Matching NLO QCD with Parton Showers
Motivation

Monte Carlo (MC) approximation to QCD dynamics and not from fixed-order calculations.

However, these models operate on partonic states with high multiplicity and

For the description of exclusive hadronic final states, perturbative calculations

usually truncated at next-to-leading order (NLO).

All systematic approaches to this problem are based on perturbation theory,

collider experiments and new particle searches.

process is important not only as a test of QCD but also for the design of

Reliable prediction of cross sections and final-state distributions for QCD
MC hadronization modules are adopted.

The matching between hard- and soft-emission regions is smooth.

Soft/collinear emissions are treated as in MC.

Hard emissions are treated as in NLO computations.

Results in NS.
\[
\frac{\text{e}^{-x} + x}{(0)OB - (x)\mathcal{R}(x)O} \mathcal{A} \int_{x=2}^{x} v + \frac{x}{\text{e}^{-x}} \mathcal{A} \int_{x=2}^{x} (0)OB^v = \mathcal{A}\langle O \rangle
\]

In subtraction method, real contribution is written as:

\[
\mathcal{B} = (x)\mathcal{R} \mathcal{A} \mathcal{I} \text{m} \mathcal{B} \text{ is complicated constant, and } \text{Im} \mathcal{B} \text{ is complicated constant, and } \lim_{x \to \infty} \mathcal{B} = \mathcal{B}_{\text{Born}, \text{Born}} \mathcal{B} = \left( \frac{xp}{\omega p} \right)
\]

where Born, virtual and real contributions are respectively:

\[
\left[ \mathcal{R} \left( \frac{xp}{\omega p} \right) + \mathcal{I} \left( \frac{xp}{\omega p} \right) + \mathcal{V} \left( \frac{xp}{\omega p} \right) \right] (x)O_{\text{e}^{-x}} \mathcal{A} \mathcal{I} \mathcal{m} \int_{x=2}^{x} = \langle O \rangle
\]

Computing the quantity

Task of predicting an infrared-safe observable to NLO amounts to \( O \) observable to NLO of MCQC, and of matching between the two.

Consider first a toy model that allows simple discussion of key features of

Toy Model
\[
\left[ \left( \frac{x}{B^p} - \Lambda^p + B \right) (0)O + \frac{x}{(x)\mathcal{H}^p} (x)O \right] \, xp \int_0^1 \mathbf{q}^p \langle O \rangle
\]

We rewrite this in a slightly different form:

\[
\left[ \frac{x}{(0)OB - (x)\mathcal{H}(x)O} \right] \, xp \int_0^1 \mathbf{q}^p \langle O \rangle + (0)O \Lambda = \mathbf{q}^s \langle O \rangle
\]

Therefore NTO prediction is:

\[
\frac{x}{(0)OB - (x)\mathcal{H}(x)O} \, xp \int_0^1 \mathbf{q}^p \langle O \rangle + (0)O \frac{\mathcal{E}}{B} \mathbf{p} - = \mathbf{u} \langle O \rangle
\]

Second integral is non-singular, so we can set \( e = 0 \).
\[ \forall x \geq x \geq 1 \]

is the probability that no photon be emitted with energy \( x \) such that

\[ 0 = (x) \hat{\Theta} \uparrow \frac{1}{x} \quad \text{and} \quad 1 = (x) \hat{\Theta} \downarrow \frac{1}{x} \quad \text{and} \quad 1 \geq (x) \hat{\Theta} \geq 0 \]

where \( (x) \hat{\Theta} \) is a monotonic function with the following properties:

\[ \left[ \frac{x}{(x) \hat{\Theta}} \int_{z \hat{\Theta}}^{1/x} \right. \right. \left. \left. \mathrm{d}x \right] = (z, x) \nabla \]

Thus the toy factor function defined for our toy model as follows:

In a treatment based on Monte Carlo methods, the system can undergo an

**Toy Monte Carlo**
\[
\left[ \frac{x}{(x)\partial B - (x)\mathcal{H}} \right]_0^\infty (x)^{\omega_{\mathcal{W}}} (1) \mathcal{L} + \left( \frac{x}{[1 - (x)\partial] B \mathcal{L} + \Lambda \mathcal{L} + \mathcal{B}} \right) \Omega^w_0 \mathcal{L} \right] xp \int_1^{0} = \text{NLO functional}
\]

We must subtracst this from second term, and add to first:

\[
\frac{x}{(x)\partial B} = \frac{x}{x} \mathcal{L} = \left( \frac{x}{\omega \mathcal{L}} \right)
\]

This is wrong: MC starting with no emissions will generate emission, with NLO distribution.

Modified subtraction:

\[
\left[ \frac{x}{(x)\partial B} (x)^{\omega_{\mathcal{W}}} (1) \mathcal{L} + \left( \frac{x}{B \mathcal{L} - \Lambda \mathcal{L} + \mathcal{B}} \right) \Omega^w_0 \mathcal{L} \right] xp \int_1^{0}
\]

So that overall generating functional is

\[
(x)^{\omega_{\mathcal{W}}} (1) \mathcal{L} \left\{ \text{Start MC with } 1 \text{ emission at } x \right\} \Leftrightarrow (x)\mathcal{O}
\]

\[
\Omega^w_0 \mathcal{L} \left\{ \text{Start MC with } 0 \text{ real emissions} \right\} \Leftrightarrow (0)\mathcal{O}
\]

We want to interface NLO to MC, Naive first try:
Note, however, that some events may have negative weight.

These resummation as \( \mathcal{L}^{(0)} \), renormalised to correct \( N\text{LO} \) cross section, gives same \( \mathcal{L}^{\text{MC}} \) and \( \mathcal{L}^{(0)} \) are now separately finite:

\((p)\mathcal{O} \rightarrow (1)\mathcal{O} \)

This prescription has several good features:

\[
\frac{x}{[(x)\mathcal{O} + (x)\mathcal{R}]v} \left( \frac{x}{[1 - (x)\mathcal{D}]} + \Lambda v + \mathcal{B} \right) \mathcal{L}^{\text{MC}} \int_{1}^{0} xp = \mathcal{L}^{\text{MC}}
\]
which must be subtracted from weight of
also generates $z \rightarrow z$, $3$ configurations,

This would be wrong because $\mathcal{F}_{\mathcal{MC} \cap \mathcal{TO}}$ to obtain functional starting from $z \rightarrow z$, $3$ hard subprocesses),

Naturally, for $\mathcal{MC} \cap \mathcal{TO}$ we would replace $\mathcal{W}_{\mathcal{MC}}^{\mathcal{W}}$ are counter-terms which cancel divergences of $\mathcal{W}_{\mathcal{MC}}^{\mathcal{W}}$

\[ \left( (\mathcal{E}^{\mathcal{M}})^{\mathcal{W}}, \mathcal{I}^{\mathcal{I}}) \mathcal{W}_{\mathcal{MC}}^{\mathcal{W}} - (\mathcal{E}^{\mathcal{M}})^{\mathcal{W}}, \mathcal{I}^{\mathcal{I}}) \mathcal{W}_{\mathcal{MC}}^{\mathcal{W}} \right) (z \rightarrow z) O + \]

\[ (\mathcal{E}^{\mathcal{M}})^{\mathcal{W}}, \mathcal{I}^{\mathcal{I}}) \mathcal{W}_{\mathcal{MC}}^{\mathcal{W}} (z \rightarrow z) O \int_{\mathcal{I}}^{\mathcal{M}} \mathcal{E}^{\mathcal{M}} \mathcal{I}^{\mathcal{I}} = q^{\mathcal{M}} \langle O \rangle \]

Subtraction method, is

pair production, schematic expression for any observable $O$, evaluated by

consider a hadron collider process which is $z \rightarrow z$ at $\mathcal{L}$, $\mathcal{O}$ or

Modified Subtraction for Real QCD
we find 10 – 20%.

Number of negative weights can be reduced by tuning coefficients. Typically

But coefficients may be negative, some events have negative weight.

provided MC does a good job in all soft and collinear limits, coefficients of

\[
\begin{aligned}
\left[ ((\phi, \mathbf{x}, \mathbf{1}, \mathbf{x}) W + (\phi, \mathbf{x}, \mathbf{1}, \mathbf{x}) W - (\phi, \mathbf{x}, \mathbf{1}, \mathbf{x}) W \right) \\
+ \left[ (\phi, \mathbf{x}, \mathbf{1}, \mathbf{x}) W - (\phi, \mathbf{x}, \mathbf{1}, \mathbf{x}) W \right]
\end{aligned}
\]

Therefore for MC@LO we define
Large-scale physics correctly handles the artihms, and yet handling the
universe, resuming large log-
MCNTO are both relevant. MCNTO
solid: NLO
Dotted: NLO
Dash: HERWIG ×

These correlations are problematic observables $M + M$
Jet Observables in $W^+W^-$ Production

For $p_T < 0$ the model $MC@NLO/MC$ factor displays the same behaviour as in the toy inclusive jet distribution. It is striking that jets have been reconstructed with...
in the $\phi$ tail and MC is enhanced by the cuts difference between MC@NLO and MC@NLO fills further the kinematic dip at $\Delta = 0$. The solid: MC@NLO.

It is interesting that the

**Dotted: NLO**

**Dashed: HERWIG**

**Solid: MC@NLO**

**W Observables**

(W+Z)

Most difficult part is calculating what MC does.


Generate 2 → 2 and 2 → 3 partition configurations and weights;

\[ W_{\text{MC}}(\phi_1, x_1; x_2; x_3) \text{ terms.} \]

Calculate analytically exactly what MC does at NLO:

Take standard subtraction formula;

Modified subtraction formula above can be used for any process.

Heavy Quark Production
\[ \prod \frac{1}{z} \left| \left( \frac{\alpha}{\pi} \right)^{2N} \right| \sum \frac{1}{z} \left| \left( \frac{\alpha}{\pi} \right)^{2N} \right| \sum \frac{1}{z} \left| \left( \frac{\alpha}{\pi} \right)^{2N} \right| = \text{etc.} \]

\[ \infty \leftarrow N \text{ according to } \prod \text{Colour structure assigned (for shower/hadronization) depending on which leg emits!} \]

\[ \text{Relation between invariants and shower variables depends on which leg emits!} \]

\[ \text{MC Heavy Quark Production} \]

\[ \text{MC Starts from } 2 \leftarrow 2 \text{ subprocess} \]

\[ \text{Emission} \]

\[ \text{Monte Carlo Reshuffling is done after real} \]

\[ \text{Limit} \]
Asymmetry at Tevatron.

New feature in MC:

\( \frac{O_T \times N_T}{O_N \times N_N} \) here.

Dotted: NLO

Dashed: HERWIG

Solid: MC@NLO

\( t, \bar{t} \) observables at colliders.
Top Rapidity Asymmetry at Tevatron
These correlations display the correctly soft-scales physics are both treated. Hard and boson pair production. Hard and same patterns as those for vector.

\( (pT)_{\phi V} \)
Dotted: NLO

Dash: HERWIG (no R-factor)

Solid: MC@NLO

Dotted: NLO

Dash: HERWIG

Solid: MC@NLO

\( \phi \) Correlations at Tevatron
All are needed to get close to data (RD Field, hep-ph/0201112):

- In parton shower MC's, 3 classes of processes can contribute:
All these problems are avoided with MC@NLO!

- GSP efficiency very poor, \( \sim 10^{-4} \)
- PDF sensitive to bottom PDF
- GSP cut-off (PTMIN) sensitivity depends on cuts and observable

GSP, PEX and FCR are complementary and all must be generated.

\( \text{GSP and PEX contributions in HERWIG} \)
emulates the effect of initial-final parton showers.

This does NOT mean that there is simply (\langle \hat{q} \rangle_{\text{GeV}} = 4 \text{ GeV} \Gamma \text{ inside proton}) it simply

\begin{align*}
\langle p_T \rangle &> 5 \text{ GeV, } |y| < 1 \\
\text{Dashed: NLO+K-FK-ICK} \\
\text{Dotted: NLO} \\
\text{Solid: MC\raise.5ex\textsuperscript{NLO}}
\end{align*}

(\langle p_T \rangle_{\text{GeV}} \sim \text{GeV\textsuperscript{NLO} (at TeVatron)})

(\text{NLO + \kappa T-Kick}) \text{ vs MC\raise.5ex\textsuperscript{NLO}}
No significant discrepancy!

Hadron-level Results on B Production

B → J/ψ f' (includes BR's)
$m_H = 115 \text{ GeV}$

Dotted: NLO
Solid: NNLO+NNLL resc
Hist: MC@NLO

SM Higgs production at LHC: Good agreement with $(N)NLO+NNLL$
Conclusions and Future Prospects

General Interface to NLO (subtraction method) Programs Feasible.

Jet Production needs a little more work.

Decay correlations implemented for $W$, $Z$, $H$ (not yet for others).

$\eta\bar{\eta}$ production, Negative weights $\sim 10\%$ to $20\%$ (qq) not a problem.

MG@NLO exists and works well for $W$, $Z$, $H$, $WW$, $ZZ$, $WWZ$, $T$ and $bb$.
