Angular ordering
Soft gluon coherence
Monte Carlo method
Sudakov form factor
Evolution of parton distributions

Lecture 1: Parton branching & showering

CDF Lectures, October 2004

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The HERWIG Event Generator
\[ \frac{z}{\theta} = \frac{z - I}{\theta} = \frac{(z - I)z}{I} = \theta \]

\[ \cos(\theta) = (\theta - I) = \theta \]

For small angles

\[ \frac{a}{E} - I = \frac{a}{E} = z \]

Consider splitting outgoing parton \( a \) into \( q + c \).

Corresponding virtual corrections can be identified and summed to all orders.

Leading soft and collinear enhanced terms in QCD matrix elements (and}
\[
0 = \vec{d} \cdot \vec{\epsilon}_{\text{out}} = \vec{\epsilon}_{\text{in}} \cdot \vec{\epsilon}_{\text{out}} \\
I - 1 = \vec{\epsilon}_{\text{out}} \cdot \vec{\epsilon}_{\text{out}} = \vec{\epsilon}_{\text{in}} \cdot \vec{\epsilon}_{\text{in}}
\]

Plane, so that

Resolve polarization vectors into \( \vec{\epsilon}_{\text{in}} \), \( \vec{\epsilon}_{\text{out}} \) in plane of branching and \( \vec{\epsilon}_{\text{in}} \) normal to

\[
-2 \gamma_{ABCD} \left[ \varepsilon (\vec{\epsilon}_{\text{in}} - \vec{\epsilon}_{\text{out}}) \cdot \vec{g} + \varepsilon (\vec{\epsilon}_{\text{in}} - \vec{\epsilon}_{\text{out}}) \cdot \vec{g} \right]
\]

Amplitude has triple-gluon vertex factor

Consider first 99 branching.
Vertex factor proportional to \( \theta \), together with propagator factor of

\[
\theta^0 \mathcal{A} z^- = \theta^q \mathcal{A}^- = q_d \cdot \frac{2}{3} \\
\theta^0 \mathcal{A} (z - 1) = \theta^q \mathcal{A}^+ = \frac{2}{3} d \cdot q_c \\
\theta^0 \mathcal{A} (z - 1) z^- = q \theta^q \mathcal{A}^- = q_d \cdot \frac{2}{3} c \\
\]

For small \( \theta \), neglecting terms of order \( \theta^2 \), we have.
polarized in plane of branching.

Branching ratio at soft $q \rightarrow z$ and to soft gluon $\langle H \rangle$ is (unpolarized) gluon splitting function.

$$\begin{bmatrix}
(z - 1)z + \frac{z - 1}{z} + \frac{z}{z - 1}
\end{bmatrix} \forall C = (z)^{66} \not\equiv \langle H \rangle \forall C$$

Sum/averaging over polarizations gives

<table>
<thead>
<tr>
<th>$(z - 1)/z$</th>
<th>out</th>
<th>in</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z/(z - 1)$</td>
<td>out</td>
<td>in</td>
</tr>
<tr>
<td>$(z - 1)z$</td>
<td>out</td>
<td>in</td>
</tr>
<tr>
<td>$(z - 1)z + (z - 1)/z + z/(z - 1)$</td>
<td>in</td>
<td>in</td>
</tr>
<tr>
<td>$(c^a, e^b, c^c)_{H}$</td>
<td>$c^a$</td>
<td>$e^b$</td>
</tr>
</tbody>
</table>

Given below are $H$ and functions $\mathcal{W}$ and $\mathcal{W}ABC$ and $\mathcal{W}AB$ come from $\mathcal{Z} = \mathcal{Z}$ where colour factor $C = \mathcal{Z}$.

$$2 \left| u \mathcal{W} \right| \langle (c^a, e^b, c^c)_{H} \rangle \mathcal{W}^{\mathcal{C}}_{\mathcal{Z}} \frac{\mathcal{Z}}{b_{\mathcal{Z}}} \sim 2 \left| 1 + u \mathcal{W} \right|$$

terms of that for parts.
opposite handedness conservation.

Vertex factor simplifies since these are associated only with gluon emission.

\[
\cdot \left[ \bar{z}(z - 1) + z \right] \mathcal{H} = (z) \mathcal{H} \equiv \langle \mathcal{H} \rangle \mathcal{H}
\]

Spin-averaged splitting function is

where \( q \) and \( \bar{q} \) are quark and antiquark spinors.

Vertex factor is

Consider next branching:

Hence branching in plane of gluon polarization preferred.

\[
\phi \mathcal{Z} \cos (z - 1)z + (z - 1)z + \frac{z - 1}{z} + \frac{z}{z - 1} \propto
\]

\[
\left| \langle \phi \mathcal{Z} \rangle \mathcal{H} \mathcal{W} \phi \sin \left( \phi \mathcal{Z} \right) \mathcal{W} \phi \cos \right| \propto \phi \mathcal{H}
\]

Correlation between polarization and plane of branching.
\[ \phi(z) \cos \frac{z - 1}{z} + \frac{z - 1}{z + 1} = \phi_H \]

Correlation being Gluon polarized in plane of branching preferred, polarization angular branch.

Helicity conservation ensures that quark does not change helicity in spin-averaged splitting function is Branching:

\[ \langle z \rangle^b \phi_H \equiv \langle \mathcal{H} \rangle \]
where \( q \) is a splitting function.

\[
(z)^{q \mu} \frac{\mu Z}{\phi p} zp \frac{\gamma}{\gamma p} u_{\Omega p} = 1 + u_{\Omega p}
\]

Introduced earlier, integrating over azimuthal angle \( \mu \) gives where and are colour factor and polarization-dependent \( z \)-distribution

\[
H \mu Z \frac{\mu Z}{\phi p} zp \frac{\gamma}{\gamma p} u_{\Omega p} = 1 + u_{\Omega p}
\]

Hence cross sections before and after branching are related by

\[
\phi p zp \frac{e(\mu Z)}{1} \frac{\nu}{\nu p} u_{\Phi p} = 1 + u_{\Phi p}
\]

Phase space factors before and after branching are related by.

Phase space
To derive evolution equation for $\mathcal{O}(x)\mathcal{D}$-dependence of $\mathcal{O}$, first introduce cross section will depend on $\mathcal{O}$ and on momentum fraction distribution of photon seen by virtual photon at this scale, finally struck by photon of virtual mass-squared and lower momentum fractions by successive small-angle emissions, and is incoming quark from target hadron, initially with low virtual mass-squared $0 \mathcal{O}$ - $h$.

Consider enhancement of higher-order contributions due to multiple small-angle emission, for example in deep inelastic scattering (DIS)
element, divided by $g^x$. This is number of paths arriving in element $(\varphi^x, \varphi^y)$ minus number leaving that.

Consider change in the partition distribution $\mathcal{D}(x,t)$ when $t$ is increased to $t + \Delta t$.

Just the $x$-distribution of paths at that scale.

Target hadron at that scale. Then distribution at that scale is $\mathcal{D}(x,t)$ of partitions at scale $t$.

At $t = 0$, paths have distribution of starting points of characteristic of target.

After the branching step downwards in $x$, at a value of $t$ equal to $t_0$ (minus) the virtual mass-squared (minus) the virtual mass-squared path in $(t', x')$-space. Each branching is a sequence of branchings by path in $(t', x')$-space. Each branching is a sequence of branchings by path in $(t', x')$-space. Each branching is a sequence of branchings by path in $(t', x')$-space.
\[
\cdot \left[(\tau' x) A - \left(\tau' x / x\right) A \frac{z}{x} \right] (z) \frac{\mu z}{\Sigma \chi} z \rho \int_{1}^{0} \frac{\tau}{\tau \emptyset} = \\
\tau_{\text{no}} D \emptyset - \tau_{\text{in}} D \emptyset = (\tau', x) D \emptyset
\]

Change in population of element is

\[
(z) \frac{\mu z}{\Sigma \chi} z \rho \int_{1}^{0} (\tau' x) A \frac{\tau}{\tau \emptyset} = \\
(xz - \tau x) \frac{\mu z}{\Sigma \chi} z \rho \int_{x}^{0} (\tau', x) A \frac{\tau}{\tau \emptyset} = (\tau', x) \tau_{\text{no}} D \emptyset
\]

For the number leaving element, must integrate over lower moments.

\[
(\tau' z / x) A(z) \frac{\mu z}{\Sigma \chi} z \rho \int_{1}^{0} \frac{\tau}{\tau \emptyset} = \\
(xz - x) \frac{\mu z}{\Sigma \chi} z \rho \int_{1}^{x} \frac{\tau}{\tau \emptyset} = (\tau', x) \tau_{\text{in}} D \emptyset
\]

All higher moments are branching probability times partition density integrated over.
remains the same.

conditions and direction of evolution are different, but evolution equation momentum fraction distribution produced by an outgoing parton. Boundary hadron produced at scale \( t \). In timelike branching, it represents instead hadron incoming hadron distribution inside incoming parton momentum fraction distribution.

\[
\begin{align*}
\int_0^1 \left[ (z) \delta \left( q \frac{z}{x} \right) d \frac{z}{x} \left( \frac{z}{x} \right) \Theta zp \int_0^1 \left( (z) \delta [(1)f - (z)f] zp \int_1^0 \right] \right.
&= \left. \int_0^1 \right. \\
&= + (z) \delta (z)f (x - z) \Theta zp \int_0^1 \left( (z) \delta (z)f zp \int_1^0 \right)
\end{align*}
\]

Here \( \int\) represents parton momentum fraction distribution inside incoming parton momentum fraction distribution produced by an outgoing parton. Boundary hadron produced at scale \( t \). In timelike branching, it represents instead hadron incoming hadron distribution inside incoming parton momentum fraction distribution.

\[
(\tau'x) D(\tau) d \frac{\tau'}{x} \frac{z}{x} \int_1^0 = (\tau'x) D(\tau) \frac{\tau'}{x}
\]

equation: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution and obtain Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation. Using this we can define regularized splitting function

\[
\begin{align*}
\int_0^1 \left( (z) \delta [(1)f - (z)f] zp \int_0^1 \right) &= + (z) \delta \left( z \right) f \left( x - z \right) \Theta zp \int_0^1 \left( (z) \delta \left( z \right) f \right) \left( x - z \right) \Theta zp \int_0^1 \\
\end{align*}
\]

Introduce plus-preservation with definition
Thus number arriving is $bb \leftrightarrow b$

or $bb \leftrightarrow b$ can arrive either from contributions or from

\[
[z(z - 1) + z^2] \mathcal{H} = (z)^{bb} \mathcal{L} = (z)^{bb}
\]

\[
+ \left( \frac{z - 1}{z^2 + 1} \right) \mathcal{J} \mathcal{C} = + (z)^{bb} \mathcal{D} = (z)^{bb}
\]

Thus plus prescription applies only to $bb \leftrightarrow b$ via a part, giving

via $bb \leftrightarrow b$ can enter element via either $bb \leftrightarrow b$ or $bb \leftrightarrow b$, but can only leave

\[
(4, z / x) \mathcal{O} \mathcal{A}(z) \mathcal{O} \mathcal{A} \frac{\mu_2}{x} \frac{z}{z} \int_1^x \mathcal{O} \mathcal{A} \mathcal{O} = (4, x) \mathcal{O} \mathcal{A} \mathcal{O} \mathcal{A}
\]

coupled DGLAP evolution equations of form

by which parton of type $l$ can enter or leave the element $(\phi, gx)$. This leads to

for several different types of partons, must take into account different processes

for quark and gluon distributions
\[
(z - 1) q \frac{\mathcal{L} f_N}{z} - \left[ (z - 1) z^\frac{q}{1} + \frac{z}{z - 1} + \left( (z - 1) z^\frac{q}{1} + \frac{z - 1}{z} \right) \right] v \mathcal{C} z = (z)^{b\delta} d
\]

After some manipulation we find

\[
\cdot \left[ z p (z)^{b\delta} d^f N + (z)^{b\delta} d \right] \frac{v z}{z p} \int_1^0 \left( \frac{q'}{x} \right)^{b\delta} d \frac{q}{4q} = \gamma_{\alpha\delta} d q
\]

Cf. can be seen by splitting into either of \( b \delta \) or \( b \delta \), so that

\[
\left\{ \left[ \left( \frac{q'}{x} \right)^{b\delta} d + \left( \frac{z}{x} \right)^{b\delta} d \right] (z - 1)^{b\delta} d + \left( \frac{q'}{x} \right)^{b\delta} d (z)^{b\delta} d \right\} \frac{v z}{z p} \int_1^0 \frac{q}{4q} = \gamma_{\alpha\delta} d q
\]
so $P_g$ and $P_{gg}$ can be written in more common forms

Using definition of the plus-prescription, can check that

\[ P_{gg}(z) = \frac{2Ca}{(1-z)^2} + \frac{3}{2\delta(1-z)} \]

\[ + \frac{1}{6(11Ca - 4N_jTR)\delta(1-z)} \]

\[ = \frac{1+z^2}{(1-z)} + \frac{3}{2\delta(1-z)} \]

\[ + \frac{1}{12\delta(1-z)} \]
\[
\cdot (t', z/x)A(z) \frac{\nu z}{s \omega} z p \int \frac{\nabla}{\mu} \frac{\mu}{\eta} \int + (0, t, x)A(\xi) \nabla = (t', x)A
\]

This is similar to DGLAP, except replaced A except replaced \(\nabla/A\) and regularized splitting function replaced by unregularized \(\nabla\).

\[
\cdot (t', z/x)A(z) \frac{\nu z}{s \omega} z p \int \frac{\nabla}{I} = \left( \frac{\nabla}{A} \right) \frac{\eta}{\Theta} \cdot t
\]

\[
\cdot (t') \frac{\eta}{\Theta} \frac{\nabla}{t} (t', x)A + (t', z/x)A(z) \frac{\nu z}{s \omega} z p \int = (t', x)A \frac{\eta}{\Theta} \cdot t
\]

Then

\[
\cdot \left[ (z) \frac{\nu z}{s \omega} z p \int \frac{\mu}{\eta} \int \nabla - \right] \text{d}x_\Theta \equiv (t) \nabla
\]

Factor Sudakov form with only one type of branching. Introduce simplified treatment with only one type of branching. Introduce structure of bound states, slightly different form is useful. Consider again DGLAP equations convenient for evolution of parton distributions. To study Sudakov form factor.
\[
\cdot (q^i z^j x^k)^i \frac{\partial^i}{\partial^i} \pi^j \int_{y^m}^{y^n} \frac{\partial^i}{\partial^i} z^p \int_{y^m}^{y^n} \frac{\partial^i}{\partial^i} = \left( \frac{\partial^i}{\partial^i} \right) \frac{\partial^i}{\partial^i} \]

which is probability of \( \text{evolving from } \theta^0 \text{ to } \theta^t \text{ without branching.} \]

Then

\[
\left[ (z)^i \int_{y^m}^{y^n} \frac{\partial^i}{\partial^i} z^p \int_{y^m}^{y^n} \frac{\partial^i}{\partial^i} \right] \exp \equiv (q)^i \nabla
\]

Sudakov form factor

\[
\text{Generalization to several species of particles straightforward. Species } i \text{ has }
\]

\[
\text{Sudakov form factor } \frac{(q)^i \nabla}{(q)^i \nabla}
\]

is probability of \( \text{evolving from } \theta^0 \text{ to } \theta^t \text{ without branching.} \]

\[
\text{contribution from paths which have their last branching at scale } \theta^t \text{. Factor of }
\]

\[
\text{probability of \( \text{evolving from } \theta^0 \text{ to } \theta^t \text{ without branching.} \text{ Second term is }
\]

\[
\text{not branch between scales } \theta^0 \text{ and } \theta \text{. Thus Sudakov form factor } \frac{(q)^i \nabla}{(q)^i \nabla}
\]

This has simple interpretation. First term is contribution from paths that do
\[ \frac{q}{t} \approx \frac{q}{t^0} - \frac{z}{t} = (\frac{q}{t})_0 \quad \text{if} \quad 0 < \frac{q}{t} < \frac{3}{z} \]

Hence for \( \frac{q}{t} \), that is,\[ 0 < \frac{3}{z} < \frac{q}{t} < \frac{3}{z} \]

\( t = \frac{t_0}{z} \) and \( \frac{q}{t} = \frac{p}{d} \)

\[ = \frac{t_0}{z} \]

\( m \) is transverse momentum in a q-\( \psi \) left 4 mass-squared, \( \frac{q}{t} \). When partition energies are much larger than virtual.

\( \text{time-like} \) branching, natural resolution limit is given by cutoff on partition virtual.

\( \text{For infrared cut-off \( e(\tau) \) depends on what we classify as resolvable emission. For infrared cut-off \( e(\tau) \) depends on what we classify as resolvable emission.} \)

\( \text{unresolvable real contributions:} \text{ both are divergent but their sum is finite.} \)

\( \text{Sudakov form facor:} \text{ sum enhanced virtual (partition loop) as well as real (partition} \)

\( \text{unresolvable:} \text{ emitted parton is too soft to detect. Sudakov form facor} \text{ with this} \)

\( \text{explicit infrared cut-off,} \text{ } \left( \frac{q}{t} \right) \text{. Branchings with} \text{ above this range} \text{ are} \)

\( \text{regularized by plus-preception. However, in above form we must introduce an} \)

\( \text{In short equation, infrared singularity of splitting functions at} \text{ are} \)

\( \text{Infrared cutoff} \text{.} \)
\[ a \text{ more restrictive effective cut-off, } \]
\[ QCD \text{ dynamics effectively reduces phase space for parton branching, leading to } \]
\[ \text{Infrared cutoff discussed here follows from kinematics. We shall see later that } \]
\[ \text{which tends to zero faster than any negative power of } t, \]
\[ \left( \frac{(0t)_{s'}}{(t)_{s'}} \right) \sim (t)^b \nabla \]
\[ \text{Then at large } t, \]
\[ t(2t - 1)z \sim \frac{Td}{z} \]
\[ \text{Careful treatment of running coupling suggests its argument should be } \]
\[ \left[ (z)^{b_0} d \frac{\mu z}{s'} z^p \int_{1/0}^{1/0} \frac{A}{dP} \int_{1/0-1}^{0} d\rho \exp \sim (t)^b \nabla \right] \]
\[ \text{Quark Sudakov form factor is then } \]
If $\tau_2$ is higher than hard process scale $\mathcal{O}$, this means branching has finished.

\[ \mathcal{U} = \frac{(1_f)\nabla}{(2_f)\nabla} \]

can be generated with the correct distribution by solving $\tau_2(1_f)\nabla/\tau_2(2_f)\nabla$ is without branching.

Since probability of evolving from $1_f$ to $\tau_2$ without branching is $0$, $\tau_2$, after the next step.

Monte Carlo branching algorithm operates as follows: Given initial mass scale and momentum fraction $1_f$, $x^1_f$, after some steps of the evolution, or as initial conditions, it generates valuses $(\tau_2, x^2_f)$ after the next step.

Monte Carlo shower" Monte Carlo programs, and is basis of "parton shower" Monte Carlo method.

Monte Carlo method
Thus branching stops when

\[ \mathcal{N} > g \]

where

\[ \mathcal{N} = \frac{(\nu \tau)\nabla}{(\tau \nabla)} \]

is now given by

From Section 2, \( \nu \to 0 \) if probability of evolving downwards without branching between \( t \) and scale 0, rather than upwards towards hard process downwards towards cutoff value \( t_0 \), rather than upwards towards hard process.

Each emitted (timelike) parton can itself branch. In that case \( t \) evolves

More generally, when exchanged parton is a gluon, azimuths must be generated with polarization angular correlations discussed earlier.

Azimuthal emission angles are then generated uniformly in the range \([0, \pi)\), from which momenta of emitted gluons can be computed.

In DIS, where

\[ \frac{d}{dz} \frac{\nu / G}{x / G} \int_{1-x/2}^{1} N = \frac{d}{dz} \frac{\nu / G}{x / G} \int_{1-x/2}^{1} N \]

is another random number and \( z \) is cutoff for resolvable branching.

Solve by solving the appropriate splitting function function, where

\[ (z) \frac{d}{dz} \frac{\nu / G}{x / G} \]

is appropriate splitting function with distribution proportional to

\[ \frac{x/2}{\frac{x}{2}} = z \]

otherwise, generate.
converted into hadrons via a hadronization model. This stage, which depends on cutoff scale $\Lambda$, outgoing partons have to be this stage, which depends on cutoff scale $\Lambda$, outgoing partons have to be this stage, which depends on cutoff scale $\Lambda$, outgoing partons have to be this stage, which depends on cutoff scale $\Lambda$, outgoing partons have to be this stage, which depends on cutoff scale $\Lambda$, outgoing partons have to be this stage, which depends on cutoff scale $\Lambda$, outgoing partons have to be this stage, which depends on cutoff scale $\Lambda$, outgoing partons have to be
where we polarization of emitted gluon.

\[
\frac{b \cdot d}{c \cdot d} = \varepsilon^{\text{soft}} \tilde{H}
\]

Spin-independent factor in amplitude.

Including numerator, soft gluon emission gives a colour factor times a universal, including numerator, soft gluon emission.

and emission angle. 

\[
\left( \frac{\cos \theta}{1} \right) \varepsilon^{\text{soft}} = \frac{b \cdot d}{c} = \frac{z \cdot m - z (b \mp d)}{1}
\]

Spin emission angle.

with momentum b, propagator factor is


\[
(\theta \cos \nu - 1) \varepsilon^{\text{soft}} = \frac{b \cdot d}{c} = \frac{z \cdot m - z (b \mp d)}{1}
\]

With momentum b, propagator factor is

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\left( \frac{\cos \theta}{1} \right) \varepsilon^{\text{soft}} = \frac{b \cdot d}{c} = \frac{z \cdot m - z (b \mp d)}{1}
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\]

Spin emission angle.

with momentum b, propagator factor is

\[
(\theta \cos \nu - 1) \varepsilon^{\text{soft}} = \frac{b \cdot d}{c} = \frac{z \cdot m - z (b \mp d)}{1}
\]

Spin emission angle.
\[
\frac{(b \theta \cos \phi \eta - 1)(b \theta \cos \phi \eta' - 1)}{b \theta \cos \phi \eta \eta' - 1} = \frac{b \cdot \theta \eta \eta'}{b \cdot \theta \eta' \eta} = \eta \text{W}
\]

Radiation function \( \eta \text{W} \) is given by

\[
\eta \text{W} \int \int \int \frac{\mathcal{W}}{\mathcal{V} \mathcal{D} \mathcal{M}} u \rho = 1 + u \rho
\]

Enhancement factor in amplitude for each external line implies cross section enhancement is sum over all pairs of external lines

\[0 \leq m \leq 0 \neq \eta \rho - \eta \] and denominator factor (b + d) is associated with mass-shell internal lines, since

\[\rho \text{soft enhancement of radiation from off-mass-shell internal lines} \]

\[\rho \text{universal factor} \rho_{\text{soft}} \text{coincides with classical eikonal formula for radiation from}
\]

\[\rho \text{universal factor} \rho_{\text{soft}} \text{coincides with classical eikonal formula for radiation from}
\]

\[\int_{\mathcal{D}} (d)n_\eta \eta' d\mathcal{Z} = (d)n(\eta \eta' + \eta \eta' - \eta \eta') =
\]

\[\int_{\mathcal{D}} (d)n(\eta \eta' + \eta \eta' - \eta \eta') = (d)n(\eta \eta' + \eta \eta' - \eta \eta')
\]

\[\text{For example, emission from quark gives numerator factor } \eta \cdot \epsilon, \text{ where}
\]
Performing azimuthal integration, we find

\[
\int_{b_i \phi}^{b_i \theta} b_i \cos p \, dp = \Omega p \text{, direction of } \theta \text{.}
\]

This function has remarkable properties of angular ordering. Write angular

\[
\left( \frac{b_i \theta - I}{I} - \frac{b_i \theta - I}{I} + \iota_i M \right) \frac{\Omega}{\iota_i M} = \iota_i M
\]

where \( \iota_i M + \iota_i M = \iota_i M = \iota_i \omega \). Then \( \Omega = \iota_i \omega \).

Radiation function can be separated into two parts containing collinear

\[
\text{process.}
\]

Color-weighted sum of radiation functions \( \iota_i M \) is antenna pattern of hard
extending to direction of \( \phi \), is contained in cone, centered on line \( \ell \). Similarly, \( W_{\ell} \) is contained in cone, centered on direction of \( \ell \), extending in angle to direc-

Thus, after azimuthal averaging, contribution
\[\frac{b_i \theta - I}{I} \]

\[\left[ f_i \left( f_i \cos b_i \theta - b_i \theta \cos f_i \theta \right) + 1 \right] \frac{(b_i \theta - I)Z}{I} = f_i \left( \frac{\nu Z}{b_i \phi p} \right) \int_0^\infty \]

Hence

\[\left. \frac{f_i \theta \cos - b_i \theta \cos}{I} \right| = \frac{z \theta - z}{I} \int = f_i \left( \frac{q}{p} \right) = \pm z\]

where \( z \) is the inside unit circle.

\[\frac{(-z - z)(+z - z)}{zp} \int \frac{q \nu r}{I} = \frac{b_i \theta \cos - b_i \theta \cos f_i \theta - I}{I} \frac{\nu Z}{b_i \phi p} \int_0^\infty \equiv f_i \left( b_i \phi f_i \right)\]

\( \exp = \text{we have} \)

\[b_i \theta \sin f_i \theta = q \quad b_i \theta \sin f_i \theta - I = a\]

Where \( b_i \phi \cos q - a = b_i \theta \cos f_i \theta - I = a \).

To prove angular ordering property, write \( a \).

Angular ordering
of pair will be $q \Delta \theta \sim q \Delta$. In this time transverse separation

$\mathcal{I} \sim \mathcal{E} \mathcal{N}$. Time available for emission is

$\theta d\zeta \sim d\zeta / \frac{d\zeta}{c} \sim \mathcal{E} \mathcal{N}$

$e \leftrightarrow e$ vertex is

transverse momentum of photon is $\mathcal{I} \gamma$, and energy imbalance at

opening angle $\theta > \theta \gg \theta e e$. For simplicity assume $\theta > 1$.

Consider emission of soft photon at angle from electron in pair with

Angular ordering is coherence effect common to all $e^+ e^- \leftrightarrow e^+ e^-$ pairs,

which has simple explanation in old-fashioned (time-ordered) perturbation

cause Chudakov effect - suppression of soft bremsstrahlung from $e^+ e^-$ pairs,
coherently by \( i \) and \( f \) and can be treated as coming directly from (colour) charge.

... then radiation outside angular-ordered cones is emitted.

More generally, if \( i \) and \( f \) come from branches of parton \( k \), with (colour) charge emission.

separately – they see only total charge of pair, which is zero, implying no separate pairs at larger angles cannot resolve electron and positron charges.

... then opening angle of pair, which is angular ordering.

and hence \( \theta_0 < \theta \). Thus soft photon emission is suppressed at angles larger.

\[
\left( \theta d\theta \right) < \left( \theta d\theta \right)_{\theta_0}^{\theta_0} \theta
\]

This implies that

\[
\left( \theta d\theta \right) \sim \theta/\gamma < q\nabla
\]

where \( \gamma \) is photon wavelength.

For non-negligible probability of emission, photon must resolve this.
For partition of energy $E$, $E^2/\nu q = \zeta$

Classify emission as unresolved. Simplest choice is coherent branching. This is to some extent arbitrary, depending on how we choose of virtual mass-squared cut-off $\theta_0$, must use angular cut-off $\zeta$. For

\[ (z)^{\nu q} \frac{\zeta p}{\nu q} z p \frac{\zeta}{\nu p} u_0 p = 1 + u_0 p \]

For successive branching, iterative formula for $n$-partition emission becomes $\zeta > \zeta$.

As evolution variable for branching a $\nu q e$, and impose angular ordering

\[ \theta \cos 1 - \frac{\nu q E}{\nu q d} = \zeta \]

Variable variable

In place of virtual mass-squared variable $t$ in earlier treatment, use angular.

Includes leading soft gluon enhancements to all orders.

Angular ordering provides basis for coherent partition branching formalism, which

Coherent branching
\[
\frac{\mathcal{Z}/0{\mathcal{E}}}{\mathcal{Z}/0{\mathcal{E}}} - 1 > z > \frac{\mathcal{Z}/0{\mathcal{E}}}{\mathcal{Z}/0{\mathcal{E}}}
\]

Thus cutoff on becomes

\[
\frac{\mathcal{Z}/z}{(z - 1)} > \frac{\mathcal{Z}/z}{(z - 1)} \quad \text{for } \mathcal{Z}/z > \mathcal{Z}/z
\]

outgoing becomes (becomes)

\[\text{Angular ordering condition}\]

\[\mathcal{Z}/0{\mathcal{E}} = \mathcal{Z}/0{\mathcal{E}}\]

... but rather

With this cut-off, most convenient definition of evolution variable is not \( S \) itself

\[ \text{... so angular distribution of radiation is cut off at } \mathcal{Z}/0{\mathcal{E}} \]

\[ \text{Angular distribution of radiation with mass-squared } \mathcal{Z}/0{\mathcal{E}} \]

For radiation from particle \( \mathcal{Z}/0{\mathcal{E}} \) with mass-squared \( \mathcal{Z}/0{\mathcal{E}} \), radiation function is
suppression of soft gluon emission by angular ordering.

At large $t$ this falls more slowly than form factor without coherence, due to the

$$\left[ (z)^{b\bar{b}} \mathcal{D}(\mathcal{A}_{z}(z-1)z) \right]_{\mathcal{S}_0} \frac{\nu \mathcal{Z}}{z \mathcal{P}} \int_{\mathcal{A}_1}^{0_{\mathcal{P}} \wedge} \int_{\mathcal{A}_1}^{0_{\mathcal{P}} \wedge} \int_{\mathcal{A}_1}^{0_{\mathcal{P}} \wedge} - \, dx \psi = (q)^b \psi$$

Thus for coherent branching Sudakov form factor of quark becomes

\[ \cdot \mathcal{A}_{z}(z-1)z = \frac{q}{z} \quad \cdot \mathcal{A}_{z}(z-1)z = q \]

momentum of branching are

Neglecting mass of $q$ and $c$, virtual mass-squared of $a$ and transverse
branching becomes disordered at small $x$.

Thus we can have either $\frac{q}{q_u} > \frac{q}{q_u}$ or $\frac{q}{q_u} > \frac{q}{q_u}$, especially at small $z$ — spacelike.

For $\left( z - \frac{q}{q_u} \right) > \frac{q}{q_u}$, $\frac{q}{q_u} > \frac{q}{q_u}$ and so for $z$ we now have

\[ \frac{q}{q_u} < \frac{q}{q_u} < \frac{q}{q_u} \]

ordering condition is

Note that for spacelike branching $q \leftarrow q$ (a incoming, $q$ spacelike), angular
Soft gluon coherence implies angular ordered parton showers.

Followed by hadronization.

Successive branching leads to parton showering, terminated by infrared cutoff.

useful for Monte Carlo implementation.

Sukerov form factor is probability of evolution without resoluble branching

DGLAP equation.

Evolution of parton distributions (or parton fragmentation) is controlled by

\( P(z) \).

Multi-parton cross sections in terms of splitting functions

Parton branching approximation describes collinear-enhanced contribution to

Summary of Lecture 1