Cavendish–HEP–95/2
LU TP 95–8
hep-ph/9504219

# Calculation of Power Corrections to Hadronic Event Shapes*

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## Abstract

We compute power corrections to hadronic event shapes in $e^+e^-$ annihilation, assuming an infrared regular behaviour of the effective coupling $\alpha_S$. With the integral of $\alpha_S$ over the infrared region as the only non-perturbative parameter, also measured in heavy quark physics, we can account for the empirical features of $1/Q$ corrections to the mean values of various event shapes.

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LU TP 95–8
March 1995

*Research supported in part by the U.K. Particle Physics and Astronomy Research Council and by the EC Programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0357 (DG 12 COMA).

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## 1. Introduction

Infrared-safe shape measures for hadronic final states in $e^+e^-$ annihilation would appear in principle to be an ideal testing ground for perturbative QCD. Such quantities are asymptotically insensitive to long-distance non-perturbative physics and can thus be computed order-by-order in perturbation theory. The large momentum scale $Q \sim M_Z$ available at existing $e^+e^-$ machines implies a small value of the running coupling $\alpha_S(Q)$ and so the perturbation series should be relatively well-behaved. Non-perturbative effects should be suppressed by inverse powers of $Q$. Thus hadronic event shapes should be capable of providing a high-precision measurement of $\alpha_S$.

Unfortunately the hoped-for precision has not yet been achieved, partly because $\mathcal{O}(\alpha_S^3)$ calculations of event shapes are still lacking, but also because non-perturbative effects turn out to be significant even at $Q \sim M_Z$. This is because they are in fact suppressed by only a single inverse power of $Q$ in many cases. Bearing in mind that $\alpha_S(M_Z) \sim 0.12$ and the non-perturbative scale is $\mathcal{O}(1 \text{ GeV})$, we see that the power correction may easily be comparable with the $\mathcal{O}(\alpha_S^2)$ next-to-leading term at present energies. Consequently it becomes essential to achieve some understanding of power corrections before embarking on any $\mathcal{O}(\alpha_S^3)$ calculations of event shapes.

In the present paper we adopt the approach, advocated in Refs. [1,2], of trying to deduce as much as possible about power corrections from perturbation theory. In particular we explore the consequences of assuming that $\alpha_S$, defined in some appropriate way, does not grow indefinitely at low scales but instead has an infrared-regular effective form. Then various moments of $\alpha_S$, integrated over the infrared region, play the rôle of non-perturbative parameters which determine the form and magnitude of power corrections. Since $\alpha_S$ is supposed to be universal, we obtain relations between the power corrections to various observables.

Our approach is related to that of Korchemsky and Sterman [3], and also to several other recent papers that discuss power corrections and the ambiguities of perturbation theory in terms of infrared renormalons [4], in the context of the Drell-Yan process [5], event shapes [6], deep inelastic scattering [7], heavy quark effective theory [8] or quark confinement [9]. From our viewpoint, infrared renormalons arise from the divergence of the perturbative expression for $\alpha_S$ at low scales, and the ambiguities associated with different ways of avoiding the renormalon poles in the Borel transform plane are resolved by specifying the infrared behaviour of $\alpha_S$. This approach implies relationships between the contributions of a given renormalon to different processes.

The quantitative results we obtain look very good in the case of the mean value of the thrust parameter [10]. The required value of the relevant moment of $\alpha_S$ is consistent with that obtained from a similar approach to heavy quark fragmentation [1]. For the other shape variables discussed here, the mean value of the $C$-parameter [11] and the longitudinal cross section [12], a comparison with LEP data is encouraging, but detailed tests must await the re-analysis of lower-energy data to establish the energy dependence of the leading power correction.

## 2. Calculations

Consider a quantity of the form

$$F=\int_0^Q dk\, f(k) \tag{1}$$

where $f(k)$ behaves like $\alpha_S(k)\,k^p$ at $k\ll Q$, say

$$f(k)\sim a_F\,\alpha_S(k)\,k^p/Q^{p+1} \qquad (k\ll Q) \tag{2}$$

where we have included the appropriate $Q$ dependence assuming $F$ is dimensionless. Suppose that $F$ has the perturbative expansion

$$F^{\rm pert}=F_1\,\alpha_S+F_2\,\alpha_S^2+\cdots\ . \tag{3}$$

More precisely, if the coefficients $F_n$ are computed in the $\overline{\rm MS}$ renormalization scheme at scale $Q$, then in terms of the $\overline{\rm MS}$ coupling at scale $\mu_R$ we have

$$F^{\rm pert}=F_1\,\alpha_S(\mu_R)+\left(F_2+\frac{\beta_0}{2\pi}\ln\frac{\mu_R}{Q}F_1\right)\alpha_S^2(\mu_R)+\cdots \tag{4}$$

where $\beta_0=(11C_A-2N_f)/3$, with $C_A=3$, for $N_f$ active flavours.

In part of the integration region of Eq. (1) the perturbative expression for $\alpha_S(k)$ is not appropriate. We may however choose an infrared matching scale $\mu_I$ such that $\Lambda\ll\mu_I\ll Q$ and assume that perturbation theory is valid above that scale. We can then introduce a non-perturbative parameter $\bar\alpha_p(\mu_I)$ to represent the portion of the integral below $\mu_I$:

$$\int_0^{\mu_I} dk\,\alpha_S(k)\,k^p\equiv\frac{\mu_I^{p+1}}{p+1}\,\bar\alpha_p(\mu_I)\ . \tag{5}$$

Before adding this contribution to $F^{\rm pert}$, we have to subtract the perturbative value of this integral, which is, to second order,

$$\frac{\mu_I^{p+1}}{p+1}\left[\alpha_S(\mu_R)+\frac{\beta_0}{2\pi}\left(\ln\frac{\mu_R}{\mu_I}+\frac{1}{p+1}\right)\alpha_S^2(\mu_R)\right]\ . \tag{6}$$

As a refinement, and for consistency with Ref. [1], we shall assume that the parameter $\bar\alpha_p(\mu_I)$ refers not to the coupling in the $\overline{\rm MS}$ scheme but rather to the scheme proposed in Ref. [13], which is expected to be more physical in the region under consideration. Thus $\alpha_S$ in Eq. (5) (only) is to be interpreted as $\alpha_S^{\rm eff}$ where in terms of the $\overline{\rm MS}$ coupling

$$\alpha_S^{\rm eff}=\alpha_S+\frac{K}{2\pi}\alpha_S^2 \tag{7}$$

with

$$K=\left(\frac{67}{18}-\frac{\pi^2}{6}\right)C_A-\frac{5}{9}N_f\ . \tag{8}$$

The only effect on Eq. (6) is that the term $\ln(\mu_R/\mu_I)$ becomes $\ln(\mu_R/\mu_I)+K/\beta_0$. The full expression for $F$ is thus

$$F=F^{\rm pert}+F^{\rm pow} \tag{9}$$

where $F^{\rm pert}$ is as given in Eq. (4) while

$$F^{\rm pow} = \frac{a_F}{p+1}\left(\frac{\mu_{\rm I}}{Q}\right)^{p+1}\left[\bar{\alpha}_p(\mu_{\rm I}) - \alpha_{\rm S}(\mu_{\rm R}) - \frac{\beta_0}{2\pi}\left(\ln\frac{\mu_{\rm R}}{\mu_{\rm I}} + \frac{K}{\beta_0} + \frac{1}{p+1}\right)\alpha_{\rm S}^2(\mu_{\rm R})\right] . \qquad (10)$$

The dependence of $F^{\rm pert}$ on the renormalization scale $\mu_{\rm R}$ is one order higher in $\alpha_{\rm S}$ than that calculated, i.e. third-order in this case. Similarly, the dependence of the power correction $F^{\rm pow}$ on both $\mu_{\rm R}$ and the infrared matching scale $\mu_{\rm I}$ is third-order, provided $\mu_{\rm I}$ is sufficiently large for $\alpha_{\rm S}(\mu_{\rm I})$ to have reached its perturbative form. Of course, the value obtained for $\bar{\alpha}_p(\mu_{\rm I})$ depends on $\mu_{\rm I}$, but this is mostly compensated by the other $\mu_{\rm I}$-dependent term.

The value of the power $p$ and the coefficient $a_F$ may be found from the infrared cutoff dependence of the lowest-order perturbative result. In this connection, it is crucial that the appropriate argument of $\alpha_{\rm S}$ for soft and/or collinear gluon emission is the gluon transverse momentum $k_\perp$ [14]. Thus the cutoff should be a $k_\perp$-cutoff.

Consider for example the mean value of the thrust $T$. The contribution to this quantity from the region $k_\perp < \mu_{\rm I}$ is

$$\delta\langle T\rangle = -\frac{C_F}{2\pi}\int_{k_\perp<\mu_{\rm I}} dx_1\,dx_2\,\alpha_{\rm S}(k_\perp)\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}\,\min\{(1-x_1),(1-x_2)\} \qquad (11)$$

where $C_F = 4/3$. Setting $1-x_{1,2} = y_{1,2}$ and considering the region $y_1 < y_2 \ll 1$, we have $k_\perp = \sqrt{y_1 y_2}Q$ and hence

$$\begin{aligned} \delta\langle T\rangle &= -4\frac{C_F}{\pi}\int_0^{\mu_{\rm I}/Q} dy_1 \int_{y_1 Q}^{\mu_{\rm I}} \frac{dk_\perp}{k_\perp}\alpha_{\rm S}(k_\perp) \\ &= -\frac{4C_F}{\pi Q}\int_0^{\mu_{\rm I}} dk_\perp\,\alpha_{\rm S}(k_\perp) \;\equiv\; -4\frac{C_F}{\pi}\frac{\mu_{\rm I}}{Q}\,\bar{\alpha}_0(\mu_{\rm I}) . \end{aligned} \qquad (12)$$

Thus in this case $p=0$ and we obtain a $1/Q$ correction, with a coefficient in Eq. (10) of $a_F = a_T$ where

$$a_T = -4\frac{C_F}{\pi} = -1.70 . \qquad (13)$$

As shown by the solid curve in Fig. 1, an excellent fit to the data on $\langle T\rangle$ over the range $14 < Q < 92$ GeV can be obtained using the perturbative prediction [15]

$$\langle T\rangle^{\rm pert} = 1 - 0.335\,\alpha_{\rm S} - 1.02\,\alpha_{\rm S}^2 \qquad (14)$$

with $\mu_{\rm R} = Q$ and $\alpha_{\rm S}(M_Z) = 0.117 \pm 0.005$ [16], plus a power correction of the form (10). For $\mu_{\rm I} = 2$ GeV, the fitted value of the non-perturbative parameter $\bar{\alpha}_0$ is

$$\bar{\alpha}_0(2\ {\rm GeV}) \equiv (2\ {\rm GeV})^{-1}\int_0^{2\ {\rm GeV}} dk\,\alpha_{\rm S}^{\rm eff}(k) = 0.53 \pm 0.04 , \qquad (15)$$

with $\chi^2 = 4.5$ for 8 degrees of freedom. Allowing both $\alpha_{\rm S}(M_Z)$ and $\bar{\alpha}_0(2\ {\rm GeV})$ to be free parameters gives

$$\alpha_{\rm S}(M_Z) = 0.120 \pm 0.004 , \quad \bar{\alpha}_0(2\ {\rm GeV}) = 0.52 \pm 0.03 , \qquad (16)$$

with $\chi^2/{\rm d.o.f.} = 3.7/7$.

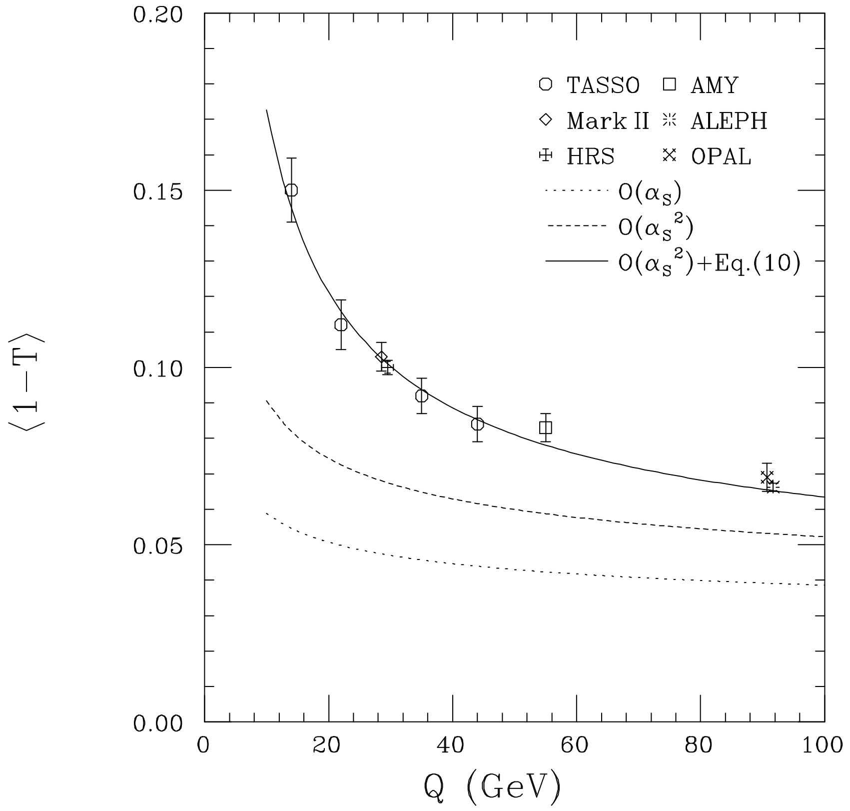


Figure 1: Mean value of $1-T$, where $T$ is the thrust.

We also obtain good fits for other values of the arbitrary infrared matching parameter $\mu_{\rm I}$. At $\mu_{\rm I} = 3$ GeV, for example, we find

$$\alpha_{\rm s}(M_Z) = 0.118 \pm 0.004 \;, \quad \bar{\alpha}_0(3 \text{ GeV}) = 0.42 \pm 0.03 \;, \tag{17}$$

with $\chi^2$/d.o.f. = 4.0/7. The change in $\bar{\alpha}_0$ implies that $\alpha_{\rm s}^{\rm eff}(2.5 \text{ GeV}) \simeq 0.2 \pm 0.1$, which is reasonable, the perturbative value being around 0.3. The change in the overall power correction is small (about 5%), since, as explained above, the $\mu_{\rm I}$-dependence mostly cancels in Eq. (10).

Using the value (15) of $\bar{\alpha}_0$, obtained by fitting the thrust data, one can now predict the power corrections to other event shapes. For the mean value of the $C$-parameter, for example, we find that the coefficient $a_F$ in Eq. (10) is

$$a_C = 6\, C_F = 8 \;. \tag{18}$$

At present there are only data on $\langle C \rangle$ at $Q = M_Z$, where Eqs. (15) and (18) imply

$$\langle C \rangle^{\rm pow} = 0.057 \pm 0.008 \;. \tag{19}$$

The second-order perturbative prediction is [15]

$$\langle C \rangle^{\rm pert} = 1.375\, \alpha_{\rm s} + 3.88\, \alpha_{\rm s}^2 = 0.214 \pm 0.011 \tag{20}$$

for $\alpha_{\rm s} = 0.117 \pm 0.005$. Thus the full theoretical prediction is

$$\langle C \rangle^{\rm th} = \langle C \rangle^{\rm pert} + \langle C \rangle^{\rm pow} = 0.271 \pm 0.014 \;, \tag{21}$$

which is consistent with the experimental result [17]

$$\langle C\rangle^{\rm exp} = 0.2587 \pm 0.0013 \pm 0.0018\ . \tag{22}$$

Note that the power correction represents over 20% of the value of this quantity.

Finally, for the longitudinal cross section fraction $\sigma_{\rm L}/\sigma_{\rm tot}$ we predict a coefficient

$$a_L = C_F = 1.33\ , \tag{23}$$

leading to the power correction at $Q = M_Z$

$$(\sigma_{\rm L}/\sigma_{\rm tot})^{\rm pow} = 0.010 \pm 0.001\ . \tag{24}$$

The first-order perturbative prediction is $\alpha_{\rm S}/\pi = 0.037$. However, the second-order correction is not yet known. The preliminary OPAL measurement [18] is

$$(\sigma_{\rm L}/\sigma_{\rm tot})^{\rm exp} = 0.067 \pm 0.008\ . \tag{25}$$

Clearly the second-order perturbative correction should be significant if there is to be satisfactory agreement between theory and experiment.

The values for $\bar\alpha_0$ obtained above from event shapes are in reasonable agreement with those deduced from heavy quark fragmentation spectra. In Ref. [1] the value $\bar\alpha_0(2\ \mathrm{GeV}) \simeq 0.6$ was obtained from fits to heavy quark energy losses in $e^+e^-$ annihilation. The same conclusion follows from an analysis of the quantity $\xi_H = -\ln\langle x_H\rangle$, where $x_H$ is the energy fraction carried by the heavy quark $H$, using the approach of the present paper. We find a quark mass ($1/M$) correction of the form (10), with $Q$ replaced by $M$, $\mu_{\rm R} \sim M$, and coefficient $a_H = C_F/2$. The perturbative prediction deduced from Ref. [1] is

$$\begin{aligned}\xi_H^{\rm pert} \;&=\; \frac{4C_F}{3\pi}\left\{\int_M^Q \frac{dk}{k}\,\alpha_{\rm S}(k) - \frac{35}{24}\alpha_{\rm S}(Q) + \frac{13}{24}\alpha_{\rm S}(M)\right.\\ &\left.\quad + \frac{1}{\beta_0}(K+\delta_2)\,[\,\alpha_{\rm S}(M) - \alpha_{\rm S}(Q)\,]\right\}, \end{aligned}\tag{26}$$

where $\delta_2$ is the (numerically negligible) 2-loop anomalous dimension correction

$$\delta_2 = \left(\frac{53}{18} - \frac{\pi^2}{3}\right) C_F + \frac{31}{36}(C_A - 2C_F) \;=\; -0.173\ . \tag{27}$$

The expression (26), which accounts for the $\alpha_{\rm S}\ln(Q/M)$, $\alpha_{\rm S}$ and $\alpha_{\rm S}^2\ln(Q/M)$ terms in $\xi_H$, but neglects $\alpha_{\rm S}^2$ terms with no large logarithm, gives $\xi_b^{\rm pert} = 0.26 \pm 0.02$ for $b$-quarks at $Q = M_Z$. Comparing with the experimental value of $0.36 \pm 0.02$ deduced from lepton spectra [19], this implies that $\xi_b^{\rm pow} = 0.10 \pm 0.03$ and hence that $\bar\alpha_0(2\ \mathrm{GeV}) \simeq 0.6 \pm 0.1$. The errors are estimated conservatively, taking into account the small scale $M_b \sim 5$ GeV and the lack of a complete $\mathcal{O}(\alpha_{\rm S}^2)$ calculation of $\xi_b^{\rm pert}$.

## 3. Conclusions

Note that the power correction coefficients $a_T$, $a_C$ and $a_L$ deduced above using a $k_\perp$ cutoff are identical to those obtained in Ref. [2] with a gluon mass cutoff. With a $k_\perp$ cutoff, however, these coefficients have a physical interpretation: they measure the contribution of the low-scale region in which $\alpha_S$ departs significantly from its perturbative form. After being used to calculate the coefficients, the cutoff is replaced by an infrared matching parameter $\mu_I$, which represents the scale below which we switch from the perturbative to the non-perturbative description of $\alpha_S$. As long as $\mu_I$ is not too small (larger than about 1 GeV) the predictions are quite insensitive to its value, indicating that the perturbative behaviour has set in at that scale.

The divergence in the perturbative expression for $\alpha_S$ at low scales is responsible for the divergence of the perturbation series for quantities like those considered here, giving rise to the so-called "renormalon ambiguity". By assuming an infrared regular form for the effective coupling, we resolve this ambiguity, at the price of introducing the non-perturbative parameters $\bar{\alpha}_p$. These parameters are, however, universal, and can be measured experimentally, like $\bar{\alpha}_0$ in Eq. (15).

Combined fits to the non-perturbative parameters $\bar{\alpha}_p$ and the perturbative parameter $\alpha_S$, using data on several different event shapes, provide the possibility of understanding something new about QCD at low scales and at the same time measuring $\alpha_S$ with improved precision. This would be useful not only for QCD but also in constraining physics beyond the Standard Model.

## Acknowledgement

This work was begun during the Cambridge Workshop on Confinement Physics, December, 1994. We thank Steven Bass for his hard work in organising such a stimulating Workshop.

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