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Power Corrections to Event Shape Distributions*

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Abstract

We estimate the effects of non-perturbative physics on the differential distributions of infrared- and collinear-safe e^+e^- event shape variables, by extending the notion of an infrared-regular effective strong coupling, which accounts for the non-perturbative corrections to the mean values of several shape variables, to their distributions. This leads to $1/Q$ power corrections over a range of values of the shape variables considered, where Q is the centre-of-mass energy. In the case of the thrust variable, the leading correction is simply a shift of the distribution, by an amount proportional to $1/Q$. We show that this gives an excellent description of the data throughout a wide range of T and Q.

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1 Introduction

The study of power-suppressed corrections to QCD observables has become a lively field of experimental and theoretical investigation. On the experimental side, the estimation of power-suppressed hadronization or higher-twist corrections is necessary for the accurate measurement of the strong coupling α_s, a crucial parameter of the Standard Model. Theoretical work on divergences of the QCD perturbation series, in particular infrared renormalons [1,2], and related attempts to define the running coupling beyond perturbation theory [3], have also led to renewed interest in power-suppressed contributions.

Power corrections to hadronic event shapes are of particular interest, because they are expected to have a characteristic $1/Q$-dependence on the hard process scale Q (the centre-of-mass energy in e^+e^- annihilation) [4-8]. Thus, in contrast to the $1/Q^2$ corrections encountered in most quantities, they are still important even at $Q \sim M_Z$, where most of the e^+e^- data have been obtained. The theory of $1/Q$-corrections is much less developed than that for higher powers; in particular, the operator product expansion does not apply in this case, since there are no relevant operators of the corresponding dimension.

In refs. [4,5,6] a treatment of $1/Q$-corrections to event shapes was proposed, based on the notion of an effective strong coupling, $\alpha_{\rm eff}$, which is approximately universal but differs from the perturbative form in the infrared region (see also [9]). Such an approach was found to be quite successful in describing the powers and approximate magnitudes of power corrections to a wide variety of QCD observables, using the low-energy moments of $\alpha_{\rm eff}$ as non-perturbative parameters. The treatment of event shapes, however, has been limited so far to the mean values only [10].

In the present paper we extend the analysis to the differential distributions of event shape variables. The central idea remains that of ref. [5]: the emission of soft gluons is assumed to be controlled by an effective coupling $\alpha_{\rm eff}$, different from the perturbative form in the infrared region but small enough for terms of higher order in $\alpha_{\rm eff}$ to be neglected as a first approximation. We combine this idea with the treatment of event shape distributions developed in refs. [11,12]. It was shown there that some shape variables have the property of exponentiation, which allows large logarithms to be resummed to all orders in perturbation theory. For such variables, the effective coupling assumption means that the leading non-perturbative corrections also exponentiate, implying a specific transformation of the whole distribution, or at least its logarithmically enhanced part.

We concentrate here, as in ref. [5], on the thrust variable, T [13]. We can show that in this case the leading non-perturbative effect over a range of thrust values is simply a shift in the distribution, by an amount proportional to $1/Q$, modulo logarithmic Q-dependence. The shift is just such that we recover the result of ref. [5] for the mean value. Remarkably, this leads to an excellent description of the data over a wide range of T and Q. The only two free parameters are the non-perturbative quantity

$$\bar{\alpha}_0(\mu_{\rm I}) \equiv \frac{1}{\mu_{\rm I}} \int_0^{\mu_{\rm I}} dq\, \alpha_{\rm eff}(q) \tag{1.1}$$

which characterises the behaviour of the effective coupling below some infrared matching scale $\mu_{\rm I}$, and the perturbative coupling $\alpha_s(M_Z)$. For these quantities we find values consistent with those obtained from other data.

After a detailed consideration of the thrust distribution, we discuss the relation between our analysis and the "dispersive approach" of ref. [6], and then comment briefly on the prospects for extending the method to other shape variable distributions.

2 Thrust distribution

It was shown in ref. [12] that the thrust distribution is given to next-to-leading logarithmic accuracy by the expression

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dT} \equiv F(T) = \frac{Q^2}{2\pi i}\int_C d\nu\, {\rm e}^{(1-T)\nu Q^2}\left[\tilde{J}^q_\nu(Q^2)\right]^2 \ , \tag{2.1}$$

where the contour C runs parallel to the imaginary axis, to the right of all singularities of the integrand, and $\tilde{J}^q_\nu(Q^2)$ represents the Laplace transform of the quark jet mass distribution at hard process scale Q. The result obtained for this function, again to next-to-leading logarithmic accuracy, was

$$\ln \tilde{J}^q_\nu(Q^2) = \int_0^1 \frac{du}{u}\left({\rm e}^{-u\nu Q^2}-1\right)\left[\int_{u^2Q^2}^{uQ^2}\frac{dq^2}{q^2}A(\alpha_{\rm S}(q)) + \frac{1}{2}B(\alpha_{\rm S}(\sqrt{u}Q))\right] \tag{2.2}$$

with

$$A(\alpha_{\rm S}) = C_F\frac{\alpha_{\rm S}}{\pi}\left(1+K\frac{\alpha_{\rm S}}{2\pi}\right) \ , \quad B(\alpha_{\rm S}) = -3C_F\frac{\alpha_{\rm S}}{2\pi} \ , \quad K = C_A\left(\frac{67}{18}-\frac{\pi^2}{6}\right)-\frac{5}{9}N_f \ , \tag{2.3}$$

and $\alpha_{\rm S}$ defined in the $\overline{\rm MS}$ scheme.

In ref. [12], the region of low q^2 in Eq. (2.2) was ignored, on the grounds that its contribution is subleading. We now include it in the following way: we first subtract the perturbative contribution from the region $q^2 < \mu_{\rm I}^2$, and then add it back again using the non-perturbative effective coupling $\alpha_{\rm eff}(q)$. Changing the order of integration, the amount added back is

$$\delta \ln \tilde{J}^q_\nu(Q^2) = \frac{2C_F}{\pi}\int_0^{\mu_{\rm I}}\frac{dq}{q}\alpha_{\rm eff}(q)\int_{q^2/Q^2}^{q/Q}\frac{du}{u}\left({\rm e}^{-u\nu Q^2}-1\right) \ . \tag{2.4}$$

Since νQ^2 is conjugate to $1-T$, for $1-T \gg \mu_{\rm I}/Q$ we can safely expand the exponential to first order, to obtain

$$\delta \ln \tilde{J}^q_\nu(Q^2) \simeq -\frac{2C_F}{\pi}\int_0^{\mu_{\rm I}} dq\, \alpha_{\rm eff}(q)\,\nu Q \equiv -\frac{2C_F}{\pi}\frac{\mu_{\rm I}}{Q}\,\bar{\alpha}_0(\mu_{\rm I})\,\nu Q^2 \ . \tag{2.5}$$

The term involving $B(\alpha_{\rm S})$ gives a correction of order $1/Q^2$, which we neglect, together with terms of order $\alpha_{\rm eff}^2$.

For the perturbative subtraction, we use the next-to-leading-order expansion of $\alpha_{\rm S}(q)$ in terms of $\alpha_{\rm S}(\mu_{\rm R})$, $\mu_{\rm R}$ being the chosen renormalization scale:

$$\alpha_{\rm S}(q) = \alpha_{\rm S}(\mu_{\rm R}) + \frac{\beta_0}{2\pi}\ln\frac{\mu_{\rm R}}{q}\,\alpha_{\rm S}^2(\mu_{\rm R}) \tag{2.6}$$

where $\beta_0 = (11C_A - 2N_f)/3$. The expression to be subtracted from the right-hand side of Eq. (2.5) is thus

$$-\frac{2C_F}{\pi}\int_0^{\mu_I} dq \left[\alpha_S(\mu_R) + \frac{\beta_0}{2\pi}\left(\ln\frac{\mu_R}{q} + \frac{K}{\beta_0}\right)\alpha_S^2(\mu_R)\right]\nu Q \ , \tag{2.7}$$

which gives

$$\delta \ln \tilde{J}_\nu^q(Q^2) = -\frac{2C_F}{\pi}\frac{\mu_I}{Q}\left[\bar{\alpha}_0(\mu_I) - \alpha_S(\mu_R) - \frac{\beta_0}{2\pi}\left(\ln\frac{\mu_R}{\mu_I} + \frac{K}{\beta_0} + 1\right)\alpha_S^2(\mu_R)\right]\nu Q^2 \ . \tag{2.8}$$

Substituting in Eq. (2.1), we see that the leading non-perturbative effect on the thrust distribution is simply to shift the perturbative prediction to lower thrust, by an amount proportional to $1/Q$:

$$F(T) = F^{\rm pert}(T - \delta T) \tag{2.9}$$

where

$$\delta T = -\frac{4C_F}{\pi}\frac{\mu_I}{Q}\left[\bar{\alpha}_0(\mu_I) - \alpha_S(\mu_R) - \frac{\beta_0}{2\pi}\left(\ln\frac{\mu_R}{\mu_I} + \frac{K}{\beta_0} + 1\right)\alpha_S^2(\mu_R)\right] \ . \tag{2.10}$$

This is precisely the formula derived in ref. [5] for the non-perturbative shift in the mean thrust, $\delta\langle T\rangle$.

We remind the reader that the simple prediction (2.9) applies only in the region $1-T \gg \mu_I/Q$, where μ_I marks the scale below which $\alpha_{\rm eff}$ starts to deviate from α_S. For $1-T \sim \mu_I/Q$ one could explore the effects of retaining more terms in the expansion of the exponential function in Eq. (2.4), but this would require a detailed parametrization of $\alpha_{\rm eff}$, and we have not tried it. In practice, we excluded the region $1 - T < 0.05$ from all our comparisons with data.

The prediction (2.9) also does not strictly apply at low values of the thrust, where terms involving powers of $\ln(1-T)$ are not dominant. In ref. [12], however, it was found that exponentiation has significant effects throughout the region in which the cross section is substantial. In earlier comparisons with data, good agreement was obtained (after hadronization corrections) by using a '$\log R$' matching scheme, in which essentially all known higher-order corrections are exponentiated. It therefore appears natural to adopt this scheme and to extend our comparisons with Eq. (2.9) rather far into the low-thrust region. In fact we find good agreement down to $T = 0.65$, which is even outside the three-jet region ($T > 2/3$).

For $F^{\rm pert}$ we take the $\log R$-matched resummed expression in ref. [12] with no modification of the logarithmic terms, i.e. with $L = -\ln(1-T)$. Initially, we set the renormalization scale $\mu_R = Q$ and the infrared matching scale $\mu_R = 2$ GeV, as in ref. [5]. The resulting predictions were compared with data on the thrust distribution in the interval $0.05 < 1 - T < 0.35$ at energies $14 < Q < 161$ GeV, as listed in Table 1.

The best fit values of the two free parameters are

$$\Lambda^{(5)}_{\overline{\rm MS}} = 0.235 \pm 0.017 \text{ GeV} \ , \quad \bar{\alpha}_0(2 \text{ GeV}) = 0.46 \pm 0.02 \tag{2.11}$$

(95% confidence level). The corresponding curves are shown in Fig. 1. The fitted values of the parameters are somewhat correlated, as shown in Fig. 2. The value of $\Lambda^{(5)}_{\overline{\rm MS}}$, corresponding to $\alpha_S(M_Z) = 0.1186 \pm 0.0013$, is in good agreement with that obtained by other

methods. The value of $\bar{\alpha}_0$ is somewhat smaller than, but within two standard deviations of, that obtained in ref. [5], $\bar{\alpha}_0(2\ \mathrm{GeV}) = 0.52 \pm 0.03$.

The quality of the overall fit is remarkable ($\chi^2/\mathrm{d.o.f.} = 134/114 = 1.18$) – much better than those typically obtained[‡] when hadronization effects are estimated from Monte Carlo models [14,15]. Furthermore, we see from Table 1 that a large contribution to the χ^2 comes from the data at 14 GeV, where our fitting region is perhaps too large ($\mu_\mathrm{I}/Q = 0.14 > 0.05$) and there may be complications due to quark mass effects, heavy quark decays and higher power corrections. However, the 14 GeV data are valuable because they provide the longest lever arm for distinguishing between inverse power and logarithmic energy dependence. If we exclude the 14 GeV data altogether, the best fit parameter values do not change, but the errors are doubled (dashed curve in Fig. 2).

To study the dependence on the infrared matching scale, we also performed a fit at $\mu_\mathrm{I} = 3$ GeV. The best fit value of $\Lambda^{(5)}_{\overline{\mathrm{MS}}}$ and the quality of the fit did not change significantly, and we obtained $\bar{\alpha}_0(3\ \mathrm{GeV}) = 0.374 \pm 0.010$, again within two standard deviations of the value obtained in ref. [5], $\bar{\alpha}_0(3\ \mathrm{GeV}) = 0.42 \pm 0.03$.

We also investigated the dependence on the the renormalization scale μ_R, in the range $Q^2/2 < \mu_\mathrm{R}^2 < 2Q^2$. The best fit parameter values varied from $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 0.204$ GeV, $\bar{\alpha}_0(2\ \mathrm{GeV}) = 0.457$ to $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 0.270$ GeV, $\bar{\alpha}_0(2\ \mathrm{GeV}) = 0.466$, respectively. Thus the error in $\Lambda^{(5)}_{\overline{\mathrm{MS}}}$ is still dominated by the systematic error due to renormalization scale dependence, and our overall estimate of this parameter is

$$\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 0.235 \pm 0.035\ \mathrm{GeV}\ , \quad \alpha_\mathrm{S}(M_Z) = 0.1185 \pm 0.0025\ . \tag{2.12}$$

Table 1: Data sets and fit results for $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 0.235$ GeV, $\bar{\alpha}_0 = 0.46$.

Collab.	Q/GeV	Ref.	Pts.	χ^2
TASSO	14	[16]	8	28.5
TASSO	22	[16]	8	11.1
TASSO	35	[16]	8	2.1
TASSO	44	[16]	8	7.6
AMY	54	[17]	6	12.5
OPAL	91.2	[18]	30	19.0
ALEPH	91.2	[19]	11	12.8
DELPHI	91.2	[20]	13	23.6
SLD	91.2	[21]	6	3.5
OPAL	133	[22]	6	2.9
DELPHI	133	[23]	6	8.4
OPAL	161	[24]	6	2.3
Total			116	134.1

[‡]Compare, for example, the best fit $\chi^2/\mathrm{d.o.f.} = 2.3$ obtained in ref. [18] at a single energy for the more restricted interval $0.06 < 1 - T < 0.30$.

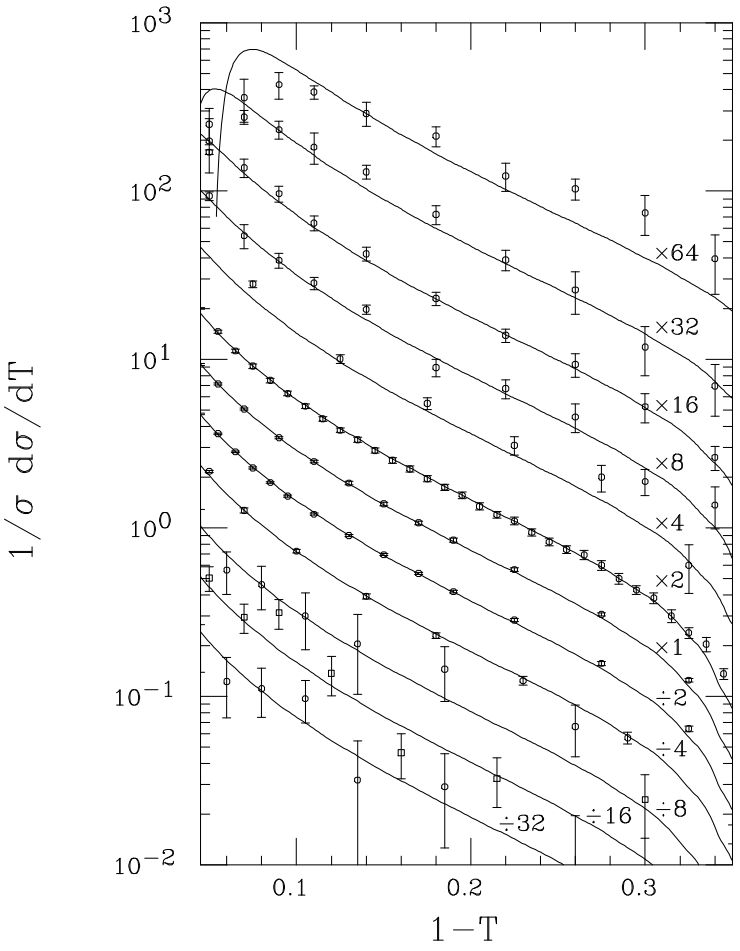

Figure 1: Data on the thrust distribution at various energies, and the prediction (2.9) for $\Lambda^{(5)}_{\overline{\rm MS}} = 0.235$ GeV, $\bar{\alpha}_0 = 0.46$. The twelve data sets are as listed in Table 1 (from top to bottom, multiplied by the factors indicated).

3 Relation to dispersive approach

Here we study the non-perturbative contribution to the thrust distribution from the viewpoint of the dispersive approach proposed in ref. [6]. As discussed there and in ref. [7], the effects of soft gluons on event shape variables are equivalent to a running effective coupling only after certain kinematic approximations, which we can clarify using this approach.

For the kinematics, we take p_+ and p_- along the initial quark and antiquark directions and set $2(p_+p_-) = 1$ (i.e. we measure all momenta in units of Q in this section). The light-cone (Sudakov) decomposition of the momentum of an object with mass m is then

$$k = zp_+ + \alpha p_- + k_\perp\ , \qquad \alpha = \frac{k_\perp^2 + m^2}{z}. \tag{3.1}$$

3.1 Exponentiation of "massive" soft gluons

Each final (massless) soft parton f contributes $\min\{\alpha_f,\, z_f\}$ to $(1-T)$. This statement is based on neglecting quark recoil: $k_{\perp f}^2$ as compared with $\alpha_f = k_{\perp f}^2/z_f$.

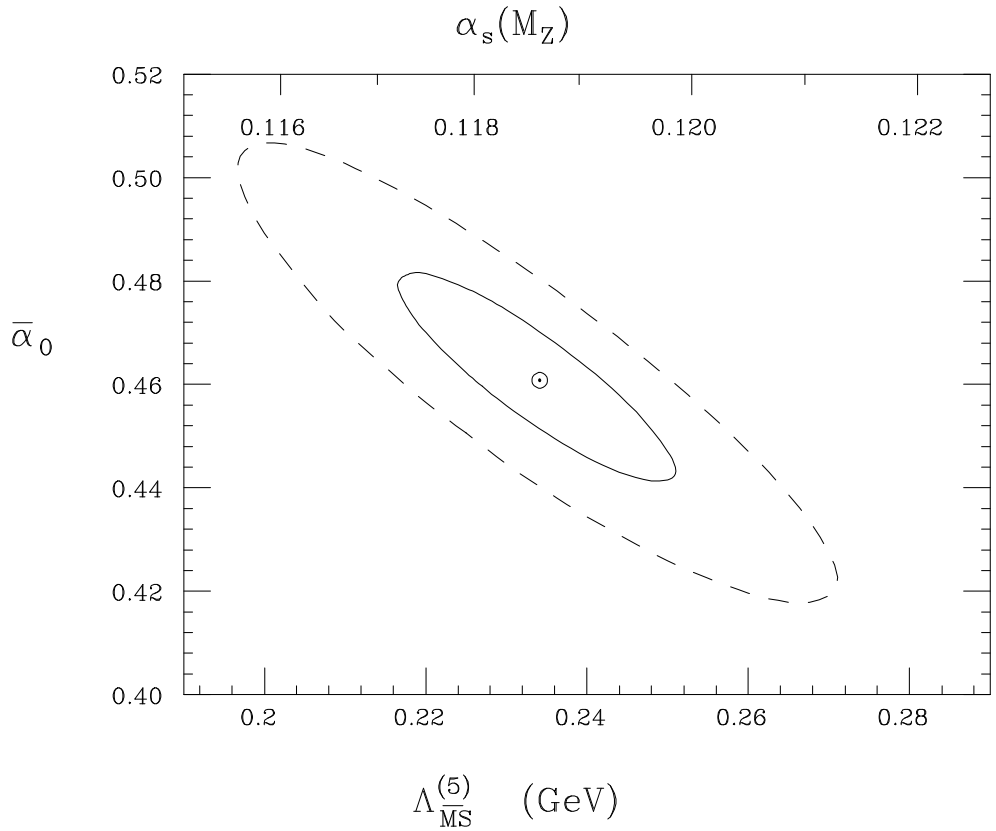

Figure 2: Best fit values and 95% confidence region for the two fitted parameters. Solid/dashed ellipses: including/excluding 14 GeV data.

The first step consists of assembling final partons into subjets generated by primary gluon radiation off the quark-antiquark line and substituting a virtual ("massive") gluon i for each subjet. It is implied in doing so that all the secondary partons belonging to a given gluon subjet have the same sign of $z_f - \alpha_f$, that is, they lie in the same hemisphere. We refer to this as the *moving-along* assumption. If it is true (say, $\alpha_f < z_f$, i.e. right-movers), the total sum $\sum_{f(i)} \alpha_f = \alpha_i$ can be attributed to the primary gluon i. (Notice that a massive parton contributes $\min\{\alpha_i, z_i\}$ to $1-T$ as well as a massless one.) This makes internal jet structure insignificant and the problem essentially Abelian.

Interchanging the definitions of z and α for the *left-moving* virtual gluons, so as to have $\min\{\alpha_i, z_i\} \equiv \alpha_i$, we apply the identity

$$\delta(1 - T - \sum_i \alpha_i) = \int \frac{d\nu}{2\pi i}\, e^{\nu(1-T)} \prod_i e^{-\nu\alpha_i} \ . \tag{3.2}$$

This, together with the factorized matrix element for multiple soft gluon radiation, results in a *factorized* i-dependence and makes exponentiation straighforward:

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = \int \frac{d\nu}{2\pi i}\, \exp\left\{\nu(1-T) + R(\nu)\right\} \ . \tag{3.3}$$

The radiator function $R(\nu)$ is obtained from the single soft gluon emission probability

$$\int \frac{dz}{z} \int \frac{d\alpha}{\alpha}\, \frac{C_F}{\pi} \int dm^2 \left\{ -\frac{1}{\pi} \Im \left[\frac{\alpha_s(-m^2)}{m^2 + i\epsilon} \right] \right\} \ , \tag{3.4}$$

with $m^2 \geq 0$ the gluon invariant mass-squared. Here for the sake of simplicity we have kept only the double-logarithmic part, corresponding to the main term $A(\alpha_s)$ in (2.2).

The crucial moving-along assumption may be correct for sufficiently *small* gluon angles (say, $z_i > \kappa^2 \alpha_i$ with $\kappa \sim e$) but obviously fails for $z_i \simeq \alpha_i$ ($k_{||i} \simeq 0$, "transversal" gluons). We have to assume these do not modify the *nature* of the leading power correction though they may contribute to its magnitude.

Hereafter we introduce the arbitrary kinematical cut κ, to quantify their effect. It should disappear after a full treatment of a large-angle gluon emission with decay products falling into opposite hemispheres.

3.2 Running coupling in the radiator

The exponent formally contains two pieces: the "real" gluon contribution and that from a cut virtual one:

$$R(\nu) = \frac{C_F}{\pi} \int_0^1 dm^2 \left\{ \alpha_s(0)\delta(m^2) + \frac{\rho(m^2)}{m^2} \right\} \int_{m\kappa}^1 \frac{dz}{z} \int_{m^2/z}^{z/\kappa^2} \frac{d\alpha}{\alpha} \left[e^{-\nu\alpha} - 1 \right] . \tag{3.5}$$

The lower limit $\alpha > m^2/z$ comes from $k_\perp^2 > 0$. The upper limit $\alpha < z/\kappa^2$ is the moving-along condition enhanced by the factor $\kappa > 1$ as discussed above, and so is the lower limit in z: $z/\kappa^2 > \alpha > m^2/z$. It is the latter that brings in non-analyticity in m^2, which is crucial for generating power corrections [2,6].

To simplify the analysis we may differentiate with respect to ν and perform the α integration:

$$-\nu R_\nu(\nu) = \frac{C_F}{\pi} \int_0^1 dm^2 \left\{ \alpha_s(0)\delta(m^2) + \frac{\rho(m^2)}{m^2} \right\} \int_{m\kappa}^1 \frac{dz}{z} \left[e^{-\nu m^2/z} - e^{-\nu z/\kappa^2} \right] . \tag{3.6}$$

The next step is to integrate with respect to m^2 by parts using (see ref. [6])

$$dm^2 \frac{\rho(m^2)}{m^2} = d\alpha_{\rm eff}(m^2) . \tag{3.7}$$

This gives

$$(\alpha_s(0) - \alpha_{\rm eff}(0)) \int_0^1 \frac{dz}{z} \left[1 - e^{-\nu z} \right] - \int_0^1 dm^2 \alpha_{\rm eff}(m^2) \int_{m\kappa}^1 \left(-\frac{\nu}{z} \right) \frac{dz}{z} e^{-\nu m^2/z} . \tag{3.8}$$

The first term vanishes identically since

$$\alpha_s(k^2) = -\int_0^\infty \frac{dm^2}{m^2 + k^2} \rho(m^2) ,$$
$$\alpha_s(0) = -\int_0^\infty \frac{dm^2}{m^2} \rho(m^2) = -\int_0^\infty d\alpha_{\rm eff}(m^2) = \alpha_{\rm s}(\infty) - \alpha_{\rm eff}(0) = \alpha_{\rm eff}(0) .$$

Performing the z-integration we finally arrive at

$$-\nu R_\nu(\nu) = \frac{C_F}{\pi} \int_0^1 \frac{dm^2}{m^2} \alpha_{\rm eff}(m^2) \left[e^{-\nu m^2} - e^{-\nu m/\kappa} \right] . \tag{3.9}$$

When evaluating (3.9) perturbatively, we do not distinguish between $\alpha_{\rm eff}$ and $\alpha_{\rm s} = \alpha_{\rm eff}(1 + \mathcal{O}(\alpha_{\rm s}^2))$. This results in the above expression (2.2) with the leading term $A(\alpha_{\rm s})$ in the *physical scheme* of refs. [9,25] for the coupling. The next-to-leading term $B(\alpha_{\rm s})$ is also easy to reproduce by keeping non-soft contributions in the elementary radiation probability (3.4).

In the non-perturbative region $m^2 < \mu_{\rm I}^2$ we can expand the exponentials to obtain the result in Eq. (2.9) with the shift expressed in terms of the first non-analytic moment of the effective coupling (A_1 in the notation of ref. [6]). The only difference from Eq. (2.5) is an overall factor of $1/\kappa$, indicating that the magnitude of the shift is sensitive to the soft, large-angle region of gluon emission.

4 Discussion

We see from the above discussion that the non-analyticity in m^2 in Eq. (3.9) can be traced back to the very kinematical region $z \sim \alpha \sim m$ which does not respect the crucial moving-along assumption. This does not affect the nature of the leading power correction but it could affect its magnitude, since reducing the contribution of this region scales down the shift δT by the factor κ which we have introduced to quantify sensitivity to large-angle gluon radiation. In the large-angle region one can expect an essential modification of the inclusive spectral density ρ due to the specific kinematics of the thrust: the "decay" products of a timelike virtual gluon in this region may make different contributions to the event shape, depending on the kinematics of the decay. In the large-N_f model studies of ref. [7], this effect was not found to be large for the thrust, but it does depend on the shape variable involved.

In the case of other jet shape variables which have the property of exponentiation, such as the heavy jet mass [26] and jet broadening [27], we expect $1/Q$ corrections to be generated by the same mechanism. However, we found that for these quantities the leading non-perturbative effect is not well represented by a simple shift in the distribution. Furthermore, the modifications due to the large-angle region discussed above will be different for different jet shape observables. Thus for an extension to other related jet observables a quantitative analysis of the large-angle region has to be pursued.

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