Preprint typeset in JHEP style - HYPER VERSION

Cavendish–HEP–02/03
CERN–TH/2001–177
DAMTP–2002–34
DCPT/01/102
IPPP/01/51
KEK–TH/812

Implementation of supersymmetric processes in the HERWIG event generator *

Stefano Moretti
Theory Division, CERN and IPPP, University of Durham
E-mail: `Stefano.Moretti@cern.ch`

Kosuke Odagiri
Theory Group, KEK
E-mail: `odagirik@post.kek.jp`

Peter Richardson
Cavendish Laboratory and DAMTP, University of Cambridge
E-mail: `richardn@hep.phy.cam.ac.uk`

Michael H. Seymour
Department of Physics & Astronomy, University of Manchester
E-mail: `M.Seymour@rl.ac.uk`

Bryan R. Webber
Cavendish Laboratory, University of Cambridge
E-mail: `webber@hep.phy.cam.ac.uk`

Abstract: We describe the implementation of supersymmetric processes in the HERWIG Monte Carlo event generator. We define relevant parameter and mixing conventions and list the hard scattering matrix elements. Our implementation is based on the Minimum Supersymmetric Standard Model, with the option of R-parity violation. The sparticle spectrum is completely general. Both hadron–hadron and lepton–lepton collisions are covered. This article supplements a separate publication in which the general features of HERWIG 6.2 are described, and updates the treatment of supersymmetry to version 6.4.

Keywords: suy.ssm.hig.

*Work supported in part by the U.K. Particle Physics and Astronomy Research Council and by the EU Fourth Framework Programme 'Training and Mobility of Researchers', Network 'Quantum Chromodynamics and the Deep Structure of Elementary Particles', contract FMRX-CT98-0194 (DG 12 - MIHT).

Contents

1. Introduction

HERWIG[1] [1] is a general-purpose event generator for high-energy processes. A major new feature is the inclusion of supersymmetric (SUSY) processes, made available for the first time in HERWIG 6.1 and extended in subsequent versions. This article describes the implementation, defines the parameter and mixing conventions, and lists the hard

[1]The latest version of the program, together with all relevant information, is always available via the official HERWIG page: `http://hepwww.rl.ac.uk/theory/seymour/herwig/`.

scattering matrix elements. For the details of the input file format and for a more general description of our implementation, as well as the description of the Standard Model (SM) physics implemented in the generator, the reader should consult Ref. [1]. This article supplements that paper and extends it to version 6.4. The exact version number of the code described in this article is 6.400.

The key features of the HERWIG 6.4 supersymmetry implementation are as follows:

- A general Minimal Supersymmetric Standard Model (MSSM) particle spectrum.
- Both hadron–hadron and lepton–lepton collisions.
- MSSM $2 \to 2$ sparticle production subprocesses.
- $2 \to 1$, $2 \to 2$ and $2 \to 3$ Two Higgs Doublet Model (2HDM) Higgs boson production subprocesses in hadron–hadron collisions, and $2 \to 2$ Higgs boson production processes in lepton–lepton collisions.
- All R-parity violating $2 \to 2$ scattering subprocesses in hadron–hadron collisions that can, but do not necessarily, have an s-channel resonance, and $2 \to 2$ scattering processes in lepton–lepton collisions, not necessarily with an s-channel resonance.
- Cascade decay of heavy objects according to branching fractions taken from an input data file, incorporating 3-body and 4-body matrix elements.
- Colour coherence between emitted partons.
- Spin correlations in $2 \to 2$ R-parity conserving production processes and all cascade decay processes.
- Beam polarisation in lepton–lepton collisions.

The masses, total decay widths and branching fractions, the mixings of gauginos, third generation sfermions and Higgs bosons, the bilinear, trilinear and R-parity violating trilinear couplings, are read in from a data file. A separate program (ISAWIG) is available (from the HERWIG web page) to convert the data from ISAJET [2] and HDECAY [3] to a form suitable for HERWIG and modify it to include the R-parity violating sector.

In our implementation, supersymmetric particles do not radiate gluons. This assumption is reasonable if the decay lifetimes of the coloured sparticles (spartons) are much shorter than the QCD confinement scale. In particular, a stable gluino Lightest Supersymmetric Particle (LSP) is not simulated. The simulation of a long-lived light top squark would neglect effects due to gluon emission and hadronization.

CP-violating phases are not included for either the mixings or the couplings.

R-parity violating lepton–gaugino and slepton–Higgs mixing is not considered.

2. Parameter and mixing conventions

There are several different conventions that are commonly adopted in the literature for the MSSM Lagrangian. While most authors have chosen to follow the conventions used in Refs. [4, 5], other conventions, for example Ref. [2], are also commonly used. The conventions we have used in general follow those of Refs. [4, 5]. In this section we present our conventions for the mixing matrices for the charginos, neutralinos and sfermions, together with the conversion between the conventions of Refs. [4, 5] and Ref. [2].

2.1 Particle content

The MSSM particle content, expressed in terms of its mass eigenstates, is given in Tab. 1.

spin 0	spin 1/2	spin 1
down-type squarks $(\tilde{d}_L, \tilde{d}_R, \tilde{s}_L, \tilde{s}_R, \tilde{b}_1, \tilde{b}_2)$	down-type quarks (d, s, b)	
up-type squarks $(\tilde{u}_L, \tilde{u}_R, \tilde{c}_L, \tilde{c}_R, \tilde{t}_1, \tilde{t}_2)$	up-type quarks (u, c, t)	
charged sleptons $(\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_1, \tilde{\tau}_2)$	charged leptons (e^-, μ^-, τ^-)	
sneutrinos $(\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$	neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$	
	gluino $\tilde{g}$	gluon g
neutral Higgs (h^0, H^0, A^0)	neutralinos $\tilde{\chi}^0_i$ $(i = 1-4)$	photon and Z^0 (γ, Z^0)
charged Higgs H^+	charginos $\tilde{\chi}^+_i$ $(i = 1-2)$	$W^\pm$ W^+
	Goldstino $\tilde{G}$	

Table 1: Particle content of the MSSM, expressed in terms of its mass eigenstates. The Goldstino $\tilde{G}$ is the spin-1/2 component of the gravitino. We neglect mixing for the first two generation sfermions.

In defining our conventions we need to write the particle content in terms of the interaction eigenstates. This is shown in Tab. 2. Superscripts c refer to charge conjugation, which, stated in the four-component notation, is defined by:

$$\psi^c = C\bar{\psi}^T = i\gamma^2\gamma^0\bar{\psi}^T. \tag{2.1}$$

We adopt the chirality basis. The two-component case, which we refer to when discussing mixing conventions, follows trivially. Acting on chirality-left and chirality-right two-component spinors, we have:

$$(\phi_L)^c = i\sigma^2\phi_L^* = \varepsilon_{ij}\phi^*_{jL}, \tag{2.2}$$

$$(\phi_R)^c = -i\sigma^2\phi_R^* = -\varepsilon_{ij}\phi^*_{jR}. \tag{2.3}$$

The Levi-Civita tensor ε_{ij} is defined by $\varepsilon_{12} = -\varepsilon_{21} = 1, \varepsilon_{11} = \varepsilon_{22} = 0$.

2.2 Parameter conventions

In terms of the scalar fields defined in Tab. 2, we write our scalar superpotential as follows:

$$W_s = \varepsilon_{ij} \left[\mu H_1^i H_2^j + f_\ell H_1^i \widetilde{L}^j \widetilde{E} + f_D H_1^i \widetilde{Q}^j \widetilde{D} - f_U H_2^i \widetilde{Q}^j \widetilde{U} \right] . \tag{2.4}$$

The above definition is consistent with Refs. [5, 6]. The sign of the term proportional to the up-type quark coupling Yukawa matrix f_U is such that the Yukawa couplings are positive.

spin 0	spin 1/2	spin 1
$\widetilde{Q} = \begin{pmatrix} \widetilde{U}_L \\ \widetilde{D}_L \end{pmatrix}$ $\widetilde{D} = \widetilde{D}_R^*$ $\widetilde{U} = \widetilde{U}_R^*$	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $D = d_R^c$ $U = u_R^c$	
$\widetilde{L} = \begin{pmatrix} \widetilde{\nu} \\ \widetilde{e}_L \end{pmatrix}$ $\widetilde{E} = \widetilde{e}_R^*$	$L = \begin{pmatrix} \nu \\ e_L \end{pmatrix}$ $E = e_R^c$	
	$\widetilde{g}$	g
  $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$ $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$	$\widetilde{B}$ $\widetilde{W} = \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{W}_3^0 \\ \widetilde{W}^- \end{pmatrix}$ $\widetilde{H}_1 = \begin{pmatrix} \widetilde{H}_1^0 \\ \widetilde{H}_1^- \end{pmatrix}$ $\widetilde{H}_2 = \begin{pmatrix} \widetilde{H}_2^+ \\ \widetilde{H}_2^0 \end{pmatrix}$	B $W = \begin{pmatrix} W^+ \\ W_3^0 \\ W^- \end{pmatrix}$
	$\widetilde{G}$	

Table 2: Field content of the MSSM, expressed in terms of its interaction eigenstates. Generation and colour indices are suppressed.

For the R-parity violating scalar superpotential, we take:

$$W_{\not R_p} = \varepsilon_{ij} \left[\kappa_a \widetilde{L}_a^i H_2^j + \frac{1}{2} \lambda_{abc} \widetilde{L}_a^i \widetilde{L}_b^j \widetilde{E}_c + \lambda'_{abc} \widetilde{L}_a^i \widetilde{Q}_b^j \widetilde{D}_c \right] + \frac{1}{2} \lambda''_{abc} \varepsilon_{c_1 c_2 c_3} \widetilde{U}_a^{c_1} \widetilde{D}_b^{c_2} \widetilde{D}_c^{c_3} . \tag{2.5}$$

a, b, c are the generation indices. In the last term, c_1, c_2, c_3 are the triplet colour indices and the totally antisymmetric tensor is defined by $\varepsilon_{123} = 1$. Summation over colour indices is implicit in the other terms. We omit the bilinear term proportional to κ_a.

We introduce our soft SUSY-breaking bilinear and trilinear terms as follows:

$$V_I = \varepsilon_{ij} \left[\mu B H_1^i H_2^j + f_\ell A_\tau H_1^i \widetilde{L}^j \widetilde{E} + f_D A_b H_1^i \widetilde{Q}^j \widetilde{D} - f_U A_t H_2^i \widetilde{Q}^j \widetilde{U} \right] + (\text{h.c.}). \tag{2.6}$$

This is consistent with the definition in Ref. [5]. We neglect A term contributions to the first two generations. The contribution of the potential to the Lagrangian is $\mathcal{L} = -V$ in general. (h.c.) stands for Hermitian conjugation.

Our soft SUSY-breaking masses are defined schematically as follows:

$$V_M = \sum_\phi M_\phi^2 \phi^\dagger \phi + \sum_\psi M_\psi \bar{\psi}\psi, \tag{2.7}$$

where ϕ are the sfermion fields and ψ are the gaugino and gluino fields in the four-component notation. Summations over colour and weak isospin indices are implicit. We write the gaugino soft-breaking masses as $M_{\widetilde{B}} = M_1$ and $M_{\widetilde{W}} = M_2$. In the notation of Refs. [4, 5], M' and M are used instead of M_1 and M_2, respectively. However, the notation M_1 and M_2 has become more common.

The above conventions for the signs of the μ and the A terms are the same as those adopted in ISAJET [2] and Haber and Kane [4]. This also agrees with the conventions of the other major SUSY event generators, SPYTHIA [7] and SUSYGEN [8]. While these conventions are the same internally, ISAJET uses a different notation which corresponds to the following:

$$2m_1 = -\mu, \tag{2.8}$$

$$\mu_1 = -M_1, \tag{2.9}$$

$$\mu_2 = -M_2, \tag{2.10}$$

where m_1, μ_1 and μ_2 are in the notation of ISAJET.

2.3 Mixing conventions

While internally HERWIG uses the conventions of Refs. [4, 5, 6], the mass and decay spectra are fed in from a data file. In particular, we have provided a separate code (ISAWIG) for the conversion of data from ISAJET [9]. This way, the masses of the sparticles and their R-parity conserving decay rates are calculated using ISAJET and the R-parity violating section is added by ISAWIG. ISAJET also calculates the mixing matrices for the electroweak gauginos and scalar fermions. ISAJET uses the conventions of Refs. [2, 10] and it is essential that the conversion between the two formalisms is done correctly.

We first consider the conversion between the two formalisms for the gauginos and then for the left–right sfermion mixing. We compare the Lagrangian of Ref. [2] against that of Refs. [4, 5] to obtain the conversions between them. We adopt the conventions of Refs. [4, 5] for the chargino and neutralino mixing matrices.

2.3.1 Charginos

In the notation of Ref. [5], we define the following doublets of two-component spinors for the charginos in their interaction eigenstates:

$$\psi^+ = \left(-i\lambda^+, \psi^+_{H_2}\right), \tag{2.11}$$

$$\psi^- = \left(-i\lambda^-, \psi^-_{H_1}\right), \tag{2.12}$$

where $\lambda^\pm$ are the charged Winos, $\psi^-_{H_1}$ is the charged Higgsino associated with the Higgs that gives mass to the down-type quarks and $\psi^+_{H_2}$ is the charged Higgsino associated with the Higgs doublet that gives mass to the up-type quarks. The mass term in the Lagrangian from eqn. (A.2) of Ref. [5] is:

$$\mathcal{L}_{\text{chargino}} = -\frac{1}{2}\left(\psi^+ \ \psi^-\right)\begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + (\text{h.c.}), \tag{2.13}$$

where the mass matrix is given by:

$$X = \begin{pmatrix} M_2 & M_W\sqrt{2}\sin\beta \\ M_W\sqrt{2}\cos\beta & \mu \end{pmatrix}. \tag{2.14}$$

In Ref. [5] this is diagonalized in the two-component notation by defining the mass eigenstates:

$$\chi_i^+ = V_{ij}\psi_j^+, \tag{2.15}$$

$$\chi_i^- = U_{ij}\psi_j^-, \tag{2.16}$$

where U and V are unitary matrices chosen such that

$$U^* X V^{-1} = M_D. \tag{2.17}$$

M_D is the diagonal chargino mass matrix. The four-component mass eigenstates, the charginos, are defined in terms of the two-component fields as:

$$\tilde{\chi}_1^+ = \begin{pmatrix} \chi_1^+ \\ (\chi_1^-)^c \end{pmatrix}, \qquad \tilde{\chi}_2^+ = \begin{pmatrix} \chi_2^+ \\ (\chi_2^-)^c \end{pmatrix}. \tag{2.18}$$

We need to express the chargino mass terms in the Lagrangian in a four-component notation so that we can compare this Lagrangian with the conventions of Ref. [2]. We define the four-component spinors as in Ref. [5]:

$$\widetilde{W}^+ = \begin{pmatrix} -i\lambda^+ \\ (-i\lambda^-)^c \end{pmatrix}, \qquad \widetilde{H}^+ = \begin{pmatrix} \psi_{H_2}^+ \\ (\psi_{H_1}^-)^c \end{pmatrix}. \tag{2.19}$$

Using this notation, we can express the Lagrangian in the four-component notation:

$$\mathcal{L}_{\text{chargino}} = -\left(\overline{\widetilde{W}^+} \ \overline{\widetilde{H}^+}\right)\left[X P_L + X^T P_R\right]\begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}^+ \end{pmatrix}. \tag{2.20}$$

The chirality projection operators are defined as $P_{L/R} = (1{-}/{+}\gamma_5)/2$. Eqn. (2.20) has a similar form to the chargino mass term[2] in eqn. (2.1) of Ref. [2]:

$$\mathcal{L}_{\text{chargino}} = -\left(\bar{\lambda} \ \ \bar{\chi}\right)\left[M_{\text{charge}} P_L + M_{\text{charge}}^T P_R\right]\begin{pmatrix} \lambda \\ \chi \end{pmatrix}, \tag{2.21}$$

[2]This Lagrangian is taken from Ref. [11] which corrects a sign error in the off-diagonal terms in Ref. [2].

where

$$M_{\text{charge}} = \begin{pmatrix} \mu_2 & -gv' \\ -gv & 2m_1 \end{pmatrix} = -\begin{pmatrix} M_2 & \sqrt{2}M_W \cos\beta \\ \sqrt{2}M_W \sin\beta & \mu \end{pmatrix}, \tag{2.22}$$

where v and v' are the vacuum expectation values for the Higgs fields that give mass to the up- and down-type quarks, respectively. We now come to the problem of comparing the notation of Ref. [5] against that of Ref. [2]. The Wino and Higgsino fields of Ref. [2] are the charge conjugates of those used in Ref. [5], so that the following transformation is required:

$$\widetilde{W}^+ = \lambda^c, \tag{2.23}$$

$$\widetilde{H}^+ = \chi^c. \tag{2.24}$$

This gives the chargino mass matrix in the form:

$$\mathcal{L}_{\text{chargino}} = \left(\overline{\widetilde{W}^+}, \overline{\widetilde{H}^+}\right) \left[M'_{\text{charge}} P_L + M'^T_{\text{charge}} P_R\right] \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}^+ \end{pmatrix}, \tag{2.25}$$

where

$$M'_{\text{charge}} = \begin{pmatrix} -\mu_2 & gv \\ gv' & -2m_1 \end{pmatrix} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & \mu \end{pmatrix}. \tag{2.26}$$

This agrees with eqn. (2.20) apart from the overall sign. We need to express the fields in terms of the ISAJET mixing matrices. The ISAJET mixing matrices are given in eqns. (2.10) and (2.11) of Ref. [2] as:

$$\begin{pmatrix} \widetilde{W}_+ \\ \widetilde{W}_- \end{pmatrix}_L = \begin{pmatrix} \theta_x \cos\gamma_L & -\theta_x \sin\gamma_L \\ \sin\gamma_L & \cos\gamma_L \end{pmatrix} \begin{pmatrix} \lambda \\ \chi \end{pmatrix}_L, \tag{2.27}$$

$$\begin{pmatrix} (-1)^{\theta_+}\widetilde{W}_+ \\ (-1)^{\theta_-}\widetilde{W}_- \end{pmatrix}_R = \begin{pmatrix} \theta_y \cos\gamma_R & -\theta_y \sin\gamma_R \\ \sin\gamma_R & \cos\gamma_R \end{pmatrix} \begin{pmatrix} \lambda \\ \chi \end{pmatrix}_R. \tag{2.28}$$

The mixing angles γ_L and γ_R, and the sign functions θ_x, θ_y, θ_+, and θ_- are defined in Ref. [2].

We can transform these equations into the notation of Ref. [4] with the identification:

$$\tilde{\chi}^+_1 = \widetilde{W}^c_-, \tag{2.29}$$

$$\tilde{\chi}^+_2 = \widetilde{W}^c_+. \tag{2.30}$$

This gives:

$$P_L \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}^+ \end{pmatrix} = P_L \begin{pmatrix} -\sin\gamma_R & -\theta_y \cos\gamma_R \\ -\cos\gamma_R & \theta_y \sin\gamma_R \end{pmatrix} \begin{pmatrix} (-1)^{\theta_-}\tilde{\chi}^+_1 \\ (-1)^{\theta_+}\tilde{\chi}^+_2 \end{pmatrix}, \tag{2.31}$$

$$P_R \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}^+ \end{pmatrix} = P_R \begin{pmatrix} -\sin\gamma_L & -\theta_x \cos\gamma_L \\ -\cos\gamma_L & \theta_x \sin\gamma_L \end{pmatrix} \begin{pmatrix} \tilde{\chi}^+_1 \\ \tilde{\chi}^+_2 \end{pmatrix}. \tag{2.32}$$

These mixing matrices are now in a form that allows us to compare them with the notation of Ref. [5], eqn. (A.13):

$$P_L \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}^+ \end{pmatrix} = P_L \begin{pmatrix} V_{11}^* & V_{21}^* \\ V_{12}^* & V_{22}^* \end{pmatrix} \begin{pmatrix} \widetilde{\chi}_1^+ \\ \widetilde{\chi}_2^+ \end{pmatrix}, \tag{2.33}$$

$$P_R \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}^+ \end{pmatrix} = P_R \begin{pmatrix} U_{11} & U_{21} \\ U_{12} & U_{22} \end{pmatrix} \begin{pmatrix} \widetilde{\chi}_1^+ \\ \widetilde{\chi}_2^+ \end{pmatrix}. \tag{2.34}$$

By comparing eqns. (2.32) and (2.34) we obtain the mixing matrices in the notation of Ref. [5]:

$$U = \begin{pmatrix} -\sin\gamma_L & -\cos\gamma_L \\ -\theta_x \cos\gamma_L & \theta_x \sin\gamma_L \end{pmatrix}, \tag{2.35}$$

$$V = \begin{pmatrix} -\sin\gamma_R(-1)^{\theta_-} & -\cos\gamma_R(-1)^{\theta_+} \\ -\theta_y \cos\gamma_R(-1)^{\theta_-} & \theta_y \sin\gamma_R(-1)^{\theta_+} \end{pmatrix}. \tag{2.36}$$

It should be noted that we adopt the opposite sign convention for the chargino masses due to the sign differences in the two Lagrangians.

2.3.2 Neutralinos

We define a quadruplet of two-component fermion fields for the neutralinos in the interaction eigenstate as:

$$\psi^0 = \left(-i\lambda', -i\lambda^3, \psi^0_{H_1}, \psi^0_{H_2}\right), \tag{2.37}$$

where λ' is the Bino, λ^3 is the neutral Wino, $\psi^0_{H_1}$ is the Higgsino corresponding to the neutral component of the Higgs doublet giving mass to the down-type quarks and $\psi^0_{H_2}$ is the Higgsino corresponding to the neutral component of the Higgs doublet giving mass to the up-type quarks. The Lagrangian for the neutralino masses from equation (A.18) of Ref. [5] is:

$$\mathcal{L}_{\text{neutralino}} = -\frac{1}{2}\left(\psi^0\right)^T Y \psi^0 + (\text{h.c.}), \tag{2.38}$$

where

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z \sin\theta_W \cos\beta & M_Z \sin\theta_W \sin\beta \\ 0 & M_2 & M_Z \cos\theta_W \cos\beta & -M_Z \cos\theta_W \sin\beta \\ -M_Z \sin\theta_W \cos\beta & M_Z \cos\theta_W \cos\beta & 0 & -\mu \\ M_Z \sin\theta_W \sin\beta & -M_Z \cos\theta_W \sin\beta & -\mu & 0 \end{pmatrix}. \tag{2.39}$$

In Ref. [5], the Lagrangian is diagonalized in this two-component notation to give the mass eigenstates. The diagonalization is performed by defining two-component fields:

$$\chi_i^0 = N_{ij}\psi_j^0, \qquad i,j = 1,\ldots,4, \tag{2.40}$$

where N is a unitary matrix satisfying $N^* Y N^{-1} = N_D$, and N_D is the diagonal neutralino mass matrix. The four-component mass eigenstates, the neutralinos, can be defined in terms of the two-component fields:

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ (\chi_i^0)^c \end{pmatrix}. \tag{2.41}$$

Rather than adopting this approach we need to express the neutralino mass terms in the four-component notation before performing the diagonalization in order to compare this Lagrangian with that of Ref. [2]. We use the standard procedure of Ref. [5] to express this in the four-component notation by defining four-component Majorana fields:

$$\widetilde{B} = \begin{pmatrix} -i\lambda' \\ (-i\lambda')^c \end{pmatrix}, \quad \widetilde{W}_3 = \begin{pmatrix} -i\lambda^3 \\ (-i\lambda^3)^c \end{pmatrix}, \quad \widetilde{H}_1 = \begin{pmatrix} \psi^0_{H_1} \\ (\psi^0_{H_1})^c \end{pmatrix}, \quad \widetilde{H}_2 = \begin{pmatrix} \psi^0_{H_2} \\ (\psi^0_{H_2})^c \end{pmatrix}. \tag{2.42}$$

As defined in Tab. 2, $\widetilde{B}$ is the Bino field, $\widetilde{W}_3$ is the neutral Wino field, $\widetilde{H}_1$ is the field for the Higgsino associated with the neutral component of the Higgs doublet that gives mass to the down-type quarks and $\widetilde{H}_2$ is the field for the Higgsino associated with the neutral component of the Higgs doublet that gives mass to the up-type quarks.

This gives the Lagrangian in the four-component notation:

$$\mathcal{L}_{\text{neutralino}} = -\frac{1}{2}\left(\overline{\widetilde{B}}\ \overline{\widetilde{W}_3}\ \overline{\widetilde{H}_1}, \overline{\widetilde{H}_2}\right)[YP_L + YP_R]\begin{pmatrix} \widetilde{B} \\ \widetilde{W}_3 \\ \widetilde{H}_1 \\ \widetilde{H}_2 \end{pmatrix}. \tag{2.43}$$

We can now compare this with the relevant Lagrangian, in the four-component notation, given in Ref. [2]. This Lagrangian[3] is from Ref. [2], eqn. (2.2):

$$\mathcal{L}_{\text{neutralino}} = -\frac{1}{2}\left(\bar{h}^0, \bar{h}'^0, \bar{\lambda}_3, \bar{\lambda}_0\right)[M_{\text{neutral}}P_L + M_{\text{neutral}}P_R]\begin{pmatrix} h^0 \\ h'^0 \\ \lambda_3 \\ \lambda_0 \end{pmatrix}. \tag{2.44}$$

h^0 is the Higgsino partner of the neutral component of the Higgs doublet that gives mass to the up-type quarks, h'^0 is the Higgsino partner of the Higgs boson that gives mass to the down-type quarks, λ_3 is the neutral Wino and λ_0 is the Bino. The mass matrix is given by:

$$M_{\text{neutral}} = \begin{pmatrix} 0 & -2m_1 & -gv/\sqrt{2} & g'v/\sqrt{2} \\ -2m_1 & 0 & gv'/\sqrt{2} & -g'v'/\sqrt{2} \\ -gv/\sqrt{2} & gv'/\sqrt{2} & \mu_2 & 0 \\ g'v/\sqrt{2} & -g'v'/\sqrt{2} & 0 & \mu_1 \end{pmatrix}. \tag{2.45}$$

[3] This Lagrangian is taken from Ref. [11] which corrects a sign error in Ref. [2].

It is easier to compare this with eqn. (2.43) after reordering the entries and reexpressing it in terms of M_Z, β and θ_W. This gives:

$$\mathcal{L}_{\rm neutralino} = \frac{1}{2}\left(\bar{\lambda}_0, \bar{\lambda}_3, \bar{h}'^0, \bar{h}^0\right)\left[M'_{\rm neutral}P_L + M'_{\rm neutral}P_R\right]\begin{pmatrix}\lambda_0\\ \lambda_3\\ h'^0\\ h^0\end{pmatrix}, \tag{2.46}$$

where:

$$M'_{\rm neutral} = \begin{pmatrix} M_1 & 0 & M_Z\sin\theta_W\cos\beta & -M_Z\sin\theta_W\sin\beta\\ 0 & M_2 & -M_Z\cos\theta_W\cos\beta & M_Z\cos\theta_W\sin\beta\\ M_Z\sin\theta_W\cos\beta & -M_Z\cos\theta_W\cos\beta & 0 & -\mu\\ -M_Z\sin\theta_W\sin\beta & M_Z\cos\theta_W\sin\beta & -\mu & 0\end{pmatrix}. \tag{2.47}$$

This equation then agrees with eqn. (2.43) up to an overall sign, provided we make the following identification:

$$\widetilde{B} = \lambda_0, \tag{2.48}$$
$$\widetilde{W}_3 = \lambda_3, \tag{2.49}$$
$$\widetilde{H}_1 = -h'^0, \tag{2.50}$$
$$\widetilde{H}_2 = -h^0. \tag{2.51}$$

The convention from ISAJET for the mixing, taken from eqn. (2.12) of Ref. [2], is

$$\begin{pmatrix}(-i\gamma_5)^{\theta_1}\widetilde{Z}_1\\ (-i\gamma_5)^{\theta_2}\widetilde{Z}_2\\ (-i\gamma_5)^{\theta_3}\widetilde{Z}_3\\ (-i\gamma_5)^{\theta_4}\widetilde{Z}_4\end{pmatrix} = \begin{pmatrix} v_1^{(1)} & v_2^{(1)} & v_3^{(1)} & v_4^{(1)}\\ v_1^{(2)} & v_2^{(2)} & v_3^{(2)} & v_4^{(2)}\\ v_1^{(3)} & v_2^{(3)} & v_3^{(3)} & v_4^{(3)}\\ v_1^{(4)} & v_2^{(4)} & v_3^{(4)} & v_4^{(4)}\end{pmatrix}\begin{pmatrix}h^0\\ h'^0\\ \lambda_3\\ \lambda_0\end{pmatrix}, \tag{2.52}$$

where $\widetilde{Z}_i$ are the mass eigenstates obtained by diagonalizing the mass matrix given in eqn. (2.45), $v_j^{(i)}$ are the elements of the mixing matrix, and θ_i is zero (one) if the mass of $\widetilde{Z}_i$ is positive (negative).

After reordering, we get:

$$\begin{pmatrix}(-i\gamma_5)^{\theta_1}\widetilde{Z}_1\\ (-i\gamma_5)^{\theta_2}\widetilde{Z}_2\\ (-i\gamma_5)^{\theta_3}\widetilde{Z}_3\\ (-i\gamma_5)^{\theta_4}\widetilde{Z}_4\end{pmatrix} = \begin{pmatrix} v_4^{(1)} & v_3^{(1)} & -v_2^{(1)} & -v_1^{(1)}\\ v_4^{(2)} & v_3^{(2)} & -v_2^{(2)} & -v_1^{(2)}\\ v_4^{(3)} & v_3^{(3)} & -v_2^{(3)} & -v_1^{(3)}\\ v_4^{(4)} & v_3^{(4)} & -v_2^{(4)} & -v_1^{(4)}\end{pmatrix}\begin{pmatrix}\widetilde{B}\\ \widetilde{W}\\ \widetilde{H}_1\\ \widetilde{H}_2\end{pmatrix}. \tag{2.53}$$

We can therefore obtain the mixing matrix in the notation of Ref. [5] by making the identification:

$$N_{i1} = v_4^{(i)}, \tag{2.54}$$
$$N_{i2} = v_3^{(i)}, \tag{2.55}$$
$$N_{i3} = -v_2^{(i)}, \tag{2.56}$$
$$N_{i4} = -v_1^{(i)}. \tag{2.57}$$

Again, we need to adopt the opposite sign convention for the neutralino masses.

2.3.3 Left–right sfermion mixing

In addition to the mixing of the neutralinos and charginos, we need to consider the left–right mixing of the sfermions. In general, as the off-diagonal terms in the mass matrices are proportional to the fermion mass, these effects are only important for the third generation sfermions, *i.e.* stop, sbottom and stau. We describe our convention for the top squarks. The others are treated analogously.

The following mass matrix for the top squarks uses the conventions[4] of Ref. [5] and is taken from eqn. (4.17) therein.

$$M^2_{\tilde{t}} = \begin{pmatrix} M^2_{\widetilde{Q}} + M^2_Z \cos 2\beta \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) + m_t^2 & m_t\left(A_t - \mu\cot\beta\right) \\ m_t\left(A_t - \mu\cot\beta\right) & M^2_{\widetilde{U}} + \frac{2}{3} m_Z^2 \cos 2\beta \sin^2\theta_W + m_t^2 \end{pmatrix}, \tag{2.58}$$

where $M_{\widetilde{Q}}$ and $M_{\widetilde{U}}$ are soft SUSY-breaking masses for the left and right top squarks, respectively. A_t is the trilinear soft SUSY-breaking term for the interaction of the left and right stop squarks with the Higgs boson. This compares with the ISAJET matrix from Ref. [10]:

$$M^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + M^2_Z \cos 2\beta \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) + m_t^2 & -m_t\left(A_t - \mu\cot\beta\right) \\ -m_t\left(A_t - \mu\cot\beta\right) & M^2_{\tilde{t}_R} + \frac{2}{3} m_Z^2 \cos 2\beta \sin^2\theta_W + m_t^2 \end{pmatrix}, \tag{2.59}$$

where $M^2_{\tilde{t}_L} = M^2_{\widetilde{Q}}$ and $M^2_{\tilde{t}_R} = M^2_{\widetilde{U}}$. There is a difference in the sign of the off-diagonal terms. This means that, as the sign conventions for the μ and A terms are the same, there is a difference in the relative phases of the two fields in the different conventions. Hence we should apply the following change to the ISAJET output:

$$\theta_t \longrightarrow -\theta_t, \tag{2.60}$$

i.e. change the sign of the stop mixing angle. The same argument also applies to the sbottom and stau mixing angles.

We adopt the following convention for the sfermion mixing matrices:

$$\begin{pmatrix} \tilde{q}_{iL} \\ \tilde{q}_{iR} \end{pmatrix} = \begin{pmatrix} \cos\theta^i_q & \sin\theta^i_q \\ -\sin\theta^i_q & \sin\theta^i_q \end{pmatrix} \begin{pmatrix} \tilde{q}_{i1} \\ \tilde{q}_{i2} \end{pmatrix} = \begin{pmatrix} Q^i_{L1} & Q^i_{L2} \\ Q^i_{R1} & Q^i_{R2} \end{pmatrix} \begin{pmatrix} \tilde{q}_{i1} \\ \tilde{q}_{i2} \end{pmatrix}, \tag{2.61}$$

where $\tilde{q}_{iL}$ and $\tilde{q}_{iR}$ are the left and right squark fields, for the ith quark, where i runs over quark flavours d, u, s, c, b, t. $\tilde{q}_{i1}$ and $\tilde{q}_{i2}$ are the squark fields for the mass eigenstates, for the quark i, and θ^i_q is the mixing angle obtained by diagonalizing the mass matrix.

Similarly, we denote the slepton mixing as above with the matrix $L^i_{\alpha\beta}$ where i runs over the charged and neutral lepton flavour. β is the mass eigenstate and α is the left/right eigenstate. As we do not include the right-handed neutrino we omit the left–right mixing for the sneutrinos. Note that our convention for the mixing matrix is opposite to that for the gauginos, in the sense that here the mixing matrices Q^i act on mass eigenstates to give the interaction eigenstates.

[4] This is the same as Ref. [12], SPYTHIA [7] and SUSYGEN [8].

2.3.4 Higgs bosons

The Higgs doublets are diagonalised as follows. As in Tabs. 1 and 2, H_i denote the $SU(2)$ doublet interaction eigenstates which form the physical eigenstates $h^0, H^0, A^0, H^\pm$ after the electroweak symmetry breaking. In the charged sector:

$$\begin{aligned} H_2^+ &= H^+ \cos\beta + G^+ \sin\beta, \\ H_1^- &= H^- \sin\beta - G^- \cos\beta, \end{aligned} \tag{2.62}$$

where $G^\pm$ are the Goldstone modes. The neutral Higgs sector is parametrised in the interaction eigenstate as follows:

$$\begin{aligned} H_1^0 &= \frac{v\cos\beta + \eta_1 + i\zeta_1}{\sqrt{2}}, \\ H_2^0 &= \frac{v\sin\beta + \eta_2 + i\zeta_2}{\sqrt{2}}. \end{aligned} \tag{2.63}$$

v is the vacuum expectation value. The diagonalisation is achieved by:

$$\begin{aligned} \zeta_2 &= A^0 \cos\beta + G^0 \sin\beta, \\ \zeta_1 &= A^0 \sin\beta - G^0 \cos\beta, \\ \eta_2 &= H^0 \sin\alpha + h^0 \cos\alpha, \\ \eta_1 &= H^0 \cos\alpha - h^0 \sin\alpha. \end{aligned} \tag{2.64}$$

Our notation is consistent with Ref. [5], but the sign of α is opposite to that of ISAJET [2].

3. Matrix elements

3.1 Notation

For $2 \to 2$ hard scattering subprocesses of the form $12 \to 34$, let us define the usual Mandelstam variables in terms of the partonic momenta as $\hat{s} = (p_1 + p_2)^2$, $\hat{t} = (p_1 - p_3)^2$ and $\hat{u} = (p_1 - p_4)^2$. For simplicity, the 'hat' will be omitted in the following. As initial state partons are taken to have zero kinematic mass, it follows that $s + t + u = m_3^2 + m_4^2$, $s + t_3 + u_4 = 0$, and $ut - m_3^2 m_4^2 = s p_T^2$ where $t_3 = t - m_3^2$, $u_4 = u - m_4^2$, and p_T is the outgoing transverse momentum.

The electromagnetic coupling is e, such that $\alpha_{\mathrm{EM}} = e^2/4\pi$. The strong coupling is g_s, such that $\alpha_s = g_s^2/4\pi$. The QCD colour factors are $N_C = 3$ and $C_F = (N_C^2 - 1)/2N_C = 4/3$.

Colour flow directions are not stated explicitly as they are trivial in most of the processes whose matrix elements are given below. For a general treatment, see Ref. [13]. Matrix elements given here are summed over the final state colour and spin, and averaged over the initial state colour and spin. As usual, this is indicated by a long overline. Statistical factors of two for identical neutralinos in the final state are written explicitly.

Let us now consider the widths $\Gamma(q^2)$ appearing in the propagators of unstable massive particles with mass $M(q^2)$. In order to simplify the notation, let us define a function $\overline{\Gamma}(q^2)$, which is defined for positive q^2 by:

$$\sqrt{q^2}\,\Gamma(q^2) \equiv M(q^2)\bar{\Gamma}(q^2). \tag{3.1}$$

For negative q^2, the width is taken to be zero. The propagator is then given in general, again for positive q^2, by:

$$\frac{1}{q^2 - M^2(q^2) + i\sqrt{q^2}\Gamma(q^2)} \equiv \frac{1}{q^2 - M^2(q^2) + iM(q^2)\overline{\Gamma}(q^2)}. \tag{3.2}$$

In the HERWIG implementation, we adopt the naive running width approximation and neglect the running of mass, so that:

$$\bar{\Gamma}(q^2) = q^2\Gamma(M^2)/M^2. \tag{3.3}$$

The following coupling notations are used in processes involving electroweak interactions:

$$Q_f = U(1)_{\rm EM} \text{ charge}; \tag{3.4}$$
$$I_f^3 = \text{weak isospin}; \tag{3.5}$$
$$L_f = (I_f^3 - Q_f \sin^2\theta_W)/\cos\theta_W \sin\theta_W; \tag{3.6}$$
$$R_f = -Q_f \sin^2\theta_W / \cos\theta_W \sin\theta_W. \tag{3.7}$$

Q_f and I_f^3 are as listed in Tab. 3. In terms of the above, we define the neutralino to sfermion coupling coefficients as:

$$\widetilde{L}_{fa} = (N_{a1}\cos\theta_W + N_{a2}\sin\theta_W)\, Q_f + (-N_{a1}\sin\theta_W + N_{a2}\cos\theta_W)\, L_f; \tag{3.8}$$
$$\widetilde{R}_{fa} = (N_{a1}\cos\theta_W + N_{a2}\sin\theta_W)\, Q_f + (-N_{a1}\sin\theta_W + N_{a2}\cos\theta_W)\, R_f. \tag{3.9}$$

Following Ref. [4], we have neglected the contribution of the Higgsino component, which is proportional to the Yukawa coupling. This affects the cross sections for sfermion–Higgsino associated production, namely the Higgsino component of the processes $gb \to \tilde{t}\tilde{\chi}^{\pm}, \tilde{b}\tilde{\chi}^0$.

f	Q_f	I_f^3
e	-1	$-1/2$
ν	0	$+1/2$
d	$-1/3$	$-1/2$
u	$+2/3$	$+1/2$

Table 3: The $U(1)_{\rm EM}$ charge Q_f and the weak isospin I_f^3 for the Standard Model fermions.

3.2 Sparton pair production

The matrix elements and the colour decompositions for $2 \to 2$ parton-to-sparton scattering are given in Ref. [13]. The term 'sparton' refers to coloured superparticles, namely the squarks and the gluino. Neglecting the squark left–right mixings, the lepton-annihilation reaction $\ell^-\ell^+ \to \tilde{q}\tilde{q}^*$ is given by:

$$\begin{aligned}\overline{|\mathcal{M}|^2}_{(L,R)} &= e^4 N_C s p_T^2 \\ &\times \left[\left|\frac{Q_\ell Q_q}{s} + \frac{L_\ell(L_q, R_q)}{s - M_Z^2 + iM_Z\overline{\Gamma}_Z}\right|^2 + \left|\frac{Q_\ell Q_q}{s} + \frac{R_\ell(L_q, R_q)}{s - M_Z^2 + iM_Z\overline{\Gamma}_Z}\right|^2\right].\end{aligned} \tag{3.10}$$

The generalization to the case with left–right mixings is given by:

$$(L_q, R_q) \to L_q Q_{Li} Q_{Lj} + R_q Q_{Ri} Q_{Rj}. \tag{3.11}$$

The indices i, j label the squarks that are produced.

3.3 Slepton pair production

The matrix elements are similar to eqns. (3.10) and (3.11). For the quark annihilation reaction $q\bar{q} \to \widetilde{\ell}\widetilde{\ell}^*$ one should interchange the indices $(q \leftrightarrow \ell)$, and change the colour factor from N_C to $1/N_C$. In addition, for the charged current reaction $q\bar{q}' \to \widetilde{\ell}_L \widetilde{\nu}^*$ we have

$$\overline{|\mathcal{M}|^2}(q\bar{q}' \to \widetilde{\ell}_L \widetilde{\nu}^*) = \frac{e^4 s p_T^2}{\left(2\sin^2\theta_W\right)^2 N_C} \left| \frac{1}{s - M_W^2 + iM_W\Gamma_W} \right|^2 . \tag{3.12}$$

For the lepton annihilation reaction the colour factor is 1, and there is an extra t-channel gaugino contribution to the production of slepton pairs of the same generation as the colliding lepton pair, namely,

$$\begin{aligned} \overline{|\mathcal{M}|^2}(e^-e^+ \to \widetilde{e}_L\widetilde{e}_L^*, \widetilde{e}_R\widetilde{e}_R^*) &= e^4 s p_T^2 \\ &\times \left[\left| \frac{1}{s} + \frac{L_e(L_e, R_e)}{s - M_Z^2 + iM_Z\Gamma_Z} + \sum_{a=1}^{4} \left(\frac{\widetilde{L}_{ea}^2}{t - M_{\widetilde{\chi}_a^0}^2}, 0 \right) \right|^2 \right. \\ &+ \left. \left| \frac{1}{s} + \frac{R_e(L_e, R_e)}{s - M_Z^2 + iM_Z\Gamma_Z} + \sum_{a=1}^{4} \left(0, \frac{\widetilde{R}_{ea}^2}{t - M_{\widetilde{\chi}_a^0}^2} \right) \right|^2 \right], \end{aligned} \tag{3.13}$$

$$\overline{|\mathcal{M}|^2}(e^-e^+ \to \widetilde{e}_L\widetilde{e}_R^*, \widetilde{e}_R\widetilde{e}_L^*) = e^4 \left(\sum_{a=1}^{4} \frac{M_{\widetilde{\chi}_a^0}\sqrt{s}}{t - M_{\widetilde{\chi}_a^0}^2} \widetilde{L}_{ea}\widetilde{R}_{ea} \right)^2 . \tag{3.14}$$

The neutralino masses appearing above are the signed masses. The production of sneutrinos of the same generation as the colliding lepton pair is given by:

$$\begin{aligned} \overline{|\mathcal{M}|^2}(e^-e^+ \to \widetilde{\nu}_{eL}\widetilde{\nu}_{eL}^*) &= e^4 s p_T^2 \\ &\times \left[\left| \frac{L_e L_\nu}{s - M_Z^2 + iM_Z\Gamma_Z} + \sum_{a=1}^{2} \frac{(V_{a1}^{\widetilde{\chi}^\pm})^2 / 2\sin^2\theta_W}{t - M_{\widetilde{\chi}_a^\pm}^2} \right|^2 \right. \\ &+ \left. \left| \frac{R_e L_\nu}{s - M_Z^2 + iM_Z\Gamma_Z} \right|^2 \right]. \end{aligned} \tag{3.15}$$

3.4 Gaugino production

We consider the following seven processes in hadronic collisions:

$$q\bar{q} \to \widetilde{\chi}_a^+ \widetilde{\chi}_b^- \tag{3.16}$$

$$q\bar{q} \to \widetilde{\chi}_i^0 \widetilde{\chi}_j^0 \tag{3.17}$$

$$q\bar{q}' \to \widetilde{\chi}_a^\pm \widetilde{\chi}_i^0 \tag{3.18}$$

$$q\bar{q} \to \widetilde{\chi}_i^0 \widetilde{g} \tag{3.19}$$

$$gq \to \widetilde{\chi}_i^0 \widetilde{q} \text{ and } c.c. \tag{3.20}$$

$$q\bar{q}' \to \widetilde{\chi}_a^\pm \widetilde{g} \tag{3.21}$$

$$gq \to \widetilde{\chi}_a^\pm \widetilde{q}' \text{ and } c.c. \tag{3.22}$$

c.c. refers to charge conjugation. The indices take the values $a, b = 1, 2$ and $i, j = 1 \ldots 4$. Only processes (3.16) and (3.17) are relevant to leptonic collisions, where the colour factors and electroweak coupling coefficient need trivial modifications. Processes (3.16), (3.17), (3.18), (3.19) and (3.21) are expressed conveniently following the notation of Ref. [14]:

$$A = \sum_{\alpha,\beta=L,R} C_{\alpha\beta} (\bar{v}_2 \gamma^\mu P_\alpha u_1)(\bar{u}_3 \gamma^\mu P_\beta v_4) = \sum_{\alpha,\beta=L,R} C_{\alpha\beta} Q_\alpha^\mu X_\beta^\mu, \tag{3.23}$$

where, as before, $P_L = (1-\gamma_5)/2$ and $P_R = (1+\gamma_5)/2$. The spin-averaged amplitudes are given as:

$$\begin{aligned}\overline{|A|^2} = {} & [|C_{LL}|^2 + |C_{RR}|^2] u_3 u_4 + [|C_{LR}|^2 + |C_{RL}|^2] t_3 t_4 + \\ & + 2\mathrm{Re}[C_{LL}^* C_{LR} + C_{RR}^* C_{RL}](m_3 m_4 s).\end{aligned} \tag{3.24}$$

The masses appearing above as $(m_3 m_4 s)$ are the signed masses.

The matrix elements squared for processes (3.20) and (3.22) have the following form, where α labels the chirality of the final state squark:

$$\overline{|\mathcal{M}|^2}_\alpha = \frac{(g_s g_\alpha)^2}{2N_C} \left[-\frac{t_3}{s} - \frac{2(m_4^2 - m_3^2) t_3}{s u_4} \left(1 + \frac{m_3^2}{t_3} + \frac{m_4^2}{u_4} \right) \right]. \tag{3.25}$$

When left–right squark mixing is considered, the above is modified as:

$$\begin{aligned}\overline{|\mathcal{M}|^2}_\alpha \propto g_\alpha^2 \longrightarrow \overline{|\mathcal{M}|^2}_i \propto g_i^2, & \text{ where} \\ g_i^2 = g_L^2 Q_{Li}^2 + g_R^2 Q_{Ri}^2. &\end{aligned} \tag{3.26}$$

Process (3.16), $q\bar{q} \to \widetilde{\chi}_a^+ \widetilde{\chi}_b^-$, is given by eqn. (3.24) with the prefactor (e^4/N_C) and the following coefficients $C_{\alpha\beta}$:

$$\begin{aligned}C_{\alpha\beta} = {} & -\delta_{ab} \frac{Q_q}{s} - \frac{1}{\sin 2\theta_W} \cdot \frac{(\delta_{\alpha L} L_q + \delta_{\alpha R} R_q)(-2O_{ab}^{\prime\beta})}{s - M_Z^2 + iM_Z \Gamma_Z} \\ & - \frac{\delta_{\alpha L}}{2\sin^2\theta_W} \left[\frac{(1-\delta_{\alpha\beta})\delta_{qu} U_{a1} U_{b1}}{t - M_{\tilde{d}_L}^2} - \frac{\delta_{\alpha\beta}\delta_{qd} V_{a1} V_{b1}}{u - M_{\tilde{u}_L}^2} \right].\end{aligned} \tag{3.27}$$

Here, following the notation of Ref. [4], we have defined:

$$-2O_{ab}^{\prime L} = -V_{a2} V_{b2} + 2\delta_{ab} \cos^2\theta_W; \tag{3.28}$$

$$-2O_{ab}^{\prime R} = -U_{a2} U_{b2} + 2\delta_{ab} \cos^2\theta_W. \tag{3.29}$$

Process (3.17), $q\bar{q} \to \widetilde{\chi}_i^0 \widetilde{\chi}_j^0$, is given by eqn. (3.23) with the prefactors (e^4/N_C) and $1/(1+\delta_{ij})$, and the following coefficients $C_{\alpha\beta}$:

$$C_{LL} = \frac{L_q}{\sin 2\theta_W} \cdot \frac{N_{i4} N_{j4} - N_{i3} N_{j3}}{s - M_Z^2 + iM_Z \Gamma_Z} + \frac{\widetilde{L}_{qi} \widetilde{L}_{qj}}{u - M_{\tilde{q}_L}^2}; \tag{3.30}$$

$$C_{LR} = \frac{L_q}{\sin 2\theta_W} \cdot \frac{N_{i3}N_{j3} - N_{i4}N_{j4}}{s - M_Z^2 + iM_Z\Gamma_Z} - \frac{\widetilde{L}_{qi}\widetilde{L}_{qj}}{t - M_{\tilde{q}_L}^2}; \tag{3.31}$$

$$C_{RL} = \frac{R_q}{\sin 2\theta_W} \cdot \frac{N_{i4}N_{j4} - N_{i3}N_{j3}}{s - M_Z^2 + iM_Z\Gamma_Z} - \frac{\widetilde{R}_{qi}\widetilde{R}_{qj}}{t - M_{\tilde{q}_R}^2}; \tag{3.32}$$

$$C_{RR} = \frac{R_q}{\sin 2\theta_W} \cdot \frac{N_{i3}N_{j3} - N_{i4}N_{j4}}{s - M_Z^2 + iM_Z\Gamma_Z} + \frac{\widetilde{R}_{qi}\widetilde{R}_{qj}}{u - M_{\tilde{q}_R}^2}. \tag{3.33}$$

Process (3.18), $u\bar{d} \to \widetilde{\chi}_a^+\widetilde{\chi}_i^0$, is given by eqn. (3.23). The prefactors are given by (e^4/N_C) and $(|V_{\rm CKM}|^2/2\sin^2\theta_W)$ where $V_{\rm CKM}$ refers to the relevant term of the Cabbibo–Kobayashi–Maskawa (CKM) matrix, and the coefficients $C_{\alpha\beta}$ are:

$$C_{LL} = \frac{1}{\sin\theta_W} \cdot \frac{N_{i2}V_{a1} - N_{i4}V_{a2}/\sqrt{2}}{s - M_W^2 + iM_W\Gamma_W} + \frac{V_{a1}\widetilde{L}_{ui}}{u - M_{\tilde{u}_L}^2}; \tag{3.34}$$

$$C_{LR} = \frac{1}{\sin\theta_W} \cdot \frac{N_{i2}U_{a1} + N_{i3}U_{a2}/\sqrt{2}}{s - M_W^2 + iM_W\Gamma_W} - \frac{U_{a1}\widetilde{L}_{di}}{t - M_{\tilde{d}_L}^2}; \tag{3.35}$$

$$C_{RL} = 0; \tag{3.36}$$

$$C_{RR} = 0. \tag{3.37}$$

Process (3.19), $q\bar{q} \to \widetilde{\chi}_i^0\widetilde{g}$, is given by eqn. (3.23) with the prefactor $(e^2g_s^2C_F/N_C)$ and the following coefficients $C_{\alpha\beta}$:

$$C_{LL} = \frac{\widetilde{L}_{qi}}{u - M_{\tilde{q}_L}^2}; \tag{3.38}$$

$$C_{LR} = -\frac{\widetilde{L}_{qi}}{t - M_{\tilde{q}_L}^2}; \tag{3.39}$$

$$C_{RL} = \frac{\widetilde{R}_{qi}}{t - M_{\tilde{q}_R}^2}; \tag{3.40}$$

$$C_{RR} = -\frac{\widetilde{R}_{qi}}{u - M_{\tilde{q}_R}^2}. \tag{3.41}$$

Process (3.20), $gq \to \widetilde{\chi}_i^0\widetilde{q}$, is given by eqn. (3.25), with $g_\alpha = e(\delta_{\alpha L}\widetilde{L}_{qi} - \delta_{\alpha R}\widetilde{R}_{qi})$.

Process (3.21), $u\bar{d} \to \widetilde{\chi}_a^+\widetilde{g}$, is given by eqn. (3.23). The prefactors are $(e^2g_s^2C_F/N_C)$ and $(|V_{\rm CKM}|^2/2\sin^2\theta_W)$, and the coefficients $C_{\alpha\beta}$ are:

$$C_{LL} = \frac{V_{a1}}{u - M_{\tilde{u}_L}^2}; \tag{3.42}$$

$$C_{LR} = -\frac{U_{a1}}{t - M_{\tilde{d}_L}^2}; \tag{3.43}$$

$$C_{RL} = 0; \tag{3.44}$$

$$C_{RR} = 0. \tag{3.45}$$

Process (3.22), $gq \to \widetilde{\chi}_a^\pm\widetilde{q}'_L$, is given by eqn. (3.25), with $g_L = e(\delta_{qu}U_{a1} + \delta_{qd}V_{a1})$.

Processes (3.18), (3.21) and (3.22) involve flavour-changing CKM matrix contributions. We assume that the CKM matrix is universal and applicable also to sfermion interactions.

Process (3.16) involves two flavour-changing vertex contributions and, in principle, we should sum over the internal squark flavours and external quark pairs. However, we believe that to a good approximation this process can be treated correctly by substituting an identity matrix for the CKM matrix and so this is done in HERWIG.

3.5 Neutral Higgs production

For hadronic collisions, the following five classes of production subprocesses are implemented:

(a) gluon-gluon fusion via loops of heavy quarks $Q = b$ and t and squarks $\widetilde{Q} = \widetilde{b}$ and $\widetilde{t}$,

(b) $W^\pm W^\mp$, $Z^0 Z^0$ fusion,

(c) associated production with $W^\pm$, Z^0 bosons,

(d) associated production with pairs of heavy quarks $Q\bar{Q}$,

(e) associated production with pairs of heavy squarks $\widetilde{Q}\widetilde{Q}^*$.

The scalar Higgs bosons h^0 and H^0 can be produced by all five subprocesses (a)–(e), whereas the pseudoscalar A^0 is only produced via the first and the last two, as the A^0VV vertex, with $V = W^\pm, Z^0$, is absent at tree level. Higgs bosons are also produced in decays of heavier sparticles.

For leptonic collisions, the following are implemented:

(f) $W^\pm W^\mp$, $Z^0 Z^0$ fusion,

(g) associated production with Z^0,

(h) $h^0 A^0$ and $H^0 A^0$ production.

Of these, (f) and (g) are expressed by formulae that are obtained by changing the Z^0 boson couplings in the corresponding hadronic subprocesses. In addition, for (g), the colour factor $1/N_C$ is replaced by 1.

In the following, let us denote the neutral Higgs bosons collectively by Φ. We define the following notation for invariant mass squared. $M^2_{[ab]}$ is the invariant mass squared of the particle pair $[ab]$, $M^2_{[a_i b_i]} = M^2_{[12]} = (p_a + p_b)^2 = s$, $M^2_{[a_f b_f]} = (p_a + p_b)^2$ and $M^2_{[a_i b_f]} = (p_a - p_b)^2$, where the subscripts i and f indicate particles in the initial and final states, respectively. $M^2_{[ab]}$ is negative only in the last of these three cases.

For the gluon-fusion subprocess class (a), the matrix elements are cast in the form seen in Ref. [15]:

$$\overline{|\mathcal{M}|^2}(gg \to h^0, H^0) = \frac{\alpha_{\rm EM}\alpha_{\rm S}^2 M^4_{h,H}}{72\pi \sin^2\theta_W (N_C^2 - 1) M_W^2} \left| \sum_f g_f^{h,H} A_f^{h,H}(\tau_f) + \sum_{\widetilde{f}} g_{\widetilde{f}}^{h,H} A_{\widetilde{f}}^{h,H}(\tau_{\widetilde{f}}) \right|^2 , \tag{3.46}$$

$$\overline{|\mathcal{M}|^2}(gg \to A^0) = \frac{\alpha_{\rm EM}\alpha_{\rm S}^2 M_A^4}{72\pi \sin^2\theta_W (N_C^2-1) M_W^2} \left| \sum_f g_f^A A_f^A(\tau_f) \right|^2 , \tag{3.47}$$

for scalar and pseudoscalar production, respectively. The couplings g_f and $g_{\tilde{f}}$ are those appearing in Tabs. 4 and 5, respectively. We have defined $\tau = 4m^2/M^2$, where M is the mass of the Higgs boson concerned and m is the mass of the particle inside the loop. The functions A are given by:

$$A_Q^{h,H}(\tau) = \frac{3}{2}\tau[1+(1-\tau)f(\tau)], \tag{3.48}$$

$$A_{\tilde{Q}}^{h,H}(\tau) = -\frac{3}{4}[1-\tau f(\tau)], \tag{3.49}$$

for the scalars and

$$A_Q^A(\tau) = \tau f(\tau), \tag{3.50}$$

for the pseudoscalar, with the function $f(\tau)$ given by

$$f(\tau) = \begin{cases} \arcsin^2 \dfrac{1}{\sqrt{\tau}} & \tau \geq 1, \\ -\dfrac{1}{4}\left[\log \dfrac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \tau < 1. \end{cases} \tag{3.51}$$

Higgs boson		$g_{u,\nu}$	$g_{d,\ell}$	$g_{W^\pm,Z^0}$
SM	ϕ	1	1	1
MSSM	h^0	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H^0	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A^0	$1/\tan\beta$	$\tan\beta$	0

Table 4: Higgs coupling coefficients in the MSSM to isospin $+1/2$ and $-1/2$ fermions and to the gauge bosons $W^\pm, Z^0$, as first appearing in eqns. (3.46) and (3.47).

Higgs boson		$g_{\tilde{f}_i}$
SM	ϕ	0
MSSM	h^0	$m_{\tilde{f}_i}^{-2}\Big[m_f^2 g_f^h + M_Z^2 \cos\theta_W \sin\theta_W (L_f Q_{Li}^2 - R_f Q_{Ri}^2)\sin(\alpha+\beta)$ $+(\mu m_f(\delta_{fu}\frac{\sin\alpha}{\sin\beta} - \delta_{fd}\frac{\cos\alpha}{\cos\beta}) + A_f m_f g_f^h)Q_{Li}Q_{Ri}\Big]$
	H^0	$m_{\tilde{f}_i}^{-2}\Big[m_f^2 g_f^H + M_Z^2 \cos\theta_W \sin\theta_W (R_f Q_{Ri}^2 - L_f Q_{Li}^2)\cos(\alpha+\beta)$ $+(\mu m_f(-\delta_{fu}\frac{\cos\alpha}{\sin\beta} - \delta_{fd}\frac{\sin\alpha}{\cos\beta}) + A_f m_f g_f^H)Q_{Li}Q_{Ri}\Big]$
	A^0	0

Table 5: MSSM Higgs coupling coefficients to sfermions $\tilde{f} = \tilde{q}, \tilde{\ell}$, as first appearing in eqn. (3.46). For the leptonic case, the mixing matrices Q_{ij} should be replaced by the corresponding slepton mixing matrices L_{ij}.

For process (b), the matrix element for the $W^{\pm}W^{\mp}$ fusion process is given by:

$$\overline{|\mathcal{M}|^2}(q_1 q_2' \to q_3'' q_4''' \Phi_5) = (g^{\Phi}_{W^\pm})^2 \left(\frac{e^2}{2\sin^2\theta_W}\right)^3 (2M_W^2) \times \frac{|V_{\rm CKM}[qq'']|^2 |V_{\rm CKM}[q'q''']|^2 M^2_{[12]} M^2_{[34]}}{(M^2_{[13]} - M_W^2)(M^2_{[24]} - M_W^2)}, \quad (3.52)$$

whereas for Z^0Z^0 fusion we have:

$$\overline{|\mathcal{M}|^2}(q_1 q_2' \to q_3 q_4' \Phi_5) = (g^{\Phi}_{Z^0})^2 \left(\frac{e^2}{4\sin^2\theta_W \cos^2\theta_W}\right)^3 (4M_Z^2) \times \frac{(L_q^2 L_{q'}^2 + R_q^2 R_{q'}^2) M^2_{[12]} M^2_{[34]} + (L_q^2 R_{q'}^2 + R_q^2 L_{q'}^2) M^2_{[14]} M^2_{[23]}}{(M^2_{[13]} - M_Z^2)(M^2_{[24]} - M_Z^2)}. \quad (3.53)$$

The coupling coefficients g^{Φ}_V are as given in Tab. 4. The formulae are identical to those for the case of leptonic collisions, with the replacement of the CKM matrix elements by unity for the $W^{\pm}W^{\mp}$ fusion subprocess and the substitution of the couplings L, R with the appropriate leptonic couplings L_e, R_e for the Z^0Z^0 fusion subprocess.

For process (c), the matrix elements are given by:

$$\overline{|\mathcal{M}|^2}(u_1 \bar{d}_2 \to W_3^+ \Phi_4) = (g^{\Phi}_{W^\pm})^2 \frac{1}{N_C} \left(\frac{e^2}{2\sin^2\theta_W}\right)^2 \left(\frac{1}{2}\right) |V_{\rm CKM}[ud]|^2 \frac{s p_T^2 + 2M_W^2 s}{(s - M_W^2)^2 + M_W^2 \Gamma_W^2}, \quad (3.54)$$

$$\overline{|\mathcal{M}|^2}(q_1 \bar{q}_2 \to Z_3 \Phi_4) = (g^{\Phi}_{Z^0})^2 \frac{1}{N_C} \left(\frac{e^2}{4\sin^2\theta_W \cos^2\theta_W}\right)^2 (L_q^2 + R_q^2) \frac{s p_T^2 + 2M_Z^2 s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}. \quad (3.55)$$

The coupling coefficients g^{Φ}_V are again as given in Tab. 4. The above formula, for the Z^0 mediated subprocess, is immediately applicable to the leptonic case with the substitution of the colour factor 1 to replace $1/N_C$. The couplings L_q, R_q are replaced by L_e, R_e in this case.

For process (d), the matrix element for the $q\bar{q}$ initiated subprocess is given by:

$$\overline{|\mathcal{M}|^2}(q_1 \bar{q}_2 \to Q_3 \bar{Q}_4 \Phi_5) = \frac{e^2 g_s^4 (g^{\Phi}_Q)^2}{2\sin^2\theta_W M_W^2 s} \left(\frac{C_F}{N_C}\right) \mathcal{G} \times \left[\frac{1}{2\mathcal{G}_5} - \frac{M^2 s \mathcal{G}}{2} + M^2(\mathcal{G}_3 p_{T3}^2 + \mathcal{G}_4 p_{T4}^2) - \mathcal{G}_5 (s + M^2) p_{T5}^2\right]. \quad (3.56)$$

Here $M^2 = M_\Phi^2$ for A^0 and $M = M_\Phi^2 - 4m_Q^2$ for h^0, H^0. The p_T's are the transverse momenta of the final state particles. These can be expressed as $p_{T_i}^2 = 4p_1 \cdot p_i \; p_2 \cdot p_i / s - m_i^2$ (where $i = 3, 4, 5$) as the incoming particles are considered massless. The propagator factors are defined by:

$$\mathcal{G}_3 = 1/(M^2_{[35]} - m_Q^2), \quad (3.57)$$

$$\mathcal{G}_4 = 1/(M^2_{[45]} - m_Q^2), \quad (3.58)$$

$$\mathcal{G} = \mathcal{G}_3 + \mathcal{G}_4, \quad (3.59)$$

$$\mathcal{G}_5 = \mathcal{G}_3 \mathcal{G}_4 / \mathcal{G}. \quad (3.60)$$

For the case $Q = b$, the gg initiated subprocess is evaluated with $2 \to 1$, $2 \to 2$ and $2 \to 3$ matrix elements. In order to avoid double counting, it is recommended that the user chooses only one of these, rather than sum over them, according to their need. For the $2 \to 1$ subprocess the matrix element is given by:

$$\overline{|\mathcal{M}|^2}(b_1 \bar{b}_2 \to \Phi_5) = \frac{e^2 M_H^2 m_b^2 (g_b^\Phi)^2}{8 \sin^2\theta_W M_W^2 N_C}. \tag{3.61}$$

The $2 \to 2$ matrix element is given by:

$$\begin{aligned}\overline{|\mathcal{M}|^2}(g_1 b_2 \to b_3 \Phi_4) &= \left(\frac{g_s^2}{2N_C}\right)\left(\frac{e^2}{2\sin\theta_W^2}\right)\left(\frac{m_b^2 (g_b^\Phi)^2}{2M_W^2}\right) \\ &\times \left(\frac{-u_4^2}{st_3}\right)\left[1 + 2\frac{m_4^2 - m_3^2}{u_4}\left(1 + \frac{m_3^2}{t_3} + \frac{m_4^2}{u_4}\right)\right].\end{aligned} \tag{3.62}$$

The inconsistency in treating the initial state bottom quark as being massless and the final state bottom quark as being massive is unavoidable. The subprocess implementation being intended for high p_T single b-jet events only, the ambiguity in the treatment of the bottom quark mass in the hard scattering matrix element is negligible.

We evaluated the $2 \to 3$ matrix element for $gg \to Q\bar{Q}\Phi$ using helicity amplitudes as follows. We first wrote down the amplitude functions using the corresponding Feynman rules. We then adopted the convention of Ref. [16] for the heavy quark helicity eigenstates, and adopted the circular polarization for the gluons. This allowed some simplification in the computation which, together with the factorization of amplitudes, made the code very much faster than any other code which is publically available.

For process (e), the matrix element for the $q\bar{q}$ initated subprocess is given by:

$$\begin{aligned}\overline{|\mathcal{M}|^2}(q_1 \bar{q}_2 \to \widetilde{Q}_3 \widetilde{Q}_4^* \Phi_5) &= \left(\frac{g_S^4 e^2 {g_{\widetilde{Q}}^\Phi}^2 M_{\widetilde{Q}}^2}{\sin^2\theta_W M_W^2 s}\right)\left(\frac{C_F}{N_C}\right) \\ &\times \left[|\mathcal{G}_3|^2 p_{T3}^2 + |\mathcal{G}_4|^2 p_{T4}^2 - 2\mathrm{Re}\left(\mathcal{G}_3^* \mathcal{G}_4\right) p_{T3} \cdot p_{T4}\right].\end{aligned} \tag{3.63}$$

The propagator factors are given by:

$$\mathcal{G}_3 = 1/\left((p_4 + p_5)^2 - m_3^2 + i m_3 \Gamma_3\right), \tag{3.64}$$

$$\mathcal{G}_4 = 1/\left((p_3 + p_5)^2 - m_4^2 + i m_4 \Gamma_4\right). \tag{3.65}$$

The dot product of two transverse momenta is two dimensional, *i.e.* $p_{T3} \cdot p_{T4} = p_{x3} p_{x4} + p_{y3} p_{y4}$.

The gg initiated subprocess is again calculated using helicity amplitudes in an optimized basis. This time the two orthogonal polarisation vectors for the initial state gluons are chosen to be in the x and y directions, yielding a relatively compact expression:

$$\begin{aligned}\overline{|\mathcal{M}|^2}(g_1 g_2 \to \widetilde{Q}_3 \widetilde{Q}_4^* \Phi_5) &= \left(\frac{g_S^4 e^2 {g_{\widetilde{Q}}^\Phi}^2 M_{\widetilde{Q}}^2 N_C}{4\sin^2\theta_W M_W^2 (N_C^2 - 1)}\right) \\ &\times \sum_{i=x,y}\sum_{j=x,y}\left[|\mathcal{A}_t(i,j)|^2 + |\mathcal{A}_u(i,j)|^2 - \frac{1}{N_C^2}|\mathcal{A}_t(i,j) + \mathcal{A}_u(i,j)|^2\right].\end{aligned} \tag{3.66}$$

The colour flow components are distributed using the method of Ref. [13], namely, the $(1/N_C^2)$ term is distributed amongst the t-channel and u-channel colour flow components by their ratio. The two amplitude functions $\mathcal{A}_t$ and $\mathcal{A}_u$ are given by:

$$\begin{aligned}\mathcal{A}_t(i,j) &= 2\left(p_{i3}p_{j4}\mathcal{G}_{24}\mathcal{G}_{13} + p_{i3}p_{j3}\mathcal{G}_{13}\mathcal{G}_{45} + p_{i4}p_{j4}\mathcal{G}_{24}\mathcal{G}_{35}\right)\\ &\quad - \frac{\delta_{ij}}{\sqrt{s}}\left(p_{z3}\mathcal{G}_{45} - p_{z4}\mathcal{G}_{35}\right) - \frac{\delta_{ij}}{2}\left(\mathcal{G}_{35} + \mathcal{G}_{45}\right), \qquad (3.67)\\ \mathcal{A}_u(i,j) &= 2\left(p_{i4}p_{j3}\mathcal{G}_{14}\mathcal{G}_{23} + p_{i3}p_{j3}\mathcal{G}_{23}\mathcal{G}_{45} + p_{i4}p_{j4}\mathcal{G}_{14}\mathcal{G}_{35}\right)\\ &\quad + \frac{\delta_{ij}}{\sqrt{s}}\left(p_{z3}\mathcal{G}_{45} - p_{z4}\mathcal{G}_{35}\right) - \frac{\delta_{ij}}{2}\left(\mathcal{G}_{35} + \mathcal{G}_{45}\right). \qquad (3.68)\end{aligned}$$

The propagator factors are given by:

$$\begin{aligned}\mathcal{G}_{13} &= 1/\left((p_1+p_3)^2 - m_3^2\right), \qquad (3.69)\\ \mathcal{G}_{23} &= 1/\left((p_2+p_3)^2 - m_3^2\right), \qquad (3.70)\\ \mathcal{G}_{14} &= 1/\left((p_1+p_4)^2 - m_4^2\right), \qquad (3.71)\\ \mathcal{G}_{24} &= 1/\left((p_2+p_4)^2 - m_4^2\right), \qquad (3.72)\\ \mathcal{G}_{35} &= 1/\left((p_3+p_5)^2 - m_4^2 + im_4\overline{\Gamma}_4\right), \qquad (3.73)\\ \mathcal{G}_{45} &= 1/\left((p_4+p_5)^2 - m_3^2 + im_3\overline{\Gamma}_3\right). \qquad (3.74)\end{aligned}$$

For process (h), the matrix element is given by:

$$\overline{|\mathcal{M}|^2}(e_1^- e_2^+ \to h_3^0 A_4^0, H_3^0 A_4^0) = e^4 s p_T^2 \cdot \frac{L_\nu^2\left(L_e^2+R_e^2\right)\left(\cos^2(\beta-\alpha), \sin^2(\beta-\alpha)\right)}{(s-M_Z^2)^2 + M_Z^2\overline{\Gamma}_Z^2}. \qquad (3.75)$$

We have used the same notation as in eqn. (3.10). The coupling factor $\cos^2(\beta-\alpha)$ is for h^0A^0 production whereas $\sin^2(\beta-\alpha)$ is for H^0A^0 production.

3.6 Charged Higgs production

For hadronic collisions, the following production modes are implemented:

$$\begin{aligned}gb &\to tH^- + c.c.; \qquad (3.76)\\ gg &\to t\bar{b}H^- + c.c.; \qquad (3.77)\\ q\bar{q} &\to t\bar{b}H^- + c.c.; \qquad (3.78)\\ gg &\to \tilde{t}\tilde{b}^*H^- + c.c.; \qquad (3.79)\\ q\bar{q} &\to \tilde{t}\tilde{b}^*H^- + c.c.; \qquad (3.80)\\ q\bar{q} &\to H^+H^-; \qquad (3.81)\\ b\bar{b} &\to W^\pm H^\mp; \qquad (3.82)\\ qb &\to q'bH^+ + c.c. \qquad (3.83)\end{aligned}$$

Charged Higgs bosons are also produced through the decay of heavier objects including gauginos and third generation sfermions. Perhaps most notably, an important potential production mode for charged Higgs bosons is through the decay of top quarks and, as with other decay processes, this is achieved by specifying this decay mode in the input file. The decay is carried out using the three-body matrix element. This matrix element is not included in the spin correlation algorithm [17] which is the default treatment from version 6.4. If this decay is required, the matrix element code `NME=200` should be used and spin correlations and supersymmetric three body decays should be switched off, *i.e.* `SYSPIN=.FALSE.` and `THREEB=.FALSE.`.

When simulating processes (3.77), (3.78) and (3.83) one should bear in mind that there is potentially a large contribution from top quark decay when this decay mode is available. In this case the user should choose to simulate either the top quark production or the relevant three-body process in order to avoid double counting. In the limit in which the bottom quark is generated by the collinear splitting $g \to b\bar{b}$, process (3.77) is described by the two body process (3.76). This contribution becomes significant if the mass of the charged Higgs boson is comparable or greater than the top quark mass. Again, the user should choose to simulate either of the two processes in order to avoid double counting.

For leptonic collisions, in addition to the top quark decay, charged Higgs bosons can be produced in a process analogous to process (3.81):

$$e^- e^+ \to H^+ H^- . \tag{3.84}$$

The matrix element is obtained immediately from the hadronic case by changing the appropriate photon and Z^0 couplings and removing the colour factor $1/N_C$.

Process (3.76) is given by:

$$\begin{aligned}\overline{|\mathcal{M}|^2}(g_1 b_2 \to t_3 H_4^-) = & \left(\frac{g_s^2}{2N_C}\right)\left(\frac{e^2}{2\sin\theta_W^2}\right)|V_{\rm CKM}[tb]|^2\left(\frac{m_b^2\tan^2\beta + m_t^2\cot^2\beta}{2M_W^2}\right) \\ & \times\left(\frac{-u_4^2}{st_3}\right)\left[1+2\frac{m_4^2-m_3^2}{u_4}\left(1+\frac{m_3^2}{t_3}+\frac{m_4^2}{u_4}\right)\right].\end{aligned} \tag{3.85}$$

Process (3.77) is evaluated using helicity amplitudes in an optimized basis, similarly to the evaluation of the neutral Higgs boson production process $gg \to Q\bar{Q}\Phi$ in Sect. 3.5.

Process (3.78) is evaluated using a more general formula for $q_1\bar{q}_2 \to Q_3\bar{Q}_4\Phi$ which is given by:

$$\begin{aligned}\overline{|\mathcal{M}|^2}(q_1\bar{q}_2 \to Q_3\bar{Q}_4\Phi) = & \left(\frac{g_s^4 C_F}{2N_C}\right)\frac{2\phi_H^2}{s}\Bigg[\left(p_{T_3}^2+p_{T_4}^2-p_{T_5}^2\right)+4p_0\cdot p_3\; p_0\cdot p_4|\mathcal{G}|^2 \\ & + 2\left(p_3\cdot p_4 - \lambda m_3 m_4\right)\left[-s|\mathcal{G}|^2 + 2{\rm Re}\left(\mathcal{G}\mathcal{G}_3^* p_{T_4}^2 + \mathcal{G}\mathcal{G}_4^* p_{T_3}^2 - \mathcal{G}_3\mathcal{G}_4^* p_{T_5}^2\right)\right] \\ & - 4{\rm Re}\left[\mathcal{G}\mathcal{G}_3^* p_{T_4}^2 + \mathcal{G}\mathcal{G}_4^* p_{T_3}^2 + \mathcal{G}\left(\mathcal{G}_3^* p_0\cdot p_4 + \mathcal{G}_4^* p_0\cdot p_3\right)\frac{p_{T_3}^2+p_{T_4}^2-p_{T_5}^2}{2}\right] \\ & - 4{\rm Im}\left[\mathcal{G}\mathcal{G}_3^* p_{z_4} - \mathcal{G}\mathcal{G}_4^* p_{z_3}\right]\lambda'\sqrt{s}\left(p_{x_4}p_{y_3} - p_{x_3}p_{y_4}\right)\Bigg],\end{aligned} \tag{3.86}$$

with $p_0 = p_1 + p_2$. Denoting the $Q_3\bar{Q}_4\Phi$ vertex as $(g_{\rm S} + g_{\rm P}\gamma_5)$, the various coupling coefficients appearing above are defined by:

$$\phi_H^2 = g_{\rm S}^2 + g_{\rm P}^2, \tag{3.87}$$

$$\lambda = \frac{g_{\rm S}^2 - g_{\rm P}^2}{\phi_H^2}, \tag{3.88}$$

$$\lambda' = \frac{2 g_{\rm S} g_{\rm P}}{\phi_H^2}. \tag{3.89}$$

Process (3.79) is evaluated using helicity amplitudes. The expressions are the same as eqns. (3.66)–(3.74) but the coupling coefficient needs to be changed. Let us consider the vertex involving an incoming H^+ and outgoing $\tilde{t}_i \tilde{b}_j^*$. We have the following replacement:

$$\frac{g_{\tilde{Q}} M_{\tilde{Q}}^2}{M_W^2} \to \frac{1}{\sqrt{2}} \left[\left(\sin 2\beta - \frac{m_b^2 \tan\beta + m_t^2 \cot\beta}{M_W^2} \right) Q_{Li}^t Q_{Lj}^b - \frac{m_t m_b}{M_W^2} (\tan\beta + \cot\beta) Q_{Ri}^t Q_{Rj}^b \right.$$
$$\left. - \frac{m_b}{M_W^2} (-\mu + A_b \tan\beta) Q_{Li}^t Q_{Rj}^b - \frac{m_t}{M_W^2} (-\mu + A_t \cot\beta) Q_{Ri}^t Q_{Lj}^b \right]. \tag{3.90}$$

Process (3.81) is given by:

$$\overline{|\mathcal{M}|^2} = \frac{e^4 s p_T^2}{N_C} \times \left[\left| \frac{Q_q Q_{H^+}}{s} + \frac{L_q L_{H^+}}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2 + \left| \frac{Q_q Q_{H^+}}{s} + \frac{R_q L_{H^+}}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2 \right]. \tag{3.91}$$

We have used the same notation as in eqn. (3.10). $Q_{H^+} = +1$, and L_{H^+} is given by eqn. (3.6) with $I_f^3 = +1/2$. For the leptonic case the subscripts q should be replaced with e, and the colour factor $1/N_C$ should be replaced by 1.

The matrix element for process (3.82) is as given in [18].

Process (3.83) is given in [19]. We use a more compact expression which sets the kinematic bottom quark mass to zero. For the process $b_1 U_2 \to b_3 D_4 H_5^+$ we have:

$$\overline{|\mathcal{M}|^2}(b_1 U_2 \to b_3 D_4 H_5^+) = \frac{1}{2} \left(\frac{e^2}{2 x_W} \right)^3 |V_{\rm CKM}[UD]|^2 \frac{1}{(M_{[24]}^2 - M_W^2)^2} \times$$
$$\times \left[|V_{\rm CKM}[tb]|^2 \overline{|A_t|^2} + \overline{|A_\Phi|^2} + {\rm Re}(V_{\rm CKM}[tb] \cdot \overline{2 A_\Phi^* A_t}) \right]; \tag{3.92}$$

$$\overline{|A_t|^2} = 2 M_{[12]}^2 |\mathcal{G}_t|^2 \left[\left(\frac{m_b t_\beta}{m_W} \right)^2 \frac{[3545]}{2} + \left(\frac{m_t t_\beta^{-1}}{m_W} \right)^2 m_t^2 M_{[34]}^2 \right]; \tag{3.93}$$

$$\overline{|A_\Phi|^2} = (\mathcal{G}_{h,H}^2 + \mathcal{G}_A^2)(-M_{[13]}^2) \frac{[2545]}{2}; \tag{3.94}$$

$${\rm Re}(\overline{2 A_\Phi^* A_t}) = \frac{m_b t_\beta}{M_W} (\mathcal{G}_{h,H} + \mathcal{G}_A) \Big[2{\rm Re}(\mathcal{G}_t)(M_{[45]}^2 - M_{H^\pm}^2)(M_{[12]}^2)(-M_{[13]}^2) -$$
$$- {\rm Re}(\mathcal{G}_t) M_{[45]}^2 [1243] + {\rm Im}(\mathcal{G}_t) M_{[45]}^2 \cdot 4 \det(1234) \Big]. \tag{3.95}$$

Here we have defined $[abcd] = 4(p_a \cdot p_b \; p_c \cdot p_d + p_a \cdot p_d \; p_b \cdot p_c - p_a \cdot p_c \; p_b \cdot p_d)$ and $\det(abcd) = \det(p_a^\mu, p_b^\nu, p_c^\rho, p_d^\sigma) = \sum_{\mu\nu\rho\sigma} \epsilon_{\mu\nu\rho\sigma} p_a^\mu p_b^\nu p_c^\rho p_d^\sigma$ where $\epsilon_{0123} = +1$.

The propagators are defined by:

$$\begin{aligned}
\mathcal{G}_t &= \frac{1}{M^2_{[35]} - m_t^2 + i m_t \overline{\Gamma}_t} = \frac{M^2_{[35]} - m_t^2 - i m_t \overline{\Gamma}_t}{(M^2_{[35]} - m_t^2)^2 + m_t^2 \overline{\Gamma}_t^2}; \\
\mathcal{G}_{h,H} &= \frac{m_b \sin\alpha\cos(\beta-\alpha)/M_W\cos\beta}{M^2_{[13]} - M_h^2} + \frac{m_b \cos\alpha\sin(\beta-\alpha)/M_W\cos\beta}{M^2_{[13]} - M_H^2}; \\
\mathcal{G}_A &= \frac{m_b\tan\beta/M_W}{M^2_{[13]} - M_A^2}.
\end{aligned}$$

Processes with antiquarks in the initial state can be obtained by exchanging the subscripts $(1 \leftrightarrow 3)$ for the subprocess $\bar{b}U \to \bar{b}DH^+$, $(2 \leftrightarrow 4)$ for the subprocess $b\bar{D} \to b\bar{U}H^+$, and both for the subprocess $\bar{b}\bar{D} \to \bar{b}\bar{U}H^+$. Note that the definitions of the invariant masses squared are such that these imply momentum substitutions of the form $(p_1 \leftrightarrow -p_3)$.

3.7 R-parity violating processes

The cross sections for the implemented R-parity violating hadron–hadron processes are presented in Ref. [20]. In hadron–hadron collisions only those $2 \to 2$ processes that can occur via a resonant s-channel sparticle exchange are included, *i.e.* processes that can only occur via a t-channel diagram or for which the s-channel resonance is not kinematically accessible are neglected.

In e^+e^- collisions we include all the possible $2 \to 2$ single sparticle production mechanisms, even if there is no s-channel resonance, and a limited set of $2 \to 2$ processes where the final state contains two SM particles. As with the hadron–hadron processes, there are four types of process: single gaugino production; single slepton production in association with a gauge boson; single slepton production in association with a Higgs boson; SM particle production.

3.7.1 Single gaugino production

In e^+e^- collisions, there are two possible production mechanisms for a gaugino and a lepton, either $e^+e^- \to \tilde{\chi}^0\nu$ or $e^+e^- \to \tilde{\chi}^+\ell^-$. The matrix elements for these processes have been calculated previously in Refs. [21, 22, 23]. We have recalculated these matrix elements and the results are in agreement with [23]. We also agree with Refs. [21, 22], which do not include sfermion mixing, if we set our mixing to zero. All matrix elements are averaged over the spins of the incoming leptons as before.

$$\begin{aligned}
&\overline{|\mathcal{M}|^2}(e_i^- e_i^+ \to \nu_j \tilde{\chi}^0_l) = \\
&\frac{\lambda_{iji}^2}{2}\left[|a(\tilde{\nu}_j^*)|^2 s R(\tilde{\nu}_j)\left(s - M^2_{\tilde{\chi}^0_l}\right) + \frac{1}{(u - M^2_{\tilde{\ell}_{iL}})^2}\left[|a(\tilde{\ell}^*_{i1})|^2 + |b(\tilde{\ell}^*_{i1})|^2\right] u \left(u - M^2_{\tilde{\chi}^0_l}\right)\right. \\
&+\frac{1}{(t - M^2_{\tilde{\ell}_{iR}})^2}\left[|a(\tilde{\ell}_{i2})|^2 + |b(\tilde{\ell}_{i2})|^2\right] t\left(t - M^2_{\tilde{\chi}^0_l}\right) \\
&+\frac{2}{(u - M^2_{\tilde{\ell}_{iL}})} a(\tilde{\nu}_j^*) a(\tilde{\ell}^*_{i1}) R(\tilde{\nu}_j)\left(s - m^2_{\tilde{\nu}_j}\right) su + \frac{2}{(u - M^2_{\tilde{\ell}_{iL}})(t - M^2_{\tilde{\ell}_{iR}})} a(\tilde{\ell}^*_{i1}) a(\tilde{\ell}_{i2}) ut
\end{aligned}$$

$$
+\frac{2}{(t-M_{\ell_{iR}}^2)}a(\widetilde{\nu}_j^*)a(\widetilde{\ell}_{i2})R(\widetilde{\nu}_j)\left(s-m_{\widetilde{\nu}_j}^2\right)st\Bigg], \tag{3.96}
$$

$$
\overline{|\mathcal{M}|^2}(e_i^- e_i^+ \to \ell_j^+ \widetilde{\chi}_l^-) =
$$

$$
\frac{g^2\lambda_{iji}^2}{4}\Bigg[R(\widetilde{\nu}_j)s\left[\left(|a(\widetilde{\nu}_j^*)|^2+|b(\widetilde{\nu}_j^*)|^2\right)\left(s-m_{\ell_j}^2-M_{\widetilde{\chi}_l^-}^2\right)-4a(\widetilde{\nu}_j^*)b(\widetilde{\nu}_j^*)m_{\ell_j}M_{\widetilde{\chi}_l^-}\right]
$$

$$
+\frac{1}{\left(u-M_{\widetilde{\nu}_i}^2\right)^2}\left(M_{\widetilde{\chi}_l^-}^2-u\right)\left(m_{\ell_j}^2-u\right)\left[|a(\widetilde{\nu}_i^*)|^2+|b(\widetilde{\nu}_i^*)|^2\right]
$$

$$
-\frac{2s}{\left(u-M_{\widetilde{\nu}_i}^2\right)}\left(s-M_{\widetilde{\nu}_j}^2\right)R(\widetilde{\nu}_j)a(\widetilde{\nu}_i^*)\left[a(\widetilde{\nu}_j^*)u+m_{\ell_j}M_{\widetilde{\chi}_l^-}b(\widetilde{\nu}_j^*)\right]\Bigg]. \tag{3.97}
$$

The couplings a, b of the sleptons to the gauginos are given in Ref. [20], and

$$
R(\tilde{a}) = \frac{1}{\left(s-M_{\tilde{a}}^2\right)^2+\overline{\Gamma}_{\tilde{a}}^2 M_{\tilde{a}}^2}. \tag{3.98}
$$

3.7.2 Slepton production in association with a gauge boson

There are a number of production processes for a slepton in association with a gauge boson. We have confirmed the cross section for these processes, which were first calculated in Ref. [23]. The sneutrino resonance is not usually accessible for these processes as the charged slepton–sneutrino mass difference is less than the $W^\pm$ mass. However, the resonance can be accessible for the third generation due to left–right mixing of the staus.

The matrix elements for these processes are given below, including left–right sfermion mixing:

$$
\overline{|\mathcal{M}|^2}(e_i^- e_i^+ \to \gamma\widetilde{\nu}_j) =
$$

$$
\frac{e^2\lambda_{iji}^2}{2}\left[\frac{u}{t}+\frac{t}{u}+\frac{2}{ut}\left(M_{\widetilde{\nu}_j}^2-u\right)\left(M_{\widetilde{\nu}_j}^2-t\right)\right], \tag{3.99}
$$

$$
\overline{|\mathcal{M}|^2}(e_i^- e_i^+ \to W^+\widetilde{\ell}_{j\alpha}) =
$$

$$
\frac{g^2\lambda_{iji}^2|L_{1\alpha}^j|^2}{2M_W^2}\Bigg[s^2p_{cm}^2R(\widetilde{\nu}_j)+\frac{1}{4u^2}\left[2M_W^2\left(ut-M_W^2M_{\widetilde{\ell}_{j\alpha}}^2\right)+u^2s\right]
$$

$$
-\frac{s}{2u}R(\widetilde{\nu}_j)\left(s-M_{\widetilde{\nu}_j}^2\right)\left[M_W^2\left(2M_{\widetilde{\ell}_{j\alpha}}^2-u\right)+u\left(s-M_{\widetilde{\ell}_{j\alpha}}^2\right)\right]\Bigg], \tag{3.100}
$$

$$
\overline{|\mathcal{M}|^2}(e_i^- e_i^+ \to Z^0\widetilde{\nu}_j) =
$$

$$
\frac{g^2\lambda_{iji}^2}{\cos^2\theta_W M_Z^2}\Bigg[|Z_{\nu_j}^{11}|^2s^2p_{\rm cm}^2R(\widetilde{\nu}_j)+\frac{Z_{e_R}^2}{t^2}\left[2M_Z^2\left(ut-M_{\widetilde{\nu}_j}^2M_Z^2\right)+st^2\right]
$$

$$
+\frac{Z_{e_L}^2}{u^2}\left[2M_Z^2\left(ut-M_{\widetilde{\nu}_j}^2M_Z^2\right)+su^2\right]+\frac{2Z_{e_L}Z_{e_R}}{ut}\left[2M_Z^2\left(M_{\widetilde{\nu}_j}^2-t\right)\left(M_{\widetilde{\nu}_j}^2-u\right)-sut\right]
$$

$$
-\frac{Z_{\widetilde{\nu}_j}^{11}Z_{e_R}}{t}sR(\widetilde{\nu}_j)\left(s-M_{\widetilde{\nu}_j}^2\right)\left[M_Z^2\left(2M_{\widetilde{\nu}_j}^2-t\right)+t\left(s-M_{\widetilde{\nu}_j}^2\right)\right]
$$

$$+\frac{Z^{11}_{\widetilde{\nu}_j} Z_{e_L}}{u} sR(\widetilde{\nu}_j) \left(s - M^2_{\widetilde{\nu}_j}\right) \left[M_Z^2 \left(2M^2_{\widetilde{\nu}_j} - u\right) + u\left(s - M^2_{\widetilde{\nu}_j}\right)\right]\Bigg], \tag{3.101}$$

where $p_{\rm cm}$ is the momentum of the final-state particles in the centre–of–mass frame. As we neglect the electron mass, the cross section for $e^-_i e^+_i \to \gamma \widetilde{\nu}_j$ is divergent as $u, t \to 0$, *i.e.* in the limit that the photon is collinear with the incoming lepton beams, and therefore a cut on the transverse momenta of the outgoing particles is used to regularize the divergence for this process. The couplings of the Z^0 to the leptons and sleptons are given in Ref. [20].

3.7.3 Slepton production in association with a Higgs boson

There are a number of processes in which a Higgs boson can be produced in association with a slepton. The cross sections for these processes tend to be small due to the small lepton masses and the inaccessibility of the sneutrino resonance as the charged slepton–sneutrino mass difference is smaller than the charged Higgs mass.

The matrix elements are given below

$$\overline{|\mathcal{M}|^2}(e^-_i e^+_i \to \widetilde{\ell}^*_{j\alpha} H^-) =$$
$$\frac{g^2\lambda^2_{iji}}{4}\left[|H^c_{\widetilde{\nu}\widetilde{\ell}_{j\alpha}}|^2 sR(\widetilde{\nu}_j) + \frac{4|L^{2j-1}_{1\alpha}|^2|E^c_j|^2}{u^2}\left(ut - M^2_{\widetilde{\ell}_{j\beta}} M^2_{H^-}\right)\right], \tag{3.102}$$

$$\overline{|\mathcal{M}|^2}(e^-_i e^+_i \to \widetilde{\nu}^*_j H^0_l) =$$
$$\frac{g^2\lambda^2_{iji}}{4}\left[|H^l_{\widetilde{\nu}_j\widetilde{\nu}_j}|^2 sR(\widetilde{\nu}_j) + \frac{|E^l_i|^2}{u^2}\left(ut - M^2_{\widetilde{\nu}_j} M^2_{H^0_l}\right) + \frac{|E^l_i|^2}{t^2}\left(ut - M^2_{\widetilde{\nu}_j} M^2_{H^0_l}\right)\right]. \tag{3.103}$$

The couplings H of the Higgs bosons to the sleptons are given in Ref. [20] and the couplings E^l_i of the leptons to the Higgs bosons can be obtained from the couplings given in Ref. [20] for the down-type quarks by replacing the quark mass with the lepton mass.

3.7.4 SM particle production by sparticle exchange

As with the hadron-hadron processes described in Ref. [20] we have only included $2 \to 2$ processes where two SM particles can be produced by a resonant s-channel exchange. Where this is possible we have included any t-channel sparticle exchanges, and the relevant interference terms. We have also included the relevant SM diagrams and the interference between the SM and the R-parity violating diagrams. We have generalized the results of Ref. [24], giving

$$\overline{|\mathcal{M}|^2}(e^-_i e^+_i \to f_k \bar{f}_l) =$$
$$\frac{e^4 s^2 C_{\rm fac}}{4}\Big\{(1+\cos\theta)^2\left[|f^s_{LR}|^2 + |f^s_{RL}|^2 + |f^t_{LR}|^2 + |f^t_{RL}|^2 + 2{\rm Re}(f^{s*}_{LR} f^t_{LR}) + 2{\rm Re}(f^{s*}_{RL} f^t_{RL})\right]$$
$$+(1-\cos\theta)^2\left[|f^s_{LL}|^2 + |f^s_{RR}|^2\right] + 4\left[|f^t_{LL}|^2 + |f^t_{RR}|^2\right]\Big\}, \tag{3.104}$$

where the coefficients are given by

$$f^s_{LL} = \delta_{lk}\left\{\frac{1}{s}+\frac{L_eR_e}{s-M_Z^2+i\Gamma_Z M_Z}\right\}+\sum_{j=1,3}\frac{\lambda_{jik}\lambda_{jil}}{2e^2\left(t-M^2_{\tilde{\nu}_j}\right)}, \tag{3.105}$$

$$f^s_{LR} = \delta_{lk}\left\{\frac{1}{s}+\frac{L_e^2}{s-M_Z^2+i\Gamma_Z M_Z}\right\}, \tag{3.106}$$

$$f^s_{RL} = \delta_{lk}\left\{\frac{1}{s}+\frac{R_e^2}{s-M_Z^2+i\Gamma_Z M_Z}\right\}, \tag{3.107}$$

$$f^s_{RR} = \delta_{lk}\left\{\frac{1}{s}+\frac{L_eR_e}{s-M_Z^2+i\Gamma_Z M_Z}\right\}+\sum_{j=1,3}\frac{\lambda_{jki}\lambda_{jli}}{2e^2\left(t-M^2_{\tilde{\nu}_j}\right)}, \tag{3.108}$$

$$f^t_{LL} = \delta_{il}\delta_{lk}\left\{\frac{1}{t}+\frac{L_eR_e}{t-M_Z^2}\right\}+\sum_{j=1,3}\frac{\lambda_{jii}\lambda_{jkl}}{2e^2\left(s-M^2_{\tilde{\nu}_j}+i\Gamma_{\tilde{\nu}_j}M_{\tilde{\nu}_j}\right)}, \tag{3.109}$$

$$f^t_{LR} = \delta_{il}\delta_{lk}\left\{\frac{1}{t}+\frac{{L_e}^2}{t-M_Z^2}\right\}, \tag{3.110}$$

$$f^t_{RL} = \delta_{il}\delta_{lk}\left\{\frac{1}{t}+\frac{{R_e}^2}{t-M_Z^2}\right\}, \tag{3.111}$$

$$f^t_{RR} = \delta_{il}\delta_{lk}\left\{\frac{1}{t}+\frac{L_eR_e}{t-M_Z^2}\right\}+\sum_{j=1,3}\frac{\lambda_{jii}\lambda_{jlk}}{2e^2\left(s-M^2_{\tilde{\nu}_j}+i\Gamma_{\tilde{\nu}_j}M_{\tilde{\nu}_j}\right)}. \tag{3.112}$$

for $e^-_i e^+_i \to e^-_k e^+_l$ and by

$$f^s_{LL} = \delta_{lk}\left\{-\frac{e_d}{s}+\frac{L_eR_d}{s-M_Z^2+i\Gamma_Z M_Z}\right\}+\sum_{j=1,3}\sum_{\alpha=1,2}\frac{\lambda'_{ijk}\lambda'_{ijl}}{2e^2\left(t-M^2_{\tilde{u}_{j\alpha}}\right)}, \tag{3.113}$$

$$f^s_{LR} = \delta_{lk}\left\{-\frac{e_d}{s}+\frac{L_eL_d}{s-M_Z^2+i\Gamma_Z M_Z}\right\}, \tag{3.114}$$

$$f^s_{RL} = \delta_{lk}\left\{-\frac{e_d}{s}+\frac{R_eR_d}{s-M_Z^2+i\Gamma_Z M_Z}\right\}, \tag{3.115}$$

$$f^s_{RR} = \delta_{lk}\left\{-\frac{e_d}{s}+\frac{R_eL_d}{s-M_Z^2+i\Gamma_Z M_Z}\right\}, \tag{3.116}$$

$$f^t_{LL} = \sum_{j=1,3}\frac{\lambda_{jii}\lambda'_{jkl}}{2e^2\left(s-M^2_{\tilde{\nu}_j}+i\Gamma_{\tilde{\nu}_j}M_{\tilde{\nu}_j}\right)}, \tag{3.117}$$

$$f^t_{LR} = 0, \tag{3.118}$$

$$f^t_{RL} = 0, \tag{3.119}$$

$$f^t_{RR} = \sum_{j=1,3}\frac{\lambda_{jii}\lambda'_{jlk}}{2e^2\left(s-M^2_{\tilde{\nu}_j}+i\Gamma_{\tilde{\nu}_j}M_{\tilde{\nu}_j}\right)}, \tag{3.120}$$

for $e^-_i e^+_i \to d_k\bar{d}_l$. The colour factor $C_{\rm fac}$ is 1 for charged lepton production and N_C for quark production. The left/right charges of the fermions are given in Sect. 3.1. This formula corrects the sign of the interference term given in [24], which is necessary both to reproduce their numerical results and to obtain the correct SM Bhabha cross section formula in the limit that the R-parity violating couplings are zero.

3.8 Decay matrix elements

The decay matrix elements are implemented for both R-parity conserving and R-parity violating three-body decays of the gluino and the gauginos. The R-parity conserving decays are performed using the helicity amplitude expressions given in Ref. [17]. The R-parity violating formulae are as given in Ref. [20].

Spin correlations in the distribution of cascade decay products are implemented in all decay processes, as well as the R-parity conserving $2 \to 2$ SUSY production processes. The implementation is as described in Ref. [17].

4. Conclusions

The Monte Carlo event generator HERWIG has been extended to include the simulation of supersymmetry. We have summarised its salient features. We have defined the parameter conventions and listed the matrix elements for all original calculations.

Acknowledgements

We thank the co-authors of HERWIG for fruitful collaboration, and the members of the Cavendish ATLAS–HERWIG working group for stimulating discussions and for testing part of our code. P.R. would like to thank H. Dreiner for many useful discussions on the implementation of R-parity violating processes. S.M. thanks M.L. Mangano, H. Dreiner and M. Krämer for many useful discussions and the latter in particular for making available an independent implementation of the leading order matrix elements for gaugino production.

A. Process codes

We summarize the process codes `IPROC` for MSSM hard scattering processes implemented in HERWIG.

`IPROC`	Process
700-99	R-parity conserving SUSY processes
700	$\ell^+\ell^- \to$ 2-sparticle processes (sum of 710–760)
710	$\ell^+\ell^- \to$ neutralino pairs (all neutralinos)
706+4`IN1`+`IN2`	$\ell^+\ell^- \to \widetilde{\chi}^0_{\rm IN1}\widetilde{\chi}^0_{\rm IN2}$ (`IN1,2`=neutralino mass eigenstate)
730	$\ell^+\ell^- \to$ chargino pairs (all charginos)
728+2`IC1`+`IC2`	$\ell^+\ell^- \to \widetilde{\chi}^+_{\rm IC1}\widetilde{\chi}^-_{\rm IC2}$ (`IC1,2`=chargino mass eigenstate)
740	$\ell^+\ell^- \to$ slepton pairs (all flavours)
736+5`IL`	$\ell^+\ell^- \to \widetilde{\ell}_{L,R}\widetilde{\ell}^*_{L,R}$ (`IL` $= 1, 2, 3$ for $\widetilde{\ell} = \tilde{e}, \tilde{\mu}, \tilde{\tau}$)
737+5`IL`	$\ell^+\ell^- \to \widetilde{\ell}_L\widetilde{\ell}^*_L$ (`IL` as above)
738+5`IL`	$\ell^+\ell^- \to \widetilde{\ell}_L\widetilde{\ell}^*_R$ (`IL` as above)
739+5`IL`	$\ell^+\ell^- \to \widetilde{\ell}_R\widetilde{\ell}^*_R$ (`IL` as above)
740+5`IL`	$\ell^+\ell^- \to \widetilde{\nu}_L\widetilde{\nu}^*_L$ (`IL` $= 1, 2, 3$ for $\widetilde{\nu}_e, \widetilde{\nu}_\mu, \widetilde{\nu}_\tau$)
760	$\ell^+\ell^- \to$ squark pairs (all flavours)
757+4`IQ`	$\ell^+\ell^- \to \widetilde{q}_{L,R}\widetilde{q}^*_{L,R}$ (`IQ` $= 1...6$ for $\widetilde{q} = \tilde{d}...\tilde{t}$)
758+4`IQ`	$\ell^+\ell^- \to \widetilde{q}_L\widetilde{q}^*_L$ (`IQ` as above)
759+4`IQ`	$\ell^+\ell^- \to \widetilde{q}_L\widetilde{q}^*_R$ (`IQ` as above)
760+4`IQ`	$\ell^+\ell^- \to \widetilde{q}_R\widetilde{q}^*_R$ (`IQ` as above)
800-99	R-parity violating SUSY processes
800	Single sparticle production, sum of 810–840
810	$\ell^+\ell^- \to \widetilde{\chi}^0\nu_i$, (all neutralinos)
810+`IN`	$\ell^+\ell^- \to \widetilde{\chi}^0_{\rm IN}\nu_i$, (`IN`=neutralino mass state)
820	$\ell^+\ell^- \to \widetilde{\chi}^- e^+_i$ (all charginos)
820+`IC`	$\ell^+\ell^- \to \widetilde{\chi}^-_{\rm IC} e^+_i$, (`IC`=chargino mass state)
830	$\ell^+\ell^- \to \widetilde{\nu}_i Z^0$ and $\ell^+\ell^- \to \widetilde{\ell}^+_i W^-$
840	$\ell^+\ell^- \to \widetilde{\nu}_i h^0/H^0/A^0$ and $\ell^+\ell^- \to \widetilde{\ell}^+_i H^-$
850	$\ell^+\ell^- \to \widetilde{\nu}_i\gamma$
860	Sum of 870 and 880
870	$\ell^+\ell^- \to \ell^+\ell^-$, via LLE only
867+3`IL1`+`IL2`	$\ell^+\ell^- \to \ell^+_{\rm IL1}\ell^-_{\rm IL2}$ (`IL1,2`=1,2,3 for e, μ, τ)
880	$\ell^+\ell^- \to \bar{d}d$, via LLE and LQD
877+3`IQ1`+`IQ2`	$\ell^+\ell^- \to d_{\rm IL1}\bar{d}_{\rm IL2}$ (`IQ1,2`=1,2,3 for d, s, b)
910	$\ell^+\ell^- \to \nu_e\bar{\nu}_e h^0 + e^+e^-h^0$
920	$\ell^+\ell^- \to \nu_e\bar{\nu}_e H^0 + e^+e^-H^0$
960	$\ell^+\ell^- \to Z^0h^0$
970	$\ell^+\ell^- \to Z^0H^0$
955	$\ell^+\ell^- \to H^+H^-$
965	$\ell^+\ell^- \to A^0h^0$
965	$\ell^+\ell^- \to A^0H^0$

Table 6: The MSSM hard scattering processes implemented in HERWIG (continued on following pages).

IPROC	Process
3000-999	R-parity conserving SUSY processes
3000	2-parton $\to$ 2-sparticle processes (sum of those below)
3010	2-parton $\to$ 2-sparton processes
3020	2-parton $\to$ 2-gaugino processes
3030	2-parton $\to$ 2-slepton processes
3100+ISQ	$gg/q\bar{q} \to \tilde{q}\tilde{q}'^{*}H^{\pm}$ (*cf.* Ref. [1] for definition of ISQ)
3200+ISQ	$gg/q\bar{q} \to \tilde{q}\tilde{q}'^{*}h, H, A$ (")
3310,3315	$q\bar{q}' \to W^{\pm}h^0, H^{\pm}h^0$
3320,3325	$q\bar{q}' \to W^{\pm}H^0, H^{\pm}H^0$
3335	$q\bar{q}' \to H^{\pm}A^0$
3350	$q\bar{q} \to W^{\pm}H^{\mp}$
3355	$q\bar{q} \to H^{\pm}H^{\mp}$
3360,3365	$q\bar{q} \to Z^0h^0, A^0h^0$
3370,3375	$q\bar{q} \to Z^0H^0, A^0H^0$
3410	$bg \to b\ h^0$ + ch. conj.
3420	$bg \to b\ H^0$ + ch. conj.
3430	$bg \to b\ A^0$ + ch. conj.
3450	$bg \to t\ H^-$ + ch. conj.
3500	$bq \to bq'H^{\pm}$ + ch. conj.
3610	$q\bar{q}/gg \to h^0$
3620	$q\bar{q}/gg \to H^0$
3630	$q\bar{q}/gg \to A^0$
3710	$q\bar{q} \to q'\bar{q}'h^0$
3720	$q\bar{q} \to q'\bar{q}'H^0$
3810+IQ	$gg + q\bar{q} \to Q\bar{Q}h^0$ (IQ for Q flavour)
3820+IQ	$gg + q\bar{q} \to Q\bar{Q}H^0$ (")
3830+IQ	$gg + q\bar{q} \to Q\bar{Q}A^0$ (")
3839	$gg + q\bar{q} \to b\bar{t}H^+$ + ch. conjg.
3840+IQ	$gg \to Q\bar{Q}h^0$ (IQ as above)
3850+IQ	$gg \to Q\bar{Q}H^0$ (")
3860+IQ	$gg \to Q\bar{Q}A^0$ (")
3869	$gg \to b\bar{t}H^+$ + ch. conjg.
3870+IQ	$q\bar{q} \to Q\bar{Q}h^0$ (IQ as above)
3880+IQ	$q\bar{q} \to Q\bar{Q}H^0$ (")
3890+IQ	$q\bar{q} \to Q\bar{Q}A^0$ (")
3899	$q\bar{q} \to b\bar{t}H^+$ + ch. conjg.

Table 6 continued.

IPROC	Process
4000-99	R-parity violating SUSY processes via LQD
4000	single sparticle production, sum of 4010–4050
4010	$\bar{u}_j d_k \to \widetilde{\chi}^0 l_i^-$, $\bar{d}_j d_k \to \widetilde{\chi}^0 \nu_i$ (all neutralinos)
4010+`IN`	$\bar{u}_j d_k \to \widetilde{\chi}^0_{\rm IN} l_i^-$, $\bar{d}_j d_k \to \widetilde{\chi}^0_{\rm IN} \nu_i$ (`IN`=neutralino mass state)
4020	$\bar{u}_j d_k \to \widetilde{\chi}^- \nu_i$, $\bar{d}_j d_k \to \widetilde{\chi}^- e_i^+$ (all charginos)
4020+`IC`	$\bar{u}_j d_k \to \widetilde{\chi}^-_{\rm IC} \nu_i$, $\bar{d}_j d_k \to \widetilde{\chi}^-_{\rm IC} e_i^+$ (`IC`=chargino mass state)
4040	$u_j \bar{d}_k \to \tilde{\tau}_i^+ Z^0$, $u_j \bar{d}_k \to \widetilde{\nu}_i W^+$ and $d_j \bar{d}_k \to \widetilde{\ell}_i^+ W^-$
4050	$u_j \bar{d}_k \to \widetilde{\ell}_i^+ h^0/H^0/A^0$, $u_j \bar{d}_k \to \widetilde{\nu}_i H^+$ and $d_j \bar{d}_k \to \widetilde{\ell}_i^+ H^-$
4060	Sum of 4070 and 4080
4070	$\bar{u}_j d_k \to \bar{u}_l d_m$ and $\bar{d}_j d_k \to \bar{d}_l d_m$, via LQD only
4080	$\bar{u}_j d_k \to \nu_j l_k^-$ and $\bar{d}_j d_k \to l_j^+ l_k^-$, via LQD and LLE
4100-99	R-parity violating SUSY processes via UDD
4100	single sparticle production, sum of 4110–4150
4110	$u_i d_j \to \widetilde{\chi}^0 \bar{d}_k$, $d_j d_k \to \widetilde{\chi}^0 \bar{u}_i$ (all neutralinos)
4110 +`IN`	$u_i d_j \to \widetilde{\chi}^0_{\rm IN} \bar{d}_k$, $d_j d_k \to \widetilde{\chi}^0_{\rm IN} \bar{u}_i$(`IN` as above)
4120	$u_i d_j \to \widetilde{\chi}^+ \bar{u}_k$, $d_j d_k \to \widetilde{\chi}^- \bar{d}_i$ (all charginos)
4120 +`IC`	$u_i d_j \to \widetilde{\chi}^+_{\rm IC} \bar{u}_k$, $d_j d_k \to \widetilde{\chi}^-_{\rm IC} \bar{d}_i$ (`IC` as above)
4130	$u_i d_j \to \widetilde{g} \bar{d}_k$, $d_j d_k \to \widetilde{g} \bar{u}_i$
4140	$u_i d_j \to \tilde{b}_1^* Z^0$, $d_j d_k \to \tilde{t}_1^* Z^0$, $u_i d_j \to \tilde{t}_i^* W^+$ and $d_j d_k \to \tilde{b}_i^* W^-$
4150	$u_i d_j \to \tilde{d}_{k1}^* h^0/H^0/A^0$, $d_j d_k \to \tilde{u}_{i1}^* h^0/H^0/A^0$, $u_i d_j \to \tilde{u}_{k\alpha}^* H^+$, $d_j d_k \to \tilde{d}_{i\alpha}^* H^-$
4160	$u_i d_j \to u_l d_m$, $d_j d_k \to d_l d_m$ via UDD.

Table 6 concluded.

References

[1] G. Corcella, I.G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M.H. Seymour and B.R. Webber, JHEP **0101**, 010 (2001); hep-ph/9912396; hep-ph/0107071; hep-ph/0201201.

[2] H. Baer, V. Barger, D. Karatas and X. Tata, Phys. Rev. **D36**, 96 (1987); hep-ph/0001086.

[3] A. Djouadi, J. Kalinowski and M. Spira, Comput. Phys. Commun. **108** 56 (1998).

[4] H.E. Haber and G.L. Kane, Phys. Rept. **117**, 75 (1985).

[5] J.F. Gunion and H.E. Haber, Nucl. Phys. **B272**, 1 (1986); Erratum-ibid. **B402**, 567 (1993).

[6] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter's Guide* (Reading USA:Addison-Wesley, 1990); Erratum (1993), hep-ph/9302272.

[7] S. Mrenna, Comput. Phys. Commun. **101**, 232 (1997); T. Sjostrand, L. Lonnblad and S. Mrenna, hep-ph/0108264.

[8] S. Katsanevas and P. Morawitz, Comput. Phys. Commun. **112**, 227 (1998).

[9] H. Baer, F.E. Paige, S.D. Protopopescu and X. Tata, (2000), hep-ph/0001086.

[10] H. Baer, J. Sender and X. Tata, Phys. Rev. **D50**, 4517 (1994).

[11] X. Tata, (1997), hep-ph/9706307.

[12] A. Bartl, W. Majerotto and W. Porod, Z. Phys. **C64**, 499 (1994).

[13] K. Odagiri, JHEP **9810**, 006 (1998).

[14] V. Barger, R.W. Robinett, W.Y. Keung and R.J.N. Phillips, Phys. Lett. **B131**, 372 (1983).

[15] M. Spira, Fortsch. Phys. **46**, 203 (1998).

[16] R. Kleiss and W. J. Stirling, Nucl. Phys. B **262**, 235 (1985).

[17] P. Richardson, JHEP **0111** (2001) 029.

[18] A. A. Barrientos Bendezu and B. A. Kniehl, Phys. Rev. D **59**, 015009 (1999).

[19] S. Moretti and K. Odagiri, Phys. Rev. D **55**, 5627 (1997).

[20] H. Dreiner, P. Richardson and M.H. Seymour, JHEP **0004**, 008 (2000).

[21] H. Dreiner and S. Lola, *Prepared for Physics with e^+e^- Linear Colliders (The European Working Groups 4 Feb – 1 Sep 1995: Session 3), Hamburg, Germany, 30 Aug-1 Sep 1995.*

[22] G.F. Giudice *et al.*, (1996), hep-ph/9602207.

[23] M. Chemtob and G. Moreau, Phys. Rev. **D59** 116012 (1999).

[24] J. Kalinowski, R. Rückl, H. Spiesberger and P.M. Zerwas, Phys. Lett. **B406**, 314 (1997).