# Hadronization: Concepts and Models

## From Perturbative to Non-Perturbative QCD

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Hadronization Workshop
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# What is Hadronization?

In practice, two rather distinct meanings:

- General concepts/models for soft QCD
  - Local parton-hadron duality (LPHD)
  - Universal low-scale effective $\alpha_S$ (ULSEA)

- Models for formation of individual hadrons
  - Monte Carlo models: string & cluster
  - Thermal/statistical models
  - AdS/QCD

# General Concepts

- Local parton-hadron duality
  - Momentum & flavour follows parton flow
  - Predicts asymptotic spectra
  - Predicts two-particle correlations
- Universal low-scale effective $\alpha_S$
  - Related to “tube” model
  - Regulates IR renormalons in PT
  - Predicts power corrections to event shapes
  - Predicts jet shapes and energy corrections

# Local parton-hadron duality

- Evolution equation for fragmentation function has extra $z^2$ due to soft gluon coherence

$$t\frac{\partial}{\partial t}F(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}P(z)F(x/z, z^2 t)$$

- Solution by moments

$$\tilde{F}(N,t) \sim \exp\left[\int_{t_0}^{t}\gamma(N,\alpha_S)\frac{dt'}{t'}\right]\tilde{F}(N,t_0)$$

$$\gamma(N,\alpha_S) = \frac{\alpha_S}{2\pi}\int_0^1 z^{N-1+2\gamma(N,\alpha_S)}P(z)$$

- Anomalous dimension dominates asymptotically

$$\gamma(N,\alpha_S) = \frac{\alpha_S}{2\pi}\int_0^1 z^{N-1+2\gamma(N,\alpha_S)} P(z)$$

$$\sim \frac{C_A\alpha_S}{\pi}\frac{1}{N-1+2\gamma(N,\alpha_S)}$$

- This is regular at N=1

$$\gamma(N,\alpha_S) = \frac{1}{4}\left[\sqrt{(N-1)^2+\frac{8C_A\alpha_S}{\pi}}-(N-1)\right]$$

$$= \sqrt{\frac{C_A\alpha_S}{2\pi}}-\frac{1}{4}(N-1)+\frac{1}{32}\sqrt{\frac{2\pi}{C_A\alpha_S}}(N-1)^2+\cdots$$

$$\int^{t} \gamma(N,\alpha_S(t'))\frac{dt'}{t'} = \int^{\alpha_S(t)} \frac{\gamma(N,\alpha_S)}{\beta(\alpha_S)}\, d\alpha_S$$

where $\beta(\alpha_S) = -b\alpha_S^2 + \cdots$. Hence

$$\tilde{F}(N,t) \sim \exp\left[\frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S}} - \frac{1}{4b\alpha_S}(N-1) + \frac{1}{48b}\sqrt{\frac{2\pi}{C_A\alpha_S^3}}(N-1)^2 + \cdots\right]_{\alpha_S=\alpha_S(t)}$$

Mean multiplicity — Position of peak — Width of peak

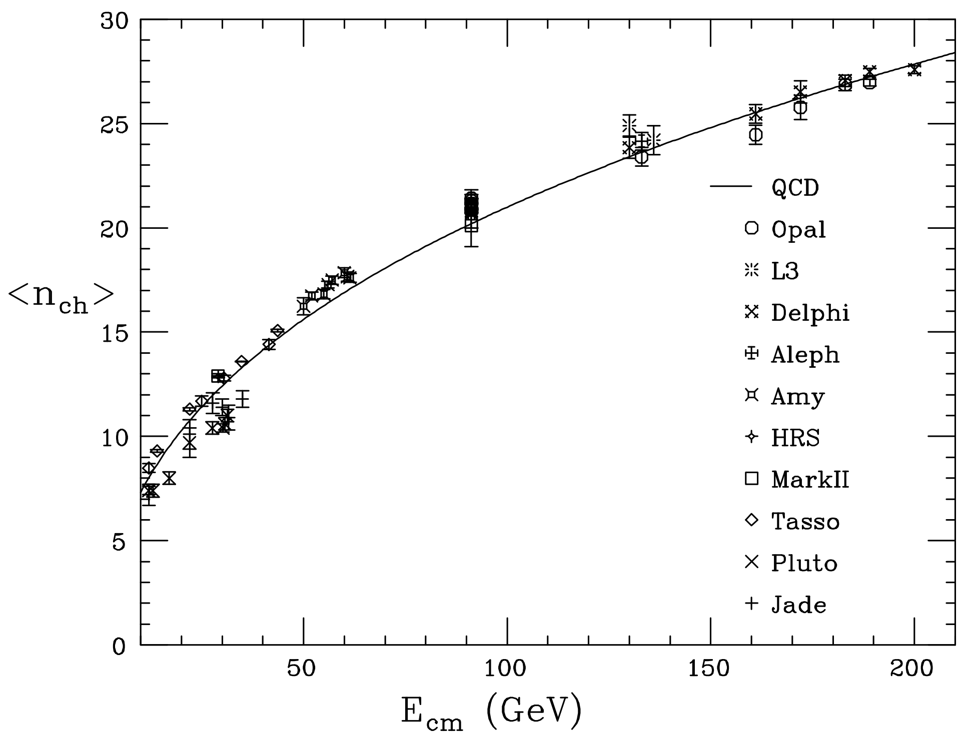
- Gaussian in $N \leftrightarrow$ Gaussian in $\xi \equiv \ln(1/x)$
- Mean multiplicity

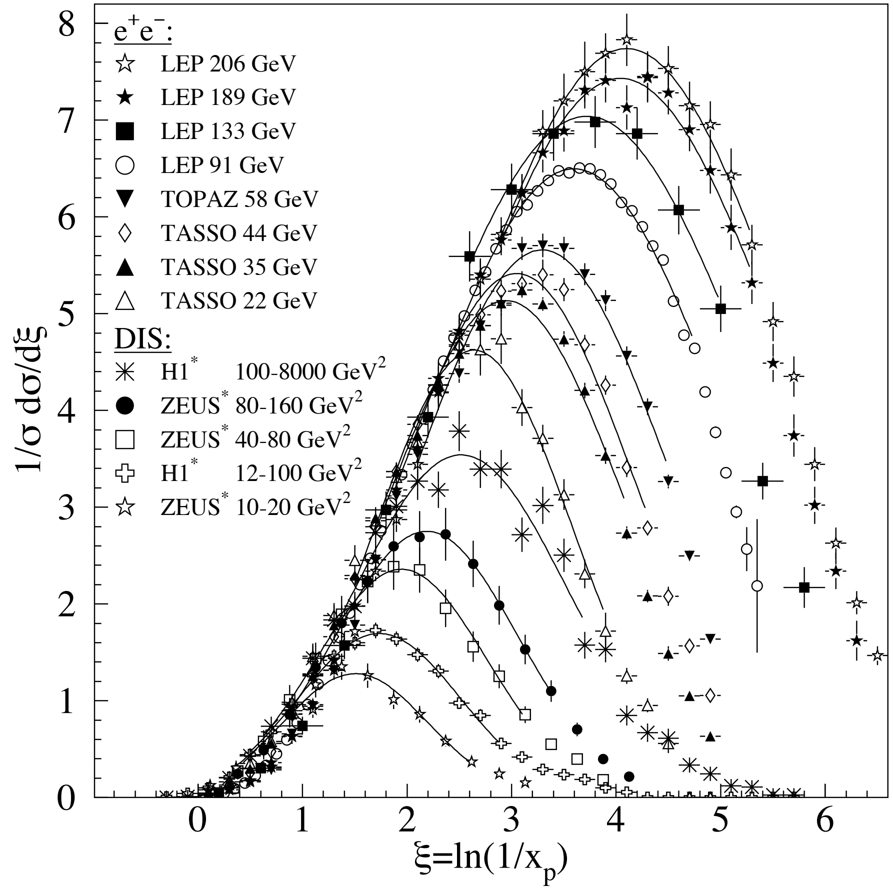
$$\langle n(s)\rangle = \int_0^1 dx\, F(x,s) = \tilde{F}(1,s)$$

$$\sim \exp\frac{1}{b}\sqrt{\frac{2C_A}{\pi\alpha_S(s)}} \sim \exp\sqrt{\frac{2C_A}{\pi b}\ln\left(\frac{s}{\Lambda^2}\right)}$$

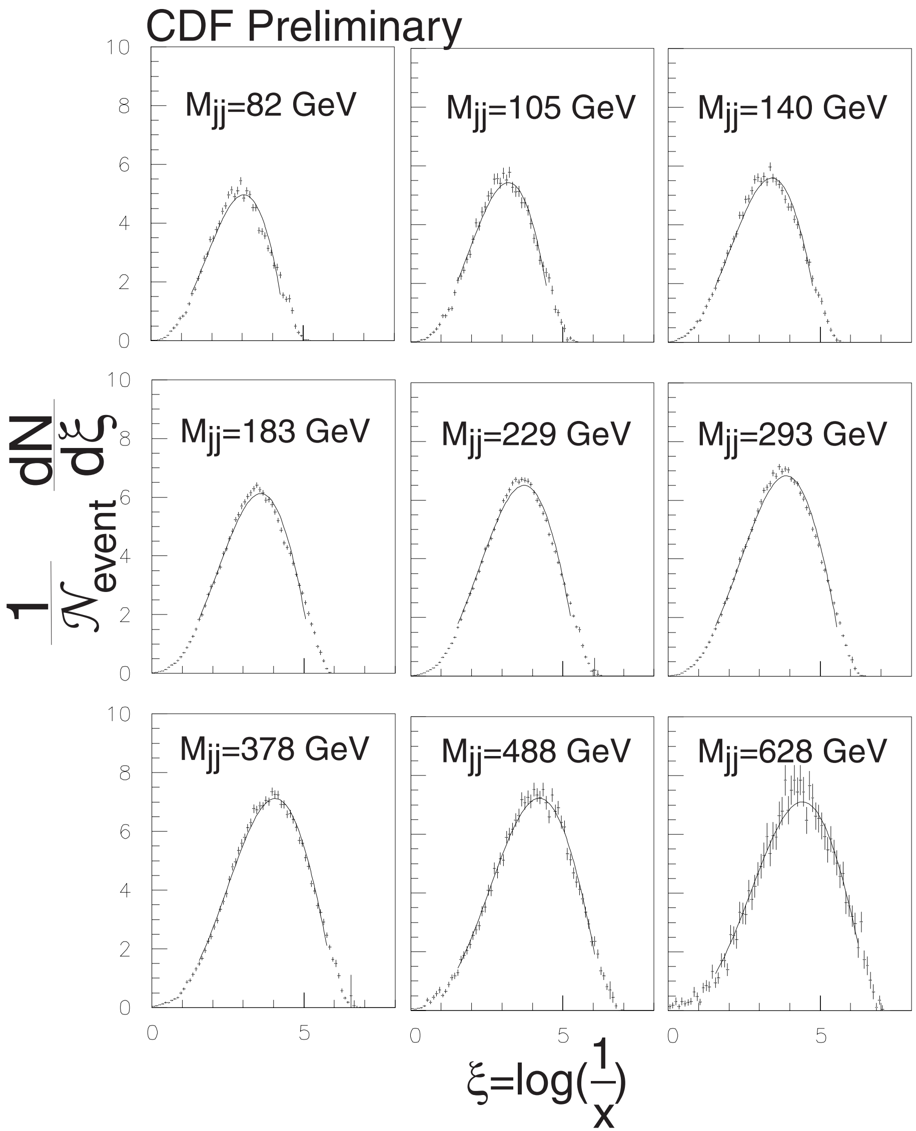
# LPHD Predictions

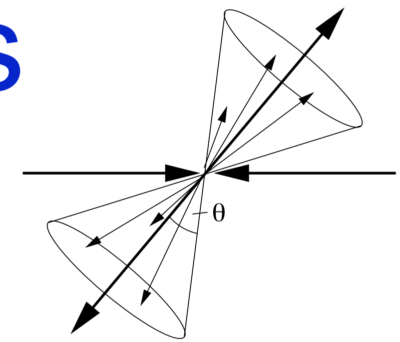
- Good agreement with data

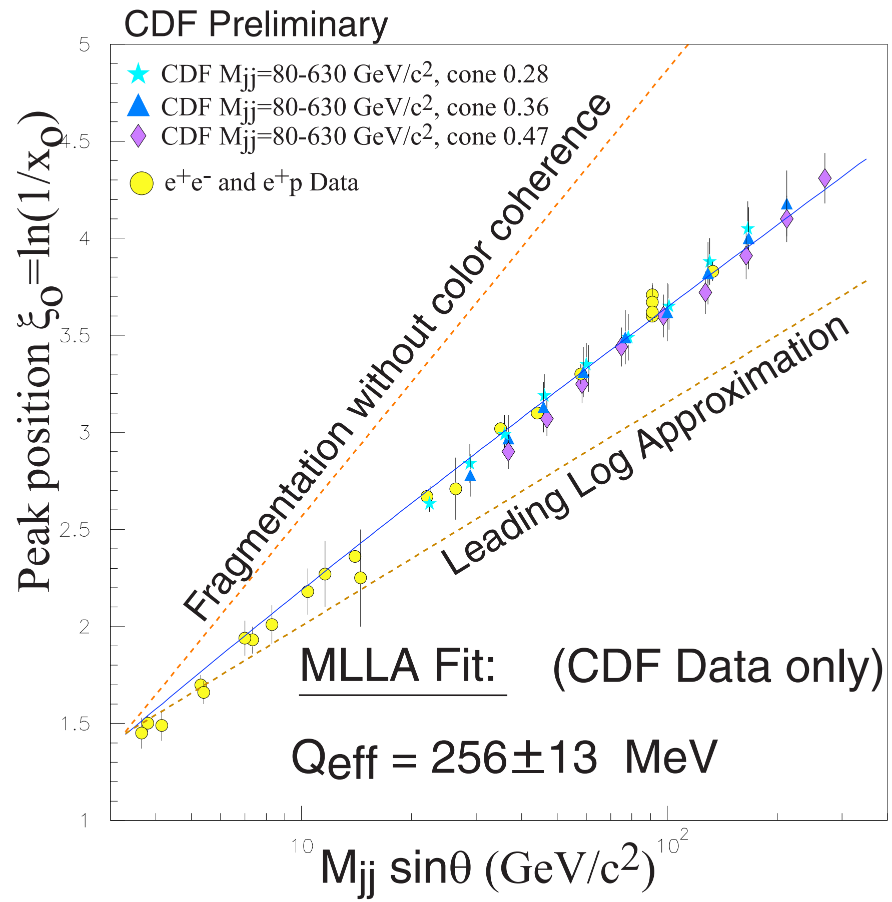


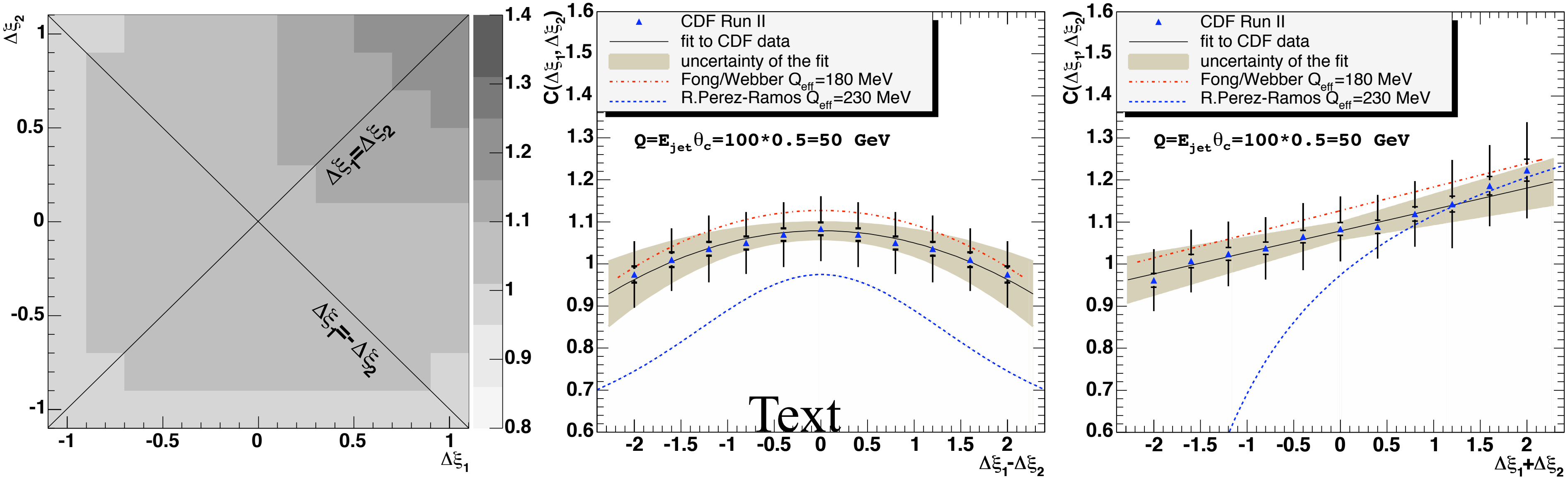
# LPHD in $p\bar{p} \to$ dijets



# Two-particle energy correlations




CDF, arXiv:0802.3182

$$R(\Delta\xi_1,\Delta\xi_2) = r_0 + r_1(\Delta\xi_1+\Delta\xi_2) + r_2(\Delta\xi_1-\Delta\xi_2)^2$$

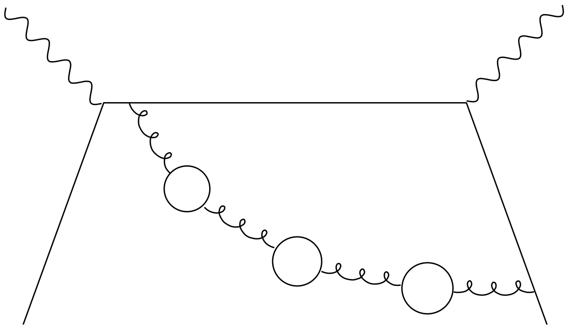
$$r_0^q = 1.75 - \frac{0.64}{\sqrt{\tau}}, \qquad r_1^q = \frac{1.6}{\tau^{3/2}}, \qquad r_2^q = -\frac{2.25}{\tau^2} \qquad \tau = \ln(Q/Q_{eff})$$

CP Fong & BW,
NP B355(1991)54

$$r_0^g = 1.33 - \frac{0.28}{\sqrt{\tau}}, \qquad r_1^g = \frac{0.7}{\tau^{3/2}}, \qquad r_2^g = -\frac{1.0}{\tau^2}.$$

# Universal low-scale effective $\alpha_S$

- Infrared renormalon



$$F \sim \int_0^Q \frac{dp_t}{Q} \alpha_S(p_t)$$

$$= \alpha_S(Q) \sum_n \int_0^Q \frac{dp_t}{Q} \left[ b\alpha_S(Q) \ln \frac{Q^2}{p_t^2} \right]^n$$

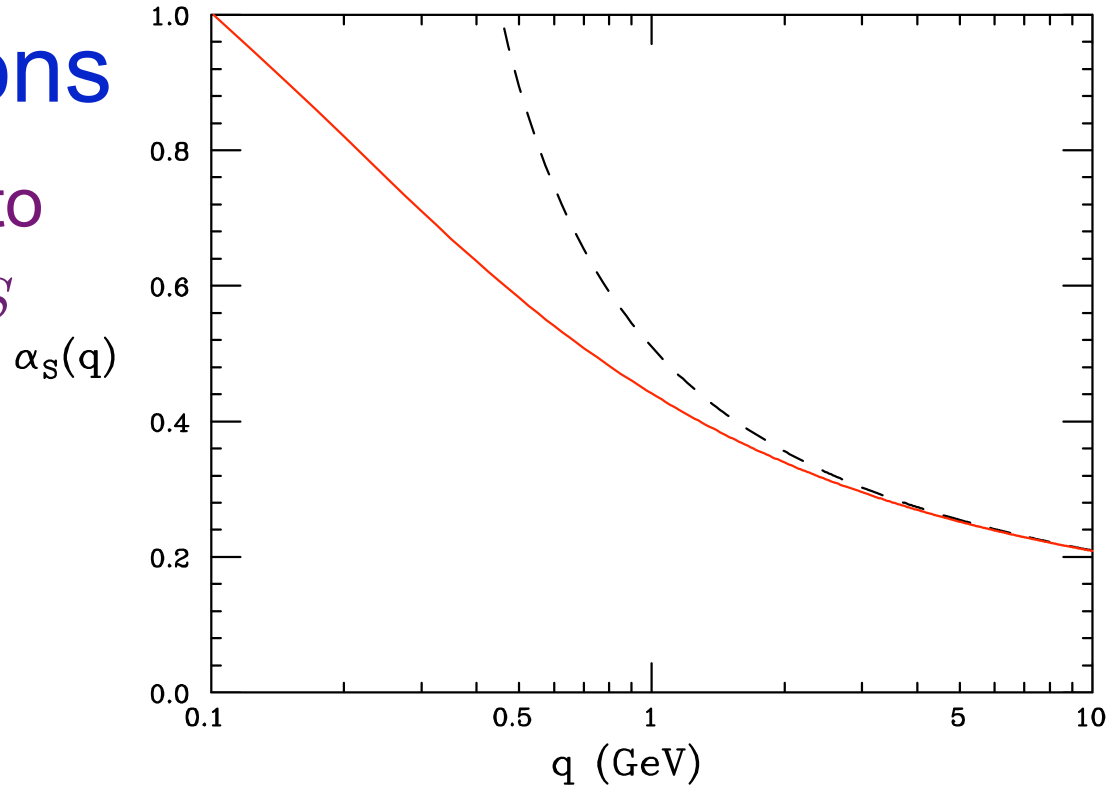
$$= \alpha_S(Q) \sum_n n! [2b\alpha_S(Q)]^n$$

- Divergent series: truncate at smallest term ( $n_m = [2b\alpha_S(Q)]^{-1}$ ) $\Rightarrow$ uncertainty

$$\delta F \sim n_m! [2b\alpha_S(Q)]^{n_m} \sim e^{-n_m} = \frac{\Lambda}{Q}$$

# Power Corrections

- Renormalon is due to IR divergence of $\alpha_S$
- Postulate universal IR-regular $\alpha_S$
- Power corrections depend on

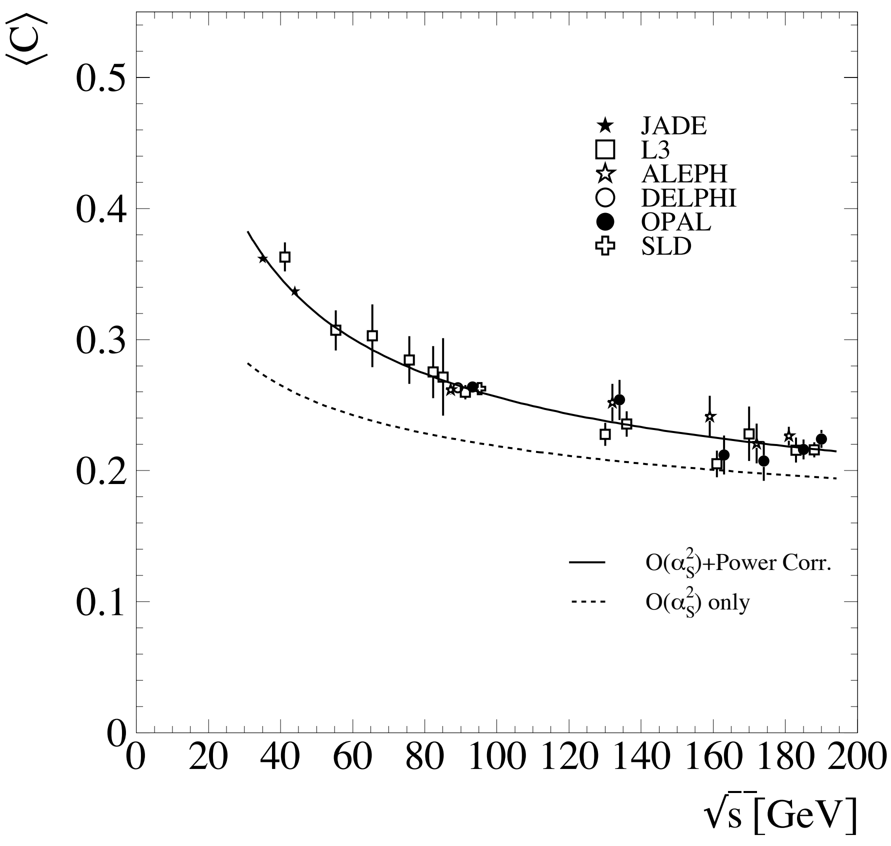


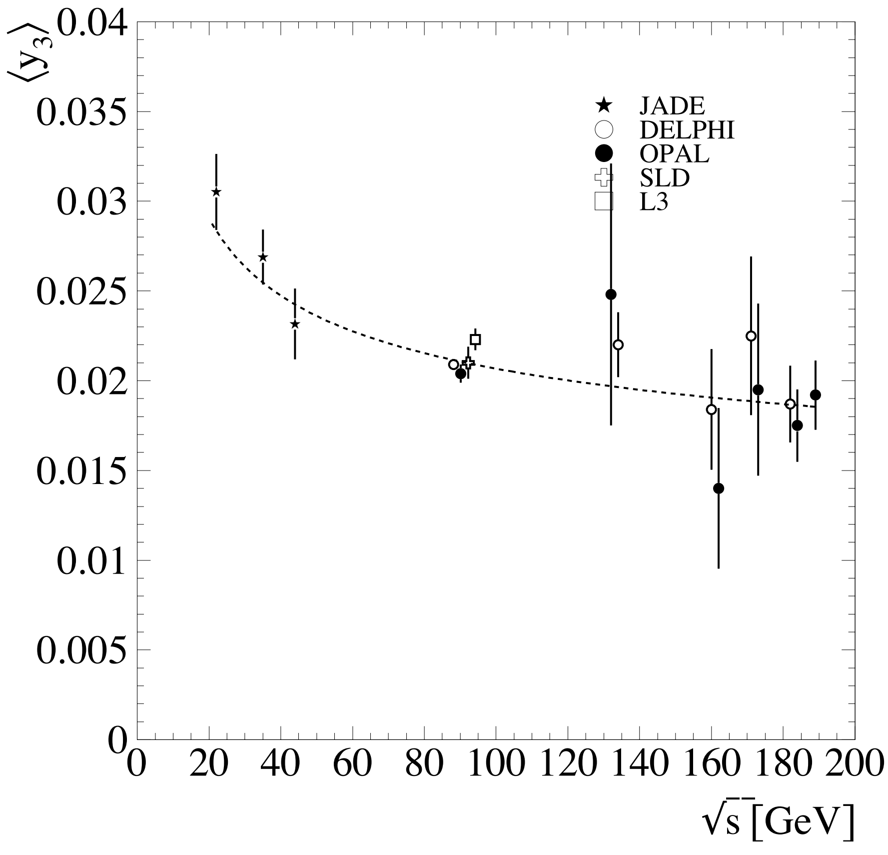

$$\alpha_0(\mu_I) = \frac{1}{\mu_I}\int_0^{\mu_I} \alpha_S(p_t)\, dp_t$$

- Match NP & PT at $\mu_I \sim 2$ GeV

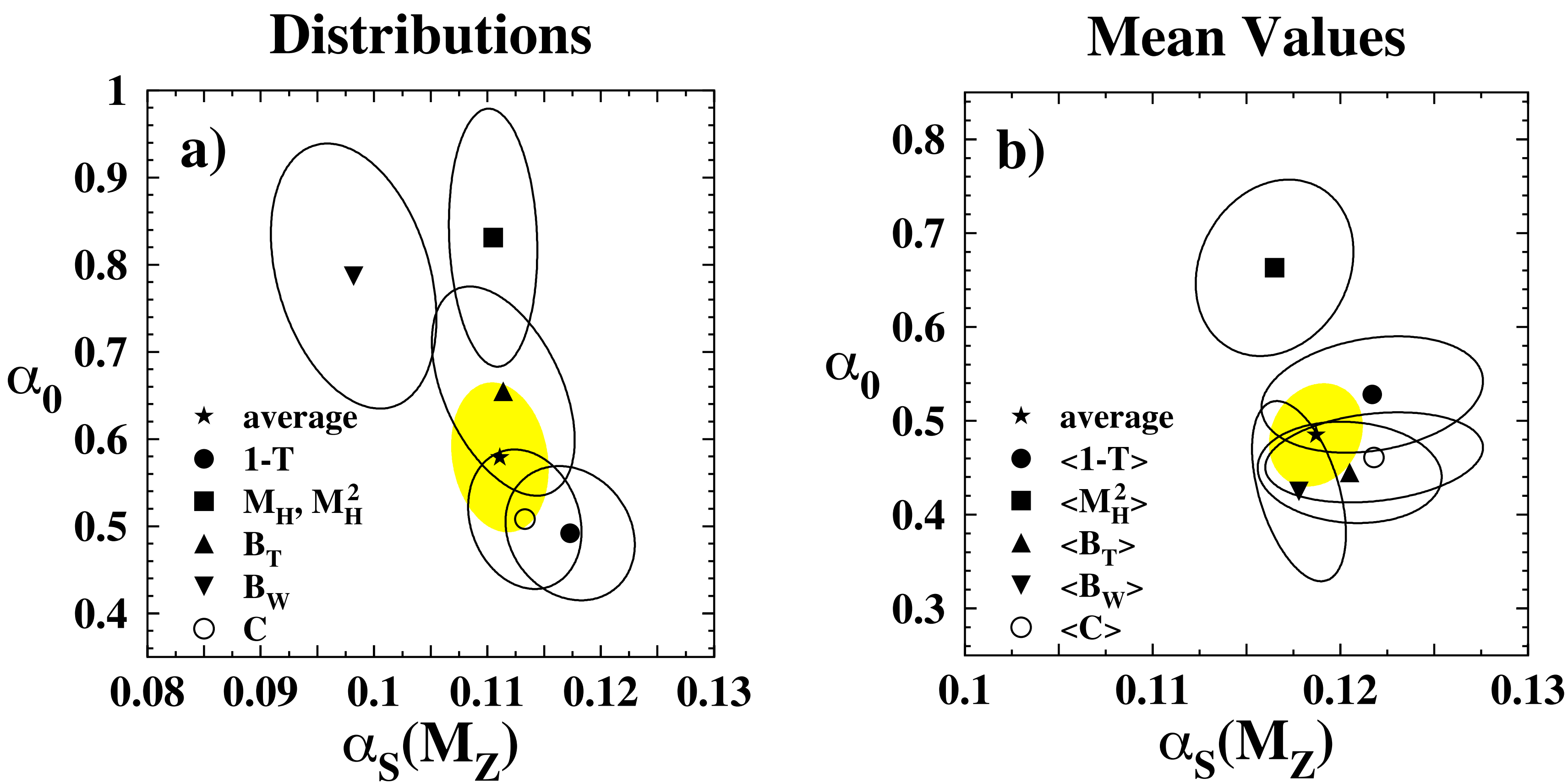
# Power corrections to event shapes

- 1/Q renormalon present in C, absent in $y_3$



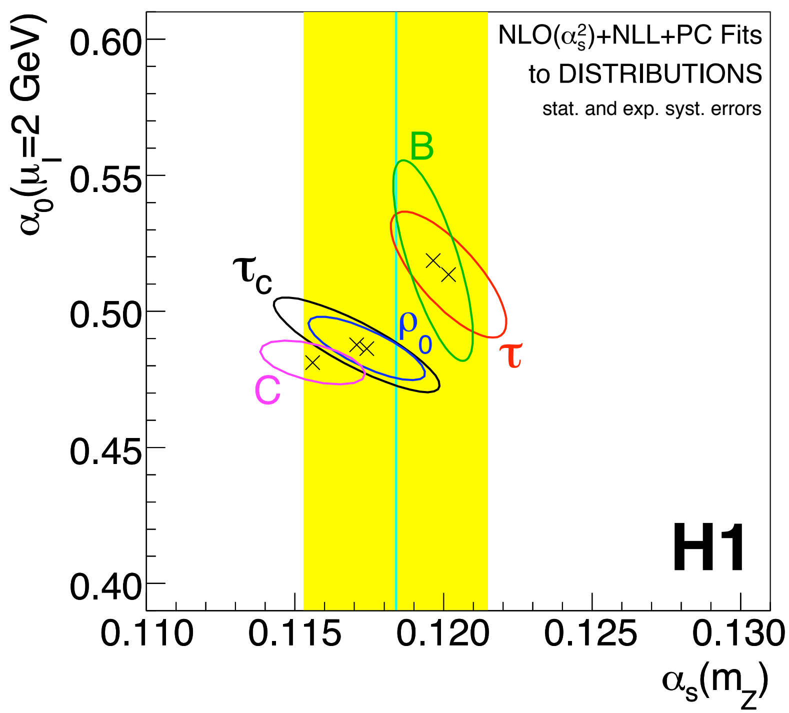
# ULSEA results from $e^+e^-$

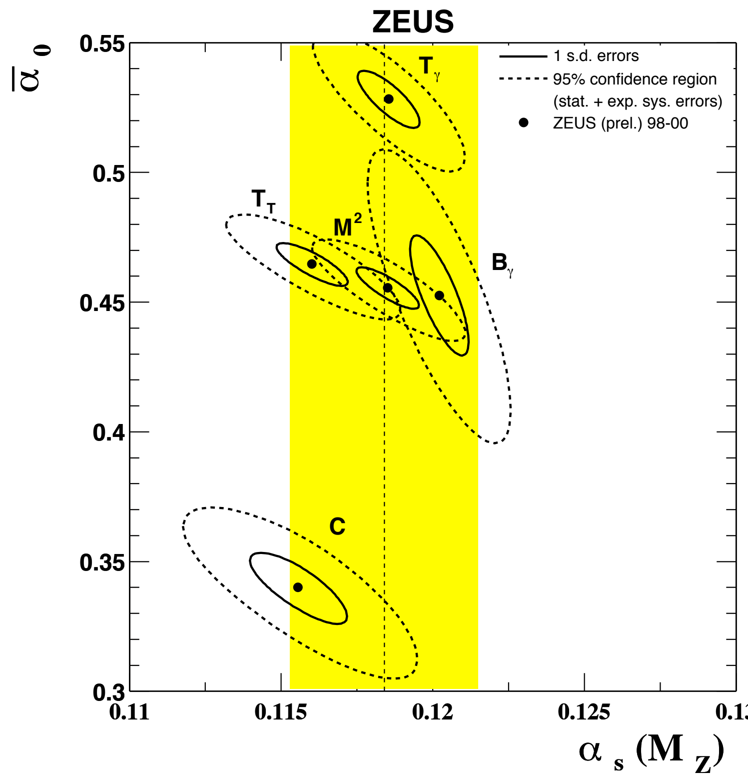



Movilla Fernandez, Bethke, Biebel & Kluth, EPJ C22(2001)1

# ULSEA results from DIS

- Consistent with $e^+e^-$



# ULSEA hadronic jet energy correction

$$\langle \delta p_t \rangle_{\mathrm{h}}^{(jr)} = C_{jr}\,\mathcal{A}(\mu_I)\left(-\frac{1}{R} - \frac{1}{4}R + \frac{1}{192}R^3 - \frac{5}{2304}R^5 + \mathcal{O}\left(R^7\right)\right)$$

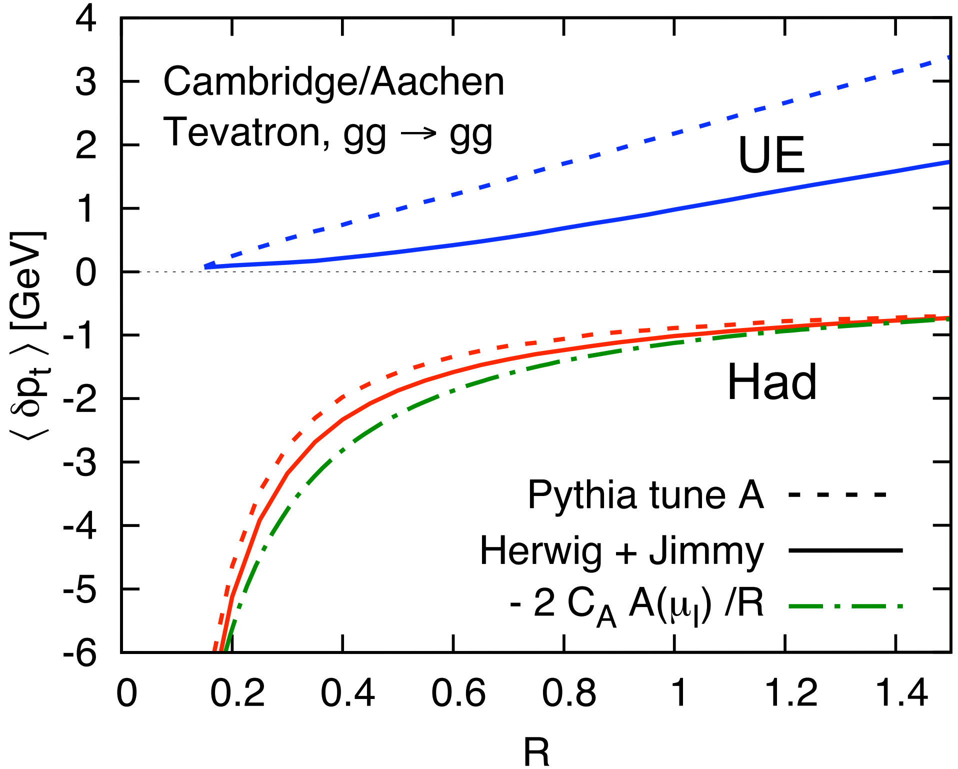
$$\mathcal{A}(\mu_I) = \frac{1}{\pi}\,\mu_I\left[\alpha_0\left(\mu_I\right) - \alpha_s(p_t) - \frac{\beta_0}{2\pi}\left(\ln\frac{p_t}{\mu_I} + \frac{K}{\beta_0} + 1\right)\alpha_s^2(p_t)\right]$$

PT subtraction

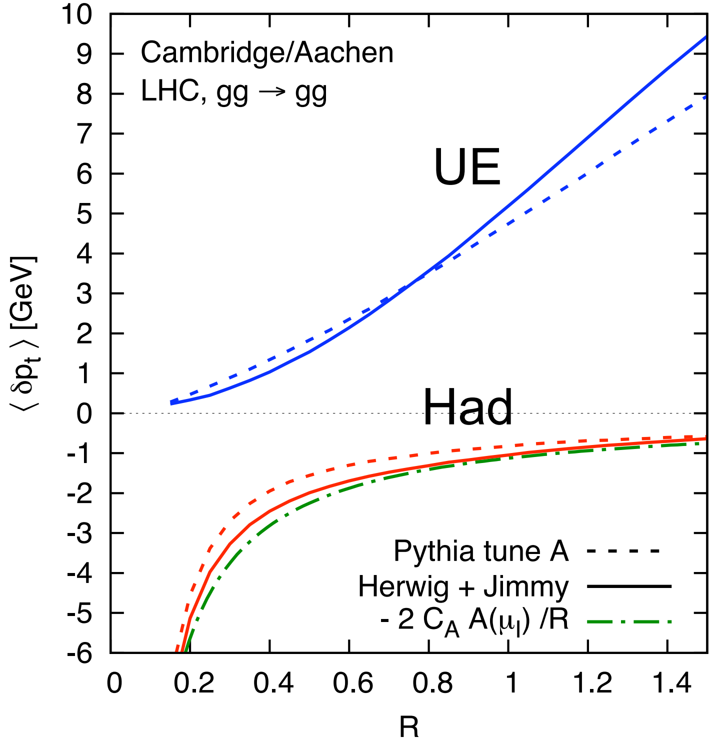
Dasgupta, Magnea & Salam, JHEP02(2008)055

# ULSEA jet energy correction

Tevatron




LHC



# Monte Carlo Models

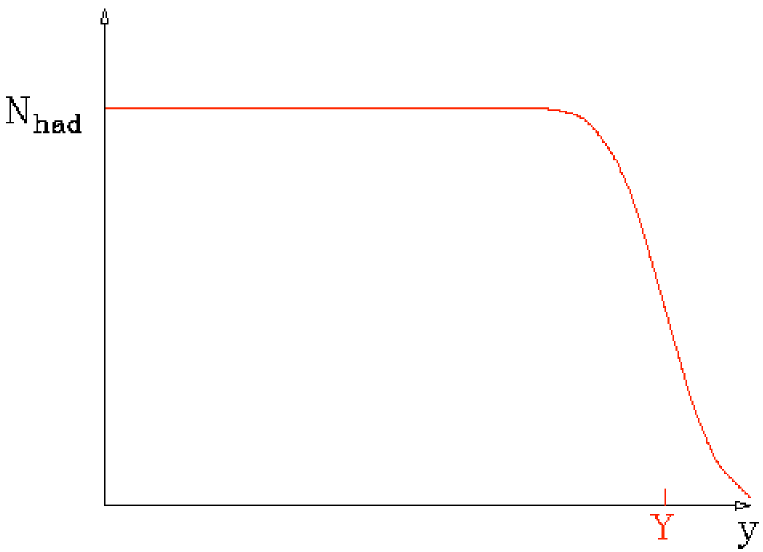
- “Tube” (longitudinal phase space) model
- Independent fragmentation model
- Confinement & Lund string model
- Preconfinement & cluster model

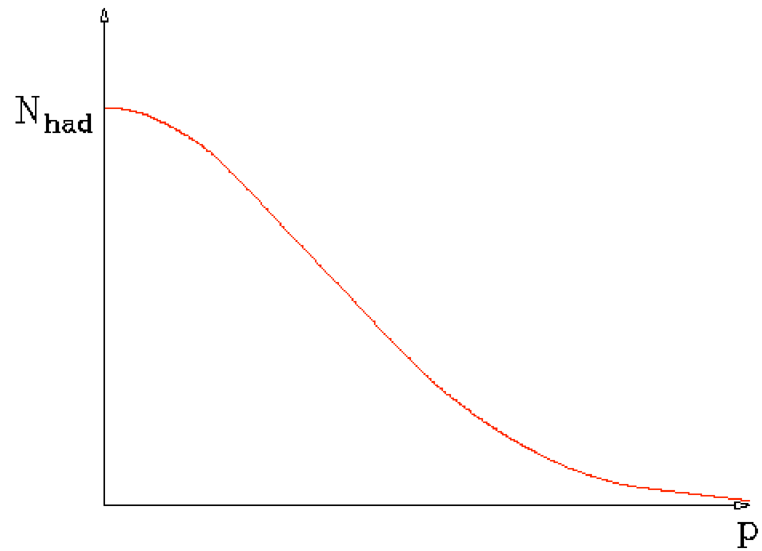
# "Tube" Model for Jet Fragmentation

- Precursor of MC models
- Shows some features of ULSEA

Experimentally, $e^+e^- \to$ two jets:

Flat rapidity plateau and limited $p_t$, $\rho(p_t^2) \sim e^{-p_t^2/2p_0^2}$



Tube model gives simple estimates of hadronization corrections to perturbative quantities.

E.g. Jet energy and momentum:

$$E = \int_0^Y dy\, d^2p_t\, \rho(p_t^2)\, p_t\, \cosh y = \lambda \sinh Y$$

$$P = \int_0^Y dy\, d^2p_t\, \rho(p_t^2)\, p_t\, \sinh y = \lambda(\cosh Y - 1) \sim E - \lambda,$$

with $\lambda = \int d^2p_t\, \rho(p_t^2)\, p_t,$ mean transverse momentum.
Estimate from Fermi motion $\lambda \sim 1/R_{had} \sim m_{had}.$

Jet acquires non-perturbative mass: $M^2 = E^2 - P^2 \sim 2\lambda E$
Large: ~ 10 GeV for 100 GeV jets.

# Independent Fragmentation Model (Field-Feynman)

MC implementation of tube model.

Longitudinal momentum distribution = arbitrary fragmentation function: parameterization of data.

Transverse momentum distribution = Gaussian.

Recursively apply $q \to q' + \mathrm{had.}$

Hook up remaining soft $q$ and $\bar{q}$.

Strongly frame dependent.

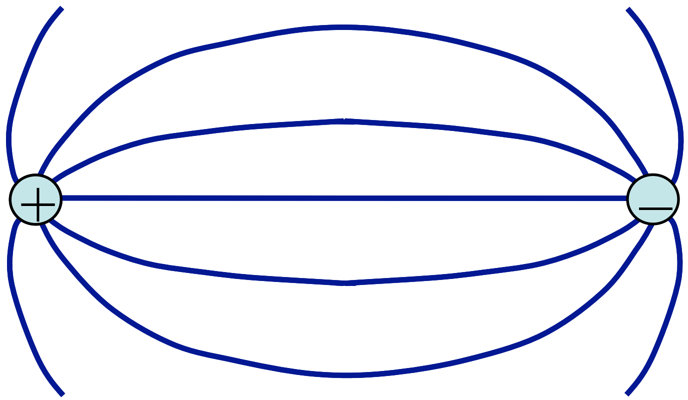
No obvious relation with perturbative emission.

Not infrared safe.

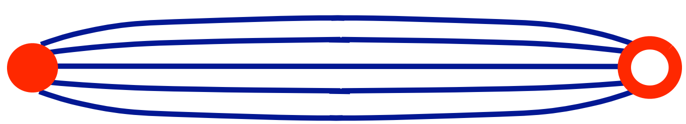
Not a model of confinement.

# Confinement

Asymptotic freedom: $Q\bar{Q}$ becomes increasingly QED-like at short distances.

QED: 

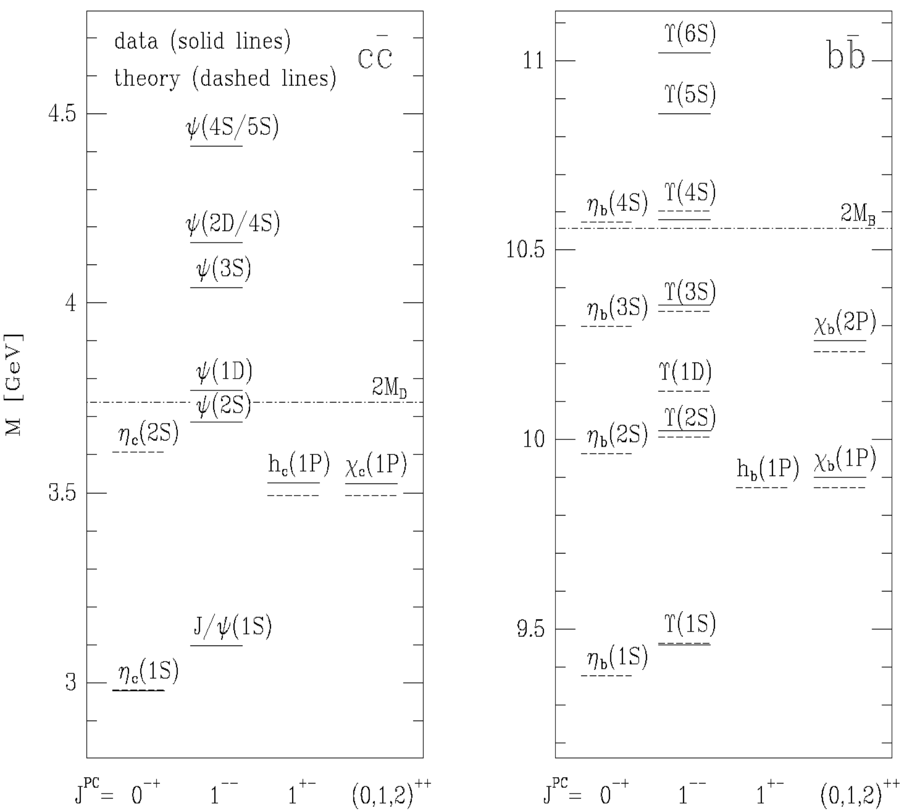

but at long distances, gluon self-interaction makes field lines attract each other:

QCD: 

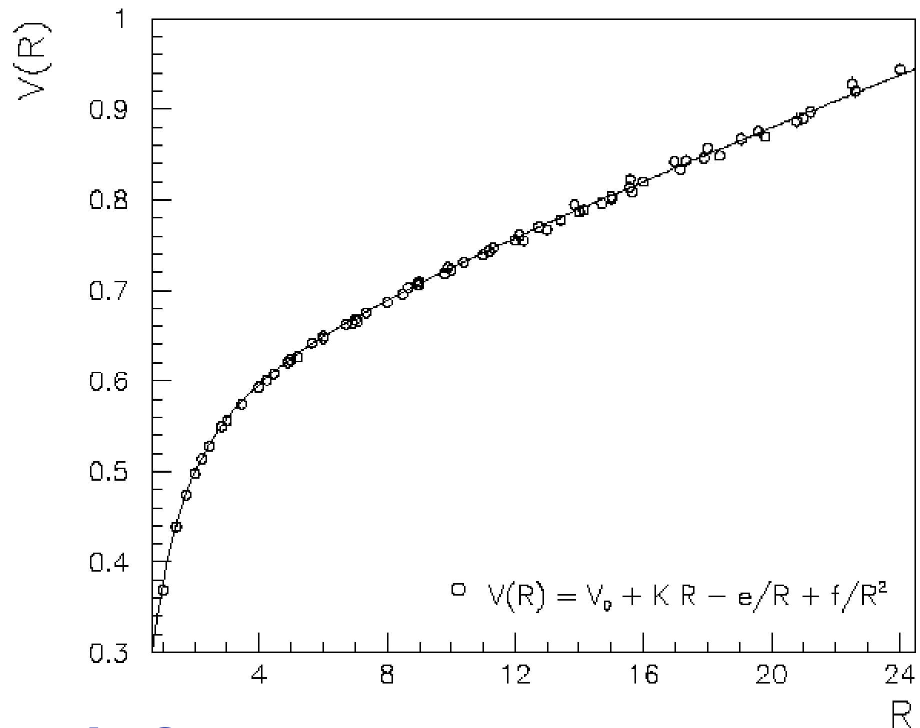
→linear potential → confinement

# Interquark Potential

Can measure from quarkonia spectra:




or from lattice QCD:



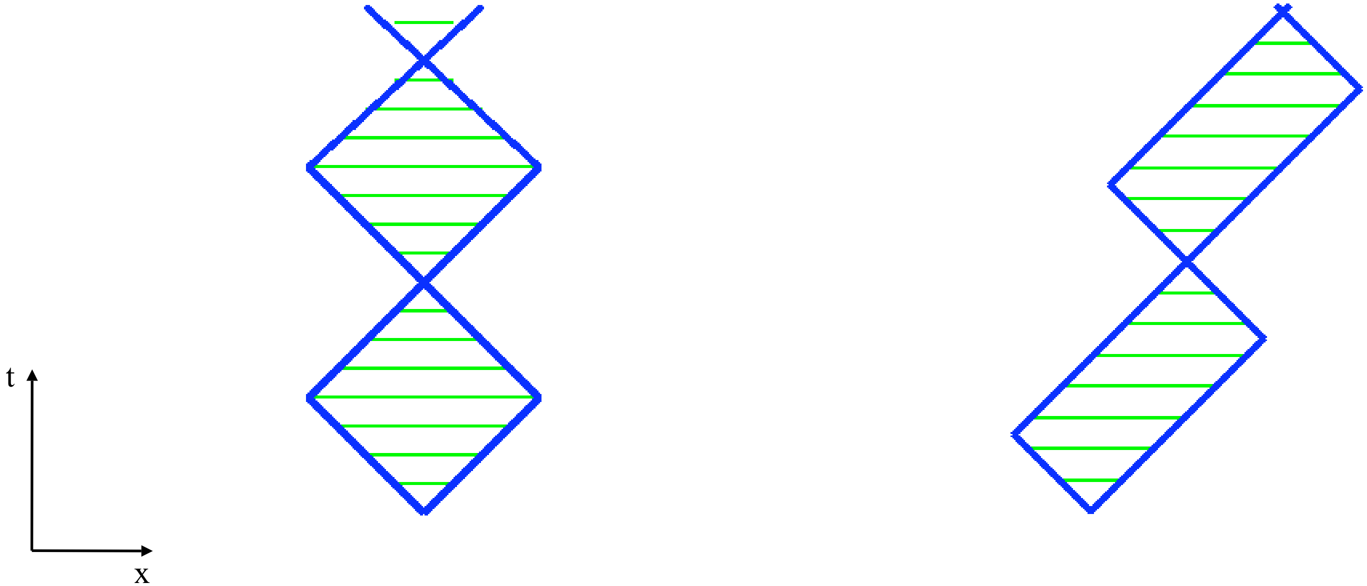

→ String tension

$$\kappa \approx 1 \text{ GeV/fm.}$$

# 2-d String Model of Mesons

Light quarks connected by string.

L=0 mesons only have 'yo-yo' modes:




Obeys area law: $m^2 = 2\kappa^2$ area

# The Lund String Model

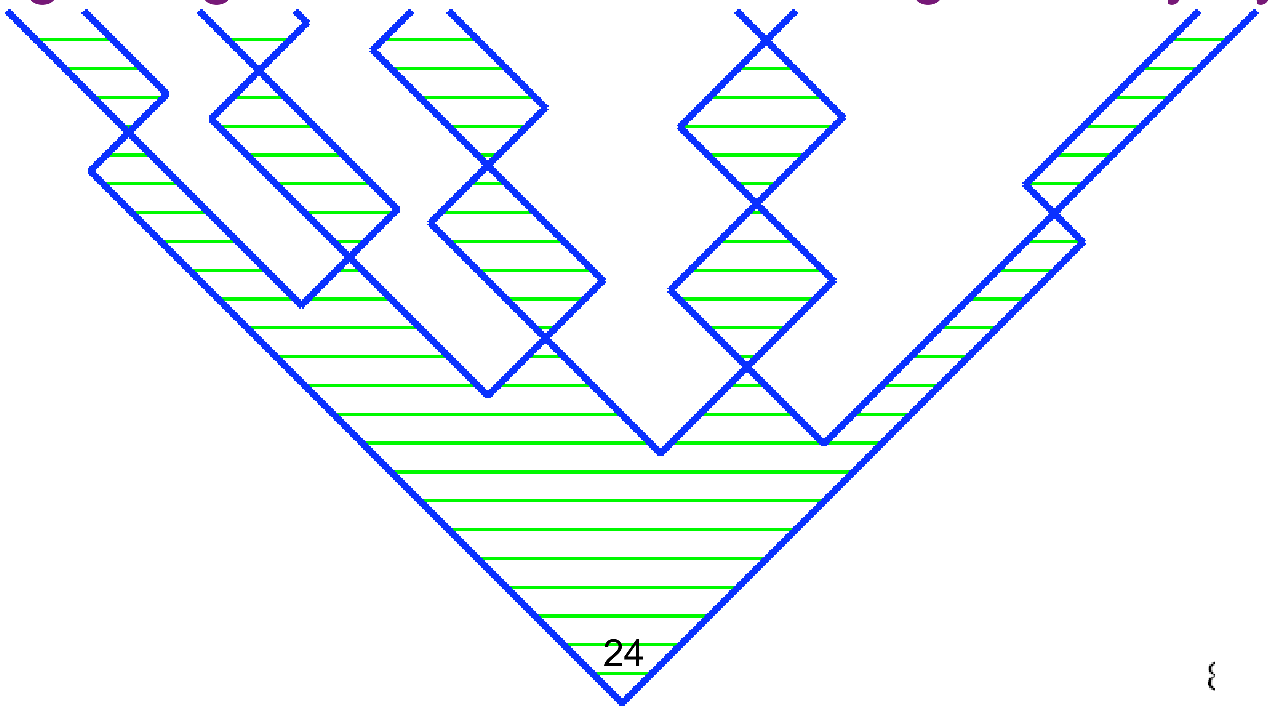
Start by ignoring gluon radiation:

$e^+e^-$ annihilation = pointlike source of $q\bar{q}$ pairs

Intense chromomagnetic field within string → $q\bar{q}$ pairs created by tunnelling. Analogy with QED:

$$\frac{d(\text{Probability})}{dx\ dt} \propto \exp(-\pi m_q^2/\kappa)$$

Expanding string breaks into mesons long before yo-yo point.



# Lund Symmetric Fragmentation Function

String picture → constraints on fragmentation function:

- Lorentz invariance
- Acausality
- Left—right symmetry

$$f(z) \propto z^{a_\alpha - a_\beta - 1}(1-z)^{a_\beta}$$

$a_{\alpha,\beta}$ adjustable parameters for quarks $\alpha$ and $\beta$.

Fermi motion → Gaussian transverse momentum.

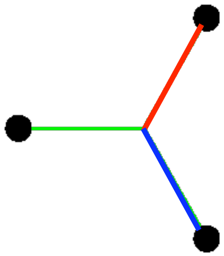
Tunnelling probability becomes

$$\exp\left[-b(m_q^2 + p_t^2)\right]$$

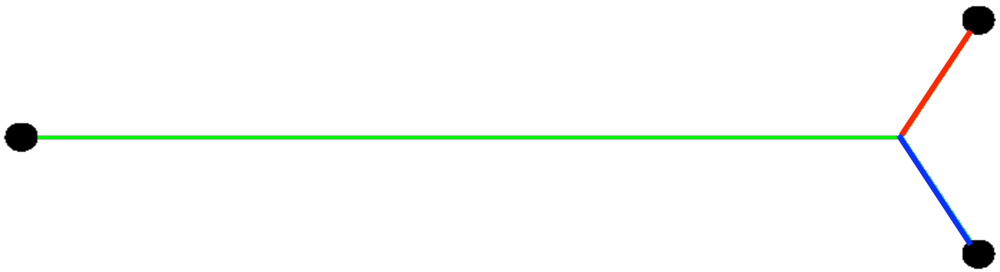
$a, b$ and $m_q^2$ = main tunable parameters of model

# Baryon Production

Baryon pictured as three quarks attached to a common centre:



At large separation, can consider two quarks tightly bound: diquark



→ diquark treated like antiquark.

Two quarks can tunnel nearby in phase space: baryon—antibaryon pair

Extra adjustable parameter for each diquark!

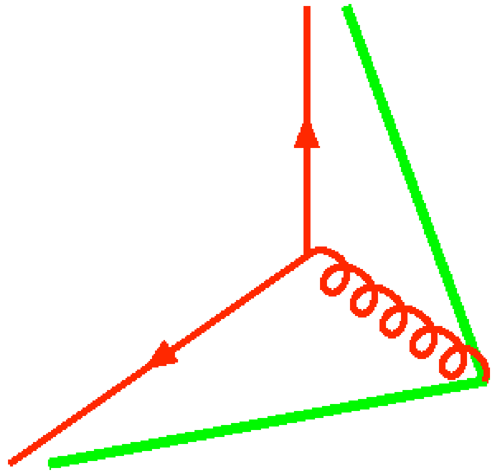
Alternative “popcorn” model:

# Three-Jet Events

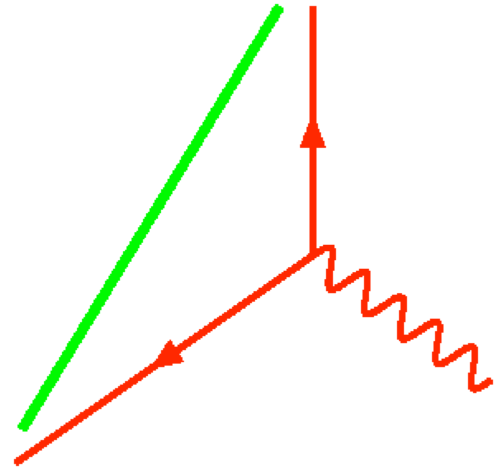
So far: string model = motivated, constrained independent fragmentation!

New feature: universal

Gluon = kink on string → the string effect



vs.



Infrared safe matching with parton shower: gluons with $k_\perp <$ inverse string width irrelevant.

# String Model Summary

- String model strongly physically motivated.
- Very successful fit to data.
- Universal: fitted to $e^+e^-$, little freedom elsewhere.

- How does motivation translate to prediction?
  ~ one free parameter per hadron/effect!

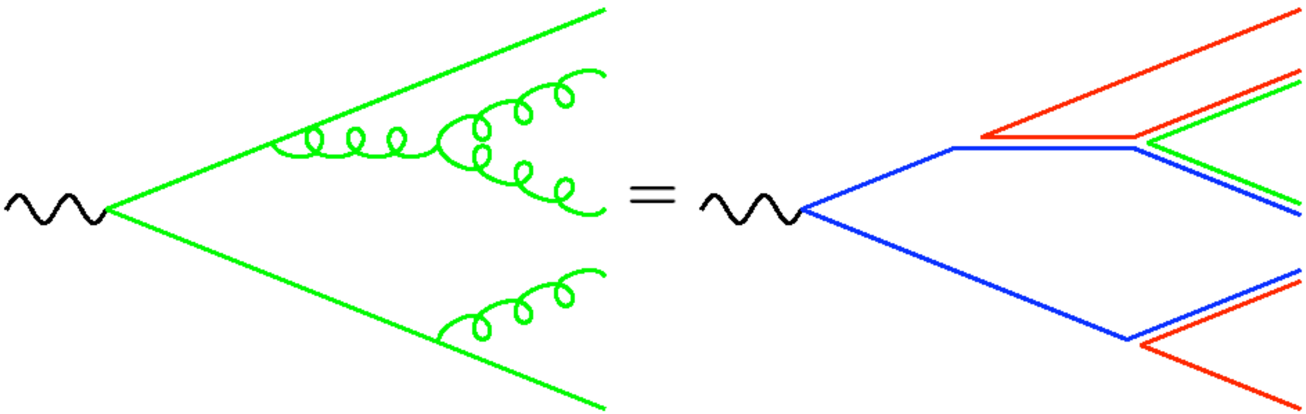
- Blankets too much perturbative information?

- Can we get by with a simpler model?

# Cluster Model: Preconfinement

Planar approximation: gluon = colour—anticolour pair.

Follow colour structure of parton shower: colour-singlet pairs end up close in phase space

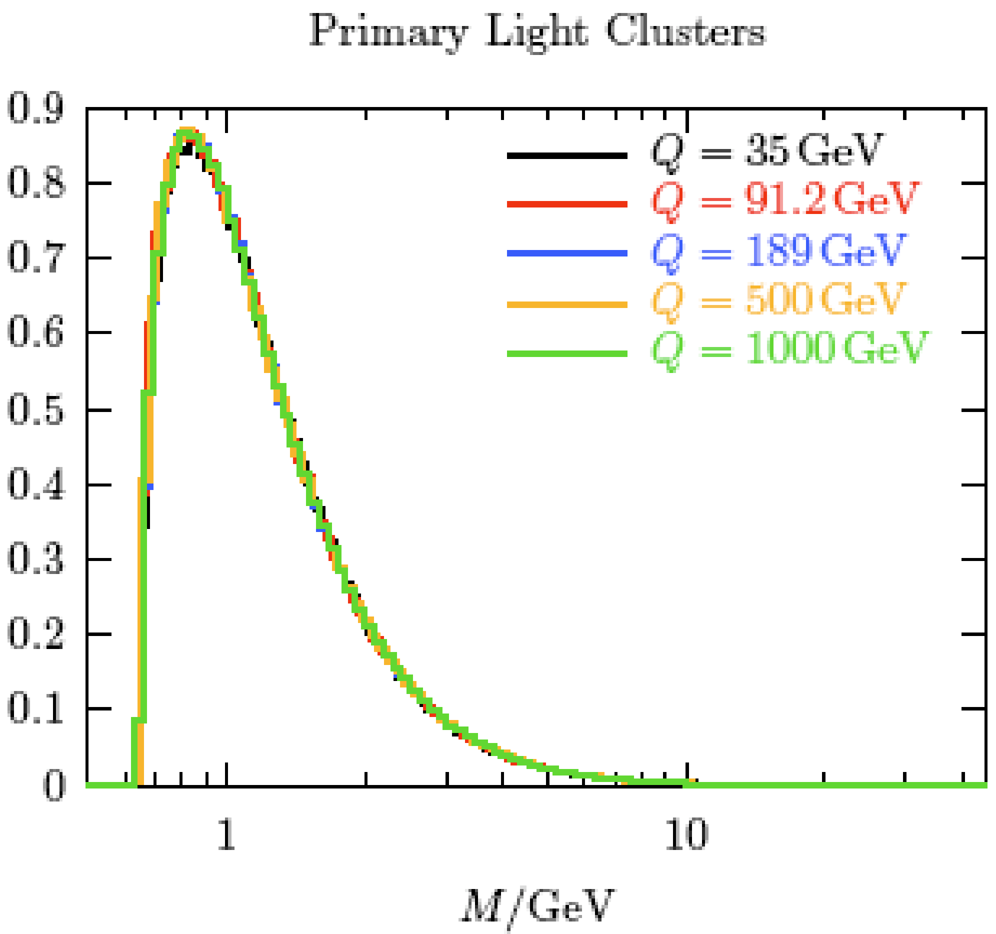


Mass spectrum of colour-singlet pairs asymptotically independent of energy, production mechanism, …

Peaked at low mass $\sim Q_0$.

# Cluster mass distribution

- Independent of shower scale $Q$
  - depends on $Q_0$ and $\Lambda$



# The Naïve Cluster Model

Project colour singlets onto continuum of high-mass mesonic resonances (=clusters). Decay to lighter well-known resonances and stable hadrons.

Assume spin information washed out:

decay = pure phase space.

→ heavier hadrons suppressed

→ baryon & strangeness suppression 'for free' (i.e. untuneable).

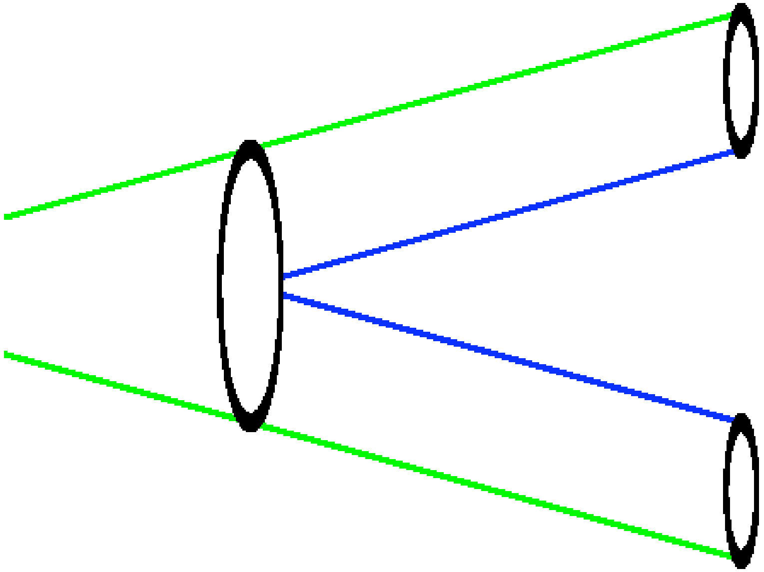
Hadron-level properties fully determined by cluster mass spectrum, i.e. by perturbative parameters.

Shower cutoff $Q_0$ becomes parameter of model.

# The Cluster Model

Although cluster mass spectrum peaked at small m, broad tail at high m.

"Small fraction of clusters too heavy for isotropic two-body decay to be a good approximation" → Longitudinal cluster fission:



Rather string-like.

Fission threshold becomes crucial parameter.

~15% of primary clusters get split but ~50% of hadrons come from them.

# The Cluster Model

"Leading hadrons are too soft"

→ 'perturbative' quarks remember their direction somewhat

$$P(\theta^2) \sim \exp(-\theta^2/2\theta_0^2)$$

Rather string-like.

Extra adjustable parameter.

## Strings

"Hadrons are produced by hadronization: you must get the non-perturbative dynamics right"

Improving data has meant successively refining perturbative phase of evolution…

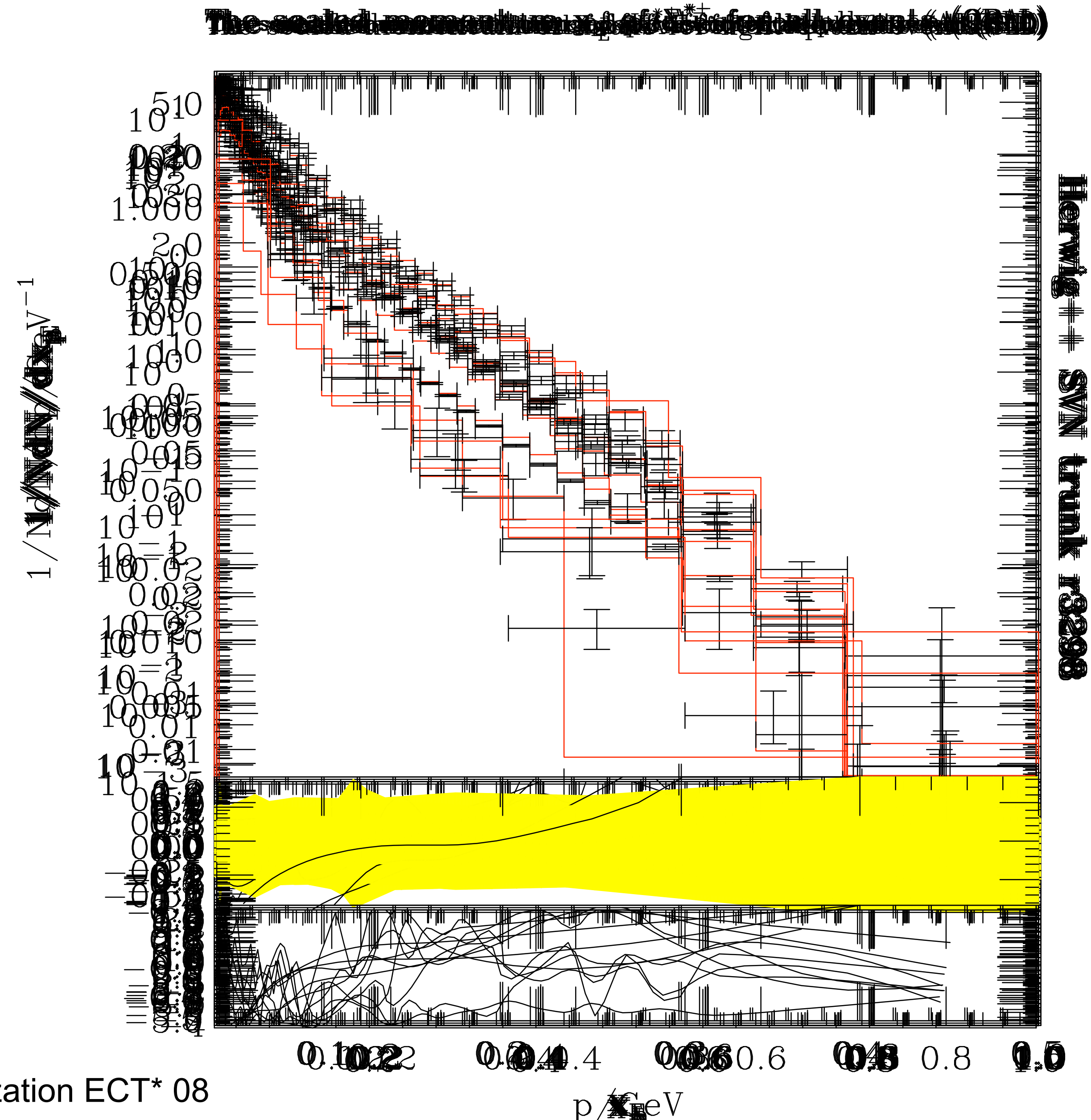
## Clusters

"Get the perturbative phase right and any old hadronization model will be good enough"

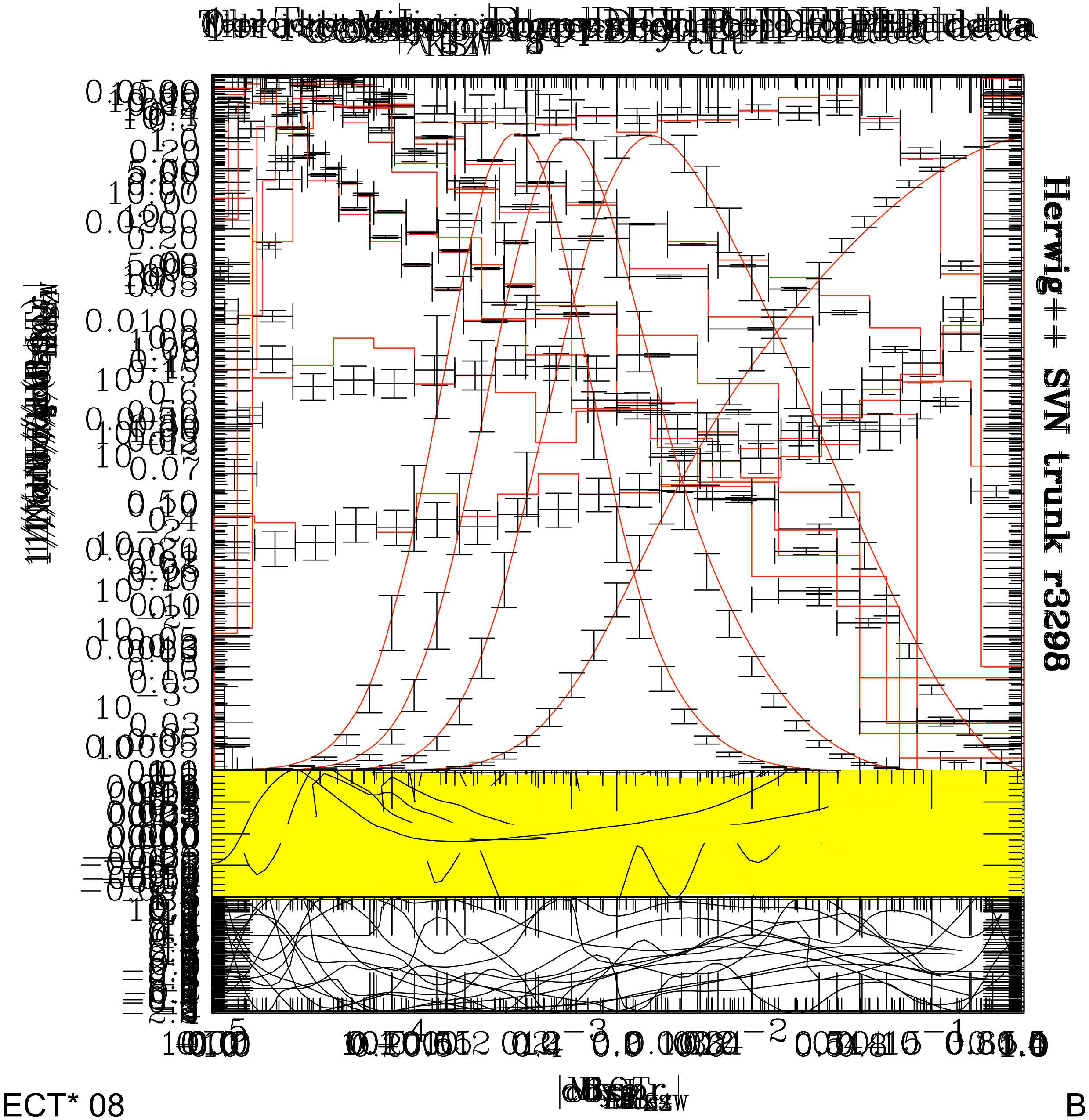
Improving data has meant successively making non-perturbative phase more string-like…

???

# Comparisons with LEP1/SLC: Spectra



# Comparisons with LEP1/SLC: Shapes



# Thermal/Statistical Model

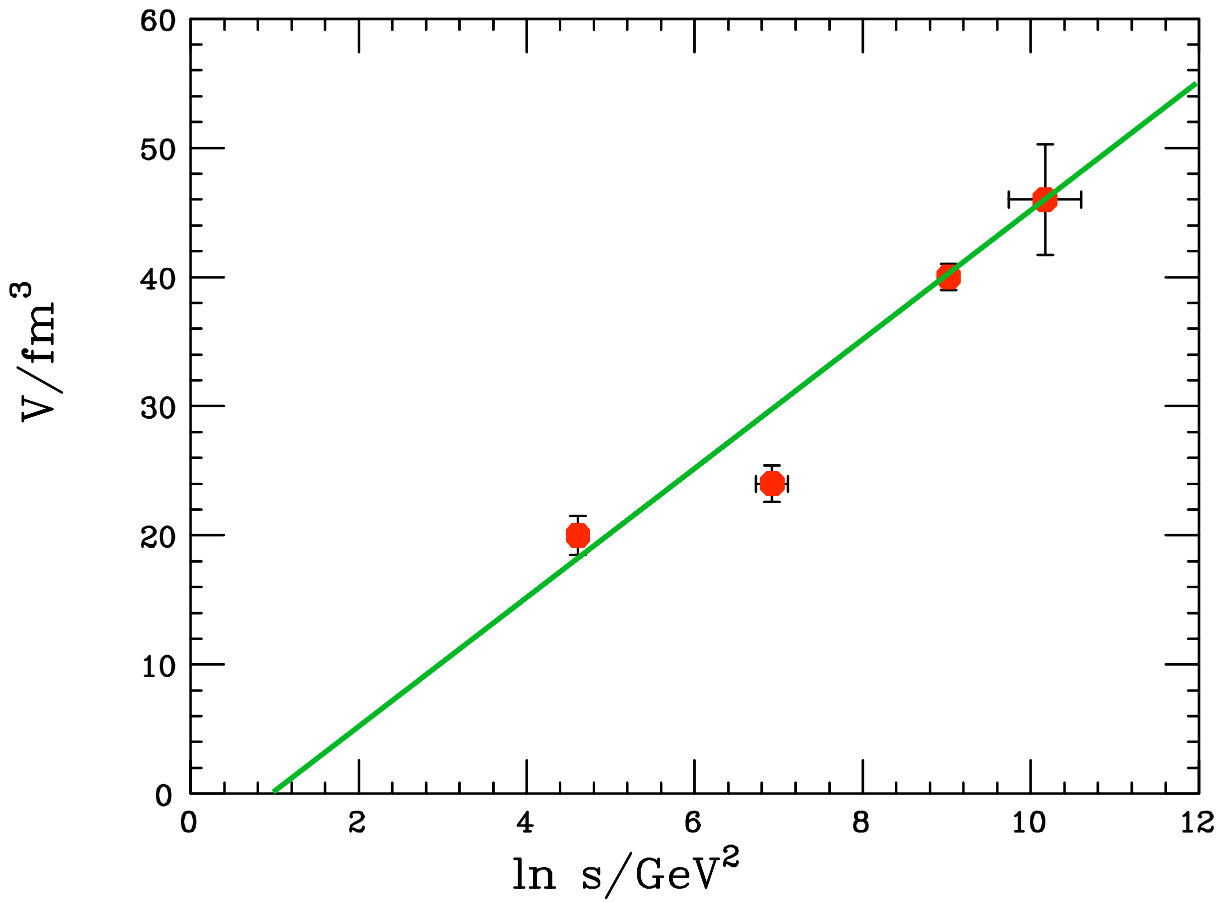
- Assume $e^+e^- \to 2$ jets in thermal equilibrium
  - 3 parameters T, V, $\gamma_s$

Values of fit parameters in $e^+e^-$ collisions at different energies.

| $\sqrt{s}$[GeV] | T[MeV] | V[fm$^3$] | $\gamma_s$ | $\chi^2$/dof |
|---|---|---|---|---|
| 10 | 152±1.7 | 20±1.5 | 0.82±0.02 | 333/21 |
| 29-35 | 156±1.7 | 24±1.4 | 0.92±0.03 | 95/18 |
| 91 | 154±0.50 | 40±1.0 | 0.76±0.007 | 631/30 |
| 130-200 | 154±2.8 | 46±4.3 | 0.72±0.03 | 12/2 |

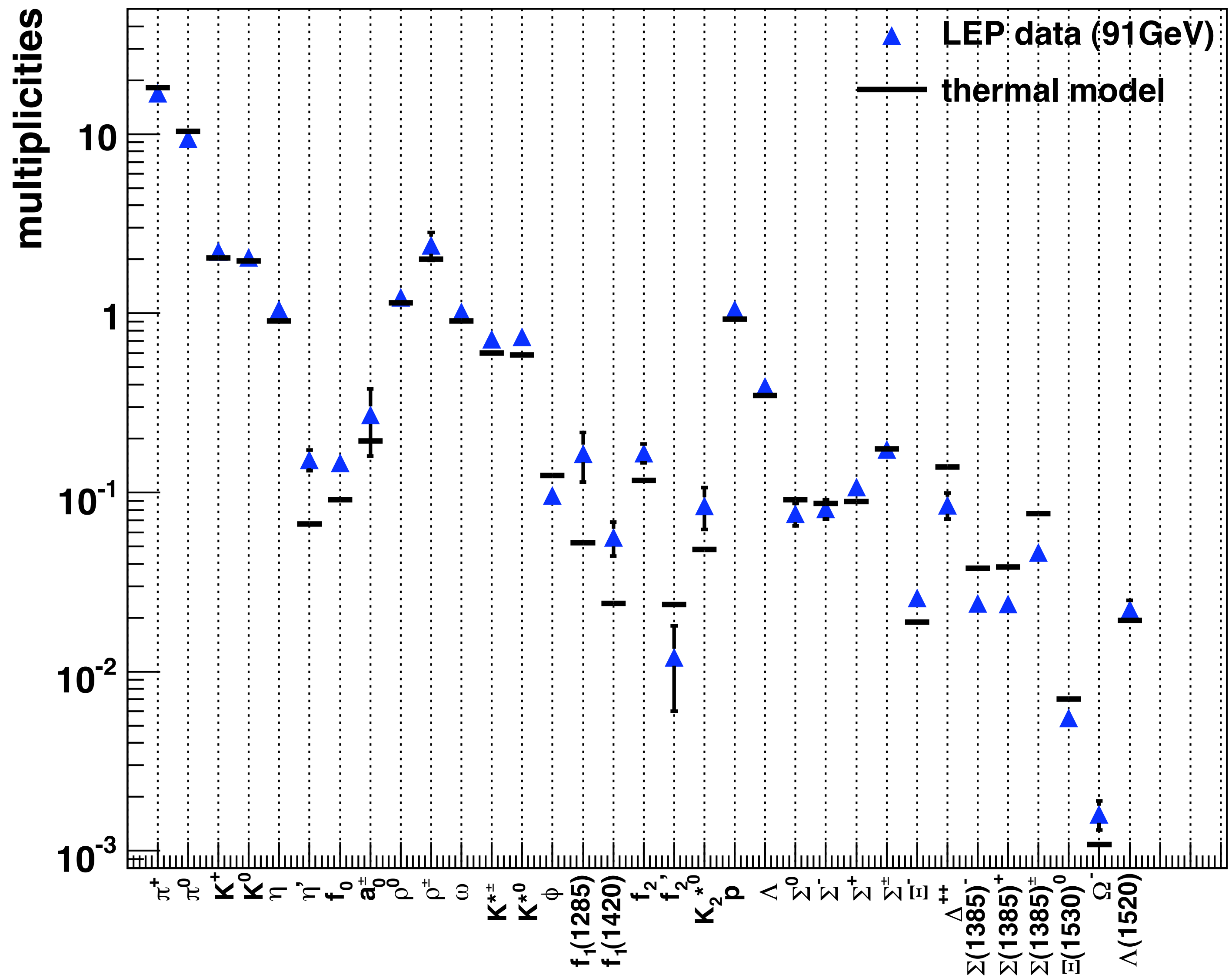
A Andronic et al., arXiv:0804.4132

# Hadronization Volume




- dV/dy ~ 5 fm$^3$

# Thermal model results

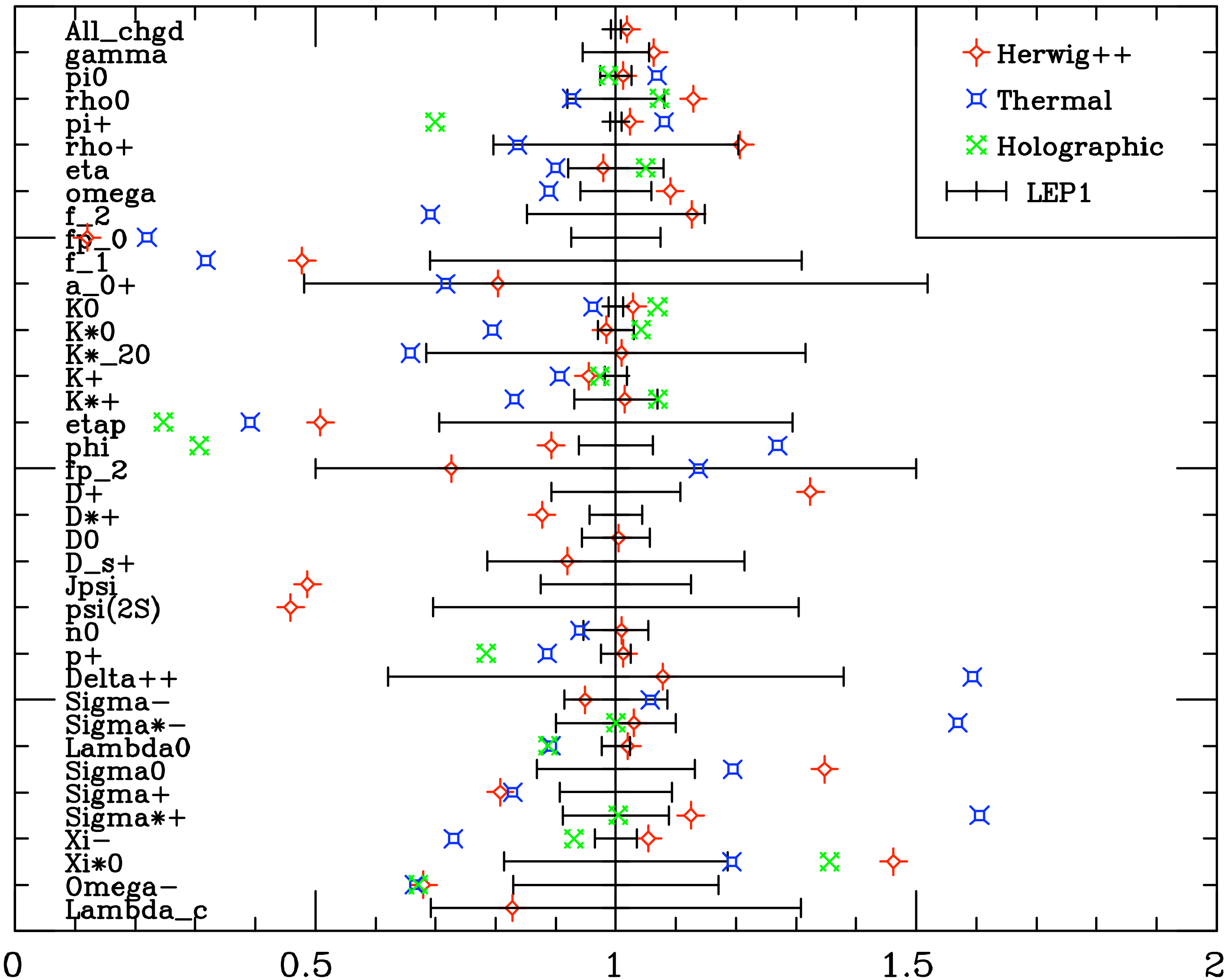


# Holographic Model

- Strongly-coupled gauge theory dual to weakly-coupled 5D gravity
  - Promising approach to IR behaviour of QCD
  - Relative hadron multiplicities given by 5D radial wave function overlap with common Gaussian
  - 4 parameters (1 energy dependent)

N Evans & A Tedder, PRL100(2008)162003

# Hadron Yields at LEP1

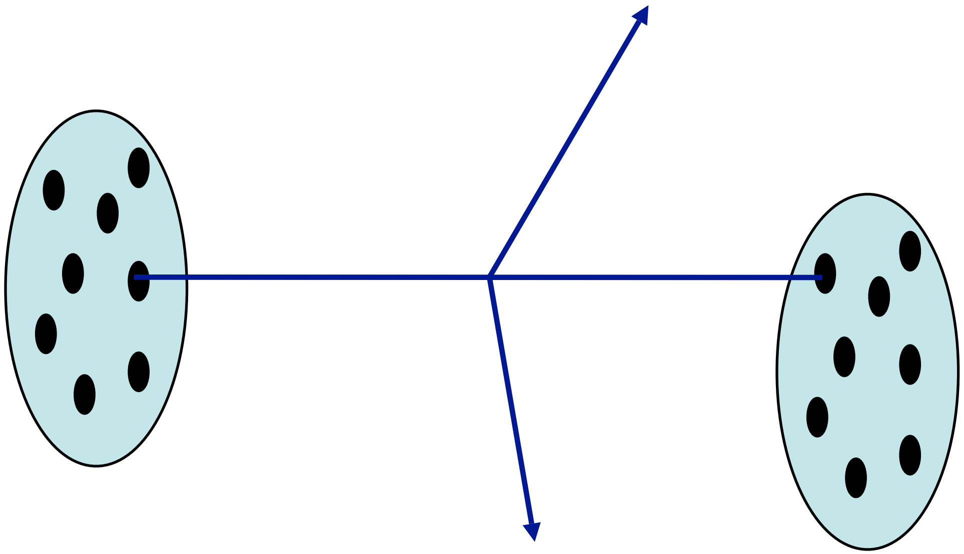


# Conclusions?

- General concepts (LPHD, ULSEA) successful at 20% level, but
  - No systematic scheme for improvement
  - Don’t say anything about hadrons
- Monte Carlo models more successful
  - Complete final states
  - Matched to perturbation theory
  - But ad hoc parameters
- Other models (thermal, holographic)
  - Fewer parameters but limited predictions
  - Match to perturbation theory at cluster/string level?

# The Underlying Event

- Protons are extended objects
- After a parton has been scattered out of each, what happens to the remnants?

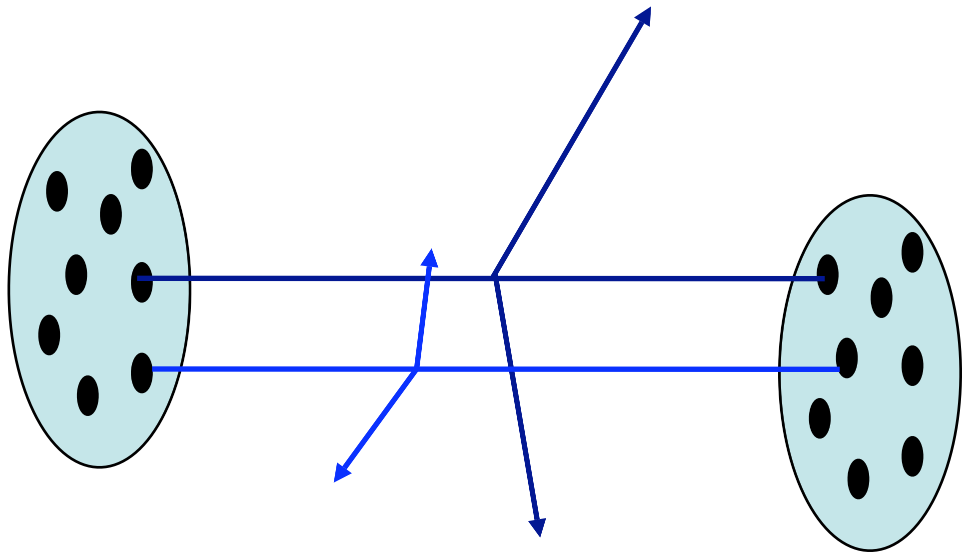


- Only viable current model: multiple parton interactions

# Multiple Parton Interaction Model (PYTHIA/JIMMY)

For small $p_{t\,min}$ and high energy inclusive parton—parton cross section is larger than total proton—proton cross section.

→ More than one parton—parton scatter per proton—proton



Need a model of spatial distribution within proton

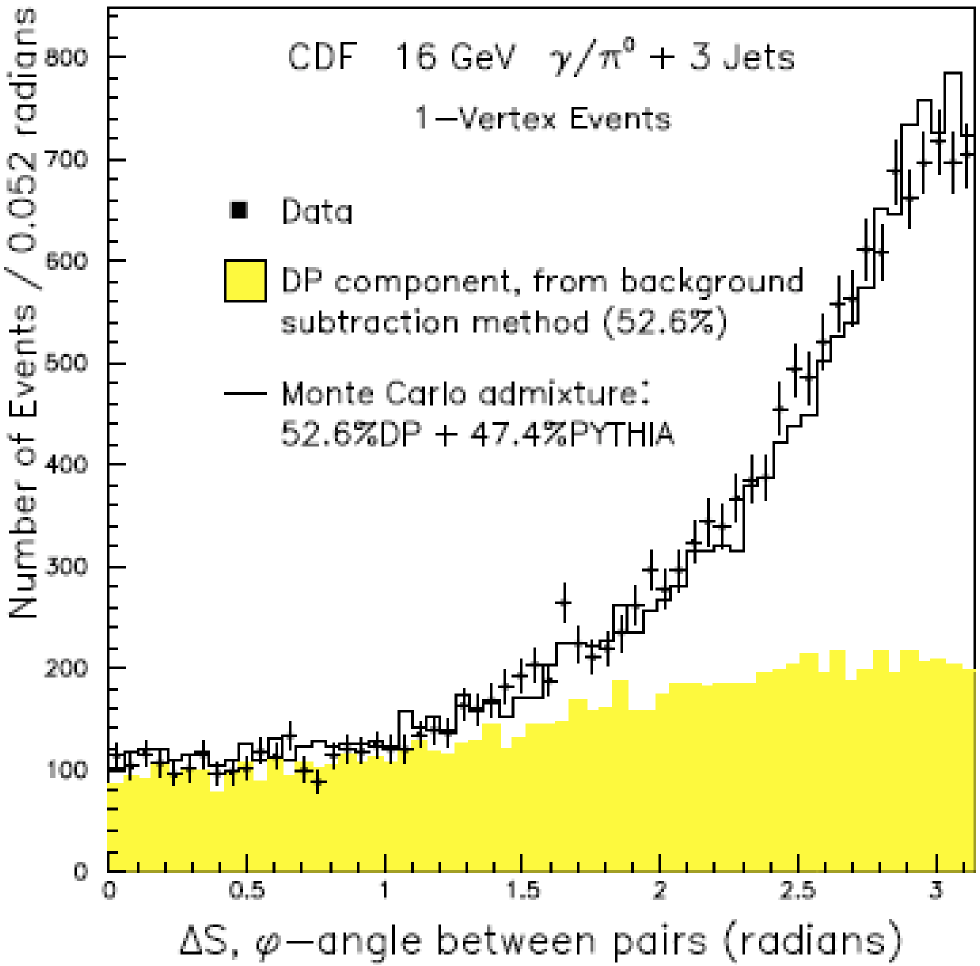
→ Perturbation theory gives n-scatter distributions

# Double Parton Scattering

- CDF Collaboration, PR D56 (1997) 3811

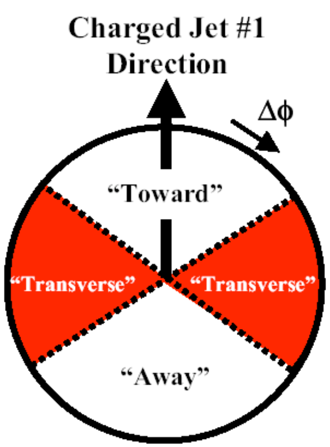
$$\sigma_{\mathrm{DPS}} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\mathrm{eff}}}$$

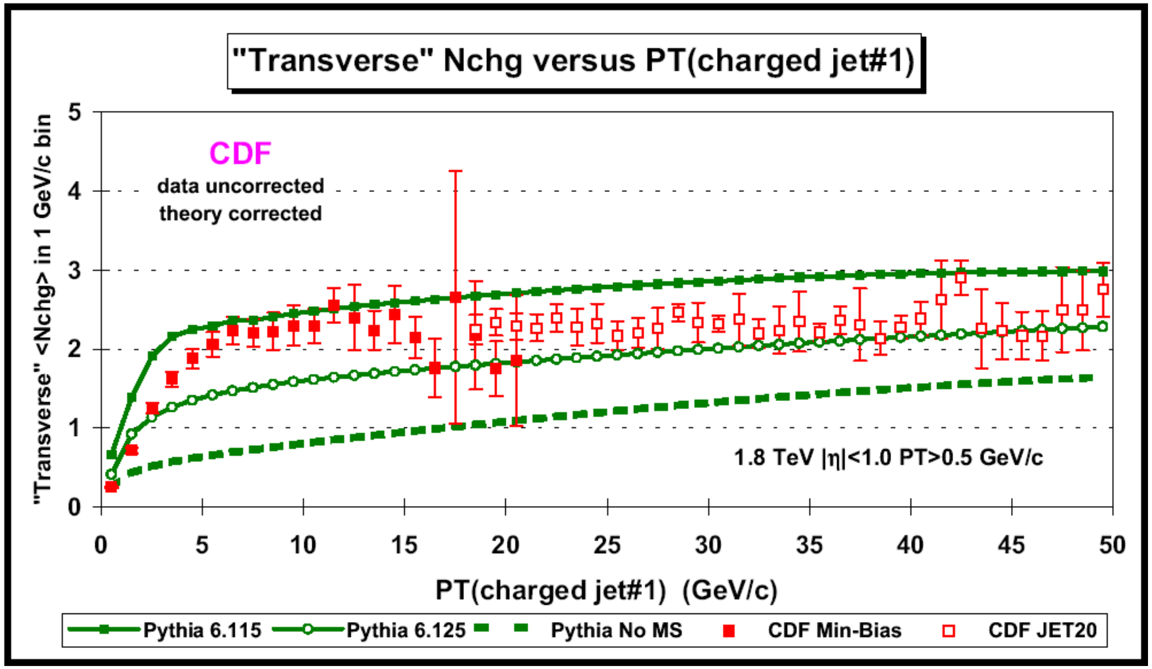
$$\sigma_{\mathrm{eff}} = 14 \pm 1.7^{+1.7}_{-2.3}\ \mathrm{mb}$$



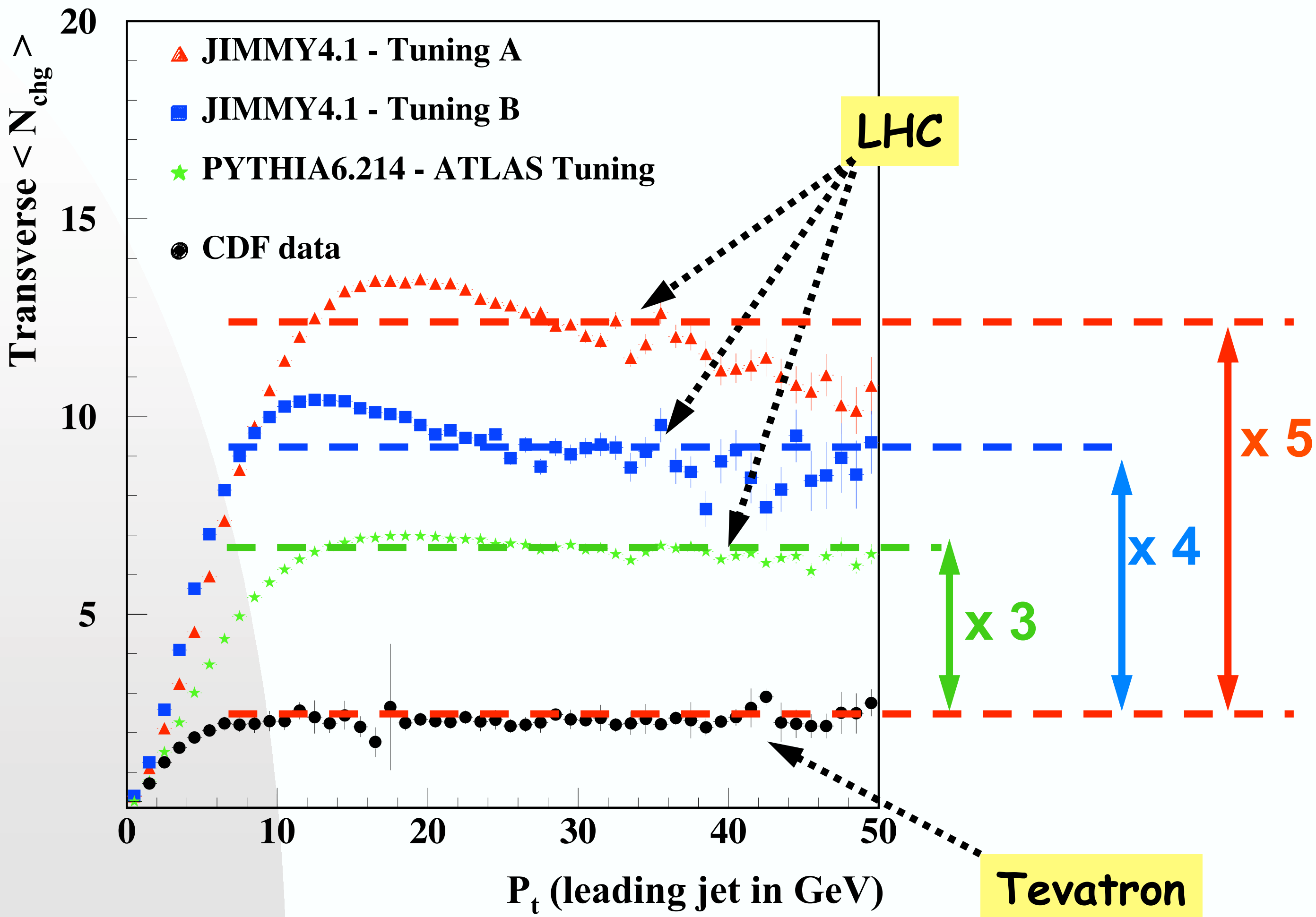
# Tuning PYTHIA to the Underlying Event

- Rick Field (CDF): keep all parameters that can be fixed by LEP or HERA at their default values. What's left?
- Underlying event. Big uncertainties at LHC...



# LHC predictions: JIMMY4.1 Tunings A and B vs. PYTHIA6.214 – ATLAS Tuning (DC2)




A. M. Moraes

Minimum-bias and the Underlying Event at the LHC

5th November 2004

# Optimal Jet Cone Size