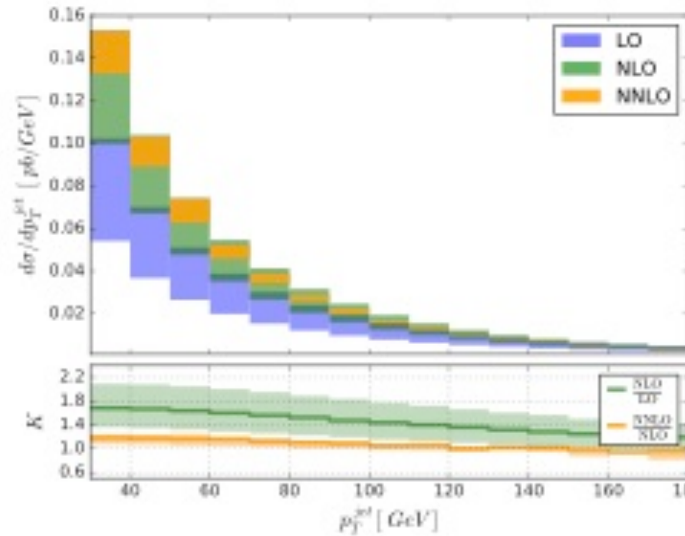


Figure 3: The transverse momentum of the leading jet at LO, NLO, and NNLO in the strong coupling constant. The lower inset shows the ratios of NLO over LO cross sections, and NNLO over NLO cross sections. Both shaded regions in the upper panel and the lower inset indicate the scale-variation errors.

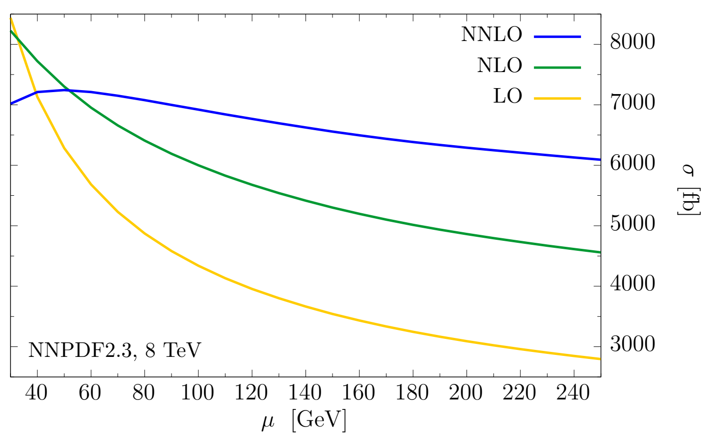


Figure 2: Dependence of the total LO, LO and NNLO cross-sections on the unphysical scale μ. See text for details.

See http://arxiv.org/pdf/1504.07922v1.pdf

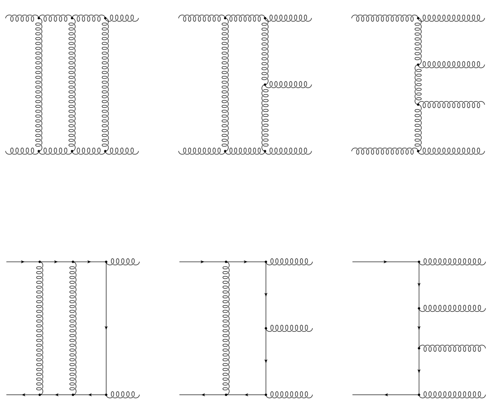
- Dramatic reduction of scale dependence

Dijet production

See http://arxiv.org/pdf/1407.5558.pdf

- Major hadron collider process:
  - bSM searches
  - PDF’s

- Now fully known at NNLO



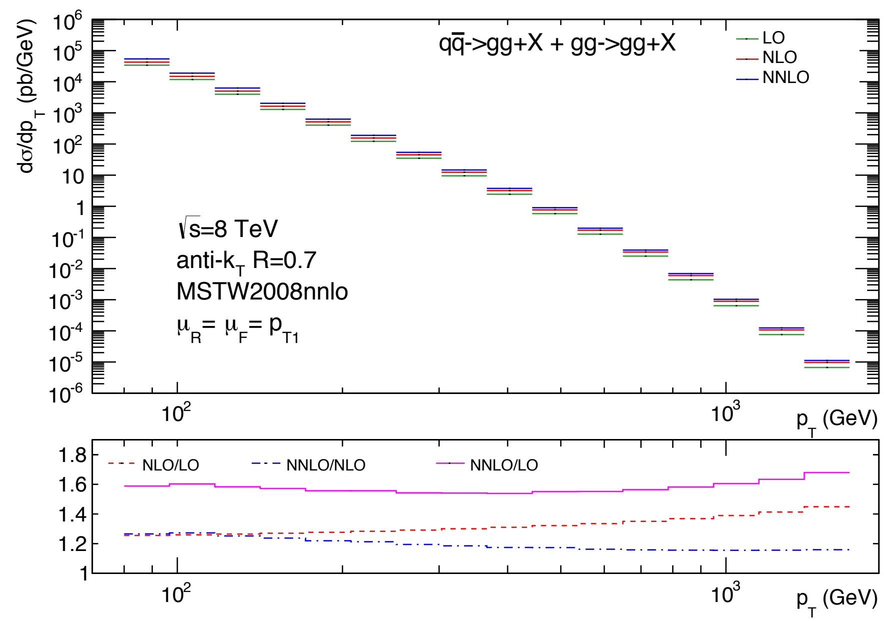
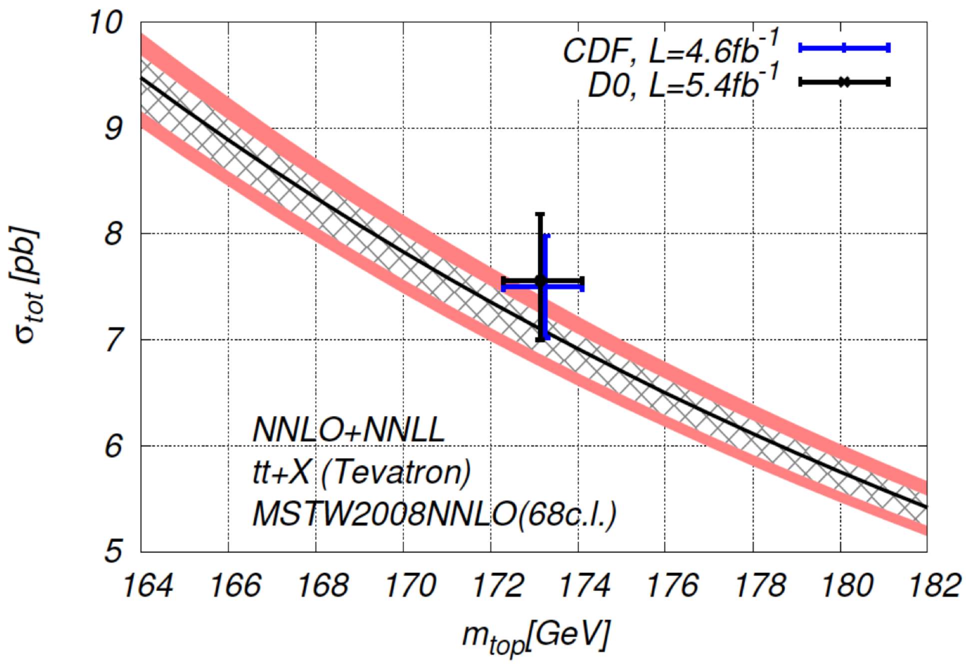
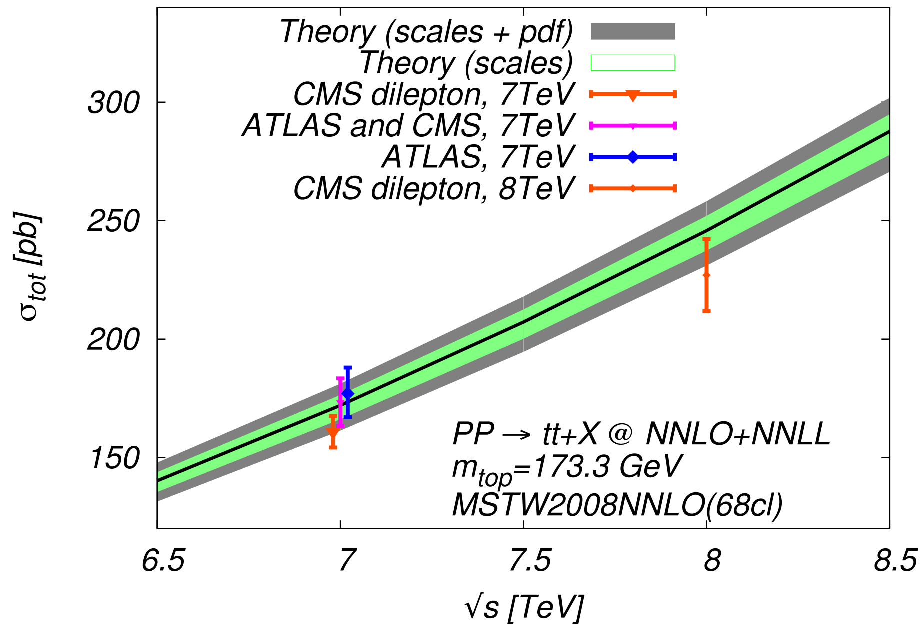


Figure 2: Inclusive jet transverse energy distribution, $d\sigma/dp_T$, for jets constructed with the anti-k_T algorithm with $R = 0.7$ and with $p_T > 80$ GeV, $|y| < 4.4$ and $\sqrt{s} = 8$ TeV at NNLO (blue), NLO (red) and LO (dark-green). The lower panel shows the ratios of different perturbative orders, NLO/LO, NNLO/LO and NNLO/NLO.

Even 3-jet production at NNLO has just been presented at a conference!

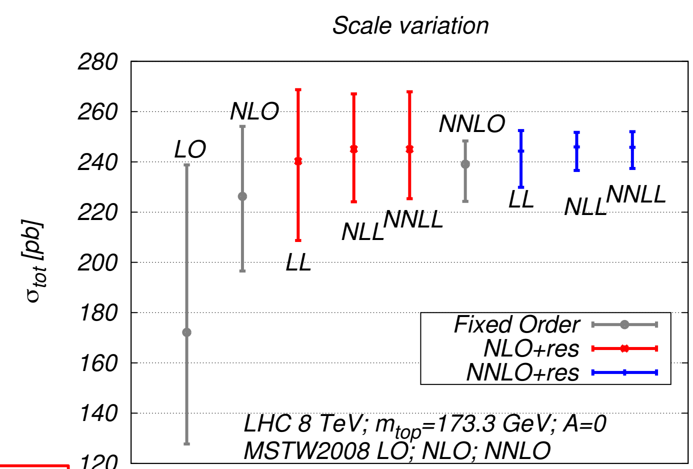
Top-pair production





Scale error at 3%; similar to parametric errors due to α_s, m_{top}, pdf

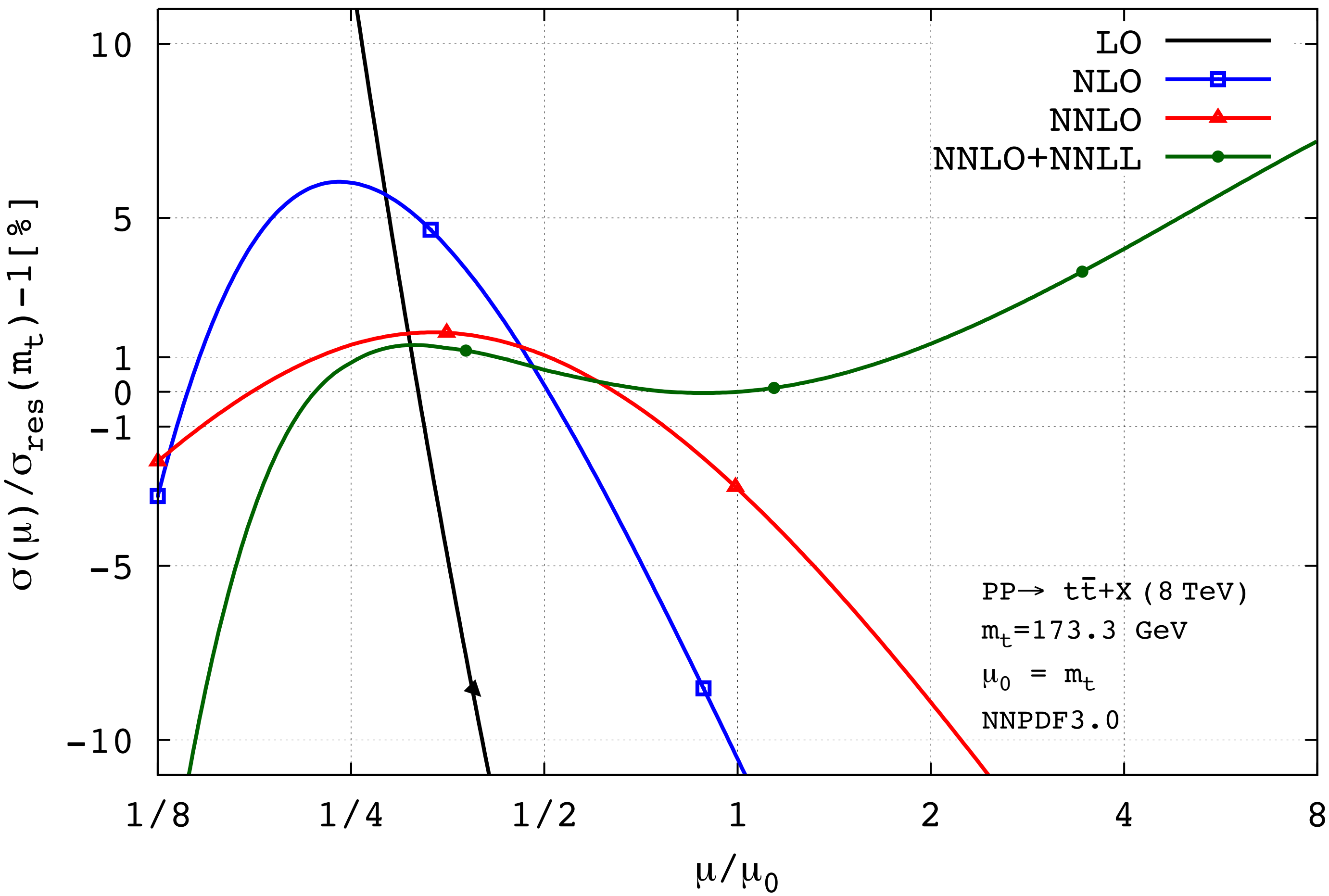
Impressive convergence of perturbation theory in this process.



For more details see arXiv:1305.3892

Top-pair production

- The scale variation in top production is small. Indicates good perturbative convergence.
- Amazing parallel to Higgs production at N^{3}LO!



From http://arxiv.org/pdf/1606.03350.pdf

Top-pair production AFB asymmetry

- An excellent example why QCD matter: for a long time it was thought that no higher order QCD corrections should be expected, thus making the discrepancy between CDF measurement / SM theory more significant.

- However, NNLO QCD brings large corrections that helped to finally resolve the long standing Tevatron AFB puzzle.
  - A new D0 measurement was also much closer to SM.

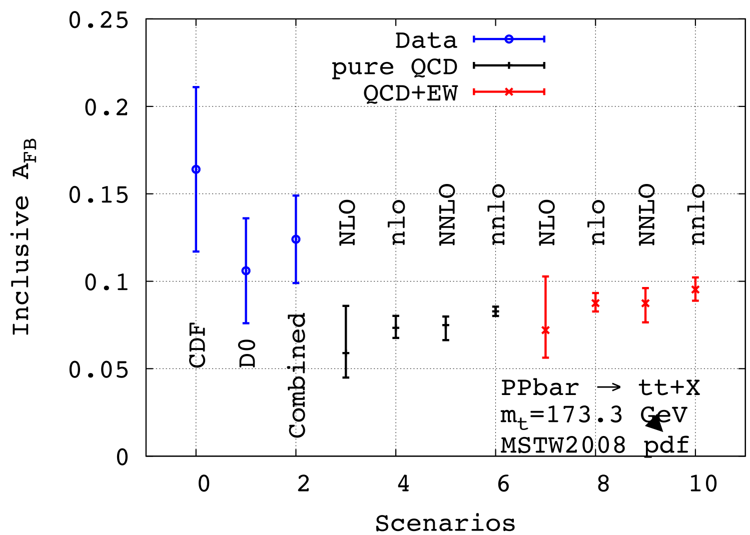
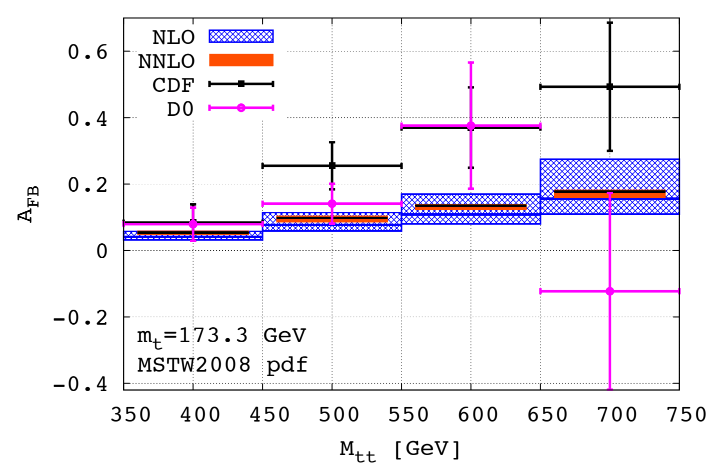


FIG. 1: The inclusive asymmetry in pure QCD (black) and QCD+EW[28] (red). Capital letters (NLO, NNLO) correspond to the unexpanded definition (2), while small letters (nlo, nnlo) to the definition (3). The CDF/DØ (naive) average is from Ref. [29]. Error bands are from scale variation only. Our final prediction corresponds to scenario 10.

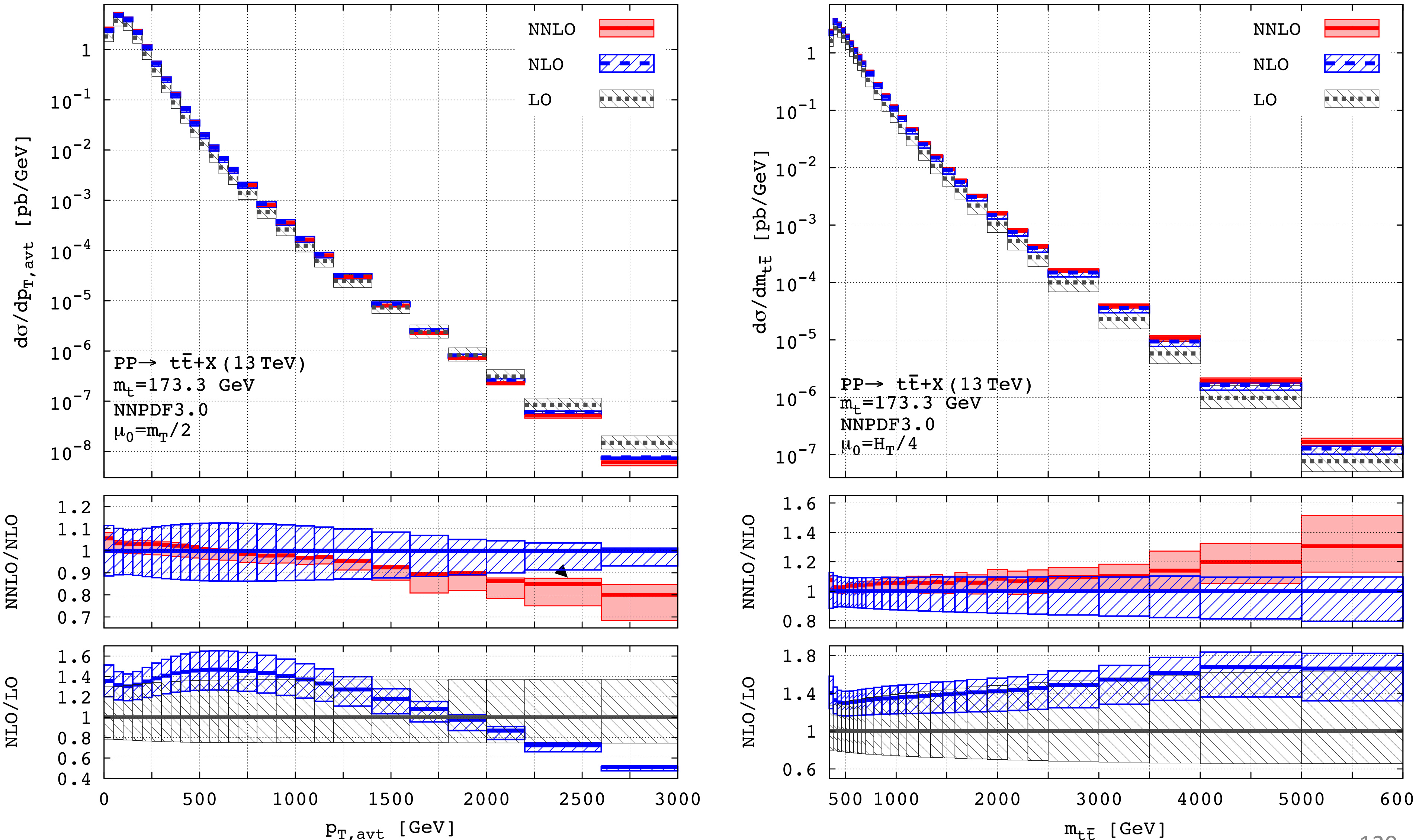
From http://arxiv.org/pdf/1411.3007.pdf



Top-pair production

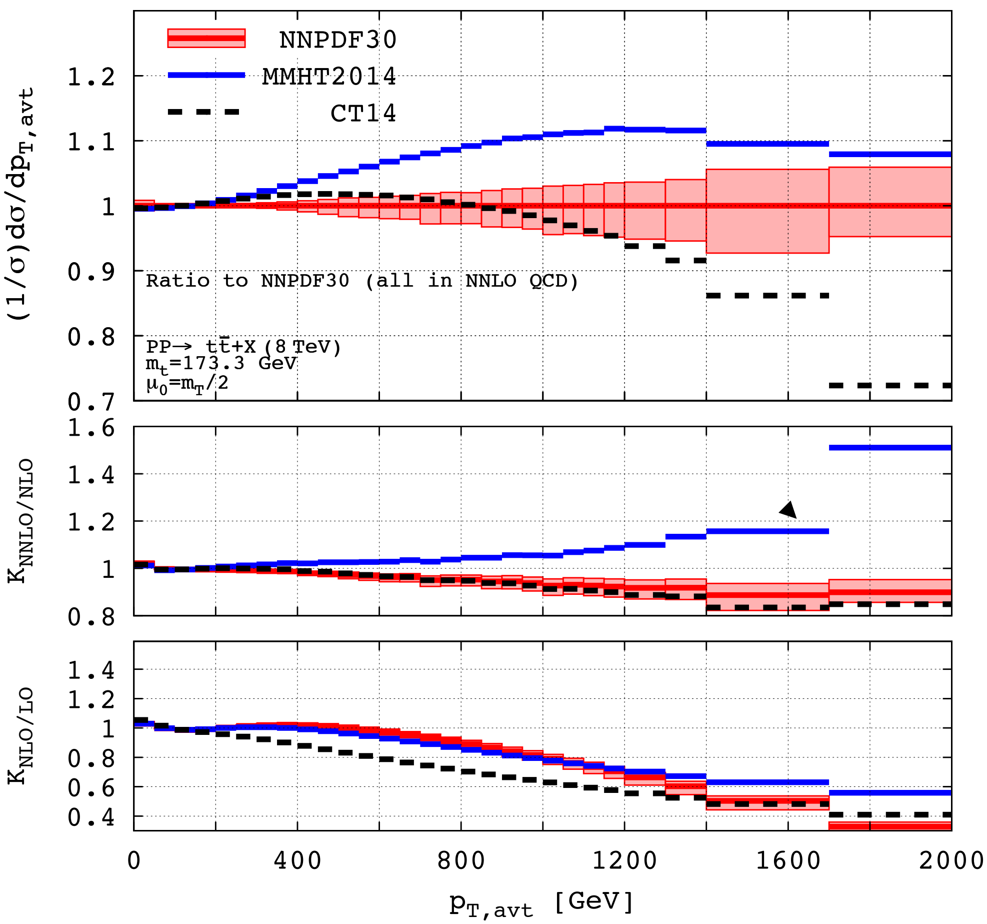
- Top pair production is now under good control all the way into the TeV regime

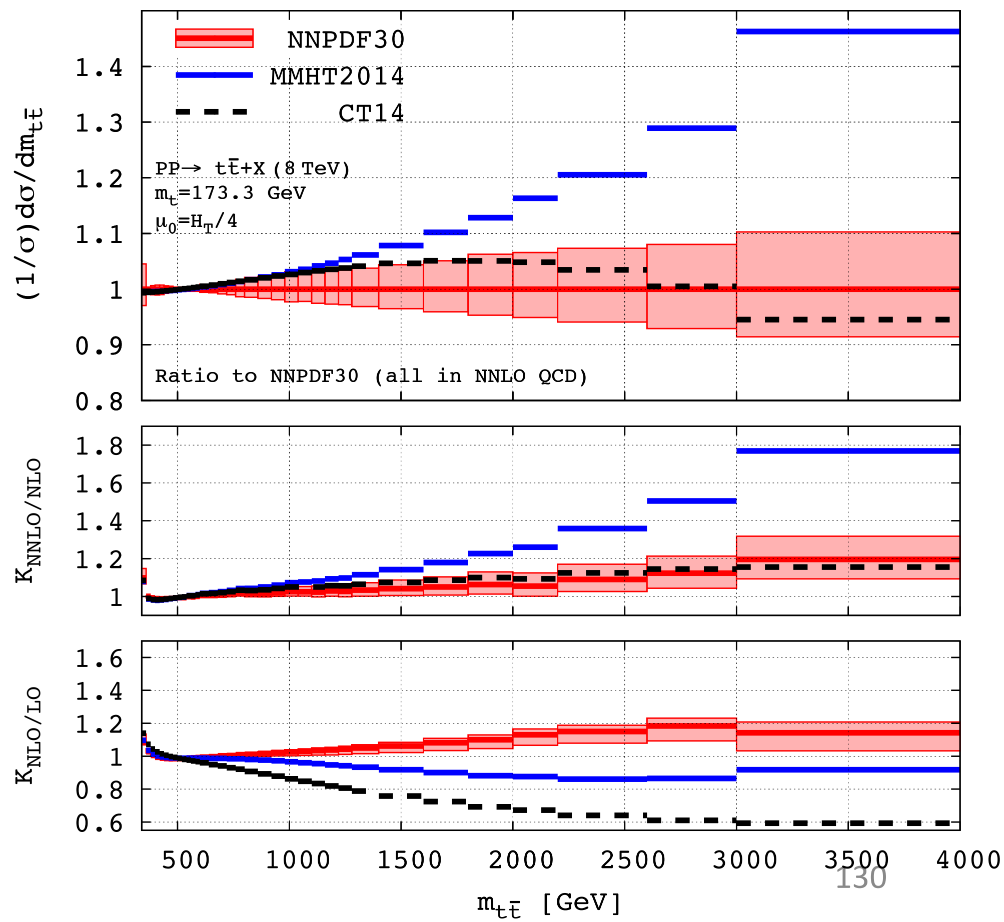
From http://arxiv.org/pdf/1606.03350.pdf



Top-pair production

- Top pair production is now under good control all the way into the TeV regime
- One could actually predict the relevant M_{tt} behaviour below 1 TeV with around O(1%)!
- Main restrictive factor for the future?
  - PDF – this will be a major concern for the future!
  - Possibly even m_{top}





Top-pair production: top-mass measurement

For more details see arXiv:1310.0799

- **The fate of the Universe might depend on 1 GeV in M_{top}!**

Cosmological implications:

Bezrukov, Shaposhnikov '07-'08
De Simone, Hertzbergy, Wilczek'08

- Higgs Inflation: Higgs = inflaton

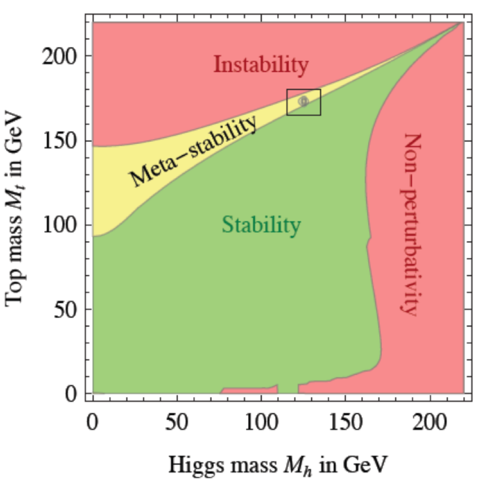
$$\mathcal{L}_h = -|\partial H|^2 + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2 + \xi H^\dagger H \,\mathcal{R}$$

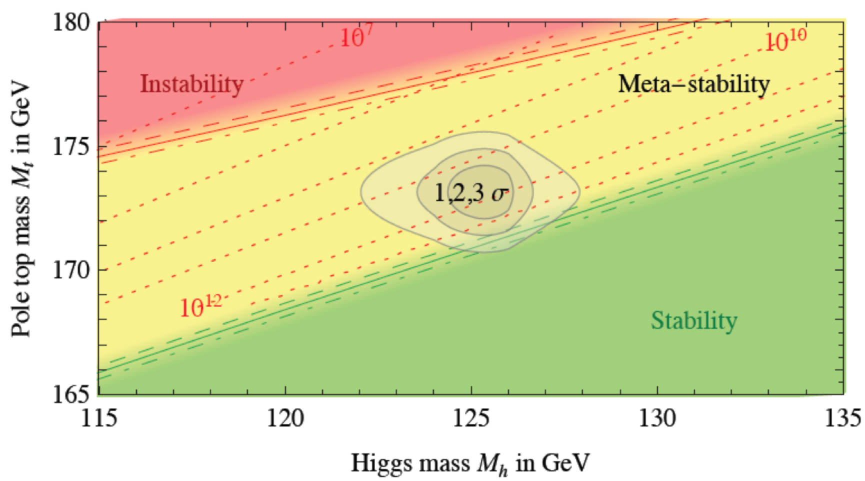
$$m_h > 125.7\,\mathrm{GeV} + 3.8\,\mathrm{GeV}\left(\frac{m_t - 171\,\mathrm{GeV}}{2\,\mathrm{GeV}}\right) - 1.4\,\mathrm{GeV}\left(\frac{\alpha_s(m_Z) - 0.1176}{0.0020}\right) \pm \delta$$

Strong dependence on the top mass!

- Higgs mass and vacuum stability in the Standard Model at NNLO.

Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia '12



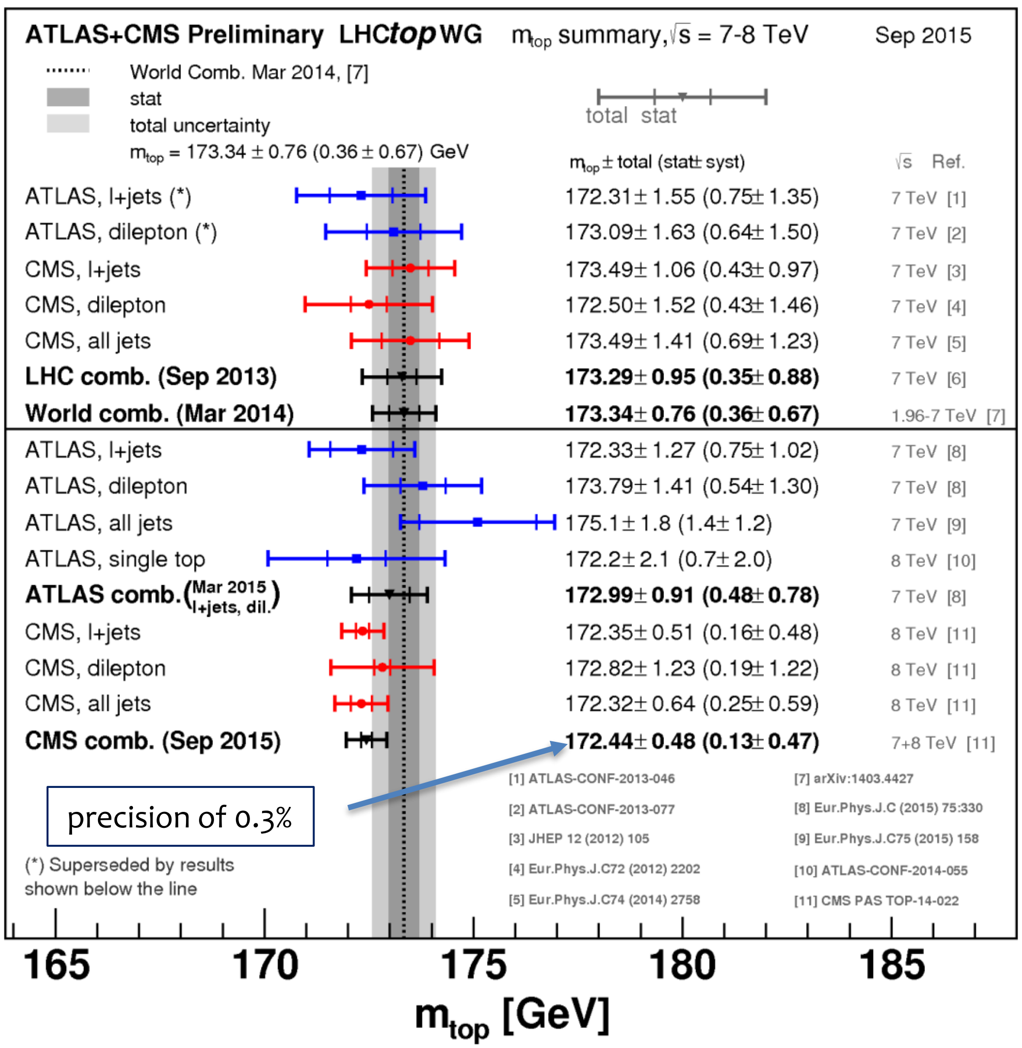


Instability scale Λ [GeV]
δM_{top} is the dominant uncertainty!

- And the current status:
  - Thanks to Javier Fernandez (LHCP2016, Lund Sweden)

CMS + ATLAS m_{top} (MC)

LHCTOPWG



Analysis combined using BLUE, accounts for correlations between all uncertainties.

CMS combination :

m_{top} = 172.44 ± 0.48 GeV

ATLAS combination :

(OLD) m_{top} = 172.99 ± 0.91 GeV

(NEW) m_{top} = 172.84 ± 0.70 GeV

(not in the combination plot)

World combination:

m_{top} = 174.34 ± 0.76 GeV

Total uncertainty is now well below 1 GeV