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$e^+e^- \to 6$ jets in parton level QCD at LEP1, LEP2 and NLC[1]

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Abstract

We study electron-positron annihilations into six jets at the parton level in perturbative Quantum Chromo-Dynamics. We use helicity amplitude methods. Results are presented for the case of the Durham and Cambridge jet clustering algorithms at three different collider energies.

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1. Introduction and motivation

Experimental collaborations at LEP have studied the physics of hadronic jets in great detail. The list of references is huge and we recommend the reader to look at any of the various high energy physics databases and exploit the appropriate keyword search, rather then trying to reproduce them here. What we would like to point out is that the phenomenology of six-jet events is not at all well known to date, from both the experimental and theoretical point of view. Indeed, this is not surprising, since the complications arising from, on the one hand, the poor event rate and large number of tracks, and, on the other hand, the complexity of the perturbative QCD calculations, are difficult to overcome.

Nonetheless, it is important that we soon examine this issue. Why ? Well, we could stand on the fact that ALEPH sees at 161 GeV a large excess of five- and especially six-jets events [1] (see also [2]) and speculate about its significance and the need for reliable perturbative calculations. We rather prefer to make a more far-seeing consideration. As for hadronic physics in electron-positron annihilations we can say the following. LEP1 was the era of the Z-peak and of its two-jet (and several higher order) decays. LEP2 is the age of the W^+W^--resonance and of its four-jet (and some higher order) decays. As we will move into the NLC epoch, we will step into a long series of resonant processes ending up with six-jet signatures, typically. One can mention top physics for a start. As this is one of the main goals of NLC, we should expect a lot of experimental studies concerned with $t\bar{t} \rightarrow b\bar{b}W^+W^- \rightarrow 6$ jet events, as the $W^\pm$'s show the 'colourful' tendency of decaying hadronically, though one would rather prefer to exploit a mixed semileptonic signature (in which one of the $W^\pm$'s accomplishes an electron or muon decay), to make things easier in terms of multi-jet resolution and mass reconstruction. Then one should not forget the new generation of gauge boson resonances, such as W^+W^-Z and ZZZ, and their favorite decays, needless to say, into six jets[2]. Indeed, for accurate studies of these kinds of events, it is worthwhile to use all the decay signatures, not only those involving leptons.

It is not our task here to remind the reader why top-antitop and three-gauge-boson production are relevant in the future of particle physics. To this end, one can find a complete compilation of motivations, procedures and expectations in Ref. [3]. What we would like to stress here is that one should be ready with all the appropriate phenomenological instruments to challenge the new kind of multi-jet experimental studies that will have to be carried out at the new generation of e^+e^- machines.

[2]One could also add photonic processes, like γW^+W^-, γZZ, $\gamma\gamma Z$ and $\gamma\gamma\gamma$ (though the photons would not be so virtual to easily comply with the tendency of eventually yielding a pair of partons) as well as those involving the Higgs particle, via $ZH \rightarrow ZW^+W^-$ and $ZH \rightarrow ZZZ$ to detect it and via $ZH \rightarrow ZHH$ to measure its self interactions.

As for the theoretical progress in this respect, studies of $e^+e^- \to$ six fermion electroweak processes are under way. A brief account of approaches and methods can be found in Ref. [4]. These kinds of reactions have already been calculated and analysed for the case involving up to four-quarks in the final state [5] (and other similar studies are in preparation [6]), while in Ref. [7] the concern was about Higgs processes with two quarks produced. One should probably expect the upgrade of the codes used for those studies to the case of six-quark production via electroweak interactions quite soon. However, a large fraction of the six-jet cross section comes from QCD interaction involving gluon propagators, gluon emissions and quark-gluon couplings.

In the present paper, we by-pass the case of $\mathcal{O}(\alpha_s^2)$ interactions. Those involving two quarks represent in fact a trivial extension of the projects carried out in Ref. [5]–[7]. Those involving two gluons have been dealt with for the case of $W^+W^- \to 6$ jet decays in Ref. [8] and it would be straightforward to extend those calculations to the case of $\gamma\gamma, \gamma Z, ZZ \to$ 6 jet decays as well. As for the other channels, they will be discussed elsewhere [9]. Here, we address the case of six-jet production through $\mathcal{O}(\alpha_s^4)$ in perturbative QCD. That is, we calculate the tree-level processes

$$e^+e^- \to q\bar{q}gggg, \qquad e^+e^- \to q\bar{q}q'\bar{q}'gg, \qquad e^+e^- \to q\bar{q}q'\bar{q}'q''\bar{q}'', \tag{1}$$

where q, q', q'' represent *massless* quarks and g a gluon. We will apply our results to the case of LEP1, LEP2 and NLC energies. A short description of the computational techniques adopted will be given in Section 2. Results and conclusions are in Section 3.

2. Matrix Elements

In order to master the large number of Feynman diagrams (of the order of several hundreds) entering in processes (1), we have used spinor techniques [10] and exploited the `HELAS` subroutines [11]. The coded helicity amplitudes have been checked for gauge invariance and integrated using `VEGAS` [12]. The three colour matrices have been calculated using the orthogonal basis method of Ref. [13], which reduces considerably the difficulty of such computation. The expressions of both the amplitudes and the colour factors will be given elsewhere [9]. In the present publication, we only mention that the number of different 'topologies' describing processes (1) is rather small in the end, and the same implementation can be exploited several times by means of recursive permutations of the momenta of the external particles. Furthermore, each common 'sub-diagram' is saved and then reused in the same numerical evaluation. In the case of two-quark-four-gluon diagrams the topologies are those shown in Fig. 1(a), eight in total. For four-quark-two-gluons, Fig. 1(b), one has ten and for six-quarks, Fig. 1(c), three topologies.

(a)
(b)
(c)

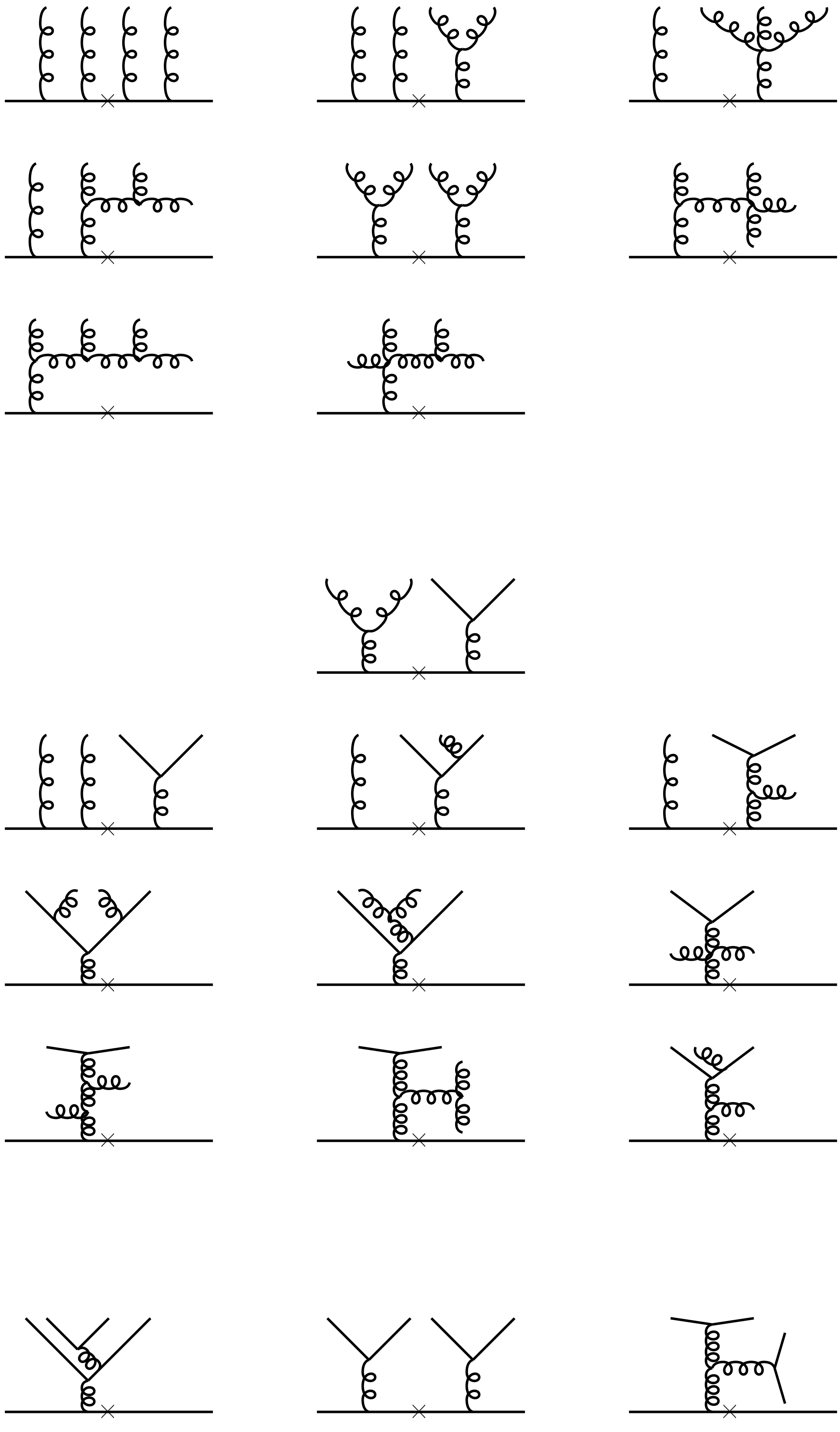

Figure 1: Tree-level Feynman topologies contributing to: (a) $e^+e^- \to q\bar{q}gggg$, (b) $e^+e^- \to q\bar{q}q'\bar{q}'gg$ and (c) $e^+e^- \to q\bar{q}q'\bar{q}'q''\bar{q}''$. The symbol $\times$ refers to the insertion of the $e^+e^- \to \gamma, Z$ current. Permutations are not shown.

We have adopted $M_Z = 91.17$ GeV, $\Gamma_Z = 2.516$ GeV, $\sin^2(\theta_W) = 0.23$, $\alpha_{em} = 1/128$ and the two-loop expression for α_s. As centre-of-mass (CM) energies representative of LEP1, LEP2 and NLC, we have used the values $\sqrt{s} = M_Z, 180$ GeV and 500 GeV, respectively.

As jet clustering algorithms we have adopted the Durham (D) [14] and Cambridge (C) [15] schemes, which are based on the same 'jet measure'

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})}{s}, \tag{2}$$

but differ in the clustering procedure. In eq. (2), E_i and E_j represent the energies of any pair of partons i and j, whereas θ_{ij} is their relative angle. A six-jet sample is selected by imposing the constraint $y_{ij} \geq y$ on all n possible parton combinations ij (with $n = 15$ for the Durham and $5 \leq n \leq 15$ for the Cambridge algorithm).

3. Results

We define the y-dependent six-jet fraction[3] by means of the relation

$$f_6(y) = \frac{\sigma_6(y)}{\sum_m \sigma_m(y)} = \frac{\sigma_6(y)}{\sigma_{\rm tot}}, \tag{3}$$

where $\sigma_6(y)$ is the actual six-parton cross section and $\sigma_{\rm tot}$ identifies the *total* hadronic rate $\sigma_{\rm tot} = \sigma_0(1 + \alpha_s/\pi + ...)$, σ_0 being the Born cross section. In perturbative QCD one can rewrite eq. (3) in terms of a series in α_s, beginning with its fourth power, as

$$f_6(y) = \left(\frac{\alpha_s}{2\pi}\right)^4 G(y) + ..., \tag{4}$$

where $G(y)$ is the lowest-order (or leading-order, LO) 'coefficient function' of the six-jet rate. We show this quantity in Fig. 2, for the case of both the Durham and Cambridge schemes, at LEP1[4].

The larger rate for the C scheme as compared to the D one is a direct consequence of the 'soft freezing' procedure of resolved jets described in Ref. [15]. The step of eliminating from the sequence of clustering the less energetic one in a resolved pair of particles (i.e., with $y_{ij} > y$), implemented in the C algorithm, tends to enhance the final jet multiplicity of the original D scheme. In fact, the procedure prevents the attraction of the remaining particles (at large angle) into new unresolved pairs, whose momenta would then be merged together, producing a lower number of final jets. For example, at $y = (0.001)[0.005]\{0.010\}$ relative differences between the two algorithms are of the order of $(10)[14]\{16\}\%$.

[3]The jet resolution parameter y is often indicated as y_{cut} in the specialised literature.

[4]Here and in the following, unless otherwise stated, the summations over the three reactions (1) and over all possible combinations of quark flavours in each of these have been performed.

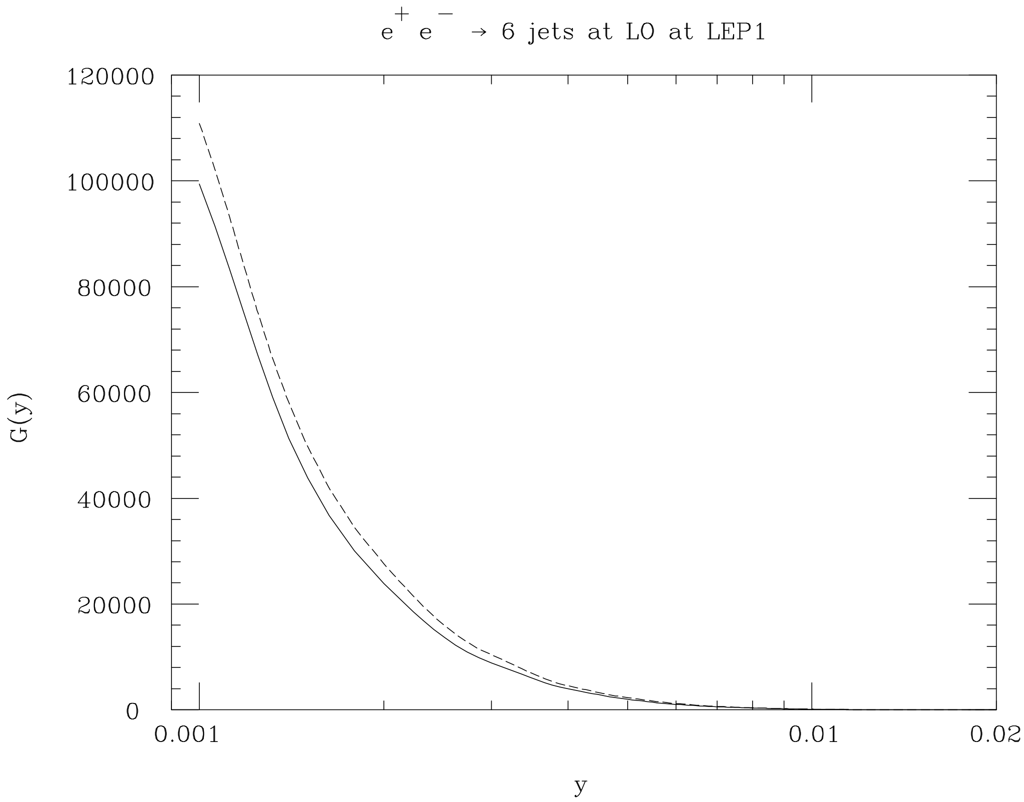

Figure 2: The parton level $G(y)$ function entering in the six-jet fraction at LO in the D (continuous line) and C (dashed line) schemes at LEP1.

The jet fraction $f_6(y)$ and cross section $\sigma_6(y)$ corresponding to the rates in Fig. 2 are given in Tab. 1, for a representative selection of y's. The value of α_s adopted is 0.120, whereas that used for $\sigma_{\rm tot}$ is 39.86 nb. Given the total luminosity collected at LEP1 in the 1989-1995 years (more than 10^7 hadronic events have been recorded by the four collaborations), the six-jet event rate is comfortably measurable. For example, for a luminosity of, say, 100 pb^{-1} per experiment, perturbative QCD predicts at LO some 52,000 original six-parton events recognised as six-jet ones, for $y = 0.001$ in the D scheme (a number that increases to approximately 58,000 if the C algorithm is adopted instead). Indeed, six-jet fractions have been studied at LEP1 in several experimental [16] and theoretical papers [17].

The rates given in Fig. 2 and Tab. 1 are LO results. Next-to-leading order (NLO) corrections proportional to $\mathcal{O}(\alpha_s^5)$ are expected to be large. In fact, the size of higher order (HO) contributions generally increases with the power of α_s and with the number

y	$f_6(y)$	$\sigma_6(y)$ (pb)
0.001	$(1.31)[1.47] \times 10^{-2}$	(523.91)[584.19]
0.002	$(3.16)[3.65] \times 10^{-3}$	(125.95)[145.44]
0.003	$(1.17)[1.37] \times 10^{-3}$	(46.81)[54.72]
0.004	$(5.29)[6.01] \times 10^{-4}$	(21.09)[23.96]
0.005	$(2.63)[3.05] \times 10^{-4}$	(10.49)[12.17]
0.006	$(1.38)[1.59] \times 10^{-4}$	(5.49)[6.35]
0.007	$(7.90)[8.85] \times 10^{-5}$	(3.15)[3.53]
0.008	$(4.80)[5.20] \times 10^{-5}$	(1.91)[2.07]
0.009	$(2.77)[3.08] \times 10^{-5}$	(1.10)[1.23]
0.010	$(1.64)[1.93] \times 10^{-5}$	(0.65)[0.77]
LEP1		

Table 1: Jet fraction and cross section rates for $e^+e^- \to 6$ jets at LEP1, for several values of y in the Durham and Cambridge schemes, in round and squared brackets, respectively. The numerical errors do not affect the significative digits shown.

of particles in the final state or, in other terms, with their possible permutations (that is, with the number of different possible attachments of both the additional real and virtual particles to those appearing in lowest-order). The highest-order corrections calculated to date in $e^+e^- \to n$ jet annihilations are the NLO ones to the four-jet rate, that is, terms proportional to α_s^3 [18]–[20]. The total four-jet cross section at NLO was found to be larger than that obtained at LO in α_s^2 by a factor 1.5 or more, depending on the value of y implemented during the clustering procedure and the algorithm used as well. Therefore, we should expect that the rates given in Tab. 1 underestimate the six-jet rates by *at least* a similar factor.

In this respect the Monte Carlo (MC) programs [21]–[25] largely exploited in experimental analyses perform better than the fixed order perturbative calculations, especially at low y-values, where the latter need to be supported by the additional contribution of perturbation series involving leading and next-to-leading powers of $\log y$ resummed to all orders (for the case of the D scheme, see Refs. [26, 27]) in order to fit the same data. However, as such MCs generally implement only the infrared (i.e., soft and collinear) dynamics of quarks and gluons in the standard 'parton-shower (PS) $+\mathcal{O}(\alpha_s)$ Matrix Element (ME)' modeling, in many cases their description of the large y-behaviour and/or that of the interactions of secondary 'branching products' is (or should be expected to be) no longer adequate. In fact, this has been shown to be the case, e.g., for some typical angular quantities of four-jet events [2, 28]. In contrast, once $\mathcal{O}(\alpha_s^2)$ MEs are inserted and properly matched to the parton shower, e.g., using the JETSET string fragmentation model [23] (see also Ref. [29])[5], then the agreement is recovered. In this context, one should however mention that ME models with 'added-on' hadronisation cannot be reliably extrapolated from one energy to another, as the fragmentation tuning is energy dependent. If one considers that four-jet events represent only the next-to-lowest order QCD interactions in e^+e^- scatterings, then it is not unreasonable to argue that further complications might well arise as the final state considered gets more and more sophisticated, such as in five- [30] and six-jet events. Under these circumstances, we believe the availability of exact perturbative calculations of the latter to be essential to model the HO parton dynamics for both future tests of QCD and QCD background studies as well.

The total hadronic cross section falls drastically when increasing the CM energy from the LEP1 to the LEP2 values, by more than three orders of magnitude, and so does the six-jet rate. Nonetheless, six-jet fractions have been measured also during the 1995-1996 runs at the CM energies of 130–136, 161 and 172 GeV, when a total luminosity of around 27 pb^{-1} was collected. Results can be found in Ref. [31]. Within a 'typical' K-factor of 2 (which quantifies the ratio between the NLO and the LO six-jet rates) the values of $f_6(y)$ as reconstructed from our rates are always well compatible with those produced by the MCs (also at $\sqrt{s} = 161$ GeV, where the six-jet excess was observed) used by, e.g., the ALEPH collaboration [1]. For reference, we present the jet rates at $\sqrt{s} = 180$ GeV in Fig. 3 in the form of the total cross section. The relative differences between the two D and C algorithms are similar to those already encountered at LEP1.

The six-jet event rate that will be collected at NLC can be rather large, despite the cross section being more than a factor 10^4 smaller than at LEP1 (e.g., at $\sqrt{s} = 500$ GeV). This is due to the large yearly luminosity expected at this machine, around 10 fb^{-1}. For such a value and assuming a standard evolution of the coupling constant with the increasing energy (that is, no non-Standard Model thresholds occur up to 500 GeV), at the minimum of the y-values considered here (i.e., $y = 0.001$), one should expect some 220 events per annum by adopting the D scheme and about 13% more if one adopts the C one. However, these rates decrease rapidly as the resolution parameter gets larger, by a factor of 50 or so at $y = 0.005$ and of approximately 800 at $y = 0.01$.

It is also interesting to look at the composition of the total rates in terms of the three processes (1). Whereas this is probably of little concern at LEP1 and LEP2, the capability of the detectors of distinguishing between jets due to quarks (and among these, bottom

[5] A special '$\mathcal{O}(\alpha_s^2)$ + PS + cluster hadronisation' version of HERWIG is also in preparation.

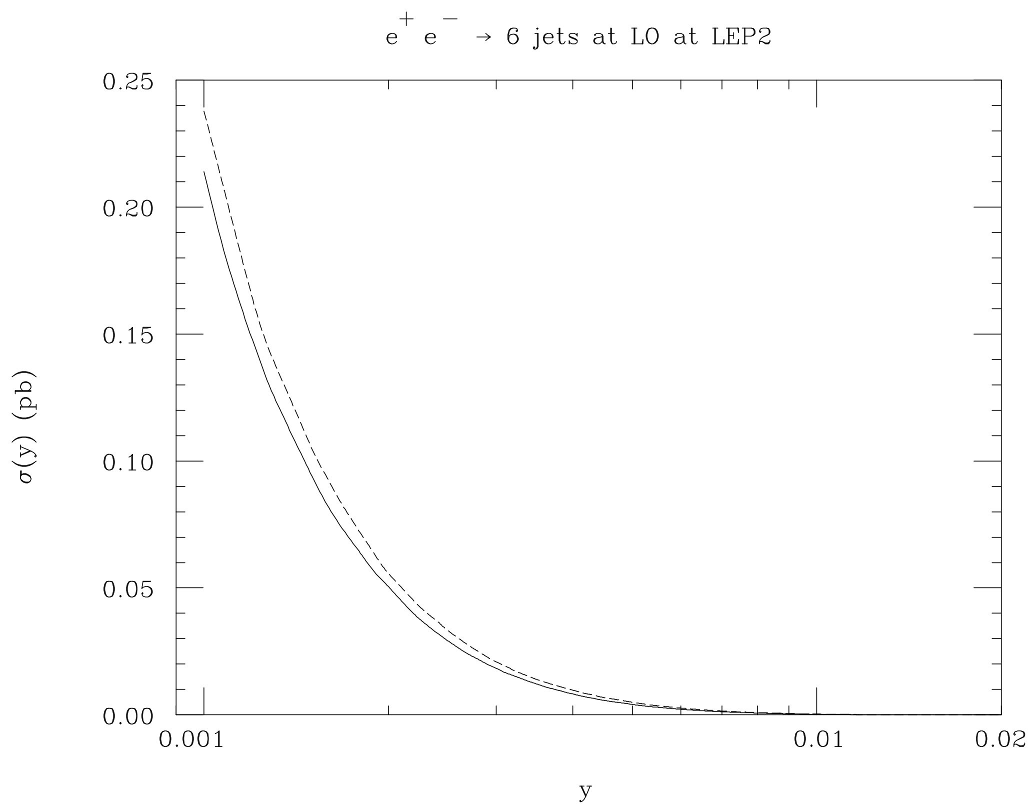

Figure 3: The total cross section of six-jet events at LO in the D (continuous line) and C (dashed line) schemes at LEP2.

ones in particular: e.g., in tagging top and/or Higgs decays) and gluons is essential at NLC, in order to perform dedicated searches for signals of both anomalous gauge couplings and new particles. The different behaviours of the three reactions in (1) can be appreciated in Fig. 4, e.g., for the case of the C scheme, in terms of total cross sections. The rates for the D algorithm follow the same pattern.

Tests of multiple electroweak self-couplings of gauge-bosons (as well as searches for new resonances) will often need to rely on the mass reconstruction of multi-jet systems (particularly di-jet ones). Therefore, it is instructive to look at the invariant mass distributions which will be produced at NLC by all the possible two-parton combinations ij in six-jet events from QCD at $\mathcal{O}(\alpha_s^4)$ (with $i = 1, ...5$ and $j = i + 1, ...6$). As usual in multi-jet analyses, we first order the jets in energy, so that $E_1 > E_2 > ... > E_5 > E_6$. Then, we construct

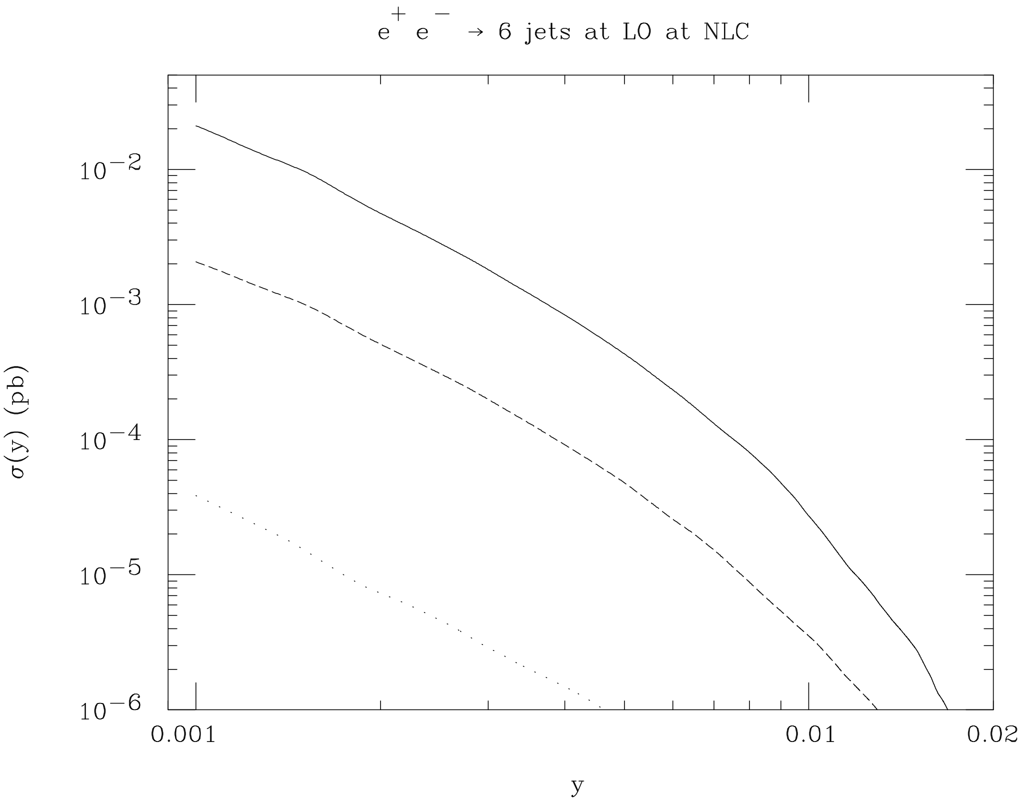

Figure 4: The total cross section of six-jet events at LO in the C scheme at NLC decomposed in terms of the three contributions $e^+e^- \rightarrow q\bar{q}gggg$ (continuous line), $e^+e^- \rightarrow q\bar{q}q'\bar{q}'gg$ (dashed line) and $e^+e^- \rightarrow q\bar{q}q'\bar{q}'q''\bar{q}''$ (dotted line).

the quantities

$$m_{ij} \equiv \frac{M_{ij}^2}{s} = \frac{2E_iE_j(1-\cos\theta_{ij})}{s}, \tag{5}$$

where M_{ij}^2 represents the Lorentz invariant (squared) mass and the equality holds for massless ij particles. These fifteen quantities are shown in Fig. 5 for the C scheme at $y = 0.001$, their shape being similar for the D algorithm. We found it convenient to plot the 'reduced' invariant masses m_{ij} rather than the actual ones M_{ij}, as energies and angles 'scale' with the CM energy in such a way that the shape of the distributions is largely unaffected by changes of the value of $\sqrt{s}$ in the energy range relevant to NLC. Therefore, from Fig. 5 one should then be able to reconstruct rather accurately the Lorentz invariant mass distributions for a given CM energy $\sqrt{s}$ by exploiting eq. (5). Note that the reduced mass spectra are similar in all three reactions (1), their integral being however rescaled according to the numbers given in Fig. 4. To allow for an easy conversion of the differential cross sections into numbers of events in a certain mass range, the spectra in Fig. 5 sum to the total cross section of processes (1) at NLC.

It is interesting to notice in Fig. 5 the 'resonant' behaviour of some of the distributions. This is particularly true for those involving the most energetic of all the partons. Using the definition (5) they translate into peak-like structures at the true invariant mass values $M_{ij} \approx (250)[212]\{177\}$ GeV, for the combinations $ij = (12)[13]\{14\}$, and, possibly, $M_{15} \approx 150$ GeV as well. In all the other cases the spectra are generally softer and do not show any distinctive kinematic feature. It is however important that all such behaviours are correctly implemented in the simulation programs that will be adopted by the NLC experiments, so that it will be possible to recognise and eventually subtract the six-jet QCD background, if not to study it as signal on its own in testing the fundamental nature of QCD. Furthermore, it should be noted that the much lower value of α_s at NLC energies (as compared to that at LEP1 and/or LEP2) in principle implies a reduced importance of the uncalculated HO strong corrections.

In summary, we have studied the parton level processes $e^+e^- \to q\bar{q}gggg$, $e^+e^- \to q\bar{q}q'\bar{q}'gg$ and $e^+e^- \to q\bar{q}q'\bar{q}'q''\bar{q}''$ (for massless quarks) at leading-order in perturbative QCD by computing their exact matrix elements. The use of helicity amplitude methods has allowed the implementation of the several hundreds of Feynman diagrams entering in such reactions in a compact form usable for high statistic MC simulations. It requires about 10^{-2} CPU seconds to evaluate a single event on a alpha-station DEC 3000 - M300X and further optimisations are under way. Some results relevant to multi-jet analyses at past, present and future high energy electron-positron colliders have been presented.

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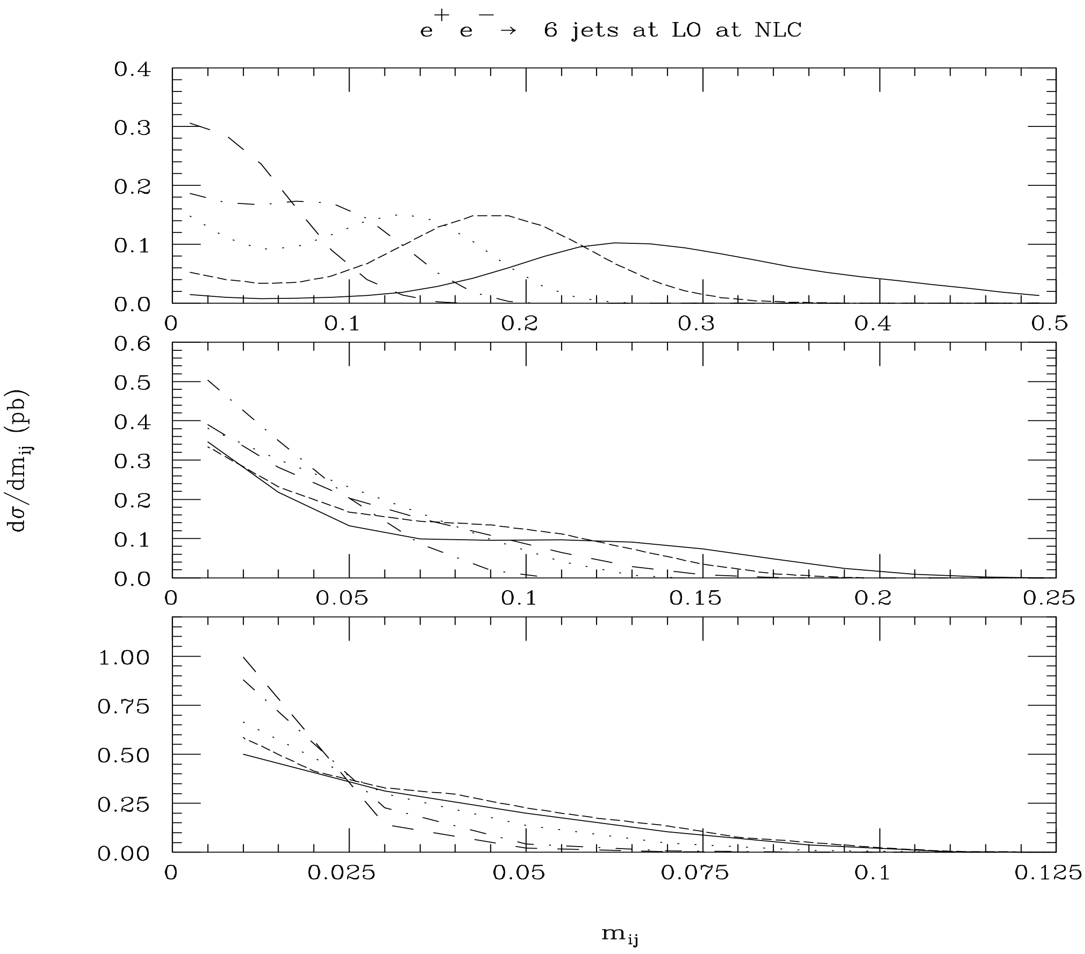

Figure 5: The distributions in the reduced invariant mass m_{ij} (5) of events of the type (1), in the C scheme with $y = 0.001$, for the following combinations of parton pairs ij: (12)[23]{35} (continuous lines), (13)[24]{36} (short-dashed lines), (14)[25]{45} (dotted lines), (15)[26]{46} (dot-dashed lines) and (16)[34]{56} (long-dashed lines), in the (upper)[central]{lower} frame.