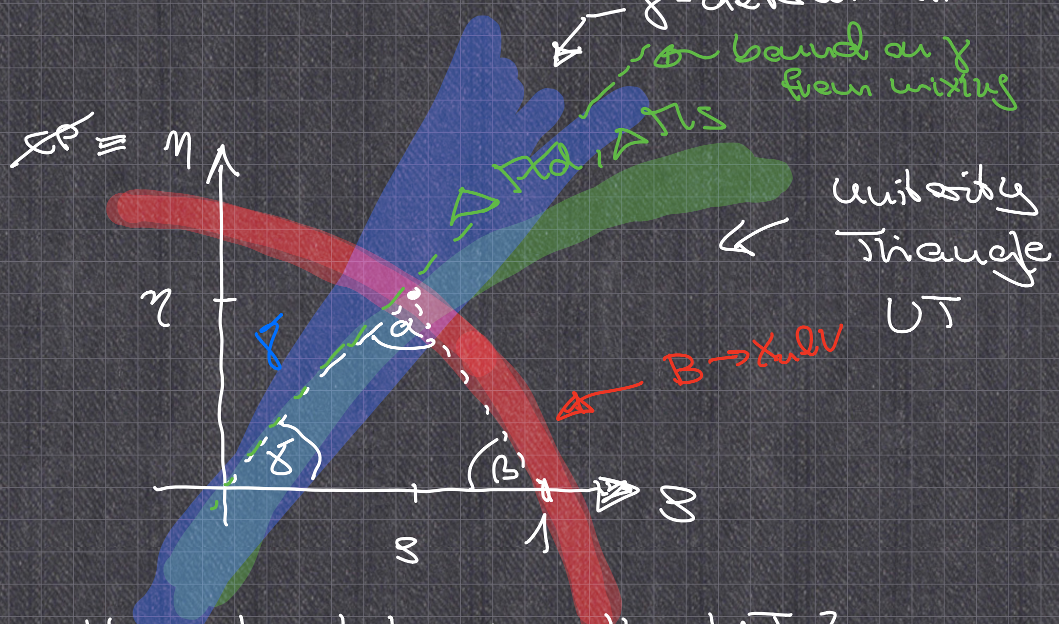
$$0 = V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^*$$

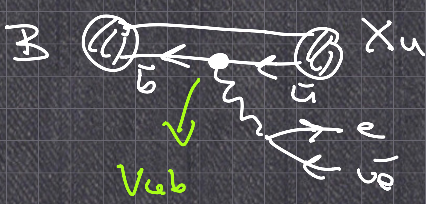
Wolfenstein

$$= A\lambda^3\left[\left(1-\frac{\lambda^2}{2}\right)(\rho+i\eta) - 1 + \left(1-(\rho+i\eta)\right)\right]$$



How to determine the UT?

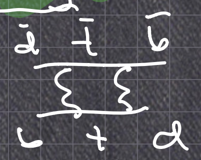
(1) $\underbrace{Br[B \to X_u e \bar{\nu}]}_{\text{Exp}\checkmark} \sim |V_{ub}|^2 \underbrace{\cdots\cdots}_{\text{Theory}\checkmark}$



$\sim \underbrace{(\rho^2+\eta^2)}\ \underbrace{\cdots\cdots}_{\text{Theory}}$

circle around (0,0)

② B-mixing

$\Delta M_d \sim$  $\sim |V_{td}|^2 \underbrace{\cdots\cdots}_{\text{theory}}$

$\sim \underbrace{(\rho-1)^2+\eta^2}_{} \underbrace{\cdots\cdots}_{\text{theory}}$

⇓ circle around (1,0)

Current status of Flavour Physics:

① CKM-picture confirmed ⇒ NP 2008

increase presision
⇒ some problems are arising

* $V_{cb}^{\text{incl.}} = (42.19 \pm 0.78)\cdot 10^{-3}$

  $V_{cb}^{\text{excl.}} = (38.79 \pm 0.69)\cdot 10^{-3}$

* unitarity of first row

  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 \neq 0 \quad 3\sigma$

  $\boxed{2101.02706}$

② $\not{CP}$
→ large $\not{CP}$ effect in B-system

$B_d \to J/\psi\, K_S \Rightarrow \sin 2\beta = 0.699(17)$
$\sim 50\%$

$\rightarrow \gamma^{\text{LHCb'20}} = (67 \pm 4)^\circ$ LHCb-Conf-2020-003

$\gamma^{\text{B-mixing}} \leq 66.9^\circ$ (at 5σ)

$\rightarrow$ B-

$\rightarrow$ 2019

Direct $\cancel{CP}$ in D-decays: ΔA_{CP}

tiny effect ~ ‰ and smaller

③ Indirect searches for BSM effects

$\rightarrow$ Impressive confirmation of SM at quantum level 𝛾

$B_s \rightarrow \mu^+\mu^- \longleftrightarrow Br \quad 3 \cdot 10^{-9}$

$B \rightarrow X_s \gamma$

$\rightarrow B_d \rightarrow K^* \mu^+ \mu^-$

Flavour Anomalies

1. Br fractions: $Br(B_d \rightarrow K^* \mu^+ \mu^-)$

$\rightarrow$ depends on form factor ☹

2. Ratios: P_5'

$\rightarrow$ part of the form factor dependence cancel

☺

③ $R_K = \frac{Br(B \to K \mu^+\mu^-)}{Br(B \to K e^+e^-)} = 1 + \varepsilon$

tiny

03/21 : $R_K = 0.846^{+0.044}_{-0.041}$ $\quad$ $\underline{3.1\sigma}$

④ Tree-level

- $R_D = \frac{B \to D \tau \nu}{B \to D \mu \nu}$

- $B_s \to D \pi^-$

$\rightsquigarrow$ combined fit of $\underbrace{① \ldots ③}_{250 \text{ obs.}}$

$\Rightarrow \underline{> 7\sigma}$

④ QCD

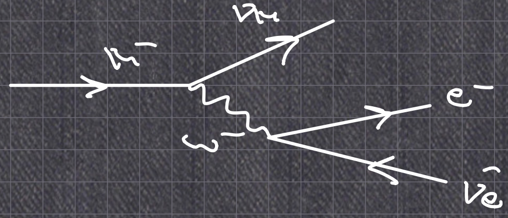
- perturbative NLO, NNLO, NNNLO
- lattice $\to$. decay constant
- sum rules . formfactors
  . B-mixing
  B-lifetimes

Lecture 2:

$\mathcal{H}_{eff}$, ...

Myon-decay!

$\longrightarrow$ [Fr. 1–3 pm
Zoom



$$\tau_\mu = \frac{1}{\Gamma_{tot}}$$

$$\Gamma_{tot} = \frac{G_F^2\, m_\mu^5}{192\pi^3}\, f\left(\frac{m_e}{m_\mu}\right)$$

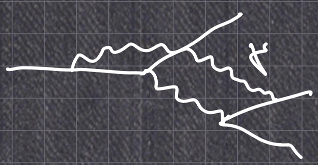
- $G_F = \dfrac{g^2}{4\sqrt{2}\, M_W^2}$ $\rightsquigarrow$ Fermi constant

- $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$

$$\Rightarrow \tau^{Theo\,1} = 2.18776 \cdot 10^{-6}\,s$$

$$\tau^{Exp} = 2.1969811(22) \cdot 10^{-6}\,s$$

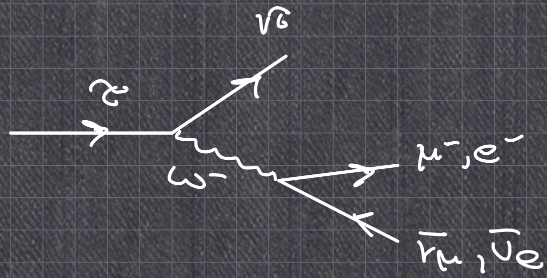
Add higher order corrections

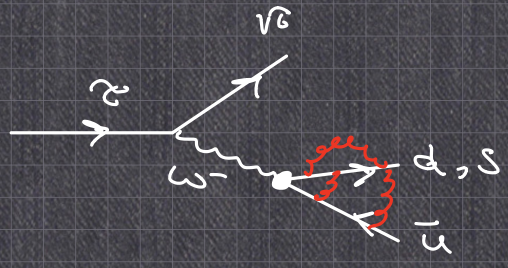


$$1 \longrightarrow 1 + \frac{\alpha}{4\pi} \cdot 2\left(\frac{25}{4} - \pi^2\right)$$

$$\Rightarrow \tau^{Theo\,2} = 2.19699 \cdot 10^{-6}\,s$$

Tau-decay:





$$\Gamma_{tot} = \frac{G_F^2 m_\tau^5}{192\pi^3}\left[f\left(\frac{m_e}{m_\tau}\right) + f\left(\frac{m_\mu}{m_\tau}\right) + N_c |V_{ud}|^2 g\left(\frac{m_u}{m_\tau}, \frac{m_d}{m_\tau}\right) + N_c |V_{us}|^2 g\left(\frac{m_u}{m_\tau}, \frac{m_s}{m_\tau}\right)\right]$$

$$\left([\ldots] \simeq 5 \right)$$

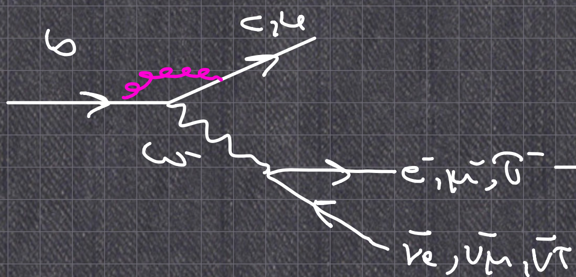
$$\tau_\tau^{\text{Theory}} = 3.267 \cdot 10^{-13}\,s$$

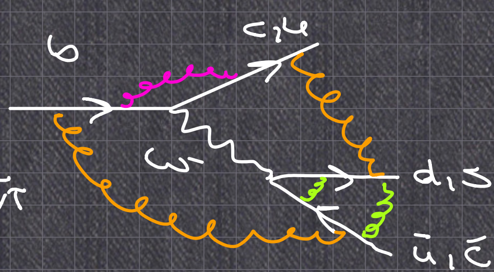
$$\tau_\tau^{\text{Exp}} = 2.906(1) \cdot 10^{-13}\,s$$

$\Longrightarrow$ QCD effects are important

$\Longrightarrow \alpha_s(m_\tau)$

b-quark decay:





$$\Gamma_{tot}^b = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \cdot \ldots$$

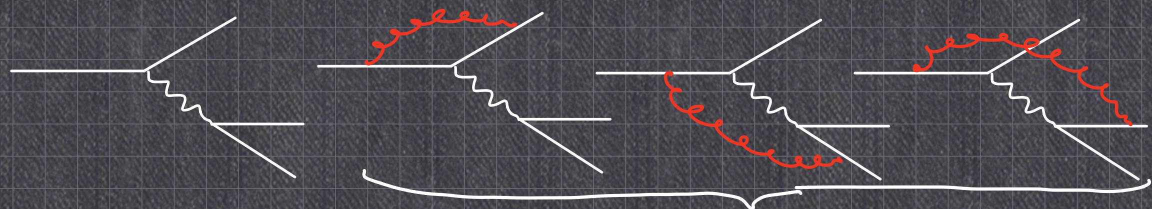
$\downarrow$

small

$\tau_b^{\text{Theo}} = 0.90 \ldots 3.7$ ps

- QCD will be crucial
- Definition of a quark mass

include QCD corrections to b-decay



Tree

$\downarrow$

$O(1)$

1-loop

$\downarrow$

$O(\alpha_s(m_b) \simeq 0.2)$

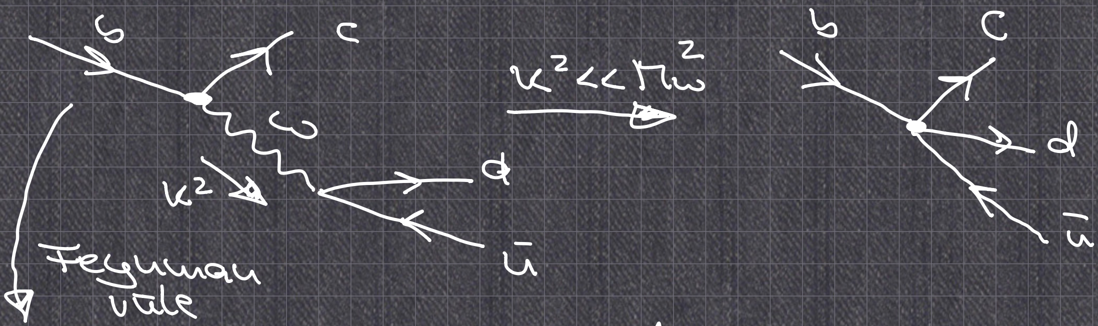
$\alpha_s(M_Z) = 0.1$

do the loop diagram: $\underbrace{\alpha_s(m_b)}_{0.2} \ldots \to \underbrace{\alpha_s(m_b) \ln \frac{m_b^2}{M_W^2}}_{1.2} \ldots$

Tree	1			
1-loop	$\alpha_s \cdot \ln$	α_s		
2-loop	$\alpha_s^2 \cdot \ln^2$	$\alpha_s^2 \ln$	α_s^2	
3-loop	$\alpha_s^3 \ln^3$	$\alpha_s^3 \ln^2$	$\alpha_s^3 \ln$	α_s^3
$\vdots$	$\vdots$			

The effective Hamiltonian

Integrate out W



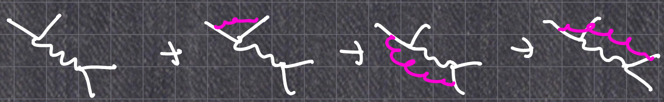
$$= \bar{c}\,\frac{i g_2}{2\sqrt{2}} V_{cb}^* \gamma_\mu (1-\gamma_5) b \cdot \frac{1}{k^2 - M_W^2} \cdot \bar{d}\,\frac{i g_2}{2\sqrt{2}} \gamma^\mu (1-\gamma_5) u$$

$$\xrightarrow{k^2 \ll M_W^2} \underbrace{\left(\frac{g_2}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2}}_{\frac{G_F}{\sqrt{2}}} \underbrace{V_{cb}^* V_{ud}}_{V_{CKM}} \cdot \underbrace{1}_{C_2} \cdot \underbrace{\bar{c}\gamma_\mu (1-\gamma_5) b \cdot \bar{d}\gamma^\mu (1-\gamma_5) u}_{Q_2}$$

$\mathcal{H}_{eff} =$

C_2 → Wilson coefficient

Q_2 → 4-quark operator

→ Add QCD: 

① New operator arises:

Q_2 : is a colour singlet

$$(\bar{c}^\alpha b^\alpha)_{V-A} (\bar{d}^\beta u^\beta)_{V-A}$$

Q_1 : colour rearranged

$$(\bar c^{\alpha} b^{\beta})_{V-A} \, (\bar d^{\beta} u^{\alpha})_{V-A}$$

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \left(C_1 Q_1 + C_2 Q_2 \right)$$

$\rightarrow C_2 = 1 + \mathcal{O}(\alpha_s) = 1.1$

$\rightarrow C_1 = 0 + \mathcal{O}(\alpha_s) = -0.2$

(2)

	LL Leading Log-approx	NLL	NNLL	
Tree	1			
1-loop	$\alpha_s \cdot \ln$	α_s		
2-loop	$\alpha_s^2 \cdot \ln^2$	$\alpha_s^2 \ln$	α_s^2	
3-loop	$\alpha_s^3 \ln^3$	$\alpha_s^3 \ln^2$	$\alpha_s^3 \ln$	α_s^3
$\vdots$	$\vdots$			

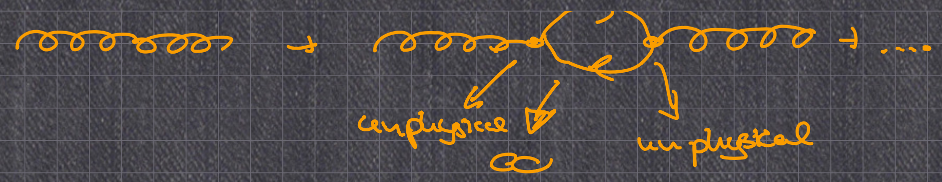
(3) Separation of LD & SD

↳ long distance   ↳ short distance

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \left[C_1(\mu) \, Q_1 + C_2(\mu) Q_2 \right]$$

μ: renormalisation scale

Loop integrals are in general divergent



Dimensional regularisation

spacetime $\quad$ 4-dim $\longrightarrow$ $D = 4 - 2\varepsilon$ dim.

loop integral $\quad \infty \longrightarrow \frac{1}{\varepsilon}$ + const.

coupling $\quad g \longrightarrow g\mu^{\varepsilon}$

↳ arbitrary mass scale

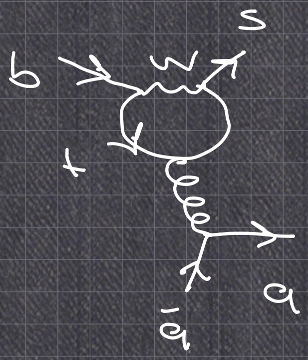
separating scale μ

short distances (large energies)	long distance (low energies)
$C_i(\mu)$	$\langle K\pi \vert Q_i \vert B\rangle(\mu)$

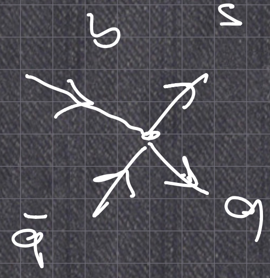
→ technical details $\mathcal{H}_{eff}$ ① ||

→ B-mixing / BSM searches ② ~~||||~~ |

at 1 loop



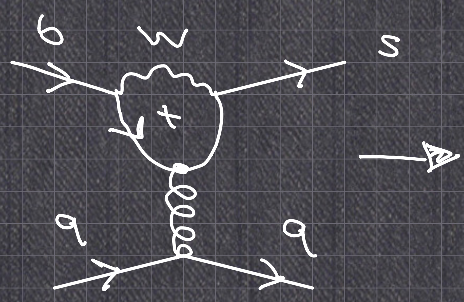
$\xrightarrow[W,\, t]{\text{integrate out}}$



86-
88

Penguin operators

QCD penguin operators



$$Q_3 = (\bar{s} b)_{V-A} \sum_{q=u,\dots,b} (\bar{q} q)_{V-A}$$

$$Q_4 = (\bar{s}^{\alpha} b^{\beta})_{V-A} \sum_{q=u,\dots,b} (\bar{q}^{\beta} q^{\alpha})_{V-A}$$

$$Q_5 = (\bar{s} b)_{V-A} \sum_{q=u,\dots,b} (\bar{q} q)_{V+A}$$

$$Q_6 = (\bar{s}^{\alpha} b^{\beta})_{V-A} \sum_{q=u,\dots,b} (\bar{q}^{\beta} q^{\alpha})_{V+A}$$

Electro-weak penguin operators

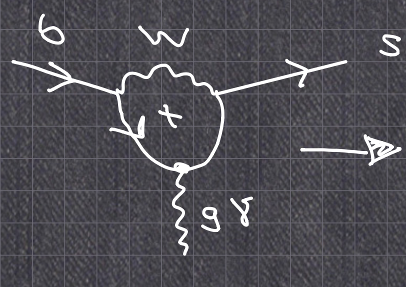
$$Q_7 = \frac{3}{2} (\bar{s} b)_{V-A} \sum_{q=u,\dots,b} e_q (\bar{q} q)_{V+A}$$

$$Q_8 = \frac{3}{2} (\bar{s}^{\alpha} b^{\beta})_{V-A} \sum_{q=u,\dots,b} e_q (\bar{q}^{\beta} q^{\alpha})_{V+A}$$

$$Q_9 = \frac{3}{2} (\bar{s} b)_{V-A} \sum_{q=u,\dots,b} e_q (\bar{q} q)_{V-A}$$

$$Q_{10} = \frac{3}{2} (\bar{s}^{\alpha} b^{\beta})_{V-A} \sum_{q=u,\dots,b} e_q (\bar{q}^{\beta} q^{\alpha})_{V-A}$$

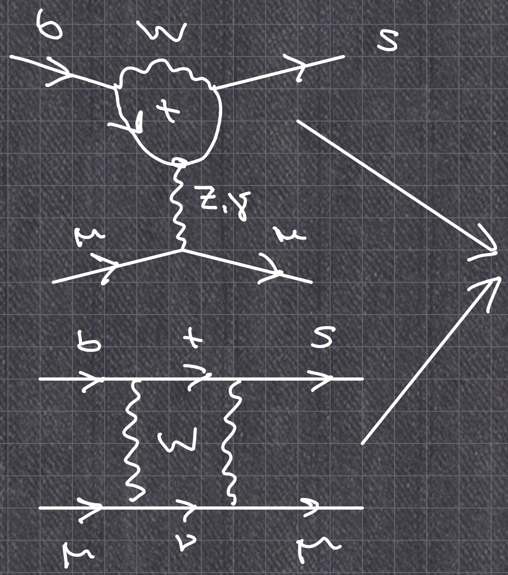
Magnetic penguin operators



$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma_5) b_\alpha F_{\mu\nu}$$

$$Q_{8g} = \frac{g}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} (1+\gamma_5) T^a_{\alpha\beta} b_\beta G^a_{\mu\nu}$$

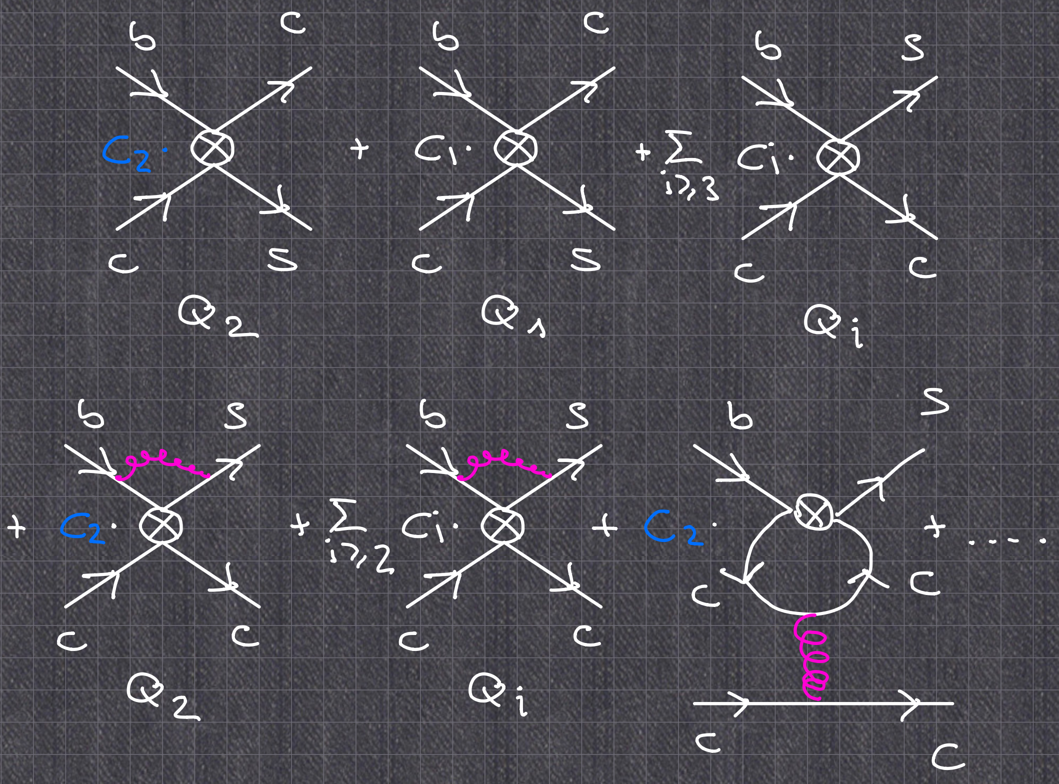
Semileptonic penguin operators



$$Q_{9V} = (\bar{s} b)_{V-A} (\bar{\mu} \mu)_V$$

$$Q_{10A} = (\bar{s} b)_{V-A} (\bar{\mu} \mu)_A$$

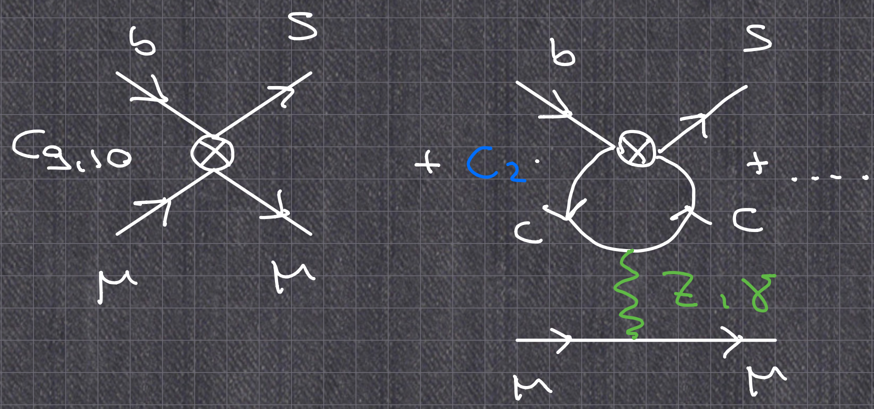
$$b \to c\bar{c}s$$



suppressed by α_s

C_2 : large Wilson coefficient

$$b \to s\, \mu^+ \mu^-$$



C_2 : large Wilson coefficient