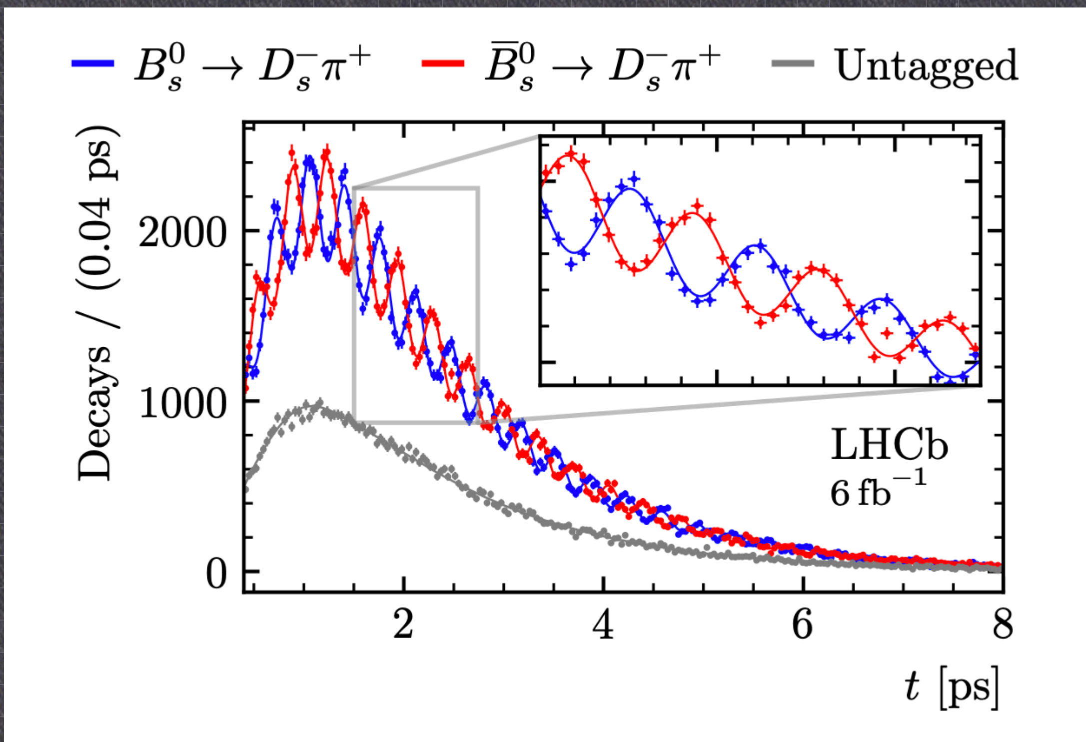
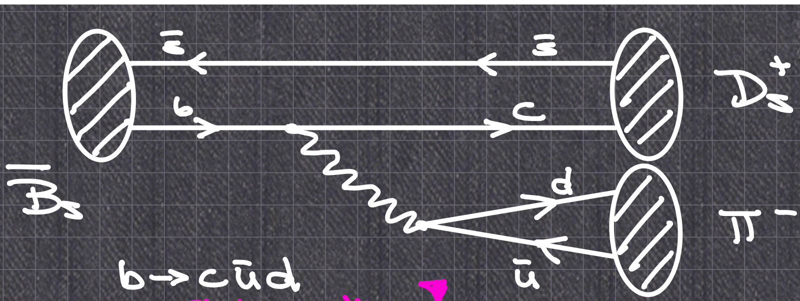
Lecture 3: B & D mixing





$b \to c\bar{u}d$

CKM leading!

Mixing in Particle physics

Mass eigenstate $\neq$ Interaction
= physical eigenstate eigenstates

a) Neutrino oscillations

b) V_{CKM}

c) Weak Boson $\underbrace{SU(2)_L}_{W_1, W_2, W_3} \times \underbrace{U(1)_Y}_{B}$

W^+, W^- Z^0, A
↳ $\sin\Theta_W$

d) Neutral Mesons

1955 K^0 ⇢ Regeneration

1986 ΔM_{B_d}

2006 ΔM_{B_s}

2012 $\Delta \Gamma_{B_s}$

2007...12 $\Delta \Gamma_D$

$\forall$ $\Delta\Gamma_{B_d}$, ΔM_D

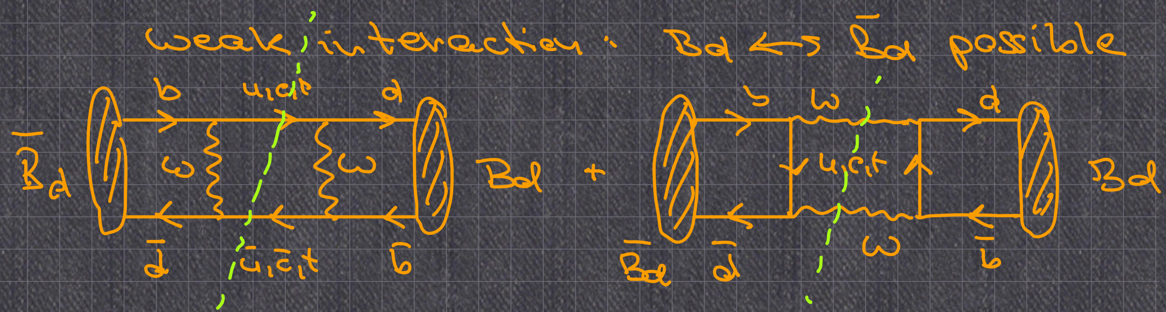
Neutral Mesons:

B_d meson : time evolution

$$\overset{(-)}{B}_d(t) = e^{-iM_d t}\, e^{-\frac{1}{2}\Gamma_d t}$$

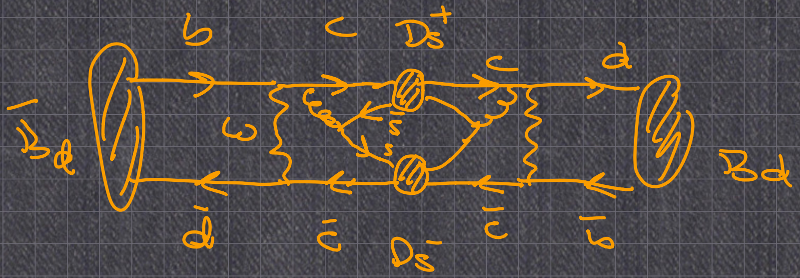
$$\Leftrightarrow \quad i\hbar\partial_t \begin{pmatrix} B_d \\ \bar{B}_d \end{pmatrix} = \begin{pmatrix} M_{11}-\frac{i}{2}\Gamma_{11} & \cancel{0} \\ \cancel{0} & M_{22}-\frac{i}{2}\Gamma_{22} \end{pmatrix} \begin{pmatrix} B_d \\ \bar{B}_d \end{pmatrix}$$

weak interaction: $B_d \leftrightarrow \bar{B}_d$ possible



M_{12}: off-shell internal particle : $W^{\pm}$, t, u, c

Γ_{12}: on-shell internal particle : – – u, c



Mixing Matrix
$$\begin{pmatrix} M_{11}-\frac{i}{2}\Gamma_{12} & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{21}-\frac{i}{2}\Gamma_{21} & M_{22}-\frac{i}{2}\Gamma_{22} \end{pmatrix}$$

$\rightarrow$ non-diagonal mass & decay rate matrix

$\Rightarrow B_d, \bar{B}_d$ are not mass eigenstates

Diagonalise Mixing Matrix

$B_{d,H} = p\, B_d - q\, \bar{B}_d$ H: Heavy

$B_{d,L} = p\, B_d + q\, \bar{B}_d$ L: Light

$\Delta M_d = M_H - M_L = \Delta M_d (M_{12}, \Gamma_{12})$

$\Delta \Gamma_d = \Gamma_L - \Gamma_H = \Delta \Gamma_d (M_{12}, \Gamma_{12})$

in B-system: $|\Gamma_{12}| \ll |M_{12}|$

(D-system: $\Delta\Gamma \simeq \Delta M$)

- $\Delta M_d \simeq 2 |M_{12}^d| + \mathcal{O}\left(\frac{\Gamma_{12}^2}{M_{12}^2}\right)$; $\frac{\Gamma_{12}}{M_{12}} \simeq 5 \cdot 10^{-3}$
- $\Delta \Gamma_d \simeq 2 |\Gamma_{12}^d| \cos \phi_{12}^d$

$\phi_{12}^d = \mathrm{Arg}\left(- \frac{M_{12}^d}{\Gamma_{12}^d}\right)$ very small in SM

Time evolution of B-mesons:

$$|B_{d\,H,L}(t)\rangle = e^{-\left(i M_{H,L}^d + \frac{\Gamma_{H,L}^d}{2}\right)t} |B_{d\,H,L}(0)\rangle$$

$$\underbrace{|B_d(t)\rangle}_{\text{Interaction ES}} = g_+(t)\, |B_d^0\rangle + \frac{q}{p} g_-(t)\, |\bar{B}_d^0\rangle$$

$$\Rightarrow g_+(t) = e^{-i M_{B_d} t}\, e^{-\frac{i \Gamma_{B_d} t}{2}} \Big[\cosh \frac{\Delta\Gamma_d \cdot t}{4} \cdot \cos \frac{\Delta M_d t}{2} - \sinh \frac{\Delta\Gamma_d t}{4} \cdot \sin \frac{\Delta M_d t}{2} \Big]$$

Experimental results

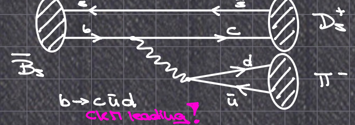
ΔM	ps^{-1}	eV	$\frac{2 \cdot \pi \cdot c}{\Delta M}$ [mm · Bq]
B_s	17.8	0.01	0.1
B_d	0.51	$3 \cdot 10^{-4}$	4
D^0	0.01	$7 \cdot 10^{-6}$	200
K^0	0.005	$3 \cdot 10^{-6}$	360

$\Delta\Gamma$	ps^{-1}
B_s	0.09
D^0	0.03
K^0	0.01
B_d	< 0.007

3rd observable: Flavour specific CP asymm.

FS decay $B^0_q \to f$:
- $\bar{B}^0_q \to f$ forbidden
- $B^0_q \to \bar{f}$ — " —
- no direct $\cancel{CP}$: $\langle f | B \rangle = \langle \bar{f} | \bar{B} \rangle$

Examples: • semi-leptonic



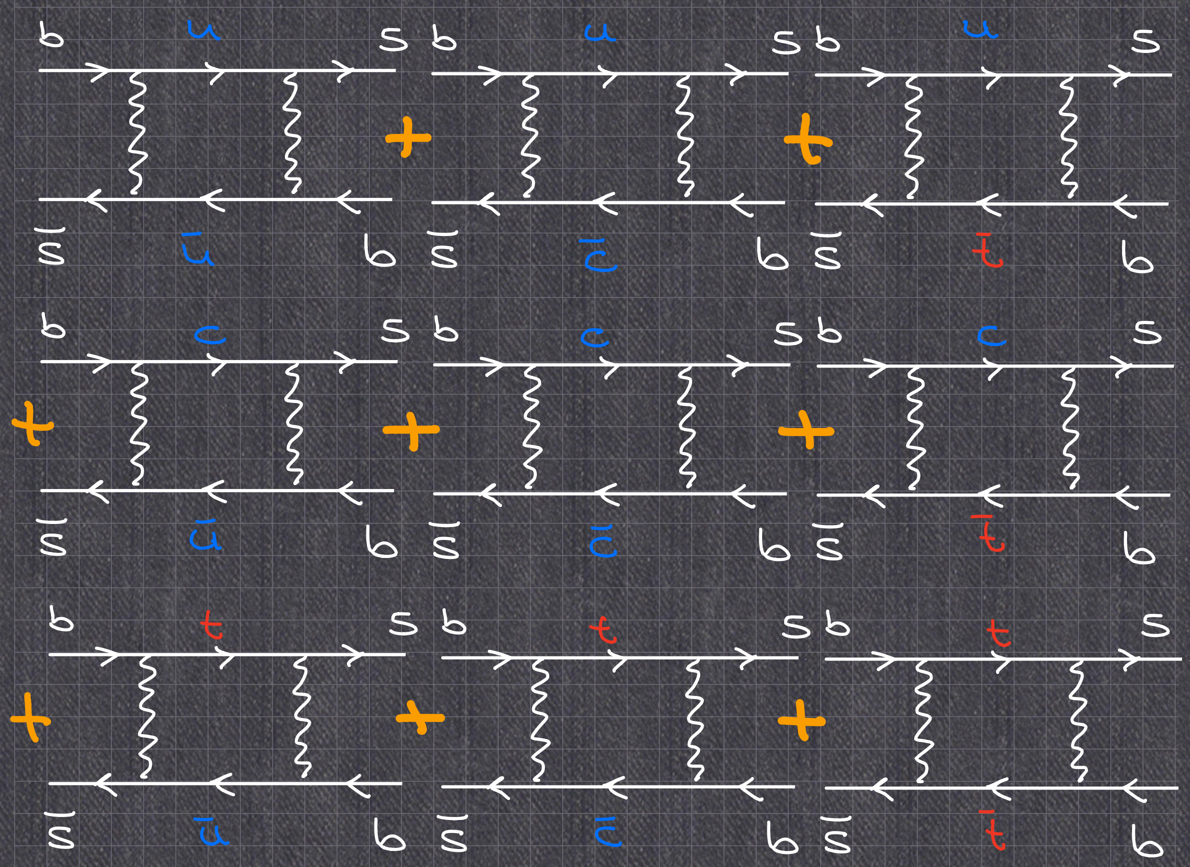
$$a_{fs} = \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to \bar{f})}{-\text{"}- + -\text{"}-}$$

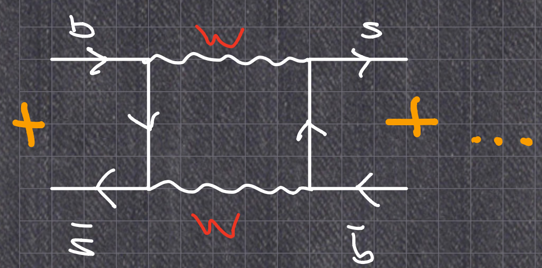
$$= \frac{\Gamma(\bar{B}_q \to B_q \to f) - \Gamma(B_q \to \bar{B}_q \to \bar{f})}{-\text{"}- + -\text{"}-}$$

$= \cancel{CP}$ in mixing $\neq 0$ if $\bar{B} \to B \neq B \to \bar{B}$

$$a_{fs}^q = \underbrace{\left|\frac{\Gamma_{12}^q}{M_{12}^q}\right|}_{5 \cdot 10^{-3}} \cdot \underbrace{\sin \phi_{12}^q}_{\phi_{12}^s \simeq \frac{1}{250}} \rightsquigarrow \underline{a_{fs}^s \simeq 2 \cdot 10^{-5}}$$

How to calculate M_{12} & Γ_{12}?





$$
\begin{aligned}
M_{12} = {} & \lambda_u^2 F(u,u) + \lambda_u \lambda_c F(u,c) + \lambda_u \lambda_t F(u,t) \\
& \lambda_c \lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c \lambda_t F(c,t) \\
& \lambda_t \lambda_u F(t,u) + \lambda_t \lambda_c F(t,c) + \lambda_t^2 F(t,t)
\end{aligned}
$$

		Bs	Bd
CKM:	$\lambda_u = V_{us}^* V_{ub}$	$\sim \lambda^{4.8}$	$\lambda^{3.8}$
	$\lambda_c = V_{cs}^* V_{cb}$	$\sim \lambda^2$	λ^3
	$\lambda_t = V_{ts}^* V_{tb}$	$\sim \lambda^2$	λ^3

(Comment : Wolfenstein $V_{ub} = A\lambda^3(\rho - i\eta)$

$\Rightarrow V_{ub} \sim \lambda^3$

CKMfitter, UTfit : $V_{ub} = \lambda^{3.8}$)

$$\lambda_u + \lambda_c + \lambda_t = 0$$

$$M_{12} = \lambda_u^2 \left[F(c,c) - 2F(u,c) + F(u,u) \right]$$
$$+ 2\lambda_u\lambda_t \left[F(c,c) - F(u,c) + F(u,t) - F(c,t) \right]$$
$$+ \lambda_t^2 \left[F(c,c) - 2F(c,t) + F(t,t) \right]$$

GIM mechanism

- Loop - integral

adding!

$$F(p,q) = \underbrace{f_0}_{\frac{1}{\varepsilon} + \dots} + f(x_q, x_p) \qquad x_q := \frac{m_q^2}{M_W^2}$$

$\rightsquigarrow$ f_0 will cancel exactly !

$\rightsquigarrow$ $x_u = 7.2 \cdot 10^{-10} \simeq 0$

$x_c = 2.4 \cdot 10^{-4} \simeq 0$

$x_t = 4 \simeq 4$

$$\Rightarrow \boxed{M_{12} \sim \lambda_t^2 \left[\underbrace{F(0,0) - 2F(0,t) + F(t,t)}_{S(x_t):\ \text{Inami-Lim function}} \right]}$$

$$S(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x \ln x}{2(1-x)^2}$$

$$\Rightarrow \frac{\Delta M_d}{\Delta M_s} = \frac{\cancel{2}|M_{12}^d|}{\cancel{2}|M_{12}^s|} = \frac{|\lambda_t^2|\,f(B_d)}{|\lambda_t^2|\,B_s)} \frac{\cancel{|S(x_t)|}}{\cancel{|S(x_t)|}} \simeq \frac{|V_{td}|^2}{|V_{ts}|^2}$$

$\frac{\Delta M_d}{\Delta M_s} \rightsquigarrow 0.0285$ $\qquad$ $\frac{|V_{td}|^2}{|V_{ts}|^2} \rightsquigarrow 0.044$

The full SM expression reads:

$$M_{12}^q = \frac{G_F^2}{12\pi^2} \lambda_t^2 \, M_W^2 \, S(x_t) \, \underbrace{f_{B_q}^2 B_{B_q}} \cdot \underbrace{\hat\eta_B}$$

$\hat\eta_B \to$ 2 loop correction

pert. 0.84

B  $\bar B$

$$\langle \bar B_q | (\bar q b)_{V-A} (\bar q b)_{V-A} | B_q \rangle = \frac{8}{3} f_{B_q}^2 B_{B_q} M_{B_q}^2$$

$f_{B_q} \to$ Decay constant (lattice)

$B_{B_q} \to$ Bag parameter (lattice, SR)

A huge theory progress up to 2019

$$\Delta\Gamma_s = \left(18.4^{+0.7}_{-1.2}\right) ps^{-1} \dashrightarrow 17.757(21)$$

$$\Delta\Gamma_d = \left(0.533^{+0.022}_{-0.036}\right) ps^{-1} \dashrightarrow$$

$$\frac{\Delta\Gamma_d}{\Delta\Gamma_s} = 0.0298^{+0.005}_{-0.009}$$

Lecture 4

BSM

vs.

Flavour

BSM searches:

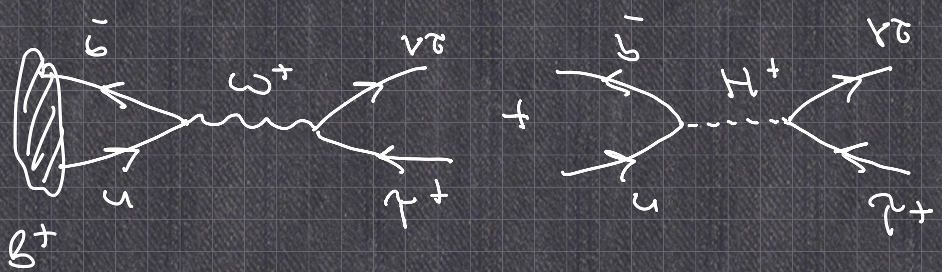
a) Model dependent

→ Guess/choose a BSM model

→ Calculate Flavour observable in this model

2HDM: SM particles + H^+, H^-, H^0, A^0

$B^+ \to \tau^+ \nu$



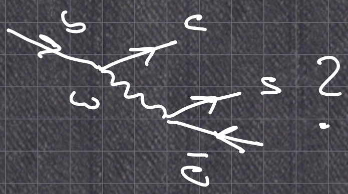
$$\underbrace{Br^{Exp}(B^+ \to \tau^+ \nu_\tau)}_{\text{known}} = \underbrace{Br^{SM}(\ldots)}_{\text{known}} + Br^{2HDM}(m_{H^+}, \underbrace{\tan\beta}_{\text{new Higgs couplings}})$$

⇒ bounds on m_{H^+}, $\tan\beta$

$\underline{b \to s\gamma}$: $m_{H^+} > 200$ GeV

b) Model independent:

① Can there be NP in tree-level decays



?

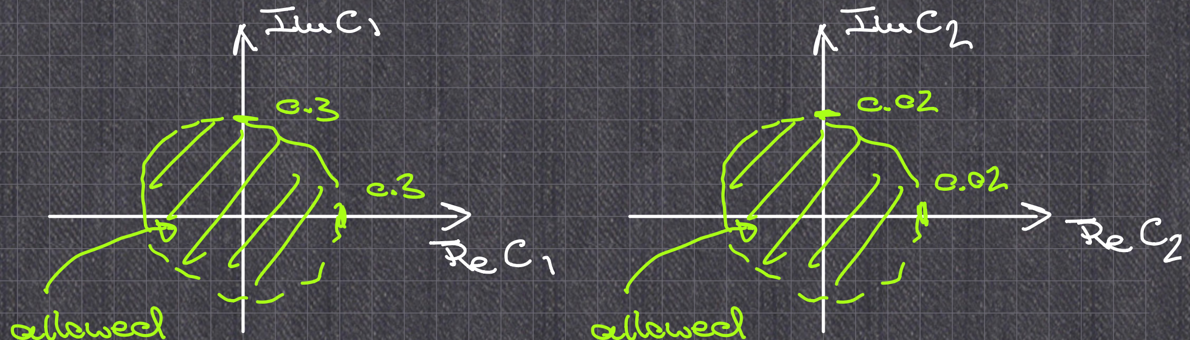
$\mathcal{H}_{eff}^{SM} = \frac{G_F}{\sqrt{2}} \left(C_1 Q_1 + C_2 Q_2 + \text{penguins} \right)$

$C_1 = C_1^{SM} + \Delta C_1$

$C_2 = C_2^{SM} + \Delta C_2$

$\Delta C_i \in \mathbb{C}$

Do all possible bounds on ΔC_i



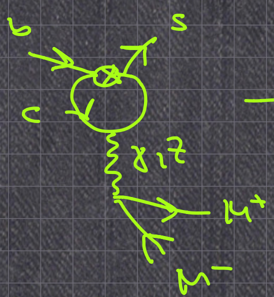
⇝ → What does $\text{Im}\, C_1 = 0.3$ mean?

γ^{CKM}-extraction could be modified by 5° ?

⇝ BSM effects in $b \to c\bar{c}s$

new effects:

→ contribution to mixing, $\Delta\Gamma_s$

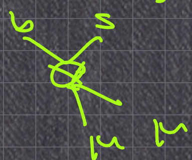


→ contribution to Flavour anomalies

$b \to c\bar{u}d$
CKM leading!

QCD factorisation predictions for $B_s \to D_s^+ \pi^-$ and friends deviate by up to 5.6 σ from Experiment

⇝ B-anomalies Q_9^{sl}, Q_{10}^{sl}

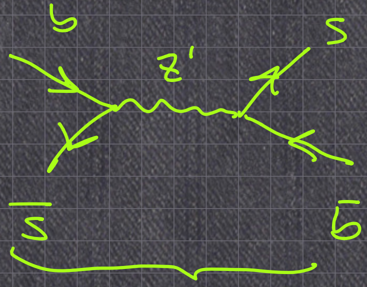


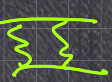
What BSM models could describe the anomalies

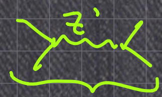
I) Z' models

II) Lepto-quark

$\Longrightarrow$



ΔM_s^{Exp} = 

\+ 

$17.757\ ps^{-1}$

$18\ ps^{-1}$ [uncertainty] ≥ 0

The end!