QCD and Collider Phenomenology

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Lecture 2: Jet Fragmentation and Hadron-Hadron Processes

- Jet Fragmentation
  - Fragmentation functions
  - Coherent parton branching
  - Small-$x$ fragmentation and average multiplicity

- Hadronization Models
  - General ideas
  - Cluster model
  - String model

- Hadron-Hadron Processes
  - Parton-parton luminosities
  - Lepton pair, jet and heavy quark production
  - Higgs boson production

- Survey of NLO Calculations for LHC
Jet Fragmentation

- **Fragmentation functions** $F^h_i(x, t)$ gives distribution of momentum fraction $x$ for hadrons of type $h$ in a jet initiated by a parton of type $i$, produced in a hard process at scale $t$.

- Like parton distributions in a hadron, $D^h_i(x, t)$, these are *factorizable* quantities, in which infrared divergences of PT can be factorized out and replaced by experimentally measured factor that contains all long-distance effects.

- In $e^+e^-$ annihilation, for example, the hard process is $e^+e^- \rightarrow q\bar{q}$ at scale equal to c.m. energy squared $s$; distribution of $x = 2p_h/\sqrt{s}$ is (for $s \ll M_Z^2$)

$$\frac{d\sigma}{dx} = 3\sigma_0 \sum_q Q^2_q \left\{ F^h_q(x, s) + F^h_{\bar{q}}(x, s) \right\}$$

where $\sigma_0$ is $e^+e^- \rightarrow \mu^+\mu^-$ cross section.

- Fragmentation functions satisfy DGLAP evolution equation

$$t \frac{\partial}{\partial t} F^h_i(x, t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ji}(z, \alpha_s) F^h_j(x/z, t) .$$

Splitting functions $P_{ji}$ have perturbative expansions of the form

$$P_{ji}(z, \alpha_s) = P_{ji}^{(0)}(z) + \frac{\alpha_s}{2\pi} P_{ji}^{(1)}(z) + \cdots$$
Leading terms $P_{ji}^{(0)}(z)$ were given earlier. Notice that splitting function is $P_{ji}$ rather than $P_{ij}$ since $F_{j}^{h}$ represents fragmentation of final parton $j$.

- Solve DGLAP equation by taking moments as explained for DIS. As in that case, scaling violation is clearly seen.
**Soft Gluon Coherence**

- Parton branching formalism discussed so far takes account of **collinear** enhancements to all orders in PT. There are also **soft** enhancements: When external line with momentum $p$ and mass $m$ (not necessarily small) emits gluon with momentum $q$, propagator factor is

$$\frac{1}{(p \pm q)^2 - m^2} = \frac{\pm 1}{2p \cdot q} = \frac{\pm 1}{2\omega E(1 - v \cos \theta)}$$

where $\omega$ is emitted gluon energy, $E$ and $v$ are energy and velocity of parton emitting it, and $\theta$ is angle of emission. This diverges as $\omega \to 0$, for any velocity and emission angle.

- Including numerator, soft gluon emission gives a colour factor times universal, spin-independent factor in amplitude

$$F_{\text{soft}} = \frac{p \cdot \epsilon}{p \cdot q}$$

where $\epsilon$ is polarization of emitted gluon. For example, emission from quark gives numerator factor $N \cdot \epsilon$, where

$$N^\mu = (\slashed{q} + \slashed{m}) \gamma^\mu u(p) \quad \omega \to 0 \quad (\gamma^\nu \gamma^\mu p_\nu + \gamma^\mu m) u(p)$$

$$= (2p^\mu - \gamma^\mu \slashed{q} + \gamma^\mu m) u(p) = 2p^\mu u(p).$$

(Using Dirac equation for on-mass-shell spinor $u(p)$).

- Universal factor $F_{\text{soft}}$ coincides with classical **eikonal formula** for radiation from current $p^\mu$, valid in long-wavelength limit.
No soft enhancement of radiation from off-mass-shell internal lines, since associated denominator factor $(p + q)^2 - m^2 \rightarrow p^2 - m^2 \neq 0$ as $\omega \rightarrow 0$.

Enhancement factor in amplitude for each external line implies cross section enhancement is sum over all pairs of external lines $\{i, j\}$:

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_s}{2\pi} \sum_{i,j} C_{ij} W_{ij}$$

where $d\Omega$ is element of solid angle for emitted gluon, $C_{ij}$ is a colour factor, and radiation function $W_{ij}$ is given by

$$W_{ij} = \frac{\omega^2 p_i \cdot p_j}{p_i \cdot q p_j \cdot q} = \frac{1 - v_i v_j \cos \theta_{ij}}{(1 - v_i \cos \theta_{iq})(1 - v_j \cos \theta_{jq})}.$$ 

Colour-weighted sum of radiation functions $C_{ij} W_{ij}$ is antenna pattern of hard process.
Radiation function can be separated into two parts containing collinear singularities along lines $i$ and $j$. Consider for simplicity massless particles, $v_{i,j} = 1$. Then $W_{ij} = W^i_{ij} + W^j_{ij}$ where

$$W^i_{ij} = \frac{1}{2} \left( W_{ij} + \frac{1}{1 - \cos \theta_{iq}} - \frac{1}{1 - \cos \theta_{jq}} \right).$$

This function has remarkable property of angular ordering. Write angular integration in polar coordinates w.r.t. direction of $i$, $d\Omega = d\cos \theta_{iq} d\phi_{iq}$. Performing azimuthal integration, we find

$$\int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} W^i_{ij} = \frac{1}{1 - \cos \theta_{iq}} \quad \text{if} \: \theta_{iq} < \theta_{ij}, \text{otherwise} \: 0.$$

Thus, after azimuthal averaging, contribution from $W^i_{ij}$ is confined to cone, centred on direction of $i$, extending in angle to direction of $j$. Similarly, $W^j_{ij}$, averaged over $\phi_{jq}$, is confined to cone centred on line $j$ extending to direction of $i$. 
To prove angular ordering property, write

\[ 1 - \cos \theta_{jq} = a - b \cos \phi_{iq} \]

where

\[ a = 1 - \cos \theta_{ij} \cos \theta_{iq} \quad b = \sin \theta_{ij} \sin \theta_{iq} . \]

Defining \( z = \exp(i\phi_{iq}) \), we have

\[ I_{ij}^i \equiv \int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} \frac{1}{1 - \cos \theta_{jq}} = \frac{1}{i\pi b} \oint \frac{dz}{(z - z_+)(z - z_-)} \]

where \( z \)-integration contour the unit circle and

\[ z_\pm = \frac{a}{b} \pm \sqrt{\frac{a^2}{b^2} - 1} . \]

Now only pole at \( z = z_- \) can lie inside unit circle, so

\[ I_{ij}^i = \sqrt{\frac{1}{a^2 - b^2}} = \frac{1}{|\cos \theta_{iq} - \cos \theta_{ij}|} . \]
Hence

\[ \int_{0}^{2\pi} \frac{d\phi_{iq}}{2\pi} W_{ij} = \frac{1}{2(1 - \cos \theta_{iq})} [1 + (\cos \theta_{iq} - \cos \theta_{ij}) I_{ij}^{i}] \]

\[ = \frac{1}{1 - \cos \theta_{iq}} \quad \text{if} \quad \theta_{iq} < \theta_{ij}, \text{otherwise} \ 0. \]

Angular ordering is coherence effect common to all gauge theories. In QED it causes Chudakov effect – suppression of soft bremsstrahlung from \( e^+ e^- \) pairs, which has simple explanation in old-fashioned (time-ordered) perturbation theory.

Consider emission of soft photon at angle \( \theta \) from electron in pair with opening angle \( \theta_{ee} < \theta \). For simplicity assume \( \theta_{ee}, \theta \ll 1 \).

Transverse momentum of photon is \( k_T \sim zp\theta \) and energy imbalance at \( e \rightarrow e\gamma \) vertex is

\[ \Delta E \sim \frac{k_T^2}{zp} \sim zp\theta^2. \]
Time available for emission is $\Delta t \sim 1/\Delta E$. In this time transverse separation of pair will be $\Delta b \sim \theta_{ee}\Delta t$.

For non-negligible probability of emission, photon must resolve this transverse separation of pair, so

$$\Delta b > \lambda/\theta \sim (zp\theta)^{-1}$$

where $\lambda$ is photon wavelength.

This implies that

$$\theta_{ee}(zp\theta^2)^{-1} > (zp\theta)^{-1},$$

and hence $\theta_{ee} > \theta$. Thus soft photon emission is suppressed at angles larger than opening angle of pair, which is angular ordering.

Photons at larger angles cannot resolve electron and positron charges separately – they see only total charge of pair, which is zero, implying no emission.

More generally, if $i$ and $j$ come from branching of parton $k$, with (colour) charge $Q_k = Q_i + Q_k$, then radiation outside angular-ordered cones is emitted coherently by $i$ and $j$ and can be treated as coming directly from (colour) charge of $k$. 

Coherent Branching

- Angular ordering provides basis for coherent parton branching formalism, which includes leading soft gluon enhancements to all orders.
- In place of virtual mass-squared variable $t$ in earlier treatment, use angular variable
  \[
  \zeta = \frac{p_b \cdot p_c}{E_b E_c} \simeq 1 - \cos \theta
  \]
  as evolution variable for branching $a \to bc$, and impose angular ordering $\zeta' < \zeta$ for successive branchings. Iterative formula for $n$-parton emission becomes
  \[
  d\sigma_{n+1} = d\sigma_n \frac{d\zeta}{\zeta} dz \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z).
  \]
- In place of virtual mass-squared cutoff $t_0$, must use angular cutoff $\zeta_0$ for coherent branching. This is to some extent arbitrary, depending on how we classify emission as unresolvable. Simplest choice is $\zeta_0 = t_0/E^2$ for parton of energy $E$.
- For radiation from particle $i$ with finite mass-squared $t_0$, radiation function becomes
  \[
  \omega^2 \left( \frac{p_i \cdot p_j}{p_i \cdot q p_j \cdot q} - \frac{p_i^2}{(p_i \cdot q)^2} \right) \simeq \frac{1}{\zeta} \left( 1 - \frac{t_0}{E^2 \zeta} \right),
  \]
  so angular distribution of radiation is cut off at $\zeta = t_0/E^2$. Thus $t_0$ can still be interpreted as minimum virtual mass-squared.
With this cutoff, most convenient definition of evolution variable is not $\zeta$ itself but rather

$$\tilde{t} = E^2 \zeta \geq t_0 .$$

Angular ordering condition $\zeta_b, \zeta_c < \zeta_a$ for timelike branching $a \rightarrow bc$ ($a$ outgoing) becomes

$$\tilde{t}_b < z^2 \tilde{t}, \quad \tilde{t}_c < (1 - z)^2 \tilde{t}$$

where $\tilde{t} = \tilde{t}_a$ and $z = E_b/E_a$. Thus cutoff on $z$ becomes

$$\sqrt{t_0/\tilde{t}} < z < 1 - \sqrt{t_0/\tilde{t}} .$$

Neglecting masses of $b$ and $c$, virtual mass-squared of $a$ and transverse momentum of branching are

$$t = z(1 - z)\tilde{t}, \quad p_t^2 = z^2(1 - z)^2 \tilde{t} .$$

Thus for coherent branching Sudakov form factor of quark becomes

$$\tilde{\Delta}_q(\tilde{t}) = \exp \left[ - \int_{\tilde{t}_0}^{\tilde{t}} \frac{dt'}{t'} \int_{\sqrt{t_0/t'}}^{1 - \sqrt{t_0/t'}} \alpha_S(z^2(1 - z)^2 t') \hat{P}_{qq}(z) \right]$$

At large $\tilde{t}$ this falls more slowly than form factor without coherence, due to the suppression of soft gluon emission by angular ordering.
Note that for spacelike branching $a \rightarrow bc$ ($a$ incoming, $b$ spacelike), angular ordering condition is

$$\theta_b > \theta_a > \theta_c .$$

However, kinematics implies $E_b \theta_b > E_a \theta_a$ at small $x$ and in this case $E_b < E_a$, so angular ordering does not impose an extra constraint on branching. Therefore gluon emission is not suppressed by coherence in spacelike branching at small $x$.

- This permits the rapid rise of structure functions at small $x$.
- We shall see that the production of low-momentum hadrons in jet fragmentation at small $x$, controlled by timelike branching, is quite different – strongly suppressed by QCD coherence.
Small-x fragmentation

- Evolution of fragmentation functions at small $x$ sensitive to moments near $N = 1$. However, anomalous dimensions $\gamma_{gq}^{(0)}$, $\gamma_{gg}^{(0)}$ are not defined at $N = 1$: moment integrals for $N \leq 1$ are dominated by small $z$, where $P_{gi}(z)$ diverges due to soft gluon emission.

- At small $z$ must take into account coherence effects. Recall evolution variable becomes $\tilde{t} = E^2 [1 - \cos \theta]$, with angular ordering condition $\tilde{t}' < z^2 \tilde{t}$. Thus, redefining $t$ as $\tilde{t}$, evolution equation in integrated form is

$$F_i(x, t) = F_i(x, t_0) + \sum_j \int_x^1 \frac{dz}{z} \int_{t_0}^{z^2 t} \frac{dt'}{t'} \frac{\alpha_S}{2\pi} P_{ji}(z) F_j(x/z, t')$$

or in differential form

$$t \frac{\partial}{\partial t} F_i(x, t) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_S}{2\pi} P_{ji}(z) F_j(x/z, z^2 t) .$$

- Only difference from DGLAP equation is $z$-dependent scale on the right-hand side — not important for most values of $x$ but crucial at small $x$.

- For simplicity, consider first $\alpha_S$ fixed and neglect sum over $j$. Taking moments as usual,

$$t \frac{\partial}{\partial t} \tilde{F}(N, t) = \frac{\alpha_S}{2\pi} \int_x^1 dz z^{N-1} P(z) \tilde{F}(N, z^2 t) .$$
Try solution of form $F(N, t) \propto t^{\gamma(N, \alpha_S)}$. Then anomalous dimension $\gamma(N, \alpha_S)$ must satisfy

$$\gamma(N, \alpha_S) = \frac{\alpha_S}{2\pi} \int_0^1 z^{N-1+2\gamma(N, \alpha_S)} P(z).$$

For $N - 1$ not small, we can neglect $2\gamma(N, \alpha_S)$ in exponent and obtain usual formula for anomalous dimension. For $N \simeq 1$, $z \to 0$ region dominates, where $P_{gg}(z) \simeq 2C_A/z$. Hence

$$\gamma_{gg}(N, \alpha_S) = \frac{C_A\alpha_S}{\pi} \frac{1}{N - 1 + 2\gamma_{gg}(N, \alpha_S)}$$

$$= \frac{1}{4} \left[ \sqrt{(N - 1)^2 + \frac{8C_A\alpha_S}{\pi}} - (N - 1) \right]$$

$$= \sqrt{\frac{C_A\alpha_S}{2\pi}} - \frac{1}{4} (N - 1) + \frac{1}{32} \sqrt{\frac{2\pi}{C_A\alpha_S}} (N - 1)^2 + \cdots$$

To take account of running $\alpha_S$, write

$$\tilde{F}(N, t) \sim \exp \left[ \int_0^t \gamma_{gg}(N, \alpha_S) \frac{dt'}{t'} \right],$$
and note that $\gamma_{gg}(N, \alpha_S)$ should be $\gamma_{gg}(N, \alpha_S(t'))$. Use

$$
\int^t \gamma_{gg}(N, \alpha_S(t')) \frac{dt'}{t'} = \int^{\alpha_S(t)} \frac{\gamma_{gg}(N, \alpha_S)}{\beta(\alpha_S)} d\alpha_S,
$$

where $\beta(\alpha_S) = -b\alpha_S^2 + \cdots$, to find

$$
\tilde{F}(N, t) \sim \exp \left[ \frac{1}{b} \sqrt{\frac{2C_A}{\pi \alpha_S}} - \frac{1}{4b \alpha_S} (N - 1) \right. \\
+ \left. \frac{1}{48b} \sqrt{\frac{2\pi}{C_A \alpha_S^3}} (N - 1)^2 + \cdots \right]_{\alpha_S = \alpha_S(t)}.
$$

- In $e^+e^-$ annihilation, scale $t \sim s$ and behaviour of $\tilde{F}(N, s)$ near $N = 1$ determines form of small-$x$ fragmentation functions. Keeping terms up to $(N - 1)^2$ in exponent gives Gaussian function of $N$ which transforms into Gaussian function of $\xi \equiv \ln(1/x)$:

$$
x F(x, s) \propto \exp \left[ -\frac{1}{2\sigma^2} (\xi - \xi_p)^2 \right],
$$
Width of distribution

\[
\sigma = \left( \frac{1}{24b} \sqrt{\frac{2\pi}{C_A \alpha_S^3(s)}} \right)^{\frac{1}{2}} \propto (\ln s)^{\frac{3}{4}}.
\]
Peak position

\[ \xi_p = \frac{1}{4b\alpha_s(s)} \sim \frac{1}{4} \ln s \]

Energy-dependence of the peak position \( \xi_p \) tests suppression of hadron production at small \( x \) due to soft gluon coherence. Decrease at very small \( x \) is expected on kinematical grounds, but this would occur at particle energies proportional to their masses, i.e. at \( x \propto m/\sqrt{s} \), giving \( \xi_p \sim \frac{1}{2} \ln s \). Thus purely kinematic suppression would give \( \xi_p \) increasing twice as fast.

In \( p\bar{p} \rightarrow \text{dijets} \), \( \sqrt{s} \) is replaced by \( M_{JJ}\sin\theta \) where \( M_{JJ} \) is dijet mass and \( \theta \) is jet cone angle.
CDF Preliminary

\[ \frac{1}{N_{\text{event}}} \frac{dN}{d\xi} \]

\[ \xi = \log\left( \frac{1}{x} \right) \]

- \( M_{jj} = 82 \text{ GeV} \)
- \( M_{jj} = 105 \text{ GeV} \)
- \( M_{jj} = 140 \text{ GeV} \)
- \( M_{jj} = 183 \text{ GeV} \)
- \( M_{jj} = 229 \text{ GeV} \)
- \( M_{jj} = 293 \text{ GeV} \)
- \( M_{jj} = 378 \text{ GeV} \)
- \( M_{jj} = 488 \text{ GeV} \)
- \( M_{jj} = 628 \text{ GeV} \)

CDF Preliminary

MLLA Fit: (CDF Data only)

\[ Q_{\text{eff}} = 256 \pm 13 \text{ MeV} \]
Average Multiplicity

Mean number of hadrons is $N = 1$ moment of fragmentation function:

$$\langle n(s) \rangle = \int_0^1 dx \, F(x, s) = \tilde{F}(1, s)$$

$$\sim \exp \frac{1}{b} \sqrt{\frac{2C_A}{\pi \alpha_s(s)}} \sim \exp \sqrt{\frac{2C_A}{\pi b} \ln \left( \frac{s}{\Lambda^2} \right)}$$

(plus NLL corrections) in good agreement with data.
Hadronization Models

General ideas

- Local parton-hadron duality
  - Hadronization is long-distance process, involving small momentum transfers. Hence hadron-level flow of energy-momentum, flavour should follow parton level.
  - Implicit in earlier discussion of jet fragmentation.
  - Results on spectra and multiplicities support this.

- Universal low-scale $\alpha_S$
  - PT works well down to very low scales, $Q \sim 1$ GeV.
  - Assume $\alpha_S(Q)$ defined (non-perturbatively) for all $Q$.
  - Good description of heavy quark spectra, event shapes.
Universal low-scale $\alpha_S$

- Infrared renormalon

\[
F \sim \int_0^Q \frac{d p_t}{Q} \alpha_S(p_t)
\]

\[
= \alpha_S(Q) \sum_n \int_0^Q \frac{d p_t}{Q} \left[ b \alpha_S(Q) \ln \frac{Q^2}{p_t^2} \right]^n
\]

\[
= \alpha_S(Q) \sum_n n! [2b\alpha_S(Q)]^n
\]

- Divergent series: truncate at smallest term ($n_m = [2b\alpha_S(Q)]^{-1}$) $\Rightarrow$ uncertainty in $F$

\[
\delta F \sim n_m! [2b\alpha_S(Q)]^{nm} \sim e^{-nm} = \frac{\Lambda}{Q}
\]

- Renormalon is due to infrared divergence of $\alpha_S$
  - Postulate universal infrared-regular $\alpha_S$. Then $1/Q$ power corrections depend on

\[
\alpha_0(\mu_I) = \frac{1}{\mu_I} \int_0^{\mu_I} \alpha_S(p_t) \, dp_t
\]

- Match PT and NP at $\mu_I \sim 2$ GeV
Event shapes in $e^+e^-$

Event shapes in DIS

$\alpha_s(M_Z)$

$\alpha_s(\mu=2\text{GeV})$

$\langle 1-T \rangle$

$\langle \sqrt{s}/s \rangle$

$\langle M_H^2/s \rangle$

$\langle B_{T} \rangle$

$\langle B_{W} \rangle$

$\langle C \rangle$

$\langle \alpha_0 \rangle$

$\langle 1-T \rangle$

$\langle M_H^2/s \rangle$

$\langle B_{T} \rangle$

$\langle B_{W} \rangle$

$\langle C \rangle$

$\langle \alpha_0 \rangle$

$\langle \alpha_s(\mu=2\text{GeV}) \rangle$

$\langle 1-T \rangle$

$\langle M_H^2/s \rangle$

$\langle B_{T} \rangle$

$\langle B_{W} \rangle$

$\langle C \rangle$

$\langle \alpha_0 \rangle$

$\langle \alpha_s(\mu=2\text{GeV}) \rangle$

$\langle 1-T \rangle$

$\langle M_H^2/s \rangle$

$\langle B_{T} \rangle$

$\langle B_{W} \rangle$

$\langle C \rangle$

$\langle \alpha_0 \rangle$

$\langle \alpha_s(\mu=2\text{GeV}) \rangle$
Specific Hadronization Models

- General ideas do not describe hadron formation. Main current models are cluster and string.
Cluster (HERWIG)

- Non-perturbative $g \rightarrow q\bar{q}$ splitting after parton shower.
- Colour singlet $q\bar{q}$ clusters have lower mass due to preconfinement property of parton shower.

- Clusters decay according to 2-hadron density of states.
- Few parameters: natural $p_T$ and heavy particle suppression
- Problems with massive clusters, baryons, heavy quarks
String (PYTHIA)

- Uses string dynamics: colour string stretched between initial $q\bar{q}$ breaks up into hadrons via $q\bar{q}$ pair production.
- String gives linear confinement potential, area law for matrix elements.
- Gluons produced in shower give 'kinks' on string.

Extra parameters for $p_T$ and heavy particle suppression.
Some problems with baryons.

Both models describe $e^+e^-$ data well . . .
Jet rates and mean number of jets

- **Jet Fraction**

  - **OPAL (91 GeV)**
  - **Durham**
  - 2-jet
  - 3-jet
  - 4-jet
  - 5-jet

  - PYTHIA
  - HERWIG

- **k_T or Durham algorithm:**
  - Define jet resolution $y_{\text{cut}}$ (dimensionless).
  - For final-state momenta $p_i, p_j$ define $y_{ij} = 2 \min\{E_i^2, E_j^2\}(1 - \cos \theta_{ij})/s$
  - If $y_{IJ} = \min\{y_{ij}\} < y_{\text{cut}}$, combine $I, J$ into one object $K$ with $p_K = p_I + p_J$.
  - Repeat until $y_{IJ} > y_{\text{cut}}$. Then remaining objects are jets.
Light quark and gluon fragmentation functions

![Graphs showing quark and gluon fragmentation functions](image)

- **udsc Quark**
  - OPAL
  - PYTHIA 6.1
  - HERWIG 6.2
  - ARIADNE 4.08

- **Gluon**
  - OPAL
  - PYTHIA 6.1
  - HERWIG 6.2
  - ARIADNE 4.08

**DATA**
- \(\langle Q_{\text{jet}} \rangle = 6.4\,\text{GeV}\)
- \(\langle Q_{\text{jet}} \rangle = 13.4\,\text{GeV}\)
- \(\langle Q_{\text{jet}} \rangle = 21.0\,\text{GeV}\)
- \(\sqrt{s}/2 = 45.6\,\text{GeV}\)
- \(\langle Q_{\text{jet}} \rangle = 46.5\,\text{GeV}\)
- \(\langle Q_{\text{jet}} \rangle = 48.5\,\text{GeV}\)
Hadron-Hadron Processes

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer $Q^2$).

- For hadron momenta $P_1, P_2$ ($S = 2P_1 \cdot P_2$), form of cross section is

$$
\sigma(S) = \sum_{i,j} \int d x_1 d x_2 D_i(x_1, \mu) D_j(x_2, \mu) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_S(\mu), Q/\mu)
$$

where $\mu$ is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types $i, j$.

- Factorization scale is in principle arbitrary: it affects only what we call part of subprocess or part of initial-state evolution (parton shower).

- Rapidity of subprocess c.m. frame $p^\mu = p_1^\mu + p_2^\mu$:

$$
y \equiv \frac{1}{2} \ln \left[ \frac{(p^0 + p_3)/p^0 - p_3}{p^0 + p_3} \right] = \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right)
$$
Unlike $e^+ e^-$ or $ep$, we may have interaction between spectator partons, leading to soft underlying event and/or multiple hard scattering.
Double Parton Scattering


- DPS has ‘best-balanced’ ($\gamma + \text{jet}$) and dijet uncorrelated in azimuth.

- They found $\sigma_{\text{DPS}} = \sigma_{\gamma j} \sigma_{jj} / \sigma_{\text{eff}}$ where $\sigma_{\text{eff}} = 14 \pm 1.7^{+1.7}_{-2.3}$ mb
Parton-Parton Luminosities

Useful to define the differential parton-parton luminosity \( dL_{ij}/d\hat{s} \, dy \) and its integral \( dL_{ij}/d\hat{s} \):

\[
\frac{dL_{ij}}{d\hat{s} \, dy} = \frac{1}{S} \frac{1}{1 + \delta_{ij}} \left[ D_i(x_1, \mu) D_j(x_2, \mu) + (1 \leftrightarrow 2) \right].
\]

Factor with Kronecker delta avoids double-counting when partons are identical.

We have \( d\hat{s} \, dy = S \, dx_1 \, dx_2 \) and hence

\[
\sigma = \sum_{i,j} \int d\hat{s} \, dy \left( \frac{dL_{ij}}{d\hat{s} \, dy} \right) \hat{\sigma}_{ij}(\hat{s})
\]

\[
= \sum_{i,j} \int d\hat{s} \left( \frac{dL_{ij}}{d\hat{s}} \right) \hat{\sigma}_{ij}(\hat{s})
\]

This can be used to estimate the production rate for subprocesses at LHC.
Figure shows parton-parton luminosities at $\sqrt{s} = 14$ TeV for various parton combinations, calculated using the CTEQ6.1 parton distribution functions and scale $\mu = \sqrt{s}$. Widths of curves estimate PDF uncertainties.

Green = $gg$, Blue = $gq + g\bar{q} + qg + \bar{q}g$, Red = $q\bar{q} + \bar{q}q$ ($q = d + u + s + c + b$).
Lepton Pair Production

- Inverse of $e^+e^- \rightarrow q\bar{q}$ is Drell-Yan process. At $O(\alpha_s^0)$, mass distribution of lepton pair is given by

$$\frac{d\hat{\sigma}}{dM^2}(q\bar{q} \rightarrow \gamma^* \rightarrow l^+l^-) = \frac{4\pi\alpha^2}{3\hat{s}} Q_q^2 \delta(M^2 - \hat{s})$$

- Factor of $1/3 = 1/N$ instead of $3 = N$ because of average over colours of incoming $q$.

- In higher orders vertex corrections (a) have $M^2 = \hat{s}$, gluon emission (b) and QCD Compton (c) diagrams give $M^2 < \hat{s}$. 
\[ \frac{d^2 \sigma}{dM \, dy} \text{ [pb/GeV]} \]

\[ \sqrt{s} = 1.8 \text{ TeV, } |y| < 1 \]

CDF data

\[ p\bar{p} \rightarrow \ell^+ \ell^- + X \]
- $W^\pm$ boson production is similar, except sensitive to different parton distributions, e.g.

$$u\bar{d} \rightarrow W^+ \rightarrow l^+ \nu_l$$

- Transverse momentum of lepton pair, $p_T$ measures net transverse momentum of colliding partons plus any intrinsic $p_T$: 

![Graph showing $W^+ + W^-$ production at large $p_T$ with $\sqrt{s} = 1.8$ TeV, CDF data]
Jet Production

- Lowest-order subprocess for purely hadronic jet production is $2 \rightarrow 2$ scattering $p_1 + p_2 \rightarrow p_3 + p_4$

$$\frac{d\hat{\sigma}}{d\Phi_{34}} \equiv \frac{E_3 E_4 d^6 \hat{\sigma}}{d^3 p_3 d^3 p_4}$$

$$= \frac{1}{32 \pi^2 \hat{s}} \sum |\mathcal{M}|^2 \delta^4(p_1 + p_2 - p_3 - p_4).$$

- Many processes even at $\mathcal{O}(\alpha_S^2)$:

![Diagram](image-url)
● Single-jet inclusive cross section obtained by integrating over one outgoing momentum:

\[
\frac{E d^3 \hat{\sigma}}{d^3 p} = \frac{d^3 \hat{\sigma}}{d^2 p_T dy} \rightarrow \frac{1}{2\pi E_T} \frac{d^3 \hat{\sigma}}{d E_T d \eta} = \frac{1}{16\pi^2 \hat{s}} \sum |\mathcal{M}|^2 \delta(\hat{s} + \hat{t} + \hat{u})
\]

where (neglecting jet mass)

\[
E_T \equiv E \sin \theta = |p_T| , \quad \eta \equiv -\ln \tan(\theta/2) = y .
\]

● Jets can be defined by the \( k_T \) algorithm:

❖ For each final-state momentum \( p_i \) and each pair of final-state momenta \( p_i, p_j \), define

\[
k_{T_i} = E_{Ti} , \quad k_{Tij} = \min\{E_{Ti}, E_{Tj}\} \Delta R_{ij} / D
\]

where \( \Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2} \) and \( D \) = dimensionless parameter for angular size of jets (\( D = 0.5 - 1.0 \))

❖ If \( k_{TI} \) is the smallest in the list of \( \{k_{T_i}, k_{Tij}\} \), define \( I \) as a jet and remove from list.

❖ If \( k_{TIJ} \) is the smallest, combine \( I, J \) into one object \( K \) with \( p_K = p_I + p_J \).

❖ Repeat until list is empty.

● Use \( \eta \) rather than \( \theta \) for invariance under longitudinal boosts: \( x_1 \rightarrow ax_1, x_2 \rightarrow x_2/a \) gives \( \eta_i \rightarrow \eta_i + \ln a \), so \( \eta_i - \eta_j \) is invariant.
NLO predictions and data agree very well:

\[ \frac{d^2 \sigma}{dy \, dp_T} \text{[nb/(GeV/c)]} \]

CDF data (L = 1.0 fb\(^{-1}\))

Systematic uncertainties

NLO: JETRAD CTEQ6.1M corrected to hadron level

\[ p_T = \mu = \max \left( \frac{p_T^{\text{jet}}}{2}, \mu_0 \right) \]

PDF uncertainties

Data/Theory

Parton to hadron level correction

Monte Carlo modeling uncertainties
Rapidity dependence:

\[ J_{\text{ET}} T \]

\[ \frac{d^2 \sigma}{dy JET dp_{T}} \ [\text{nb}/(\text{GeV}/c)] \]

CDF data (L = 1.0 fb⁻¹)

Systematic uncertainties

NLO: JETRAD CTEQ6.1M corrected to hadron level

\[ \mu_R = \mu_F = \max p_{TJET} / 2 = \mu_0 \]

PDF uncertainties

\[ |y^{JET}| < 0.1 \times 10^6 \]

\[ 0.1 < |y^{JET}| < 0.7 \times 10^3 \]

\[ 0.7 < |y^{JET}| < 1.1 \]

\[ 1.1 < |y^{JET}| < 1.6 \times 10^3 \]

\[ 1.6 < |y^{JET}| < 2.1 \times 10^6 \]

\[ \mu_{T} D=0.7 \]

\[ K_{T} \]

\[ \mu_{T} = \text{max} p_{TJET} / 2 = \mu_0 \]
Contribution of different parton combinations determined by subprocess cross sections and parton distributions.

Quarks dominate at large $E_T$ since this selects large $x_{1,2}$:

$$\hat{s} = x_1 x_2 S > 4E_T^2$$
Heavy Quark Production

- Lowest-order subprocesses for heavy quark production are (a) light quark-antiquark annihilation (10% at LHC) and (b) gluon-gluon fusion (90% at LHC)

- NLO top quark cross section = $840 \pm 30$(scale)$\pm 20$(pdf) pb at LHC
Standard Model Higgs Boson Production

- Lowest-order subprocesses for Higgs boson production at hadron colliders:
  - (a) Gluon-gluon fusion (via top loop)
  - (b) Vector boson fusion
  - (c) Associated production with $W, Z$ boson
  - (d) Associated production with $t\bar{t}$.
NLO Higgs cross sections

\[ p+p\to H+X \]
\[ \sqrt{s}=14 \text{ TeV} \]
Discovery decay channels depend on Higgs mass
2 → 2 parton processes — all available, e.g. in MCFM (CaEl*)

2 → 3 parton processes

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<tr>
<th>Final State</th>
<th>Authors*</th>
<th>Comments</th>
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<td>3 jets</td>
<td>KuSiTr,BerDixKo,GiKi,Na</td>
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<td>V + 2 jets</td>
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<tr>
<td>V b b</td>
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NLO Update (Glover, LP2009)

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<td>$W + 3$ jets</td>
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<td>vHPP$^b$</td>
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<td>BOPP$^g$</td>
<td>$WZZ$, $WWW$, $WWW$</td>
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<td>multijets</td>
<td>GZ$^h$</td>
<td>$gg \rightarrow \text{up to 20 gluons}$</td>
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</table>

$^a$Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre

$^b$van Hameren, Papadopoulos, Pittau

$^c$Figy, Hankele, Zeppenfeld

$^d$Bredenstein, Denner, Dittmaier, Pozzorini

$^e$Bevilacqua, Czakon, Papadopoulos, Pittau, Worek

$^f$Lazopoulos, McElmurry, Melnikov, Petriello

$^g$Binoth, Ossa, Papadopoulos, Pittau

$^h$Giele, Zanderighi
Les Houches 2007 wish list of “feasible” NLO calculations

<table>
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<tr>
<th>Final State</th>
<th>Relevance</th>
<th>Progress?</th>
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<td>$VV$ jet</td>
<td>$t\bar{t}H$, new physics</td>
<td>$VV = \gamma\gamma, WW$</td>
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<td>VBF $\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics</td>
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<td>VBF $\rightarrow H \rightarrow VV$</td>
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<td>various new physics signatures</td>
<td>$W + 3$ jets</td>
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<tr>
<td>4 jets</td>
<td>various new physics signatures</td>
<td>$gg \rightarrow gggg$</td>
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“Done” does not necessarily mean a (parton-level) event generator exists

- Time for matrix element generation?
- Sum over spins and colours?
- Decays of unstable particles (with spin correlations)?
- Efficient phase space generation and unweighting?
- Interfacing to parton showers and hadronization?
Summary of Lecture 2

- Jet fragmentation functions also obey DGLAP evolution equations.
  - Scaling violation seen in $e^+e^-$.
  - Soft gluon coherence $\Rightarrow$ angular-ordered branching.
  - Small-$x$ fragmentation sensitive to coherence effects.
  - Gaussian peak in $\ln(1/x)$, peak position shows coherence.
  - Average hadron multiplicity predicted.

- Hadronization models needed for simulation of full final states.
  - General ideas describe spectra and event shapes.
    - Local parton-hadron duality $\Rightarrow$ small-$x$ hadron spectra.
    - Universal low-scale $\alpha_s \Rightarrow \langle \alpha_s(q < 2 \text{ GeV}) \rangle \sim 0.5$.
  - Specific models needed for hadron distributions.
    - String model (PYTHIA).
    - Cluster model (HERWIG).

- In hadron-hadron processes, factorization permits cross section calculations.
  - Parton-parton luminosities important: uncertainties $\sim 10 - 20\%$.
  - Lepton-pair, jet, top and Higgs production reliably predicted (NLO or NNLO).
  - All $2 \rightarrow 2$ and many $2 \rightarrow 3$ subprocesses predicted to NLO.