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UNITARITY AND SATURATION IN THE DIPOLE FORMULATION[a]

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This talk reviews briefly some of the main results of the small-x dipole formulation with regards to unitarity corrections. It illustrates the correspondence between unitarity and saturation corrections in the dipole approach and multiple t-channel pomeron exchange in the traditional BFKL view, and discusses how one can estimate and understand the effects of saturation.

The dipole formulation[1,2] is an approach to small-x physics which for total cross sections is equivalent to the BFKL approach[3] and which has been used in a variety of phenomenological studies.[4] It is reviewed in the talk by Webber.[5] in the limit of a large number of colours, the small-x heavy quarkonium (onium) light-cone wave function can be represented as a chain of colour dipoles stretching in impact parameter between the quark and antiquark. The interaction between onia is then due to the independent scattering of the dipoles in the two wave functions. To a first approximation, in any given collision only one dipole from each onium is involved in the scattering. This leads to an amplitude which violates the unitarity bound. The solution is to take into account multiple scatterings[1] (or equivalently multiple pomeron exchange) and the resulting amplitude does satisfy the unitarity bound. The unitarity corrections are much stronger for the elastic cross section than for the total cross section,[6] because the former is dominated by more central impact parameters, where the amplitude, and therefore the multiple scattering corrections, are largest. Other significant points are (i) that the multiple-pomeron series does not give a convergent sum — it is necessary to sum over multiple scatterings *before* averaging over onium configurations — and (ii) that the eikonal approximation fails very badly, because mean properties of the onium wave function are very unrepresentative of the configurations which are typically involved in scattering.[1,6]

[a] Talk at DIS96, International Workshop on Deep Inelastic Scattering and Related Phenomena, Rome, April 1996.

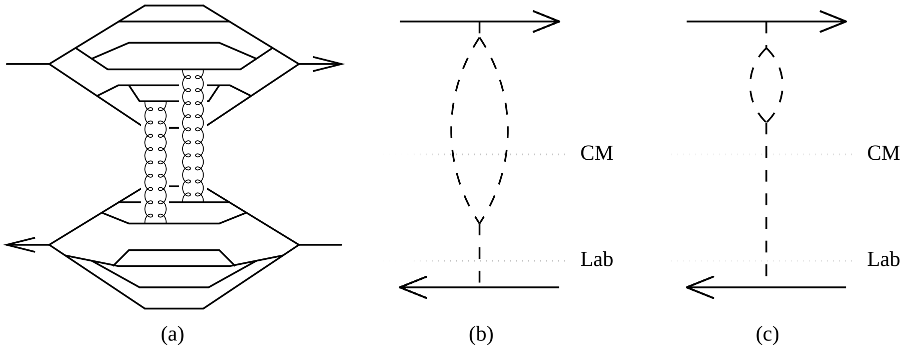

Figure 1: Two-pomeron contributions to onium-onium scattering in the dipole (a) and t-channel (b,c) views. Solid lines are dipoles, dashed lines are pomerons and dotted lines represent the division of rapidity (increasing from top to bottom) in different frames.

The connection between the dipole and the traditional t-channel pictures of high-energy scattering is illustrated for centre-of-mass (CM) scattering in fig. 1; (a) shows the evolution of the dipole structure (solid lines) for the squared wave functions of two onia. Single scattering would be caused by the exchange of a colour-neutral pair of gluons between a dipole in each onium. Here, double scattering occurs. The equivalent t-channel diagram is illustrated in (b). Everything in (b) above the dotted line labelled CM relates to evolution of the upper onium of (a), and things below it are associated with the lower onium. In the upper onium, the two dipoles involved in the scattering have a common origin early in the branching, associated with the early $1 \to 2$ pomeron vertex in (b). The dipoles involved in the scattering in the lower onium have a common origin midway through the branching, so the $2 \to 1$ vertex in (b) occurs midway down the lower part of the diagram. The critical point is that going from the top down, the 'branching out' ($1 \to 2$ vertex) is associated with the upper onium, and the 'branching in' ($2 \to 1$ vertex) with the lower onium. Diagrams such as (c) where the 'branching in' occurs in the upper onium are not included — they would correspond to a pair of dipoles in the upper onium interacting with each other and are referred to as saturation.[7]

One can argue that saturation contributions can be neglected:[1] the only difference between (b) and (c) is the rapidity range over which the two pomerons evolve. If the total rapidity is $Y = \ln s$, then there is a range of order $Y/2$ over which (c) has only one pomeron whereas (b) has two; so (c) should be smaller than (b) by a factor or order $\mathrm{e}^{-(1-\alpha_{\mathcal{P}})Y/2}$, where $(1-\alpha_{\mathcal{P}}) = 4\ln 2\alpha_s N_c/\pi$ is the usual BFKL power. This is a very qualitative argument and it would be

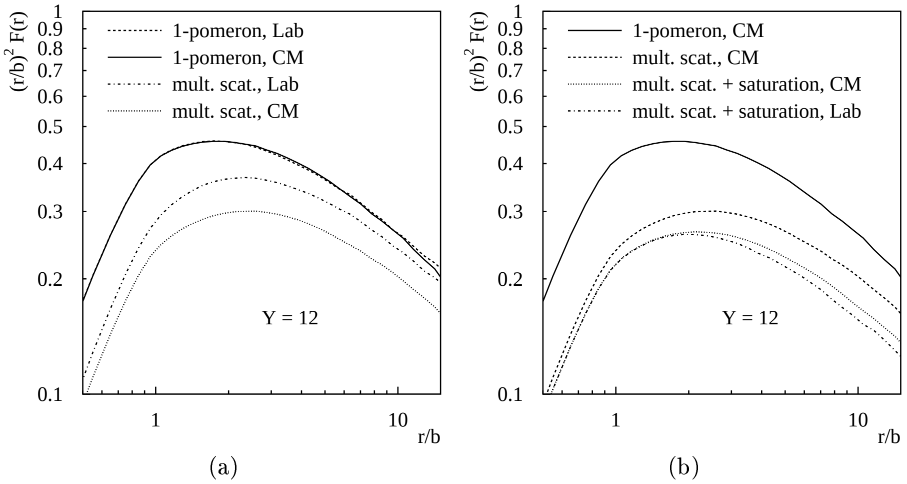

Figure 2: The elastic scattering amplitude $F(r)$ for two onia of size b, separated in transverse position by a distance r. Results obtained using OEDIPUS.[8,9]

interesting to check its validity and also to obtain some understanding of the behaviour of the wave functions once saturation is taken into account. Unfortunately an equivalent of the GLR equation[7] does not currently exist for the dipole approach. But a technique which allows one to extract considerable information about saturation is to make use of the requirement of Lorentz invariance of the scattering amplitude. In a (almost) lab frame, where the lower onium is (almost) stationary, the upper onium contains most of the evolution and graphs such as (b) of fig. 1 (saturation in the lab frame) are not included by a calculation of multiple scattering. So one expects the multiple-scattering corrections to be smaller in the lab frame than in the CM frame. This is borne out by fig. 2a (note that for 1-pomeron exchange, the amplitude is frame-invariant). Roughly, the effect of saturation must just be to change the lab frame result so that it is equal to the CM result.

More precisely one can make a guess for the effect of saturation on the dipole evolution and then tune it so as to obtain a scattering amplitude which is frame independent for any onium-onium scattering. To simulate the effect of saturation, one multiplies the evolution rate of a dipole i by a factor $S_i(\rho_i)$, where ρ_i is the local density of dipoles. S_i is chosen so as to ensure the frame-independence of the scattering amplitude, and a suitable form is

$$S_i = \frac{1 - \mathrm{e}^{-2\Omega\alpha_s^2\rho_i}}{2\Omega\alpha_s^2\rho_i} = 1 - \Omega\alpha_s^2\rho_i + O(\alpha_s^4\rho_i^2), \tag{1}$$

where Ω is a constant of order 1. Note that for small dipole densities, the effect of saturation is linear in the local dipole density (other forms which are linear work equally well and give similar results — forms which are non-linear in the dipole density fail). Examining fig. 2(b) one sees that with the inclusion of saturation, the amplitude is now the same in lab and CM frames (though there is a small discrepancy at large r).

At small r, as expected, the effect of saturation is negligible in the CM frame. But at large r, the effect of saturation is as large as that of multiple scattering: in dipole language, what occurs is that multiple scattering tends to happen on the large scale where dipoles are dilute, whereas saturation is also affected by the earlier stages of the evolution, on small and moderate scales, where dipole densities are higher. The argument for the suppression of saturation given earlier did not take into account the complications that can arise when there is an interplay between two different transverse scales (in this case, r and b).

Nevertheless for the total cross section, which is dominated by moderate impact parameters, these studies suggest that saturation can be neglected compared to multiple scattering. But in situations where two different scales play a rôle, such as large-impact parameter scattering, or DIS, it is to be expected that saturation effects may be comparable to those of multiple scattering.

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