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Power Corrections and Renormalons in e^+e^- Fragmentation Functions*

M. Dasgupta and B.R. Webber
Cavendish Laboratory, University of Cambridge,
Madingley Road, Cambridge CB3 0HE, U.K.

Abstract

We estimate the power corrections (infrared renormalon contributions) to the coefficient functions for the transverse, longitudinal and asymmetric fragmentation functions in e^+e^- annihilation, using a method based on the analysis of one-loop Feynman graphs containing a massive gluon. The leading corrections have the expected $1/Q^2$ behaviour, but the gluonic coefficients of the longitudinal and transverse contributions separately have strong singularities at small x, which cancel in their sum. This leads to $1/Q$ corrections to the longitudinal and transverse parts of the annihilation cross section, which cancel in the total cross section.

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1 Introduction

Experimental studies of the single-hadron inclusive spectrum in the e^+e^- annihilation process $e^+e^- \to \gamma^*/Z^0 \to hX$ have been performed with high precision over a wide range of energies for various types of produced hadrons h (see Refs. [1–4] and references therein). These studies have mostly concerned the total fragmentation function,

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dx}(e^+e^- \to hX) \equiv F_{\rm tot}(x,Q^2)\;, \tag{1.1}$$

where $x = 2Q\cdot p_h/Q^2$ is the energy fraction of the observed hadron and Q^μ is the virtual boson four-momentum, equal to the overall centre-of-mass momentum in this process. The most precise data are those for the total fragmentation function into charged hadrons. The scaling violations (logarithmic Q^2-dependence) in this quantity predicted by perturbative QCD [5–8] have been observed and used to measure the strong coupling constant [2,3].

The ALEPH [3] and OPAL [4] collaborations at LEP have also presented results on the joint distribution in the energy fraction x and the angle θ between the observed hadron h and the incoming electron beam. We can write [8]

$$\frac{1}{\sigma_{\rm tot}}\frac{d^2\sigma}{dx\,d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\,F_{\rm T}(x,Q^2) + \frac{3}{4}\sin^2\theta\,F_{\rm L}(x,Q^2) + \frac{3}{4}\cos\theta\,F_{\rm A}(x,Q^2)\;, \tag{1.2}$$

where $F_{\rm T}$, $F_{\rm L}$ and $F_{\rm A}$ are respectively the transverse, longitudinal and asymmetric fragmentation functions. As their names imply, $F_{\rm T}$ and $F_{\rm L}$ represent the contributions from virtual bosons polarized transversely or longitudinally with respect to the direction of motion of the observed hadron. $F_{\rm A}$ is a parity-violating contribution which comes from the interference between the Z^0 and photon contributions. Integrating over all angles, we obtain the total fragmentation function (1.1), $F_{\rm tot} = F_{\rm T} + F_{\rm L}$.

According to the factorization theorems of QCD [9], each of the functions $F_{\rm P}$ (P = tot,T,L or A) can be represented as a convolution of universal parton fragmentation functions D_i ($i = q, \bar{q}$ or g) with perturbatively calculable coefficient functions $C^i_{\rm P}$ [6–8]:

$$F_{\rm P}(x,Q^2) = \sum_i \int_x^1 \frac{dz}{z} C^i_{\rm P}(z)\,D_i(x/z,Q^2)\;. \tag{1.3}$$

The parton fragmentation functions themselves cannot be computed perturbatively, although their logarithmic Q^2-dependence, which is the main source of scaling violation, is predicted by the QCD evolution equations [5]. The evolution kernels (splitting functions) differ in non-leading orders [6,7] from those for deep inelastic structure functions. Thus scaling violation in fragmentation provides an important independent test of QCD. The longitudinal and transverse fragmentation functions can be used to measure the gluon fragmentation function [3,4], while the asymmetric part may be useful for the measurement of electroweak couplings [10]. Once measured and parametrized [3,8,11,12], the parton fragmentation functions can also be used to test QCD in jet fragmentation in other processes such as lepton-hadron [13] and hadron-hadron [14] collisions.

In addition to the logarithmic Q^2-dependence predicted by perturbative QCD, it is expected that fragmentation functions will exhibit process-dependent power corrections, i.e. contributions proportional to inverse powers of Q, analogous to the higher-twist contributions found in deep inelastic scattering [15]. An understanding of power corrections is crucial for precision tests of scaling violation. In the deep inelastic case such contributions are related to the hadronic matrix elements of local operators, and it is well established that the leading corrections should be of order $1/Q^2$. The machinery of the local operator product expansion is not applicable to fragmentation, but a study of the relevant non-local operator matrix elements again suggests a $1/Q^2$ behaviour [16]. The current data on the total e^+e^- fragmentation function $F_{\rm tot}$ are consistent with either a $1/Q$ or $1/Q^2$ form for the leading power correction [3]. Data on the separate functions $F_{\rm P}$ for P=T, L and A are at present limited to a single energy $Q = M_Z$ [3,4], and so predictions concerning their scaling violation and power corrections remain to be tested.

In the present paper we estimate the power corrections to fragmentation functions using a recently developed 'dispersive' method based on the infrared properties of Feynman graphs and some assumptions about the strong coupling at low scales [17]. The techniques involved are similar to those involved in the study of infrared renormalons [18–21], which correspond to unsummable divergences of the perturbative expansion. Here we use them as a more general probe of the influence of soft regions of integration on hard process observables. The results obtained by applying these techniques to deep inelastic scattering [17,22,23] are consistent with those of the operator product expansion and look promising phenomenologically.

A calculation of the leading power correction to the total fragmentation function $F_{\rm tot}$ using the dispersive approach was presented in Ref. [17]. A $1/Q^2$ correction was found and the corresponding quark coefficient function was computed. In the present paper we extend these results to subleading $(1/Q^4)$ power corrections, and to the transverse, longitudinal and asymmetric quark fragmentation functions separately. We also compute (in a certain approximation) the corresponding gluonic coefficient functions.

We shall be particularly concerned to clarify an apparent paradox which arises when one considers sum rules for fragmentation. Summed over all particle types, the total fragmentation function satisfies the energy sum rule, which we may write as

$$\frac{1}{2}\int_0^1 dx\, x F_{\rm tot}(x,Q^2) = 1\ . \tag{1.4}$$

Similarly the integrals

$$\frac{1}{2}\int_0^1 dx\, x F_{\rm T,L}(x,Q^2) \equiv \frac{\sigma_{\rm T,L}}{\sigma_{\rm tot}} \tag{1.5}$$

give the transverse and longitudinal fractions of the total cross section. The perturbative prediction is [24]

$$\begin{aligned}\frac{\sigma_{\rm L}}{\sigma_{\rm tot}} = 1-\frac{\sigma_{\rm T}}{\sigma_{\rm tot}} &= \frac{\alpha_s}{\pi} + \left(\frac{601}{40}-\frac{6}{5}\zeta(3)-\frac{37}{36}n_f\right)\left(\frac{\alpha_s}{\pi}\right)^2 + O(\alpha_s^3)\\ &\simeq \frac{\alpha_s}{\pi} + (13.583-1.028n_f)\left(\frac{\alpha_s}{\pi}\right)^2\end{aligned} \tag{1.6}$$

where $\zeta(3) = 1.202$ and n_f is the number of active flavours. Note that the whole of the $\mathcal{O}(\alpha_s)$ correction to $\sigma_{\rm tot}$ comes from the longitudinal part, while the $\mathcal{O}(\alpha_s^2)$ correction receives both longitudinal and transverse contributions.

The OPAL data point [4] for $\sigma_{\rm L}/\sigma_{\rm tot}$ is shown, together with the perturbative predictions, in Fig. 1. The data lie somewhat above the next-to-leading-order prediction (dashed), which suggests that higher-order and/or non-perturbative corrections are significant. An estimate of the latter is provided by the difference between the JETSET Monte Carlo [25] hadron-level prediction (dot-dashed) and the perturbative result. This difference shows a clear $1/Q$ behaviour, with a coefficient

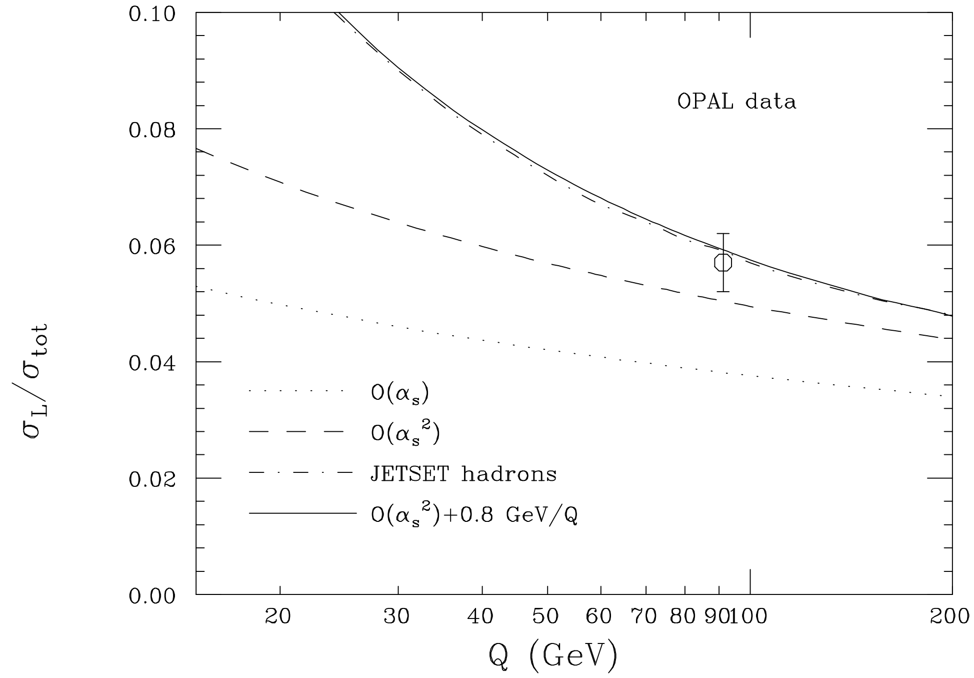

Figure 1: Longitudinal fraction of the e^+e^- hadronic cross section.

of about 1 GeV. Assuming that the same behaviour will be manifest in the data, we have an apparent paradox: the function xF_L has only a $1/Q^2$ correction, but its integral has a $1/Q$ correction.

Power corrections proportional to $1/Q$, usually referred to as hadronization corrections, are typical of hadronic event shapes in e^+e^- annihilation, where they are clearly seen in the data and are predicted by a variety of approaches [26–28]. String-like models of hadronization, for example, in which the energy of a jet is redistributed with proper density $\lambda \sim 500$ MeV per unit rapidity and limited transverse momentum relative to the jet axis, imply a correction to the longitudinal cross section of [8][†]

$$\frac{\delta\sigma_L}{\sigma_{tot}} = \frac{\pi\lambda}{2Q} \simeq \frac{0.8\ \mathrm{GeV}}{Q}\ . \tag{1.7}$$

Adding this to the next-to-leading-order prediction gives the solid curve in Fig. 1, which agrees well with the JETSET prediction. The correction arises from mixing between the transverse and longitudinal angular dependences in Eq. (1.2) due to hadronization. The transverse cross section receives an equal and opposite correction, and so there is no $1/Q$ term in the total cross section.

We shall see that dispersive approach of Ref. [17] leads to the following resolution of the paradox. The $1/Q$ terms arise from soft gluon fragmentation. The gluonic coefficient functions of the $1/Q^{2p}$ power corrections are highly singular at small x. Upon integration they are all 'promoted' to a $1/Q$ behaviour and have to be resummed. The result is a $1/Q$ correction to σ_L/σ_{tot}, with a coefficient similar to that in the hadronization models.

In the remainder of the paper, we first give a brief summary of the relevant assumptions and results from Ref. [17]. The dispersive method is based on the evaluation of one-loop Feynman graphs containing a gluon of finite mass μ. This yields the 'characteristic functions' for the relevant

[†]Note that there is a misprint in the corresponding equation (3.35) of Ref. [8].

quantities, which are given in Sect. 3. We extract the power corrections from the behaviour of the characteristic functions as $\mu \to 0$. The rules for taking this limit are also explained in Sect. 3. In Sect. 4 we give the expressions thus obtained for the power corrections to the fragmentation functions, and to the transverse and longitudinal cross sections. Finally in Sect. 5 we discuss the results and give some numerical predictions.

2 Dispersive method

We do not repeat the discussion of Ref. [17] but simply summarize the results required here. The basic assumption is that the dominant non-perturbative contributions to the coefficient functions C_{P}^i in Eq. (1.3) are of the form

$$\delta C_{\mathrm{P}}^i(x,Q^2) = \int_0^\infty \frac{d\mu^2}{\mu^2}\,\delta\alpha_{\mathrm{eff}}(\mu^2)\dot{\mathcal{F}}_{\mathrm{P}}^i(x,Q^2;\mu^2)\,. \tag{2.1}$$

Here $\delta\alpha_{\mathrm{eff}}(\mu^2)$ is a non-perturbative modification to the effective strong coupling, restricted to the region of low μ^2. $\mathcal{F}_{\mathrm{P}}^i(x,Q^2;\mu^2)$ is the relevant *characteristic function*, that is, the parton fragmentation function computed at order α_s using a non-zero value μ for the gluon mass in the Feynman denominators of the contributing graphs. $\dot{\mathcal{F}}_{\mathrm{P}}^i$ is minus the logarithmic derivative of $\mathcal{F}_{\mathrm{P}}^i$ with respect to μ^2. Since $\mathcal{F}_{\mathrm{P}}^i$ depends only on dimensionless ratios, we may write

$$\mathcal{F}_{\mathrm{P}}^i(x,Q^2;\mu^2) = \mathcal{F}_{\mathrm{P}}^i(x,\epsilon)\,, \qquad \dot{\mathcal{F}}_{\mathrm{P}}^i(x,\epsilon) \equiv -\epsilon\frac{\partial}{\partial\epsilon}\mathcal{F}_{\mathrm{P}}^i(x,\epsilon)\,, \qquad \epsilon \equiv \frac{\mu^2}{Q^2}\,. \tag{2.2}$$

The non-perturbative contributions δC_{P}^i thus depend on the small-ϵ behaviour of $\mathcal{F}_{\mathrm{P}}^i$, which is of the generic form

$$\mathcal{F}_{\mathrm{P}}^i(x,\epsilon) = -P_{\mathrm{P}}^i(x)\ln\epsilon + C_{0,\mathrm{P}}^i(x) - C_{2,\mathrm{P}}^i(x)\epsilon\ln\epsilon - \tfrac{1}{2}C_{4,\mathrm{P}}^i(x)\epsilon^2\ln\epsilon - \cdots\,, \tag{2.3}$$

where the dots indicate terms that are either $\mathcal{O}(\epsilon^3\ln\epsilon)$ or analytic and vanishing at $\epsilon = 0$. Here $P_{\mathrm{P}}^i(x)$ (contributing for P=T,A only) is the $q \to i$ splitting function, $C_{0,\mathrm{P}}^i(x)$ is the relevant perturbative coefficient function (in the gluon mass regularization scheme), and $C_{2,\mathrm{P}}^i(x)$ etc. will be related to non-perturbative corrections.

A crucial point is that, for consistency with the operator product expansion, the integer μ^2-moments of the coupling modification should vanish:

$$\int_0^\infty \frac{d\mu^2}{\mu^2}\mu^{2p}\delta\alpha_{\mathrm{eff}}(\mu^2) \;=\; 0\,, \tag{2.4}$$

at least for the first few moments $p = 1,\ldots,p_{\max} \sim 9$. As a consequence, only those terms in the small-ϵ behaviour of $\dot{\mathcal{F}}_{\mathrm{P}}^i(x,\epsilon)$ that are non-analytic at $\epsilon = 0$ lead to non-perturbative contributions [19]. Thus from the small-ϵ behaviour (2.3) we find

$$\delta C_{\mathrm{P}}^i(x,Q^2) = C_{2,\mathrm{P}}^i(x)\frac{A_2'}{Q^2} + C_{4,\mathrm{P}}^i(x)\frac{A_4'}{Q^4} + \cdots\,, \tag{2.5}$$

where, following Ref. [17], we have defined the log-moment integrals

$$A_{2p}' \;=\; \frac{C_F}{2\pi}\int_0^\infty \frac{d\mu^2}{\mu^2}\,\mu^{2p}\,\ln(\mu^2/\mu_0^2)\;\delta\alpha_{\mathrm{eff}}(\mu^2)\,. \tag{2.6}$$

Notice that since integer μ^2-moments of $\delta\alpha_{\rm eff}$ vanish, these quantities are independent of the scale μ_0^2. For convenience, we extract a universal factor of $C_F/2\pi$ from the characteristic function.

Instead of interpreting the coefficients A'_{2p} in terms of a universal low-energy effective coupling, one may treat them more generally as process-dependent parameters to be determined experimentally. Eq. (2.5) still has predictive power because the coefficient functions $C^i_{2,{\rm P}}(x)$ etc. specify the x-dependence of the power corrections. This dependence is supposed to reflect the relative sensitivity of different regions of x to soft dynamics.

The technique we use to evaluate the coefficient functions of power corrections, viz. extraction of the non-analytic terms in the massive-gluon expressions for observables as $\mu^2 \to 0$, is the same as that applied in studies of infrared renormalons [19]. In the language of renormalons, the terms computed are ambiguities in the perturbative prediction for the observable in question, which have to cancel against corresponding ambiguities in power-suppressed non-perturbative contributions. Here we argue that the non-perturbative contributions themselves should display the same power behaviour and x-dependence, since the small-μ^2 limit probes the sensitivity of an observable to the soft non-perturbative region as a function of x and Q^2.

We expect quark and gluon fragmentation to contribute to power corrections on an equal basis, since the dominant contributions are assumed to be determined at first order in $\alpha_{\rm eff}$. The application of the dispersive method to compute the quark contribution is straightforward since in that case we sum inclusively over all gluon fragmentation products. The inclusive sum generates a contribution which is equivalent to that of a massive gluon, as discussed in ref. [17]. We shall also use the dispersive approach to calculate gluonic contributions, but its application there is more questionable. By definition we observe the fragmentation products of the gluon in that case, which spoils the equivalence to a massive gluon. The situation becomes similar to that for an event shape variable: the correction has the same leading power behaviour as that due to a massive gluon, but the coefficient may be modified. This can be investigated in the large-n_f limit, in which the gluon fragments only into quark-antiquark pairs [28,29]. Although the question requires further study, we assume here that the massive-gluon technique does provide a reasonable estimate of contributions from gluon as well as quark fragmentation.

3 Characteristic functions

The object of central importance in the dispersive method is the characteristic function $\mathcal{F}^i_{\rm P}(x,\epsilon)$ for the emission of a gluon with mass-squared $\mu^2 = \epsilon Q^2$ at the hard scale Q^2. This is computed from the relevant one-loop graphs with a modified gluon propagator. The characteristic function for the total fragmentation function is obtained by contracting the resulting hadronic tensor with the tensor representing a sum over virtual-boson polarization states,

$$\sum_{\rm P} \varepsilon^\mu_{\rm P}\varepsilon^\nu_{\rm P} = -g^{\mu\nu} + \frac{Q^\mu Q^\nu}{Q^2}\,. \tag{3.1}$$

The corresponding tensor for the longitudinal part is $\varepsilon^\mu_{\rm L}\varepsilon^\nu_{\rm L}$ where $\varepsilon^\mu_{\rm L}$ is the polarization vector along the direction of $\mathbf{p}_h$, the three-momentum of the observed hadron in the virtual-boson rest frame:

$$\varepsilon^\mu_{\rm L} = \frac{1}{|\mathbf{p}_h|}\left(p^\mu_h - \frac{Q\cdot p_h\, Q^\mu}{Q^2}\right)\,. \tag{3.2}$$

The characteristic function for the transverse fragmentation function is then obtained as the difference between the total and the longitudinal part. For the asymmetric fragmentation function we take the vector-axial interference term and contract with the tensor

$$-\frac{i}{2}\epsilon_{\mu\nu\sigma\rho}\,\frac{p_h^\sigma Q^\rho}{|\mathbf{p}_h|\,Q}\,. \tag{3.3}$$

The resulting expressions for $\mathcal{F}_{\rm P}^i(x,\epsilon)$ are given below. The power corrections are deduced from the non-analytic terms in the small-ϵ behaviour of the logarithmic derivative $\dot{\mathcal{F}}_{\rm P}^i(x,\epsilon)$. When taking the small-$\epsilon$ limit, one should be careful with the phase-space boundaries, and in particular with functions that are singular on or near these boundaries. We give the rules for obtaining the correct limiting behaviour.

3.1 Quark fragmentation

The phase space for fragmentation into a quark of negligible mass with emission of a gluon of mass-squared ϵQ^2 is $0 < x < 1-\epsilon$. The characteristic function is therefore of the form

$$\mathcal{F}_{\rm P}^q(x,\epsilon) = \mathcal{F}_{\rm P}^{(r)}(x,\epsilon)\,\Theta(1-x-\epsilon) + \mathcal{F}^{(v)}(\epsilon)\,\delta(1-x) \tag{3.4}$$

where $\mathcal{F}^{(v)}$ represents the virtual contribution. For transverse quark fragmentation the real gluon emission part is

$$\begin{aligned}\mathcal{F}_{\rm T}^{(r)}(x,\epsilon) = &\left[\frac{2(1+\epsilon)^2}{1-x} - 1 - x - 2\epsilon + 4\frac{\epsilon}{x} + 6\frac{\epsilon^2}{x^2}\right]\ln\left[\frac{(x+\epsilon)(1-x)}{\epsilon}\right] \\ &- \frac{3-5\epsilon^2}{2(1-x)} + \frac{\epsilon}{(1-x)^2} + \frac{\epsilon^2}{2(1-x)^3} + \frac{\epsilon}{x+\epsilon} - 6\frac{\epsilon(1-\epsilon)}{x} - \frac{1}{2}(1-x) + 3\epsilon\,.\end{aligned} \tag{3.5}$$

The virtual contribution is

$$\mathcal{F}^{(v)}(\epsilon) = 2(1+\epsilon)^2\left[\mathrm{Li}_2(-\epsilon) + \ln\epsilon\ln(1+\epsilon) - \frac{1}{2}\ln^2\epsilon + \frac{\pi^2}{6}\right] - \frac{7}{2} - (3+2\epsilon)\ln\epsilon - 2\epsilon\,, \tag{3.6}$$

where

$$\mathrm{Li}_2(-\epsilon) = -\int_0^\epsilon \frac{dt}{t}\ln(1+t)\,. \tag{3.7}$$

The characteristic function for longitudinal quark fragmentation has only a contribution from real gluon emission, which is

$$\begin{aligned}\mathcal{F}_{\rm L}^q(x,\epsilon) = &-2\frac{\epsilon}{x}\left(2+3\frac{\epsilon}{x}\right)\ln\left[\frac{(x+\epsilon)(1-x)}{\epsilon}\right] \\ &- 2\frac{\epsilon(1+2\epsilon)}{1-x} + \frac{\epsilon^2}{(1-x)^2} + 6\frac{\epsilon(1-\epsilon)}{x} + 1 - 2\epsilon\,.\end{aligned} \tag{3.8}$$

The characteristic function for asymmetric quark fragmentation is of the form (3.4) with the same virtual contribution, but the real gluon emission part becomes

$$\begin{aligned}\mathcal{F}_{\rm A}^{(r)}(x,\epsilon) = &\left[\frac{2(1+\epsilon)^2}{1-x} - 1 - x - 2\epsilon + 2\frac{\epsilon(2+\epsilon)}{x}\right]\ln\left[\frac{(x+\epsilon)(1-x)}{\epsilon}\right] \\ &- \frac{3(1-\epsilon^2)}{2(1-x)} + \frac{\epsilon}{(1-x)^2} + \frac{\epsilon^2}{2(1-x)^3} - \frac{\epsilon}{x+\epsilon} - \frac{3}{2}(1-x) + \epsilon\,.\end{aligned} \tag{3.9}$$

In taking the small-ϵ limits of Eqs. (3.5), (3.8) and (3.9), we must remember that the phase-space boundary is at $x = 1-\epsilon$. Now for any function $F(x)$ that is analytic in a neighbourhood of $x=1$ and any test function $f(x)$, we have

$$\int_0^{1-\epsilon} F(x)\, f(x)\, dx = \int_0^1 F(x)\, f(x)\, dx - \epsilon F(1) f(1) + \tfrac{1}{2}\epsilon^2 [F'(1)f(1) + F(1)f'(1)] + \cdots \; . \tag{3.10}$$

Recalling that

$$\int_0^1 \delta^{(n)}(1-x)\, f(x)\, dx = f^{(n)}(1)\;, \tag{3.11}$$

we can make the ϵ-dependence explicit, up to terms of order ϵ^2, by replacing any expression $F(x)$ analytic at $x=1$ as follows:

$$F(x) \to F(x) - \epsilon[F(1) - \tfrac{1}{2}\epsilon F'(1)]\,\delta(1-x) + \tfrac{1}{2}\epsilon^2 F(1)\,\delta'(1-x)\;. \tag{3.12}$$

For expressions that are singular at $x=1$, we define '+', '++' and '+++' prescriptions such that, for any test function $f(x)$,

$$\begin{aligned} \int_0^1 F(x)_+\, f(x)\, dx &= \int_0^1 F(x)\,[f(x) - f(1)]\, dx \\ \int_0^1 F(x)_{++}\, f(x)\, dx &= \int_0^1 F(x)\,[f(x) - f(1) + (1-x)f'(1)]\, dx \\ \int_0^1 F(x)_{+++}\, f(x)\, dx &= \int_0^1 F(x)\,[f(x) - f(1) + (1-x)f'(1) - \tfrac{1}{2}(1-x)^2 f''(1)]\, dx\;. \end{aligned} \tag{3.13}$$

Using Eq. (3.11), we can now replace the singular terms, up to terms of order ϵ^2, as follows:[‡]

$$\begin{aligned} \frac{1}{1-x} &\to \frac{1}{(1-x)_+} - \ln\epsilon\,\delta(1-x) + \epsilon\,\delta'(1-x) - \tfrac{1}{4}\epsilon^2\,\delta''(1-x) \\ \frac{\ln(1-x)}{1-x} &\to \left(\frac{\ln(1-x)}{1-x}\right)_+ - \tfrac{1}{2}\ln^2\epsilon\,\delta(1-x) + \epsilon(\ln\epsilon - 1)\,\delta'(1-x) - \tfrac{1}{4}\epsilon^2(\ln\epsilon - \tfrac{1}{2})\,\delta''(1-x) \\ \frac{\epsilon}{(1-x)^2} &\to \frac{\epsilon}{(1-x)^2_{++}} + (1-\epsilon)\delta(1-x) + \epsilon\ln\epsilon\,\delta'(1-x) - \tfrac{1}{2}\epsilon^2\,\delta''(1-x) \\ \frac{\epsilon^2}{(1-x)^3} &\to \frac{\epsilon^2}{(1-x)^3_{+++}} + \tfrac{1}{2}(1-\epsilon^2)\,\delta(1-x) - \epsilon(1-\epsilon)\,\delta'(1-x) - \tfrac{1}{2}\epsilon^2\ln\epsilon\,\delta''(1-x)\,. \end{aligned} \tag{3.14}$$

Applying these rules to Eqs. (3.5) etc., we obtain expressions of the form (2.3). The coefficient of $-\ln\epsilon$ in $\mathcal{F}^{(r)}_{\mathrm{T,A}}$ is the quark splitting function $P_{qq}(x) = (1+x^2)/(1-x)$, which is singular for $x\to 1$. The singularity is regularized by including the virtual contribution. As explained in Ref. [17], this term produces the logarithmic scaling violation in the quark fragmentation function. The second term $C^q_{0,\mathrm{P}}(x)$ is the perturbative coefficient function (in the gluon mass regularization scheme). The remaining terms generate power corrections of the form (2.5), which we list in Sect. 4.

3.2 Gluon fragmentation

The characteristic functions for gluon fragmentation depend on the quantity $\rho = \sqrt{x^2 - 4\epsilon}$ and are defined in the phase-space region $2\sqrt{\epsilon} \le x \le 1+\epsilon$. Naturally, they have only real gluon emission

[‡]Note that these rules are different from those given in Ref. [22] for the deep inelastic case, since the phase space is different.

contributions. For transverse gluon fragmentation we find

$$\begin{aligned}\mathcal{F}_{\rm T}^{g}(x,\epsilon) = \frac{4}{x}\left[(1-x+2\epsilon-\epsilon x+\epsilon^2)\left(1+\frac{2\epsilon}{\rho^2}\right)+\frac{1}{2}x^2\right]\ln\left[\frac{(x+\rho)^2}{4\epsilon}\right] \\ -\frac{4}{\rho}(1-x+2\epsilon-\epsilon x+\epsilon^2)-4\rho\ ,\end{aligned} \tag{3.15}$$

and for longitudinal gluon fragmentation

$$\mathcal{F}_{\rm L}^{g}(x,\epsilon) = \frac{4}{\rho}(1-x+2\epsilon-\epsilon x+\epsilon^2)\left(1-\frac{2\epsilon}{\rho x}\ln\left[\frac{(x+\rho)^2}{4\epsilon}\right]\right)\ . \tag{3.16}$$

There is no gluonic contribution to the asymmetric fragmentation function.

Since the upper phase-space boundary is now at $x = 1+\epsilon$, the rule (3.12) for obtaining the small-ϵ behaviour of expressions that are analytic at $x = 1$ becomes

$$F(x) \to F(x) + \epsilon[F(1) + \tfrac{1}{2}\epsilon F'(1)]\,\delta(1-x) + \tfrac{1}{2}\epsilon^2 F(1)\,\delta'(1-x)\ . \tag{3.17}$$

There are no gluonic contributions that are singular at $x = 1$. Instead, the lower phase-space boundary, $x = 2\sqrt{\epsilon}$, is ϵ-dependent, and there are terms that are singular at or near this boundary. However, for any finite x the region of integration in Eq. (1.3) does not extend to the lower phase-space boundary, and so it and the nearby singularities are irrelevant. Thus for any finite x we can safely expand $\rho = x - 2\epsilon/x - \cdots$ and use Eq. (3.17) to obtain the small-ϵ limits of Eqs. (3.15) and (3.16).

When taking moments of the fragmentation functions, on the other hand, we integrate all the way down to $x = 0$. For sufficiently high moments the contribution of the small-x region is suppressed and the above procedure will still be reliable. For lower moments, the x-integration must be performed first, and then the small-ϵ limit can be taken. We shall see that the phase-space boundary and singularities at small x can play a crucial rôle in this case.

4 Power corrections

4.1 Quark fragmentation

The coefficients in Eq. (2.5) for the first two power corrections to the transverse quark coefficient function are found from Eqs. (3.4)–(3.6) to be

$$\begin{aligned}C_{2,\rm T}^{q}(x) &= \frac{4}{(1-x)_+} - 2 + \frac{4}{x} + 2\,\delta(1-x) - \delta'(1-x) \\ C_{4,\rm T}^{q}(x) &= \frac{4}{(1-x)_+} + \frac{12}{x^2} + 5\,\delta(1-x) + \frac{1}{2}\,\delta''(1-x)\ .\end{aligned} \tag{4.1}$$

The corresponding expressions in moment space, defined by

$$\tilde{C}(N) = \int_0^1 x^{N-1}\,C(x)\,dx \tag{4.2}$$

are

$$\tilde{C}^q_{2,\mathrm{T}}(N) = -N + 3 - \frac{2}{N} + \frac{4}{N-1} - 4S_1$$
$$\tilde{C}^q_{4,\mathrm{T}}(N) = \frac{1}{2}N^2 - \frac{3}{2}N + 6 + \frac{12}{N-2} - 4S_1\ , \tag{4.3}$$

with

$$S_1 = \sum_{j=1}^{N-1}\frac{1}{j} = \psi(N) + \gamma_E = \ln N + \mathcal{O}(1/N)\,. \tag{4.4}$$

For the longitudinal quark contribution, the corresponding results are

$$C^q_{2,\mathrm{L}}(x) = -\frac{4}{x} - 2\,\delta(1-x)$$
$$C^q_{4,\mathrm{L}}(x) = -\frac{12}{x^2} - 8\,\delta(1-x) - 2\delta'(1-x)\ , \tag{4.5}$$

$$\tilde{C}^q_{2,\mathrm{L}}(N) = -2 - \frac{4}{N-1}$$
$$\tilde{C}^q_{4,\mathrm{L}}(N) = -2N - 6 - \frac{12}{N-2}\ . \tag{4.6}$$

For the asymmetric coefficient function, which receives only a quark contribution,

$$C^q_{2,\mathrm{A}}(x) = \frac{4}{(1-x)_+} - 2 + \frac{4}{x} + 2\,\delta(1-x) - \delta'(1-x)$$
$$C^q_{4,\mathrm{A}}(x) = \frac{4}{(1-x)_+} + \frac{4}{x} + 3\,\delta(1-x) + \frac{1}{2}\,\delta''(1-x)\ , \tag{4.7}$$

$$\tilde{C}^q_{2,\mathrm{A}}(N) = -N + 3 - \frac{2}{N} + \frac{4}{N-1} - 4S_1$$
$$\tilde{C}^q_{4,\mathrm{A}}(N) = \frac{1}{2}N^2 - \frac{3}{2}N + 4 + \frac{4}{N-1} - 4S_1\ . \tag{4.8}$$

Thus the $1/Q^2$ corrections to the transverse quark and asymmetric coefficient functions are the same, but the $1/Q^4$ corrections (and higher power corrections) are slightly different.

Note that the expressions given above for the moment coefficients $\tilde{C}^q_{2p,\mathrm{P}}$ with P=T,L are only correct for $N > p$. As discussed earlier, for lower moments the low-x singularities of the characteristic functions have to be taken into account, and the singularity structure in ϵ becomes different. An important case is that of the $N = 2$ moments, which define the contributions to the transverse and longitudinal cross sections. This will be examined more fully in Sect. 4.3.

4.2 Gluon fragmentation

The coefficients in Eq. (2.5) for the first two power corrections to the transverse gluon coefficient function are found from Eq. (3.15) to be

$$C^g_{2,\mathrm{T}}(x) = -4 + \frac{8}{x} - \frac{8}{x^2} + \frac{8}{x^3} + 2\,\delta(1-x)$$
$$C^g_{4,\mathrm{T}}(x) = \frac{8}{x} - \frac{16}{x^2} + \frac{32}{x^3} - \frac{64}{x^4} + \frac{64}{x^5} + 6\,\delta(1-x) + 2\,\delta'(1-x)\ , \tag{4.9}$$

$$
\begin{aligned}
\tilde{C}^g_{2,\mathrm{T}}(N) &= 2 - \frac{4}{N} + \frac{8}{N-1} - \frac{8}{N-2} + \frac{8}{N-3} \\
\tilde{C}^g_{4,\mathrm{T}}(N) &= 2N + 4 + \frac{8}{N-1} - \frac{16}{N-2} + \frac{32}{N-3} - \frac{64}{N-4} + \frac{64}{N-5} \,.
\end{aligned}
\tag{4.10}
$$

For longitudinal gluon fragmentation, the corresponding results are

$$
\begin{aligned}
C^g_{2,\mathrm{L}}(x) &= \frac{8}{x^2} - \frac{8}{x^3} \\
C^g_{4,\mathrm{L}}(x) &= \frac{16}{x^2} - \frac{32}{x^3} + \frac{64}{x^4} - \frac{64}{x^5} \,,
\end{aligned}
\tag{4.11}
$$

$$
\begin{aligned}
\tilde{C}^g_{2,\mathrm{L}}(N) &= \frac{8}{N-2} - \frac{8}{N-3} \\
\tilde{C}^g_{4,\mathrm{L}}(N) &= \frac{16}{N-2} - \frac{32}{N-3} + \frac{64}{N-4} - \frac{64}{N-5} \,.
\end{aligned}
\tag{4.12}
$$

The above expressions for the gluonic moment coefficients $\tilde{C}^g_{2p,\mathrm{P}}$ are only valid for $N > 2p+1$. For gluonic moments with $N \le 2p+1$ the phase-space boundary and low-x singularities of the characteristic functions become important and change the singularity structure in ϵ, as will be illustrated in detail for $N = 2$ below.

4.3 Transverse and longitudinal cross sections

Defining the characteristic functions for the various contributions to the cross section as $\mathcal{R}^i_{\mathrm{P}}(\epsilon)$ for P=T,L and $i = q, \bar{q}, g$, such that

$$
\begin{aligned}
\sigma_{\mathrm{P}} &= \sigma^q_{\mathrm{P}} + \sigma^{\bar{q}}_{\mathrm{P}} + \sigma^g_{\mathrm{P}} \;=\; 2\sigma^q_{\mathrm{P}} + \sigma^g_{\mathrm{P}} \,, \\
\sigma^i_{\mathrm{P}} &= \sigma_0 \left[\frac{1}{2}(\delta_{iq} + \delta_{i\bar{q}})\,\delta_{\mathrm{PT}} + \frac{C_F}{2\pi} \int_0^\infty \frac{d\mu^2}{\mu^2}\, \alpha_{\mathrm{eff}}(\mu^2)\, \dot{\mathcal{R}}^i_{\mathrm{P}}(\epsilon = \mu^2/Q^2) \right]
\end{aligned}
\tag{4.13}
$$

where σ_0 is the Born cross section, we would expect from Eq. (2.5) that the corresponding power corrections would take the form

$$
\delta\sigma^i_{\mathrm{P}} = \frac{1}{2}\sigma_0 C_F \frac{\alpha_s}{2\pi} \int_0^\infty \frac{d\mu^2}{\mu^2} \ln\mu^2 \,\delta\alpha_{\mathrm{eff}}(\mu^2) \left[\tilde{C}^i_{2,\mathrm{P}}(2)\frac{\mu^2}{Q^2} + \tilde{C}^i_{4,\mathrm{P}}(2)\frac{\mu^4}{Q^4} + \cdots \right]
\tag{4.14}
$$

where the coefficients $\tilde{C}^i_{2,\mathrm{P}}(2)$ and $\tilde{C}^i_{4,\mathrm{P}}(2)$ are the $N = 2$ moment coefficients given above. This works more or less as expected for the quark contributions, yielding $1/Q^{2p}$ power corrections, modulo logarithms. The expressions given for $\tilde{C}^q_{4,\mathrm{P}}(N)$ in Eqs. (4.3) and (4.6) have a pole at $N = 2$; this singularity corresponds to the logarithmic divergence of the x-moments of the expressions for $C^q_{4,\mathrm{P}}(x)$ in Eqs. (4.1) and (4.5). Performing the x-integrations first, and then taking the small-ϵ limit, we find that the quark characteristic functions are in fact

$$
\begin{aligned}
\mathcal{R}^q_{\mathrm{T}}(\epsilon) &= \frac{2}{3}\ln\epsilon + \frac{22}{9} + \frac{3}{2}\epsilon^2\ln\epsilon(\ln\epsilon - 1) + \frac{1}{3}\epsilon^3\ln\epsilon + \cdots \,, \\
\mathcal{R}^q_{\mathrm{L}}(\epsilon) &= \frac{1}{4} + 3\epsilon\ln\epsilon - \frac{3}{2}\epsilon^2\ln\epsilon(\ln\epsilon - 3) + \cdots \,, \\
\mathcal{R}^q_{\mathrm{tot}}(\epsilon) &= \frac{2}{3}\ln\epsilon + \frac{97}{36} + 3\epsilon\ln\epsilon + 3\epsilon^2\ln\epsilon + \frac{1}{3}\epsilon^3\ln\epsilon + \cdots \,.
\end{aligned}
\tag{4.15}
$$

In Eqs. (4.15)–(4.18), the dots represent terms that vanish as $\epsilon \to 0$, and are either analytic or $\mathcal{O}(\epsilon^4 \ln\epsilon)$ at $\epsilon = 0$. Note that $\mathcal{R}^q_{\rm T}$ has a $\ln\epsilon$ divergence, since the separation of $\sigma^q_{\rm T}$ and $\sigma^g_{\rm T}$ is not collinear safe.

The $\mathcal{O}(\epsilon\ln\epsilon)$ term in $\mathcal{R}^q_{\rm L}$ gives a leading power correction of order $1/Q^2$ in $\sigma^q_{\rm L}$. The effect of the logarithmic divergence of the x-moment of $C^q_{4,{\rm P}}(x)$ is to 'promote' the $\mathcal{O}(\epsilon\ln\epsilon)$ terms to $\mathcal{O}(\epsilon\ln^2\epsilon)$, resulting in a $\ln Q^2$ enhancement of the $1/Q^4$ corrections to $\sigma^q_{\rm T}$ and $\sigma^q_{\rm L}$.

In the case of the gluon contributions, the relevant coefficients, given by Eqs. (4.9) and (4.11), are so singular as $x \to 0$ that the $1/Q^{2p}$ power corrections are promoted to $1/Q$. In the case of $\sigma^g_{\rm L}$, for example, we have

$$\frac{\mu^2}{Q^2}\int_{2\mu/Q}^{1+\mu^2/Q^2} x\, C^g_{2,{\rm L}}(x)dx \sim 8\frac{\mu^2}{Q^2}\int_{2\mu/Q}^{1}\frac{dx}{x^2} \sim 4\frac{\mu}{Q}\,. \tag{4.16}$$

Similarly, the most singular parts of $C^g_{4,{\rm L}}(x)$ and all the higher coefficients give contributions of order $1/Q$. Hence these terms have to be resummed, and the true behaviour of the characteristic functions for $\sigma^g_{\rm L,T}$ involves $\sqrt{\epsilon}$ singularities. Again performing the x-integrations first, and then taking the small-ϵ limit, we find

$$\begin{aligned}
\mathcal{R}^g_{\rm T}(\epsilon) &= -\frac{4}{3}\ln\epsilon - \frac{44}{9} + \pi^2(1+\epsilon)^2\sqrt{\epsilon} - \epsilon\ln\epsilon(\ln\epsilon+4) - \epsilon^2\ln\epsilon\left(\ln\epsilon+\frac{8}{3}\right) + \frac{2}{15}\epsilon^3\ln\epsilon + \cdots \\
\mathcal{R}^g_{\rm L}(\epsilon) &= 1 - \pi^2(1+\epsilon)^2\sqrt{\epsilon} + \epsilon\ln\epsilon(\ln\epsilon-2) + \epsilon^2\ln\epsilon\left(\ln\epsilon-\frac{10}{3}\right) - \frac{22}{15}\epsilon^3\ln\epsilon + \cdots \\
\mathcal{R}^g_{\rm tot}(\epsilon) &= -\frac{4}{3}\ln\epsilon - \frac{35}{9} - 6\epsilon\ln\epsilon - 6\epsilon^2\ln\epsilon - \frac{4}{3}\epsilon^3\ln\epsilon + \cdots\,.
\end{aligned} \tag{4.17}$$

Notice that the $\sqrt{\epsilon}$ singularities cancel in the total gluonic contribution. Adding the quark and gluon contributions together gives

$$\begin{aligned}
\mathcal{R}_{\rm T}(\epsilon) &= \pi^2(1+\epsilon)^2\sqrt{\epsilon} - \epsilon\ln\epsilon(\ln\epsilon+4) + 2\epsilon^2\ln\epsilon\left(\ln\epsilon-\frac{17}{6}\right) + \frac{4}{5}\epsilon^3\ln\epsilon + \cdots\,, \\
\mathcal{R}_{\rm L}(\epsilon) &= \frac{3}{2} - \pi^2(1+\epsilon)^2\sqrt{\epsilon} + \epsilon\ln\epsilon(\ln\epsilon+4) - 2\epsilon^2\ln\epsilon\left(\ln\epsilon-\frac{17}{6}\right) - \frac{22}{15}\epsilon^3\ln\epsilon + \cdots\,, \\
\mathcal{R}_{\rm tot}(\epsilon) &= \frac{3}{2} - \frac{2}{3}\epsilon^3\ln\epsilon + \cdots\,,
\end{aligned} \tag{4.18}$$

corresponding to

$$\begin{aligned}
\sigma_{\rm T} &\simeq \sigma_0\left(1 - \frac{\pi^2}{2}\frac{A_1}{Q}\right) \\
\sigma_{\rm L} &\simeq \sigma_0\left(\frac{3}{2}C_F\frac{\alpha_s}{2\pi} + \frac{\pi^2}{2}\frac{A_1}{Q}\right) \\
\sigma_{\rm tot} &\simeq \sigma_0\left(1 + \frac{3}{2}C_F\frac{\alpha_s}{2\pi} + 2\frac{A'_6}{Q^6}\right)\,,
\end{aligned} \tag{4.19}$$

where A'_6 is defined by Eq. (2.6) with $p = 3$ and

$$A_1 = \frac{C_F}{2\pi}\int_0^\infty \frac{d\mu^2}{\mu^2}\,\mu\;\delta\alpha_{\rm eff}(\mu^2) = \frac{C_F}{\pi}\int_0^\infty d\mu\;\delta\alpha_{\rm eff}(\mu^2)\,. \tag{4.20}$$

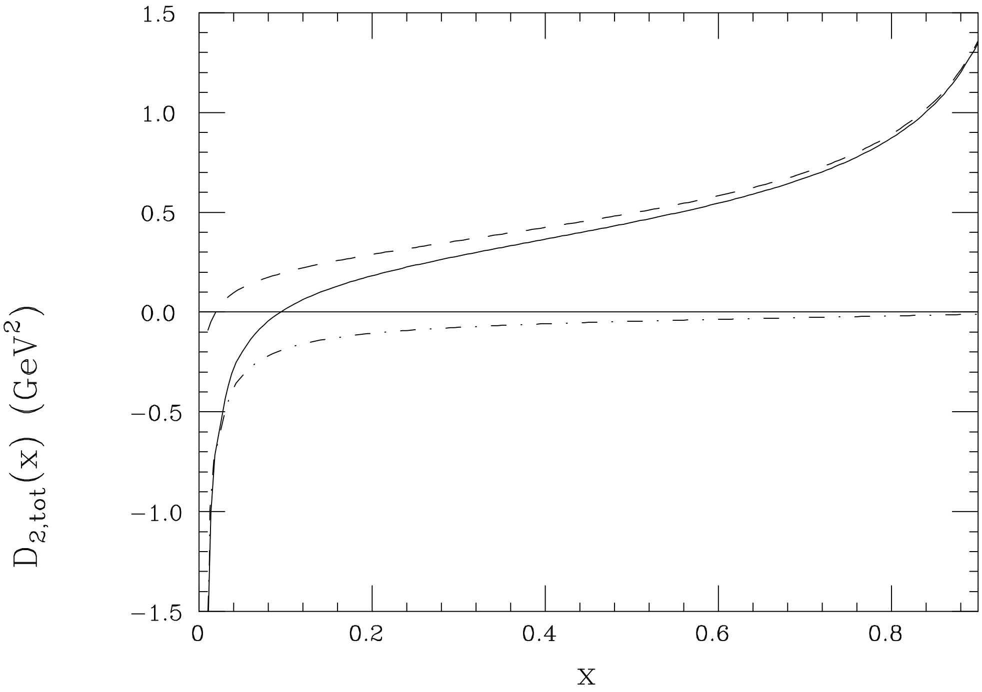

Figure 2: Coefficient of $1/Q^2$ correction to the total fragmentation function (solid), with quark (dashed) and gluon (dot-dashed) contributions shown separately.

5 Discussion

To illustrate the above results, we have computed the predicted $1/Q^2$ contributions to the total, transverse and longitudinal fragmentation functions using the parametrizations provided by the ALEPH collaboration [3] for the parton fragmentation functions $D_i(x, Q^2)$. We express the results in terms of the coefficients $D_{2,\mathrm{P}}$ where

$$\begin{aligned} F_{\mathrm{P}}(x, Q^2) &\simeq F_{\mathrm{P}}^{\mathrm{pert}}(x, Q^2)\left(1 + \frac{D_{2,\mathrm{P}}(x, Q^2)}{Q^2}\right) , \\ D_{2,\mathrm{P}}(x, Q^2) &= \frac{A_2'}{F_{\mathrm{P}}^{\mathrm{pert}}(x, Q^2)} \sum_i \int_x^1 \frac{dz}{z} C_{2,\mathrm{P}}^i(z)\, D_i(x/z, Q^2) , \end{aligned} \tag{5.1}$$

the coefficient functions $C_{2,\mathrm{P}}^i$ being as given above. In each case, $F_{\mathrm{P}}^{\mathrm{pert}}(x, Q^2)$ represents the relevant lowest-order perturbative prediction. We assumed the value $A_2' = -0.2$ GeV^2 for the non-perturbative parameter defined by Eq. (2.6) with $p = 1$. This value is suggested by deep inelastic data [22]. Calculations were performed at $Q = 22$ GeV, but the Q-dependence of the coefficients $D_{2,\mathrm{P}}(x, Q^2)$ is small.

Figs 2, 3 and 4 show the results for F_{tot}, F_{T} and F_{L}, respectively. Note that the longitudinal coefficient $D_{2,\mathrm{L}}$ is much larger than the others simply because we divide by $F_{\mathrm{L}}^{\mathrm{pert}}$, which is of order α_s relative to $F_{\mathrm{tot}}^{\mathrm{pert}}$ and $F_{\mathrm{T}}^{\mathrm{pert}}$. We see that, as discussed above, the contributions from gluon fragmentation give large opposing corrections to F_{T} and F_{L} at small x, which tend to cancel in F_{tot}.

Turning to the results on the transverse and longitudinal cross sections, Eq. (4.19) together with the empirical value (1.7) for the $1/Q$ correction to σ_{L} would suggest a value $A_1 \simeq 0.2$ GeV for

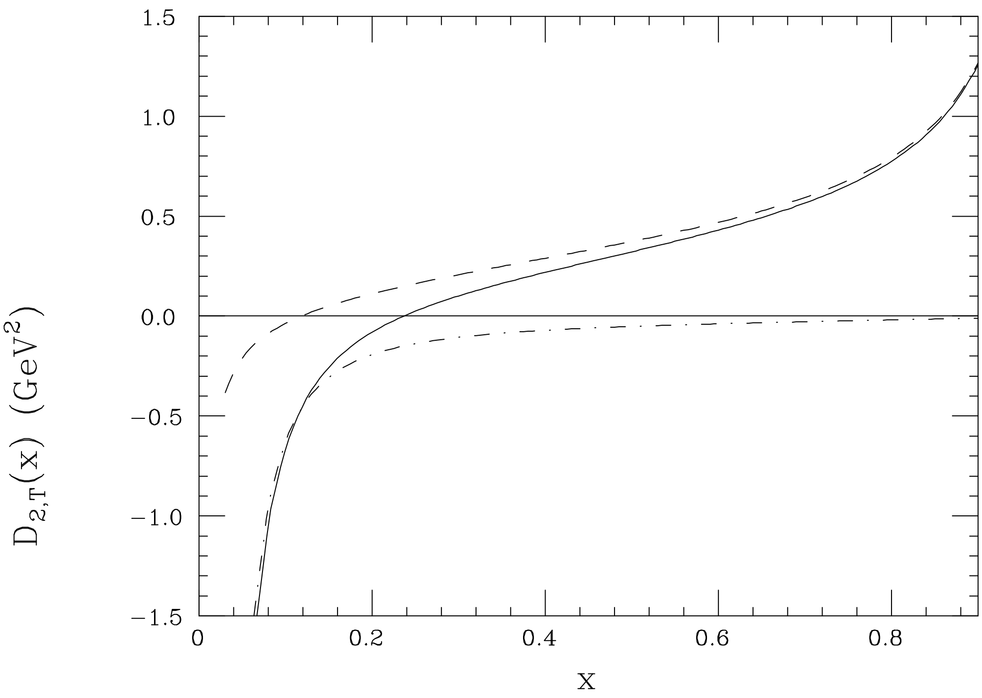

Figure 3: Coefficient of $1/Q^2$ correction to the transverse fragmentation function (solid), with quark (dashed) and gluon (dot-dashed) contributions shown separately.

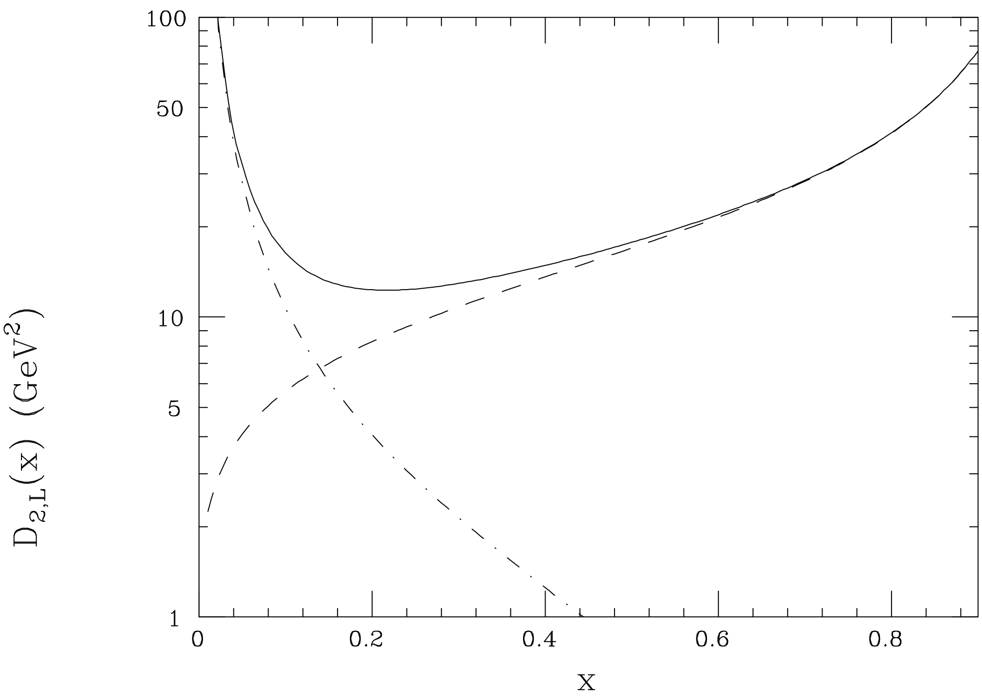

Figure 4: Coefficient of $1/Q^2$ correction to the longitudinal fragmentation function (solid), with quark (dashed) and gluon (dot-dashed) contributions shown separately.

the non-perturbative parameter (4.20). This compares fairly well with the value $A_1 \simeq 0.25$ GeV deduced event shape data in Refs. [17,27]. Note that in Ref. [27] the coefficient of A_1/Q in Eq. (4.19) was given as $2\pi \simeq 6.3$ rather than $\pi^2/2 \simeq 4.9$. This is because the calculation was performed there using the massive-gluon phase space with a massless matrix element, while here we include a gluon mass throughout. As discussed above, the fact that the correction to $\sigma_{\rm L}$ is controlled by gluon fragmentation makes the calculation of its coefficient by the massive gluon technique less reliable than that of quark-dominated quantities. Nevertheless the technique is useful in revealing the existence of the $1/Q$ correction and its cancellation in $\sigma_{\rm tot}$, and it is reassuring that the calculations with and without mass effects in the matrix element give numerically similar results.

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References

1. TASSO collaboration, W. Braunschweig *et al.*, *Zeit. Phys.* C41 (1988) 359.

2. DELPHI collaboration, P. Abreu *et al.*, *Phys. Lett.* 311B (1993) 408.

3. ALEPH collaboration, D. Buskulic *et al.*, *Phys. Lett.* 357B (1995) 487; *ibid.* 364B (1995) 247 (erratum).

4. OPAL collaboration, R. Akers *et al.*, *Zeit. Phys.* C68 (1995) 203.

5. V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* 15 (1972) 78;
G. Altarelli and G. Parisi, *Nucl. Phys.* B126 (1977) 298;
Yu.L. Dokshitzer, *Sov. Phys. JETP* 46 (1977) 641.

6. G. Curci, W. Furmanski and R. Petronzio, *Nucl. Phys.* B175 (1980) 27;
E.G. Floratos, C. Kounnas and R. Lacaze, *Nucl. Phys.* B192 (1981) 417.

7. W. Furmanski and R. Petronzio, CERN preprint TH.2933 (1980);
Phys. Lett. 97B (1980) 437.

8. P. Nason and B.R. Webber, *Nucl. Phys.* B421 (1994) 473.

9. J.C. Collins and D.E. Soper, *Ann. Rev. Nucl. Part. Sci.* 37 (1987) 383.

10. P. Nason and B.R. Webber, *Phys. Lett.* 332B (1994) 405.

11. M. Greco and S. Rolli, *Zeit. Phys.* C60 (1993) 169;
P. Chiappetta *et al..*, *Nucl. Phys.* B412 (1994) 3;
M. Greco and S. Rolli, *Phys. Rev.* D52 (1995) 3853;
M. Greco, S. Rolli and A. Vicini, *Zeit. Phys.* C65 (1995) 277.

12. J. Binnewies, B.A. Kniehl and G. Kramer, *Zeit. Phys.* C65 (1995) 277; *Phys. Rev.* D52 (1995) 4947.

13. EMC collaboration, M. Arneodo *et al.*, *Zeit. Phys.* C36 (1987) 527;
H1 collaboration, S. Aid *et al.*, *Nucl. Phys.* B445 (1995) 3;
ZEUS collaboration, M. Derrick *et al.*, *Zeit. Phys.* C67 (1995) 93.

14. CDF collaboration, F. Abe *et al.*, *Phys. Rev. Lett.* 65 (1990) 968.

15. M. Virchaux and A. Milsztajn, *Phys. Lett.* 274B (1992) 221;
A.V. Sidorov, Dubna preprint JINR-E2-96-254 [hep-ph/9607275].

16. I.I. Balitsky and V.M. Braun, *Nucl. Phys.* B361 (1991) 93.

17. Yu.L. Dokshitzer, G. Marchesini and B.R. Webber, CERN preprint TH.95-281, to be published in *Nucl. Phys.* B [hep-ph/9512336].

18. For reviews and classic references see:
V.I. Zakharov, *Nucl. Phys.* B385 (1992) 452;
A.H. Mueller, in *QCD 20 Years Later*, vol. 1 (World Scientific, Singapore, 1993).

19. M. Beneke, V.M. Braun and V.I. Zakharov, *Phys. Rev. Lett.* 73 (1994) 3058;
P. Ball, M. Beneke and V.M. Braun, *Nucl. Phys.* B452 (1995) 563;
M. Beneke and V.M. Braun, *Nucl. Phys.* B454 (1995) 253.

20. D.J. Broadhurst and A.L. Kataev, *Phys. Lett.* 315B (1993) 179.

21. C.N. Lovett-Turner and C.J. Maxwell, *Nucl. Phys.* B452 (1995) 188.

22. M. Dasgupta and B.R. Webber, *Phys. Lett.* 382B (1996) 273.

23. E. Stein, M. Meyer-Hermann, L. Mankiewicz and A. Schäfer, *Phys. Lett.* 376B (1996) 177;
M. Meyer-Hermann, M. Maul, L. Mankiewicz, E. Stein and A. Schäfer, Frankfurt preprint UFTP-414-1996 [hep-ph/9605229].

24. P.J. Rijken and W.L. van Neerven, Leiden preprint INLO-PUB-4/96 [hep-ph/9604436].

25. T. Sjöstrand, *Comp. Phys. Commun.* 39 (1984) 347;
M. Bengtsson and T. Sjöstrand, *Comp. Phys. Commun.* 43 (1987) 367.

26. B.R. Webber, in *Proc. Summer School on Hadronic Aspects of Collider Physics, Zuoz, Switzerland, 1994* [hep-ph/9411384].

27. B.R. Webber, *Phys. Lett.* 339B (1994) 148;
Yu.L. Dokshitzer and B.R. Webber, *ibid.* 352B (1995) 451.

28. P. Nason and M.H. Seymour, *Nucl. Phys.* B454 (1995) 291.

29. M. Beneke, V.M. Braun and L. Magnea, paper in preparation;
V.M. Braun, talk at "QCD 96", Montpellier, July 1996;
M. Beneke, talk at Workshop on Renormalons and Power Corrections in QCD, NORDITA, Copenhagen, August 1996.