Monte Carlo Methods in Particle Physics

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Monte Carlo Event Generation

- Basic Principles
- Event Generation
- Parton Showers
- Hadronization
- Underlying Event
- Event Generator Survey
- Matching to Fixed Order
- Beyond Standard Model
ME-PS Matching

• Two rather different objectives:
  • Matching parton showers to NLO matrix elements, without double counting
    – MC@NLO
    – POWHEG
  • Matching parton showers to LO n-jet matrix elements, minimizing jet resolution dependence
    – CKKW
    – Dipole
    – MLM Matching
    – Comparisons
Recall simple one-dim. example from lecture 1:

\[ |\mathcal{M}_{m+1}|^2 \equiv \frac{1}{x} \mathcal{M}(x) \]

x = gluon energy or two-parton invariant mass.

Divergences regularized by \( d = 4 - 2\epsilon \) dimensions.

\[ |\mathcal{M}_{m\text{--loop}}|^2 \equiv \frac{1}{\epsilon} \mathcal{V} \]

Cross section in d dimensions is:

\[
\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + \frac{1}{\epsilon} \mathcal{V} F_0^J
\]

Infrared safety: \( F_1^J(0) = F_0^J \)

KLN cancellation theorem: \( \mathcal{M}(0) = \mathcal{V} \)
Subtraction Method

Exact identity:

\[ \sigma J = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_0^J \]

\[ + \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_0^J + \frac{1}{\epsilon} \mathcal{V} F_0^J \]

\[ = \int_0^1 \frac{dx}{x} \left( \mathcal{M}(x) F_1^J(x) - \mathcal{V} F_0^J \right) + \mathcal{O}(1) \mathcal{V} F_0^J. \]

Two separate finite integrals.
Modified Subtraction

\[ \sigma^J = \int_0^1 \frac{dx}{x} \left( \mathcal{M}(x) F^J_1(x) - \mathcal{V} F^J_0 \right) + \mathcal{O}(1) \mathcal{V} F^J_0 \]

Now add parton shower:

\[ F^J_{0,1} \Rightarrow \text{result from showering after 0,1 emissions.} \]

But shower adds \( \mathcal{M}_{MC}/x \) to 1 emission. Must subtract this, and add to 0 emission (so that \( F^\text{tot}_{0,1} = 1 \Rightarrow \sigma^\text{tot} \) fixed)

\[ \sigma^J = \int_0^1 \frac{dx}{x} \left( \{ \mathcal{M}(x) - \mathcal{M}_{MC}(x) \} F^J_1(x) - \{ \mathcal{V} - \mathcal{M}_{MC}(x) \} F^J_0 \right) + \mathcal{O}(1) \mathcal{V} F^J_0 \]

MC good for soft and/or collinear \( \Rightarrow \mathcal{M}_{MC}(0) = \mathcal{M}(0) \)

0 & 1 emission contributions separately finite now!

(But some can be negative “counter-events”)
MC@NLO Results

- WW production at LHC

- Interpolates between MC & NLO in $p_T^{(WW)}$
- Above both at $\Delta\phi^{(WW)} \approx 0$

S Frixione & BW, JHEP 06(2002)029
$W^+W^-$: MC@NLO vs Resummations

Plots from M. Grazzini JHEP 0601(2006)095

- Highly non-trivial test (of both computations) for shapes and rates!

- $M_{TWW} = \sqrt{(E_{Tll} + \overline{E}_T)^2 - (p_{Tll} + \overline{p}_T)^2}$ where $E_{Tll} = \sqrt{p_{Tll}^2 + m_{ll}^2}$
  and $\overline{E}_T \equiv \sqrt{\overline{p}_T^2 + m_{ll}^2}$ (Rainwater & Zeppenfeld)

- Cuts involved in definition of $M_{TWW}$: $\Delta \phi_{l+l-} < \pi/4$, $M_{l+l-} > 35$ GeV,
  $p_{T_{\text{min},l-}} > 25$ GeV, $35 < p_{T_{\text{max},l-}} < 50$ GeV, $p_T^{WW} < 30$ GeV
$W^+W^-$ Spin Correlations

Plots from W. Quayle (preliminary)
**b** Production: PS MC vs MC@NLO

- In parton shower MC’s, 3 classes of processes can contribute:
  - FCR
  - GSP
  - FEX

- All are needed to get close to data (RD Field, hep-ph/0201112):
GSP and FEX contributions in HERWIG PS MC

- GSP, FEX and FCR are complementary and all must be generated
  - GSP cutoff (PTMIN) sensitivity depends on cuts and observable
  - FEX sensitive to bottom PDF
  - GSP efficiency very poor, $\sim 10^{-4}$

- All these problems are avoided with MC@NLO!
MC@NLO: B Production at Tevatron

- $B \rightarrow J/\psi$ results from Tevatron Run II $\Rightarrow$ B hadrons

Good agreement (and MC efficiency)

S Frixione, P Nason & BW, JHEP 0308(2003)007
M Cacciari et al., JHEP 0407(2004)033
These observables are very involved (b-jets at hadron level) and cannot be computed with analytical techniques;

The underlying event in Pythia is fitted to data; default Herwig model (used in MC@NLO) does not fit data well (lack of MPI).
MC@NLO b-Jets: Improved Underlying Event

- The JIMMY underlying event model includes multiple parton interactions and interfaces to Herwig ⇒ interfaces to MC@NLO

- The importance of the underlying event shows the necessity of embedding precise computations in a Monte Carlo framework.

Monte Carlo Methods 5  
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MC@NLO: Higgs Production at LHC

Good agreement with state-of-the-art resummation

V Del Duca, S Frixione, C Oleari & BW, in prep.
POWHEG

Positive Weight Hardest Emission Generator

- Method to generate hardest emission first, with NLO accuracy, independent of PSEG
- Can be interfaced to any PSEG
- No negative weights
- Inaccuracies only affect next-to-hardest emission
- In principle, needs ‘truncated showers’

P Nason & G Ridolfi, JHEP08(2006)077
S Frixione, P Nason & G Ridolfi, arXiv:0707.3088
How it works (roughly)

In words: works like a standard Shower MC for the hardest radiation, with care to maintain higher accuracy.

Inclusive cross section $\implies$ NLO inclusive cross section. Positive if NL $<$ LO

$$
\Phi_n = \text{Born variables} \quad \tilde{B}(\Phi_n) = B(\Phi_n) + \left\{ \begin{array}{c} \text{INFINITE} \\ \text{FINITE!} \end{array} \right\} V(\Phi_n) + \int R(\Phi_n, \Phi_r) d\Phi_r
$$

Sudakov form factor for hardest emission built from exact NLO real emission

$$
\Delta_t = \exp \left\{ - \int \theta(t_r - t) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} d\Phi_r \right\}
$$

with $t_r = k_T(\Phi_n, \Phi_r)$, the transverse momentum for the radiation.
POWHEG and MC@NLO comparison:
Top pair production

Good agreement for all observable considered
(differences can be ascribed to different treatment of higher order terms)
POWHEG for $e^+e^- \rightarrow \text{hadrons}$

Truncated Shower

- In angular-ordered shower, hardest emission is not necessarily the first
- Need to add softer, wider-angle emissions
- Checked for up to one such emission in $e^+e^-$
## Effect of truncated shower

<table>
<thead>
<tr>
<th>Observable</th>
<th>Herwig++ ME</th>
<th>Nason@NLO with truncated shower</th>
<th>Nason@NLO w/o truncated shower</th>
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<tr>
<td>$1 - T$</td>
<td>36.52</td>
<td>9.03</td>
<td>9.81</td>
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<tr>
<td>Thrust Major</td>
<td>267.22</td>
<td>36.44</td>
<td>37.65</td>
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<tr>
<td>Thrust Minor</td>
<td>190.25</td>
<td>86.30</td>
<td>90.59</td>
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<tr>
<td>Oblateness</td>
<td>7.58</td>
<td>6.86</td>
<td>6.28</td>
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<tr>
<td>Sphericity</td>
<td>9.61</td>
<td>7.55</td>
<td>9.01</td>
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<td>Aplanarity</td>
<td>8.70</td>
<td>22.96</td>
<td>25.33</td>
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<td>Planarity</td>
<td>2.14</td>
<td>1.19</td>
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<td>$C$ Parameter</td>
<td>96.69</td>
<td>10.50</td>
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<tr>
<td>$D$ Parameter</td>
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<td>8.89</td>
<td>10.88</td>
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<td>$M_{\text{high}}$</td>
<td>14.70</td>
<td>5.31</td>
<td>6.61</td>
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<td>$M_{\text{low}}$</td>
<td>7.82</td>
<td>12.90</td>
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<tr>
<td>$M_{\text{diff}}$</td>
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<td>2.09</td>
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<td>$B_{\text{max}}$</td>
<td>39.50</td>
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<tr>
<td>$N_{\text{ch}}$</td>
<td>43.33</td>
<td>1.58</td>
<td>10.08</td>
</tr>
<tr>
<td>$\langle \chi^2 \rangle / \text{bin}$</td>
<td>56.47</td>
<td>16.96</td>
<td>18.49</td>
</tr>
</tbody>
</table>

**Table 2:** $\chi^2/\text{bin}$ for all observables we studied.

Small but beneficial effect
CKKW Matching

- Use Matrix Elements down to scale $Q_1$
- Use Parton Showers below $Q_1$
- Correct ME by reweighting
- Correct PS by vetoing
- Ensure that $Q_1$ cancels (to NLL)

Example: $e^+e^- \rightarrow \text{hadrons}$

- 2- & 3-jet rates at scale $Q_1$:

\[
R_2(Q, Q_1) = [\Delta_q(Q, Q_1)]^2,
\]

\[
R_3(Q, Q_1) = 2\Delta_q(Q, Q_1) \int_{Q_1}^Q dq \frac{\Delta_q(Q, Q_1)}{\Delta_q(q, Q_1)} \Gamma_q(Q, q) \times \Delta_q(q, Q_1) \Delta_g(q, Q_1)
\]

\[
= 2 [\Delta_q(Q, Q_1)]^2 \int_{Q_1}^Q dq \Gamma_q(Q, q) \Delta_g(q, Q_1)
\]

\[
\Gamma_q(Q, q) = \frac{2C_F}{\pi} \frac{\alpha_S(q)}{q} \left( \ln \frac{Q}{q} - \frac{3}{4} \right)
\]
CKKW reweighting

- Choose $n$ according to $R_n(Q, Q_1)$ (LO)
  - use $[\alpha_S(Q_1)]^n$
- Use exact LO ME to generate $n$ partons
- Construct “equivalent shower history”
  - preferably using $k_T$-type algorithm
- Weight vertex at scale $q$ by $\frac{\alpha_S(q)}{\alpha_S(Q_1)} < 1$
- Weight parton of type $i$ from $Q_j$ to $Q_k$ by
  $$\frac{\Delta_i(Q_j, Q_1)}{\Delta_i(Q_k, Q_1)}$$
CKKW shower veto

- Shower n partons from “creation scales”
  - includes coherent soft emission
- Veto emissions at scales above $Q_1$
  - cancels leading (LL&NLL) $Q_1$ dependence

$Q \rightarrow$ shower from $Q$

$q \rightarrow$ shower from $q$

$Q_1 \rightarrow$ shower from $Q$, not $q$
Comparisons with Tevatron data

from JM Campbell, JW Huston & WJ Stirling, Rept.Prog.Phys. 70(2007)89

M.E. + PYTHIA CKKW looks good

Monte Carlo Methods

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Dipole Matching

- Implemented in ARIADNE dipole MC
- Dipole cascade replaces parton shower
- Construct equivalent dipole history \( \{ p_{Ti} \} \)
- Rejection replaces Sudakov weights
  - cascade from \( p_{Ti} \), reject if \( p_T > p_{Ti+1} \)

L Lönnblad, JHEP05(2002)046
MLM Matching

- Use cone algorithm for jet definition:
  \[ R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2 \]
  \[ E_{Ti} > E_{Tmin}, \quad R_{ij} > R_{min} \]

- Generate n-parton configurations with \( E_{Ti} > E_{Tmin}, \quad R_{ij} > R_{min} \) (no Sudakov weights)

- Generate showers (no vetos)

- Form jets using same jet definition

- Reject event if \( n_{jets} \neq n_{partons} \)
Comparisons

• ALPGEN: MLM matching
• ARIADNE: Dipole matching
• HELAC: MLM matching
• MadEvent: hybrid MLM/CKKW
• SHERPA: CKKW matching

J. Alwall et al., arXiv:0706.2569
W + Multijets (Tevatron)
W + Multijets (Tevatron)

(a) $d\sigma/dE_1$ (pb/GeV)
(b) $d\sigma/dE_2$ (pb/GeV)
(c) $d\sigma/dE_3$ (pb/GeV)
(d) $d\sigma/dE_4$ (pb/GeV)
W + Multijets (Tevatron)

(a) (1/!) d/ d"1
Alpgen
Ariadne
Helac
MadEvent
Sherpa

(b) (1/!) d/ d"2

(c) (1/!) d/ d"3

(d) (1/!) d/ d"4
W + Multijets (LHC)

\[
\frac{\sigma(W^+ + \geq N \text{ jets})}{\langle \sigma \rangle}
\]

Encoder models:
- Alpgen
- Ariadne
- Helac
- MadEvent
- Sherpa
W + Multijets (LHC)

(a) Alpgen 
Ariadne 
Helac 
MadEvent 
Sherpa

(b) 

(c) 

(d)
Summary

• Matching Parton Showers to Matrix Elements comes in different forms:
  – matching to NLO for better precision
  – matching to LO for multijets

• MC@NLO is main scheme for NLO matching
  – newer POWHEG method looks promising

• Several options for LO multijets
  – reasonably consistent
  – spread indicates uncertainties (?)

• Field still very active
  – NLO matching for jets, spin correlations,…
  – building multijet matching into OO generators