Spin Correlations in Monte Carlo Simulations

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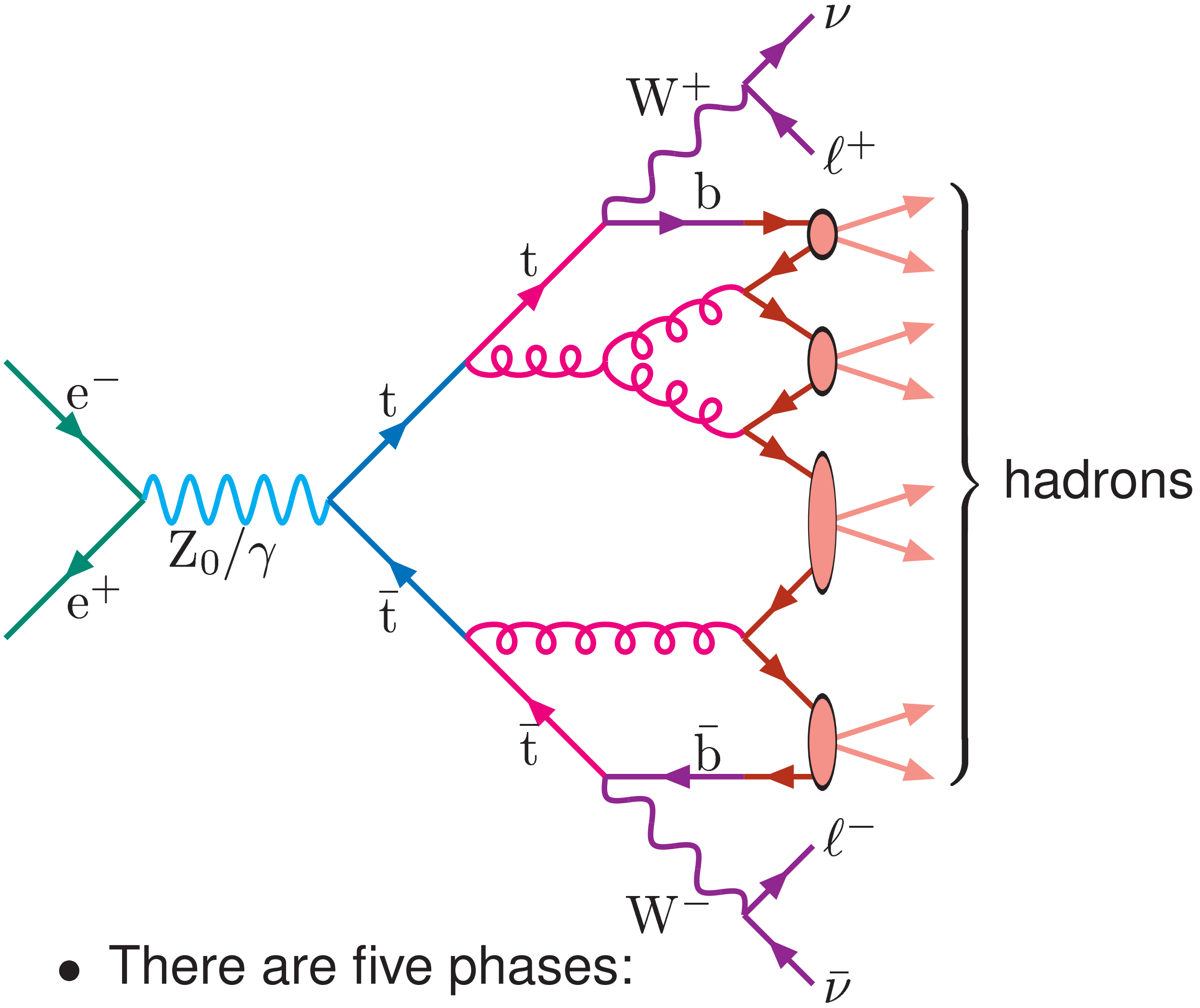
- Introduction to Monte Carlo Simulations
- Why are Spin Correlations important
- Spin Correlations in the parton shower
- Spin Correlations in particle decays
- Examples
- Combining the correlations
- Conclusions

Introduction to Monte Carlo Simulations

- Monte Carlo simulations are used by all experimental collaborations both to compare their data and theoretical predictions, and in data analysis.
- They are often treated as black boxes by experimentalists.
- It's important to understand the assumptions and approximations involved in these simulations.
- Is the result of an experiment really physics or the result of the simulation used?

Monte Carlo Event Generation



- There are five phases:
  - Hard Process;
  - Parton Shower;
  - Heavy particle decays;
  - Hadronization;
  - Hadron decays.

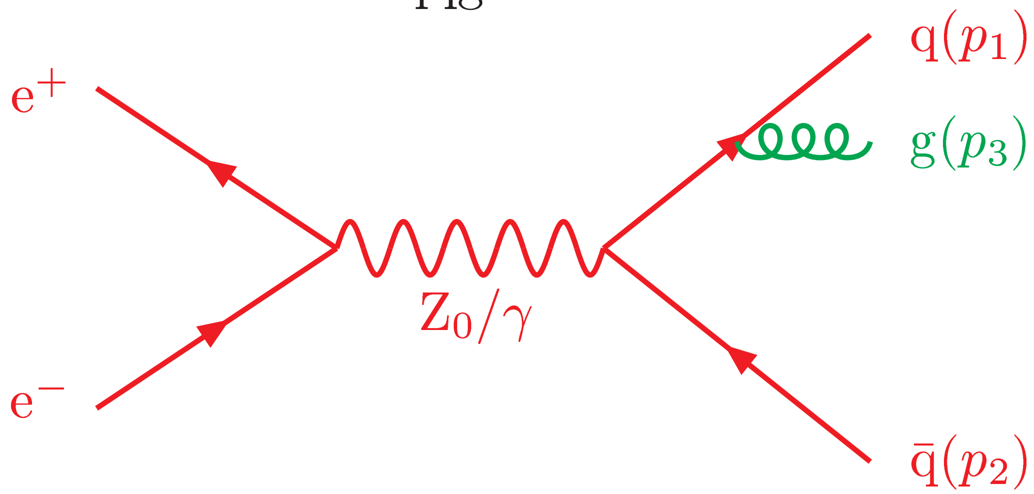
- I will mainly discuss the details of the HERWIG event generator.

QCD Radiation

- It is impossible to calculate and integrate the matrix elements for processes involving large numbers of partons.

- Instead we treat the regions where the emission of QCD radiation is enhanced.

- Consider $\mathrm{e^+e^-} \to \mathrm{q\bar{q}g}$



- The cross section is given by

$$\frac{d\sigma_{q\bar{q}g}}{dx_1 dx_2} = N_c \sigma_0 Q_q^2 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- This diverges as $x_{1,2} \to 1$ where $x_i = 2E_i/\sqrt{s}$.

QCD Radiation

- Physically this happens when

1. The gluon and one of the quarks are collinear

$$(1 - x_1) = \frac{E_2 E_3}{Q^2}(1 - \cos\theta_{2g})$$

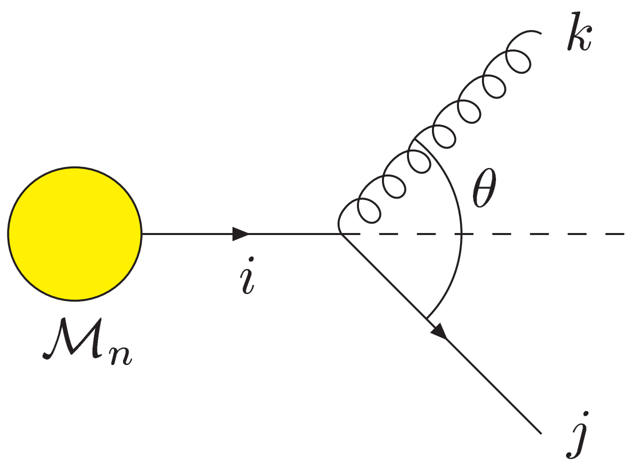
Collinear singularity

2. The energy of the gluon goes to zero

$$x_1 + x_2 + x_3 = 2, x_1, x_2 \to 1 \Rightarrow x_3 \to 0$$

Soft Singularity

Collinear Singularities



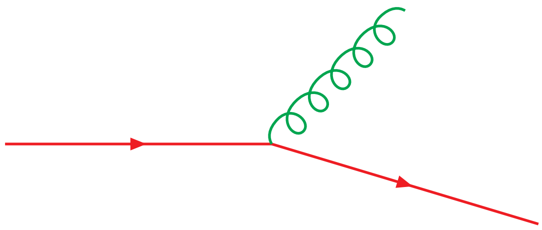
- In the collinear limit the cross section for a process factorizes

$$d\sigma_{n+1} = d\sigma_n \frac{d\theta^2}{\theta^2} dz \frac{\alpha_s}{2\pi} P_{ji}(z),$$

- P_{ji} is the DGLAP splitting function.
- This is singular as $\theta \to 0$.
- What is a parton? (difference between collinear pair and a parton)

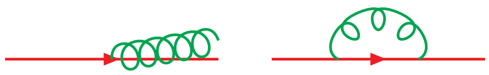
Collinear Singularities

- Introduce resolution criterion *e.g.* $k_T > Q_0$.
- Combine virtual corrections & unresolvable emission.



Resolvable Emmission

Finite



Unresolvable Emmission

Finite

- Unitary: Unresolved + Resolved = 1.

Monte Carlo Procedure

- Probability of Emission between $q^2 + dq^2$ and q^2

$$d\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{dq^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz P(z) \equiv \frac{dq^2}{q^2}\bar{P}(q^2)$$

- Define probability of no emission between Q^2 and q^2 $\Delta(Q^2, q^2)$

$$\frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2)\frac{d\mathcal{P}}{dq^2}$$

$$\Rightarrow \Delta(Q^2, q^2) = \exp\left[-\int_{q^2}^{Q^2}\frac{dk^2}{k^2}\bar{P}(k^2)\right]$$

- $\Delta(Q^2, Q_0^2) \equiv \Delta(Q^2)$ The Sudakov Form Factor probability of emitting no resolvable radiation.

Monte Carlo Procedure

- We can implement this in the following way:
  - Generate a random number $0 < \rho < 1$;
  - If $\rho < \Delta(Q^2)$ no resolvable emission;
  - Otherwise solve

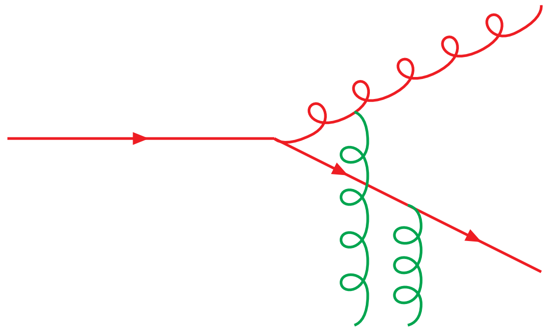
$$\rho = \Delta(Q^2, q^2)$$

  to give the emission scale q^2.

- A number of formally free choices
  - Evolution scale: $q^2/k_\perp/\theta$
  - z: Energy, light-cone momentum.

Soft Emission

- There are two regions where the emission of QCD radiation is enhanced.
- The soft limit also factorizes but at the amplitude level.

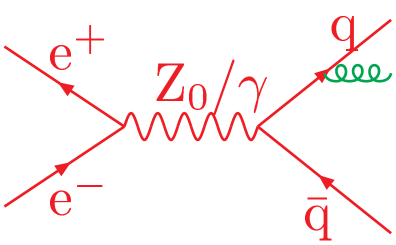


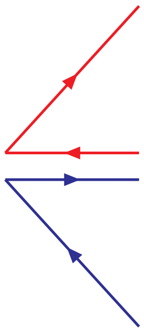
- Soft gluons come from everywhere in the event.
- There is quantum interference between these gluons.
- Does this spoil the parton shower picture?

Angular Ordering

- HERWIG uses the angular-ordered parton shower to resum both the collinear and soft logarithms.

- *e.g.* $e^+e^- \longrightarrow q\bar{q}g$ with radiation of an extra gluon

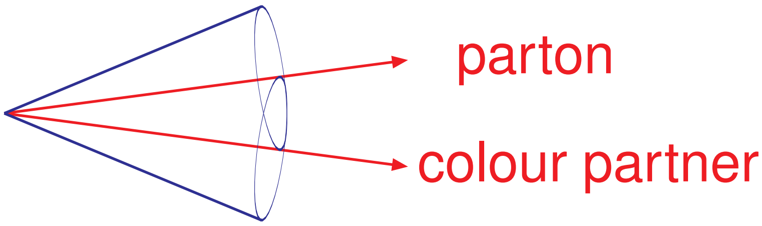




Colour Flow

- We then get that
  - q and the g are colour connected.
  - $\bar{q}$ and the g are anticolour connected.

- QCD radiation only occurs in a cone up to the direction of the colour partner. (Marchesini & Webber)



Heavy Particle Decays

- After the parton shower phase those particles which decay before hadronization, *e.g.* the top quark and SUSY particles, are decayed.

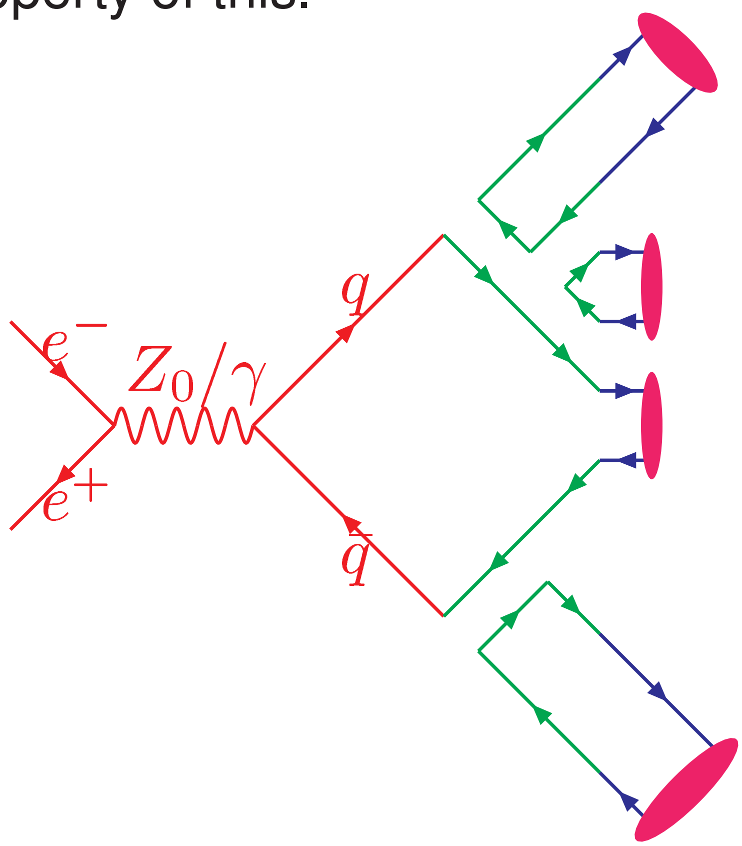
- These decays are usually performed using matrix elements to give the distributions of the decay products.

- However the production and decay of the particles are performed independently.

- This neglects spin correlation effects which may be important.

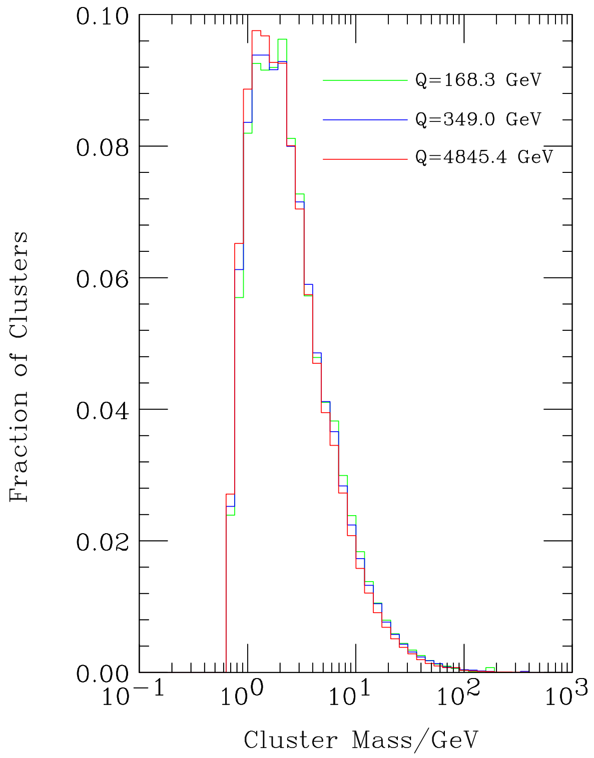
Cluster Hadronization Model

- HERWIG includes angular-ordered parton showers.
- The hadronization model is based on one property of this.



- First split the gluons into $\mathrm{q}\bar{\mathrm{q}}$ pairs.
- Each quark is then uniquely paired with an antiquark in a colour singlet.
- Form these into colour singlet clusters.

Cluster Hadronization Model



- The mass spectrum of pairs of colour singlet clusters after gluon splitting.
- Called colour 'pre-confinement'.
- Assume these clusters are superpositions of resonances.
- Decay according to phase space to hadrons.
- The hadrons decays are then performed using phase space and the measured branching ratios.

Spin Correlations

- In the production of heavy particles there are correlations between the production and decay of the particles due to their spin.

- These correlations are known to be important in tau production.

- Run II of the Tevatron should have sufficient statistics to measure the correlations in $\mathrm{t\bar{t}}$ production.

- These correlations are also important for $\mathrm{t\bar{t}}$ production at a future linear collider.

- One potentially important application is in gaugino production at a future linear collider.
  - Here both spin and polarization effects are important.
  - This affects the distribution of the decay products.
  - These distributions can be used to measure the fundamental parameters of the model.

Spin Correlations

- There have been many studies of spin correlations
  - Gaugino Production in $\mathrm{e^+e^-}$ collisions
    G. Moortgat-Pick, *et.al*, Phys. Rev. D59 (1999) 015016, Eur. Phys. J. C7 (1999) 113, Eur. Phys. J. C9 (1999) 521, Eur. Phys. J. C18 (2000) 379.
  - Top production in hadron collisions
    A. Brandenburg Phys. Lett. B338 (1996) 626.

- Most of these studies used the same formalism.

- In the limit that the width of a particle is small relative to its mass we can neglect the width and keep the particles on mass shell.

- We can write the matrix element for a $2 \to 2$ process, including decays, as

$$\mathcal{M}_{\mathrm{total}} = \frac{1}{m_a\Gamma_a}\frac{1}{m_b\Gamma_b}\sum_{\lambda_c\lambda_d}\mathcal{M}^{\lambda_c\lambda_d}_{ab\to cd}\mathcal{M}^{\lambda_c}_{c\ \mathrm{decay}}\mathcal{M}^{\lambda_d}_{d\ \mathrm{decay}}$$

Spin Correlations

- This means the amplitude squared can be written as

$$|\mathcal{M}_{\mathrm{total}}| = \frac{1}{m_a^2\Gamma_a^2}\frac{1}{m_b^2\Gamma_b^2}\rho^{\lambda_c\lambda'_c\lambda_d\lambda'_d}D_c^{\lambda_c\lambda'_c}D_d^{\lambda_d\lambda'_d}$$

where

$$\rho_{\mathrm{prod}}^{\lambda_c\lambda'_c\lambda_d\lambda'_d} = \mathcal{M}_{ab\to cd}^{\lambda_c\lambda_d}\mathcal{M}_{ab\to cd}^{*\lambda'_c\lambda'_d},$$

is the unnormalized spin density matrix, and

$$D_c^{\lambda_c\lambda'_c} = \mathcal{M}_{c\ \mathrm{decay}}^{\lambda_c}\mathcal{M}_{c\ \mathrm{decay}}^{*\lambda'_c},$$

the unnormalized decay matrix.

- Most authors then chose to evaluate these matrices analytically.

- However it will be easier for our purposes to evaluate the matrix elements using helicity amplitude techniques and then calculate the matrices numerically.

Spin Correlations

- These approaches require the production and decay of the particle to be generated at the same time.

- This is difficult if:
  - We want to generate QCD radiation from the particles before they decay;
  - The particle has many decay modes;
  - There are many sequential decays;
  - There are many final state particles.

- For a Monte Carlo event generator we need an algorithm which allows
  - The production and decay to be generated separately,
  - Has a complexity which only grows linearly with the number of final-state particles.

Helicity Amplitudes

- We will calculate all the cross sections and decays we will need as helicity amplitudes.

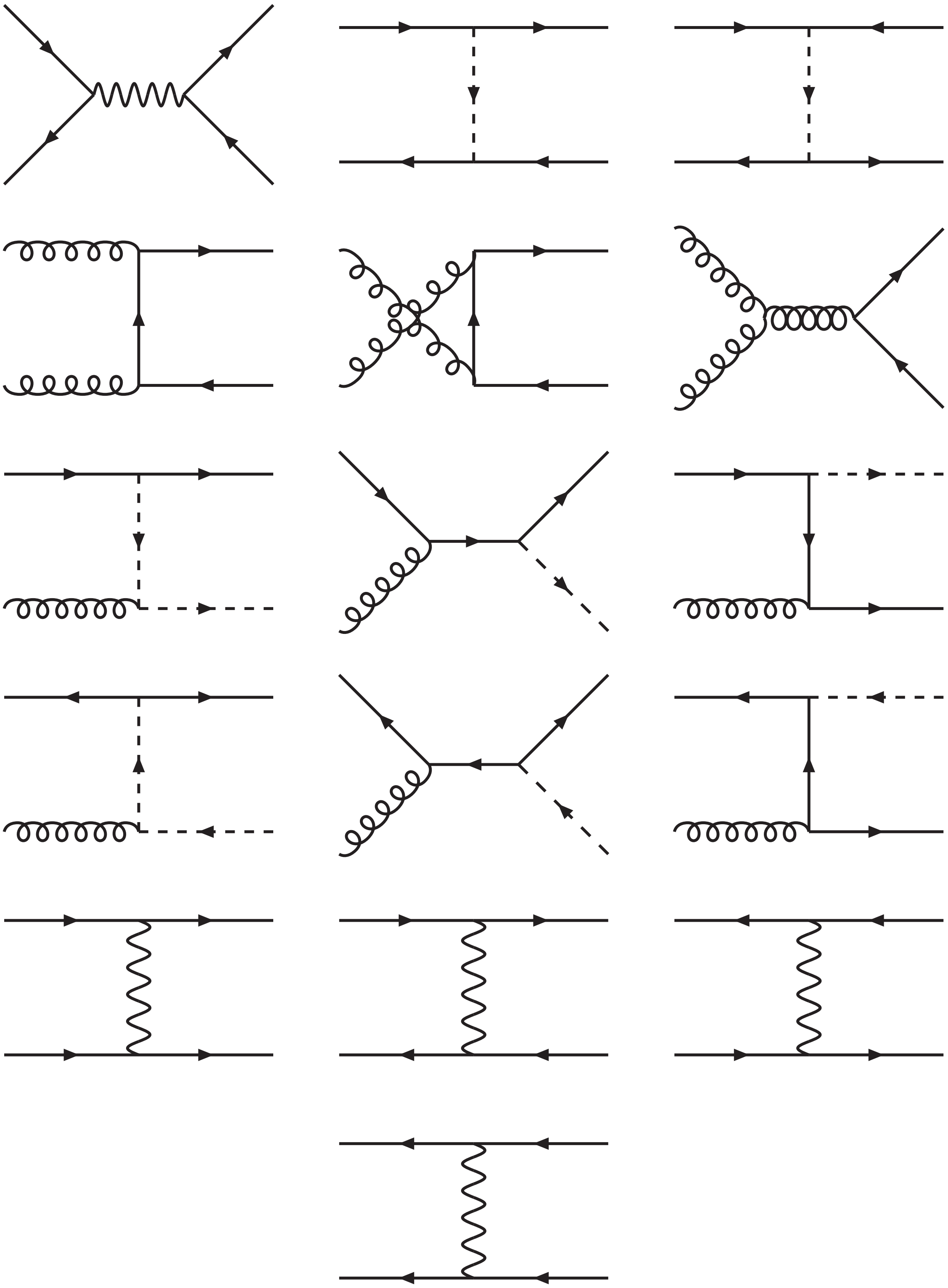
- The massive fermions will be defined as follows

$$
\begin{aligned}
u(p,\lambda) &= \frac{1}{\sqrt{2p\cdot\ell}}\,(\not{p}+m)\,u_{-\lambda}(l),\\
v(p,\lambda) &= -\frac{1}{\sqrt{2p\cdot\ell}}\,(\not{p}-m)\,u_{-\lambda}(l).
\end{aligned}
$$

- Massive gauge bosons will not be included but replaced by their decay products.

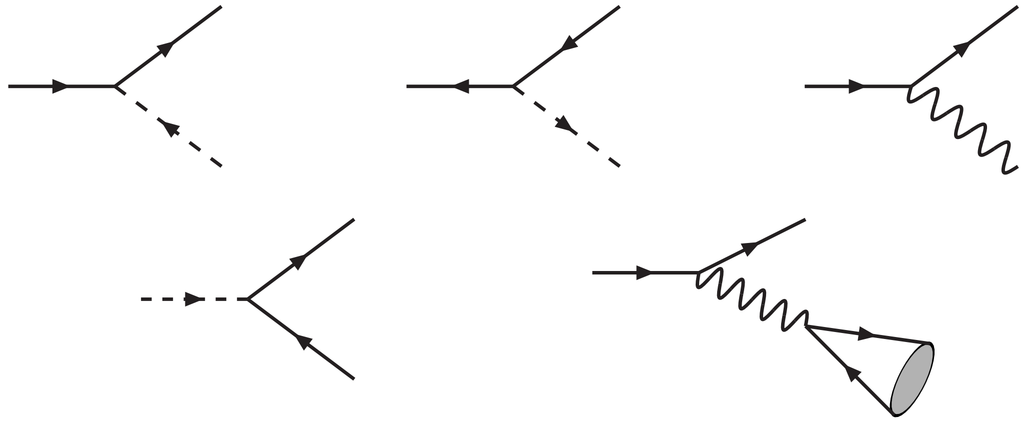
- This enables us to include the correlations by calculating a small number of helicity amplitudes for the production and decay of the particles.

Production Feynman Diagrams

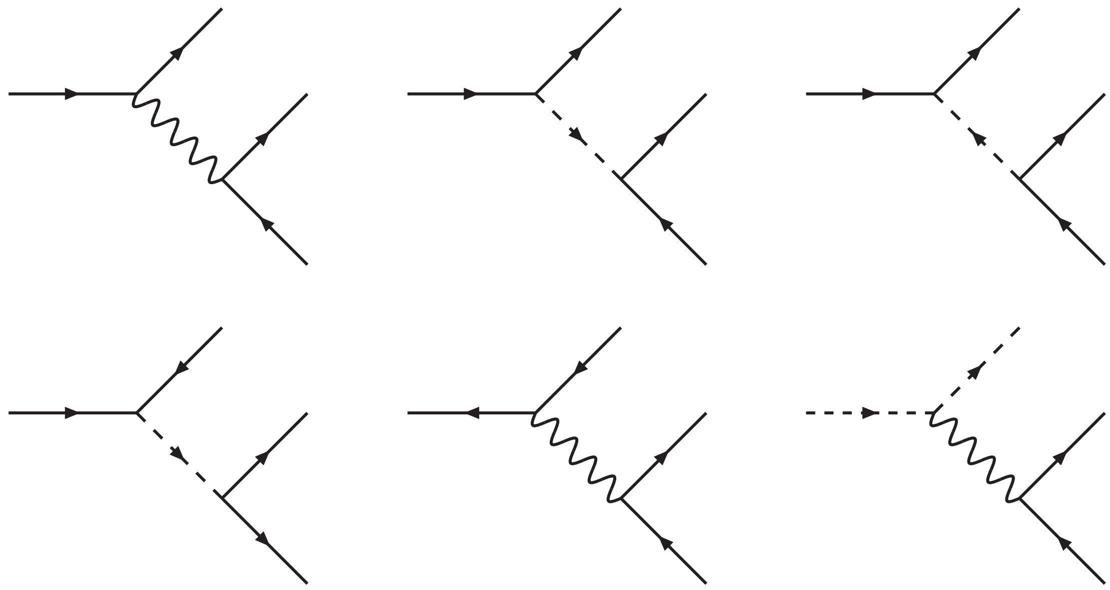


Decay Feynman Diagrams

- Two body decays



- Three body decays



Spin Correlations in the Parton Shower

- HERWIG already includes correlations in and between jets using the algorithm of Collins and Knowles

  J. C. Collins, Nucl. Phys. B304, (1988) 794.

  I. G. Knowles, Nucl. Phys. B304, (1988) 767.

  I. G. Knowles, Nucl. Phys. B310, (1988) 571.

  I. G. Knowles, Comput. Phys. Commum. 58 (1990) 271.

- The algorithm for heavy particle decays is virtually the same so it's helpful to review this algorithm.

- The algorithm is as follows

- First the momenta of the particles in the hard collision are generated using

$$\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_n}\prod_{i=1,n}D^i_{\lambda_i\lambda'_i}$$

where $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.

Spin Correlations in the Parton Shower

- One of the outgoing partons is selected at random and a spin density matrix calculated.

$$\rho_{\lambda_j\lambda'_j} = \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\prod_{i\neq j} D^i_{\lambda_i\lambda'_i} \mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_j\ldots\lambda'_n}$$

  In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
  The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

- The type of branching $i \to jk$, azimuthal angle ϕ and momentum fraction z are generated using

$$\rho^i_{\lambda_i\lambda'_i} V^{jk}_{\lambda_i\lambda_j\lambda_k}(z,\phi) V^{jk*}_{\lambda'_i\lambda'_j\lambda'_k}(z,\phi) D^j_{\lambda_j\lambda'_j} D^k_{\lambda_k\lambda'_k},$$

  where $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.

Spin Correlations in the Parton Shower

- One of the partons produced in the branching is selected and a spin density matrix calculated using

$$\rho^j_{\lambda_j\lambda'_j} = \frac{1}{N_{\rho\mathrm{QCD}}}\rho^i_{\lambda_i\lambda'_i}V^{jk}_{\lambda_i\lambda_j\lambda_k}(z,\phi)V^{jk*}_{\lambda'_i\lambda'_j\lambda'_k}(z,\phi)D^k_{\lambda_k\lambda'_k},$$

  In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
  The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

- This is repeated until the parton to be developed reaches the cut-off scale. The decay matrix of this parton is set to $\delta_{\lambda_i\lambda'_i}$ and another parton selected.

- When all of the partons in a branching have been developed a decay matrix is calculated using

$$D^i_{\lambda_i\lambda'_i} = \frac{1}{N_{D\mathrm{QCD}}}V^{jk}_{\lambda_i\lambda_j\lambda_k}(z,\phi)V^{jk*}_{\lambda'_i\lambda'_j\lambda'_k}(z,\phi) D^j_{\lambda_j\lambda'_j}D^k_{\lambda_k\lambda'_k}$$

- The normalization is chosen so that $\mathrm{Tr}D = 1$.

Spin Correlations in the Parton Shower

- A another parton produced in the previous branching is then selected and the procedure repeated using the calculated decay matrix rather than the identity.

- Eventually the hard process is reached, another parton selected and a spin density matrix calculated using the calculated decay matrix for the jet which has already been developed

- There is a similar algorithm for correlations in the initial-state parton shower.

Spin Correlations in the Decay of heavy particles

- The algorithm is as follows
- First the momenta of the particles in the hard collision are generated using

$$\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_n}\prod_{i=1,n}D^i_{\lambda_i\lambda'_i}$$

where $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.

- One of the outgoing partons is selected at random and a spin density matrix calculated.

$$\begin{aligned}\rho_{\lambda_j\lambda'_j} &= \frac{1}{N_\rho}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\prod_{i\neq j}D^i_{\lambda_i\lambda'_i}\\ &\quad \mathcal{M}_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_j\ldots\lambda'_n}\end{aligned}$$

In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

Spin Correlations in the Decay of heavy particles

- The decay mode of the particle is selected using the branching ratios.

- The momenta of the particles produced in the n-body decay are generated using

$$\rho_{\lambda_0\lambda'_0}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda'_1\ldots\lambda'_n}\prod_{i=1,n}D^i_{\lambda_i\lambda'_i}$$

where $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.

- One of the particles is selected and a spin density matrix calculated

$$\rho_{\lambda_j\lambda'_j} = \frac{1}{N_{D\rho}}\rho_{\lambda_0\lambda'_0}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_j\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda'_1\ldots\lambda'_j\ldots\lambda'_n}\prod_{i\neq j}D^i_{\lambda_i\lambda'_i}$$

In the initial step we take $D^i_{\lambda_i\lambda'_i} = \delta_{\lambda_i\lambda'_i}$.
The normalization is chosen so that $\mathrm{Tr}\rho = 1$.

Spin Correlations in the Decay of heavy particles

- These steps are repeated until a stable particle is selected. The decay matrix of this particle is set to $\delta_{\lambda_i\lambda'_i}$ and another particle from the same decay selected to be decayed next.

- When all the particles in a decay have been developed a decay matrix is calculated

$$D_{\lambda_0\lambda'_0} = \frac{1}{N_D}\mathcal{M}_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^*_{\lambda'_0;\lambda'_1\ldots\lambda'_n}\prod_{i=1,n} D^i_{\lambda_i\lambda'_i}$$

The normalization is chosen so that $\mathrm{Tr}D = 1$.

- We then select another particle from the decay which produced this particle and repeat the procedure and decay it.

- Eventually the hard process is reached and another particle selected. The whole procedure is repeated for this particle using the calculated decay matrices for those particles which have been decayed rather than the identity.

Examples

- Neutralino production in squark decay.
- Neutralino production in $\mathrm{e^+e^-}$ collisions.
- Top quark pair production in $\mathrm{e^+e^-}$ collisions.
- Top quark pair production in hadron-hadron collisions.
- Chargino production in $\mathrm{e^+e^-}$ collisions.

Neutralino Production in squark decay

- In most SUSY models the production of electroweak gauginos and sleptons at the LHC is dominated by production in squark decays rather than direct production.

- In many models the decay chain

$$\tilde{\mathrm{q}}_{\boldsymbol{L}} \to \mathrm{q}\tilde{\boldsymbol{\chi}}^{\mathbf{0}}_{\mathbf{2}} \to \mathrm{q}\tilde{\boldsymbol{\ell}}^{\pm}_{\boldsymbol{R}}\boldsymbol{\ell}^{\mp} \to \mathrm{q}\boldsymbol{\ell}^{+}\boldsymbol{\ell}^{-}\tilde{\boldsymbol{\chi}}^{\mathbf{0}}_{\mathbf{1}}$$

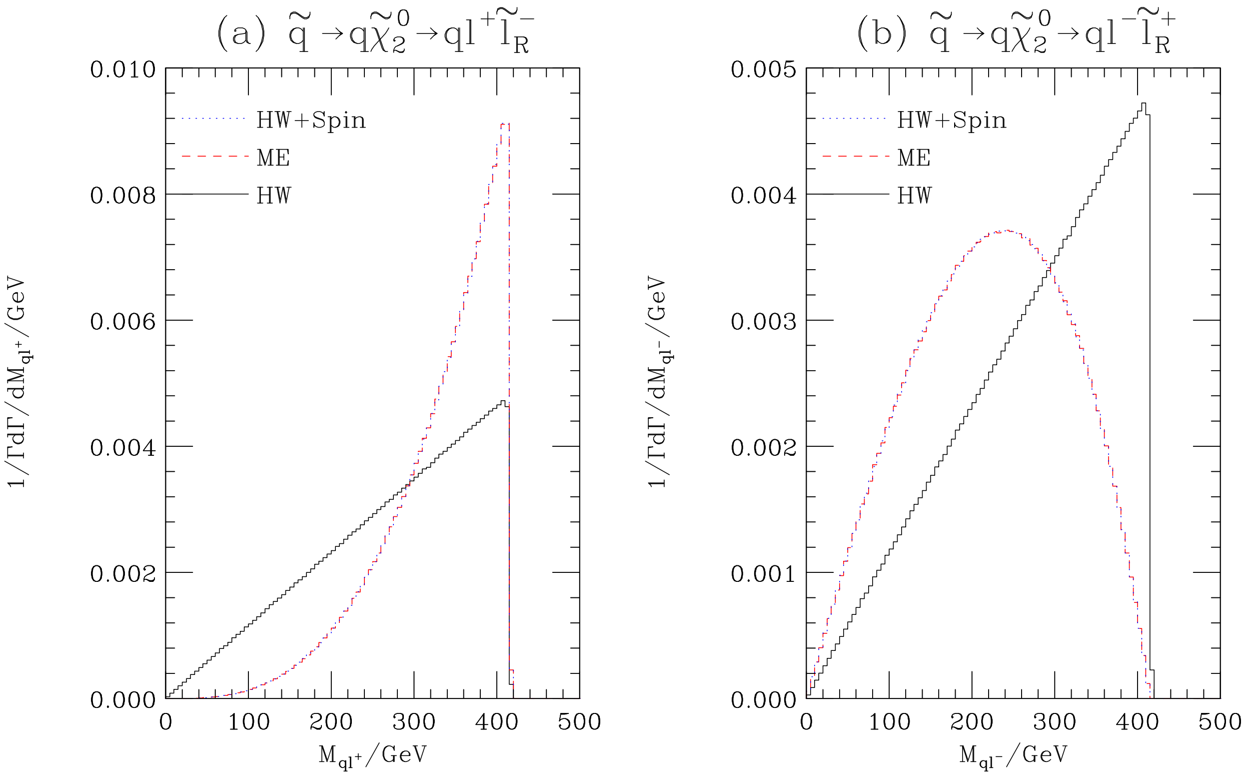
  is important as by measuring edges in the $\ell^+\ell^-$, $\mathrm{q}\ell$ and $\mathrm{q}\ell^+\ell^-$ distributions the masses of $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\ell}_R$ and $\tilde{\mathrm{q}}$ can be measured.

- Here the spin correlation algorithm must calculate the spin density matrix for $\tilde{\mathrm{q}}_L \to \mathrm{q}\tilde{\chi}^0_2$ and then use this to perform the decay $\tilde{\chi}^0_2 \to \tilde{\ell}^{\pm}_R\ell^{\mp}$.

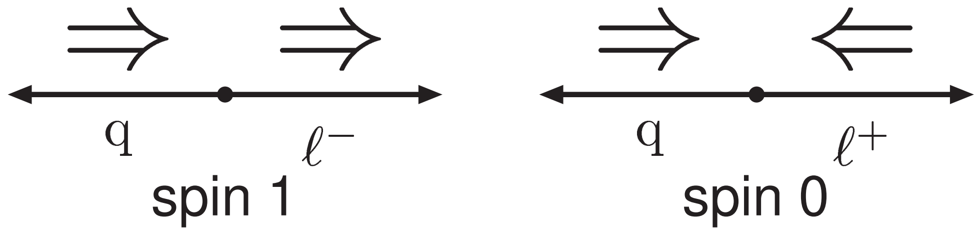
- The effects are most visible in the $\mathrm{q}\ell$ mass distribution.

- Used LHC SUGRA point 5 $M_0 = 200$ GeV, $M_{1/2} = 300$ GeV, $A_0 = 300$ GeV, $\tan\beta = 2.1$ and $\mathrm{sgn}\,\mu+$.

Neutralino Production in squark decay

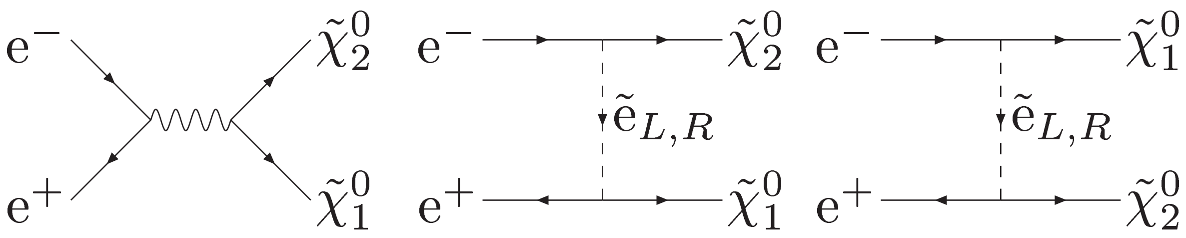


- The quark will always be left-handed and the lepton right-handed

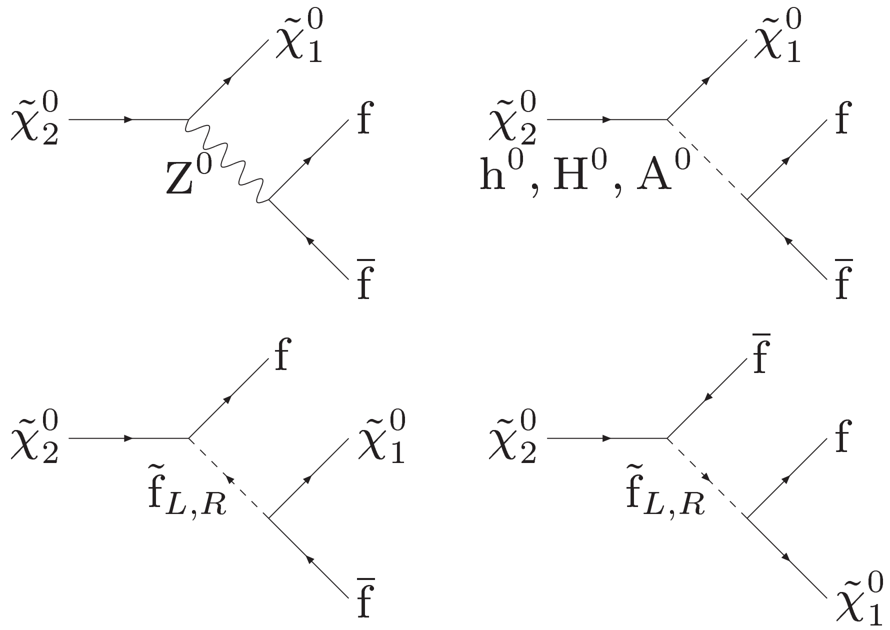


- As we can't experimentally distinguish quarks and antiquarks the average is observed.

Neutralino Production in e^+e^- collisions



- This is a more complicated example.
- As the lightest neutralino is stable in R-parity conserving SUSY models the SUSY final state which requires the lowest centre-of-mass energy to produce it may be $\tilde{\chi}^0_2\tilde{\chi}^0_1$.
- The $\tilde{\chi}^0_2$ then decays $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 f\bar{f}$.



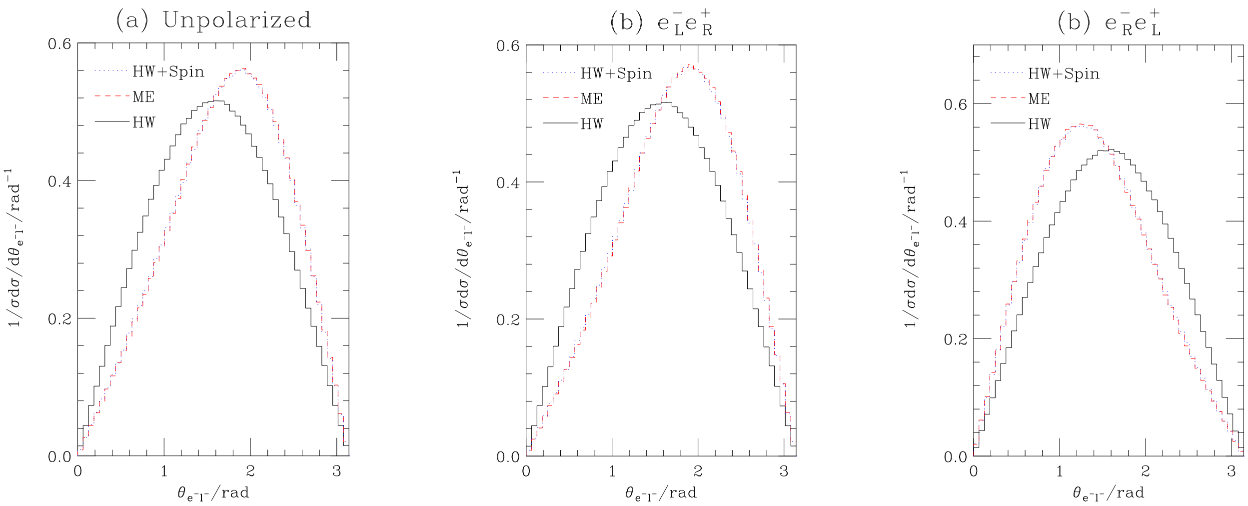
- The leptonic decay modes are the most important.

Neutralino Production in e^+e^- collisions

- Here the neutralino is produced in a $2 \to 2$ process, with the possibility of polarized beams so the calculation of the spin density matrix is more complicated.

- The decay of the neutralino, via a 2 or 3 body decay, is then performed using this decay matrix.

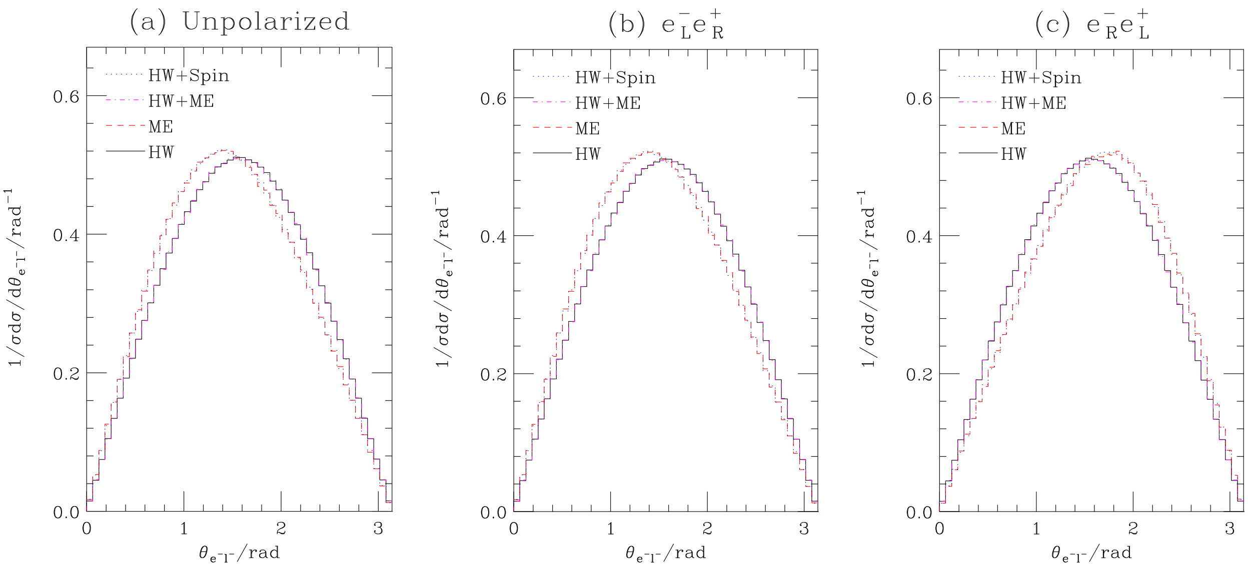
- First consider the two body decay $\tilde{\chi}^0_2 \to \tilde{\ell}^+_R \ell^-$ and study the angle between the lepton and the beam, again using SUGRA point 5.



Neutralino Production in $\mathrm{e}^+\mathrm{e}^-$ collisions

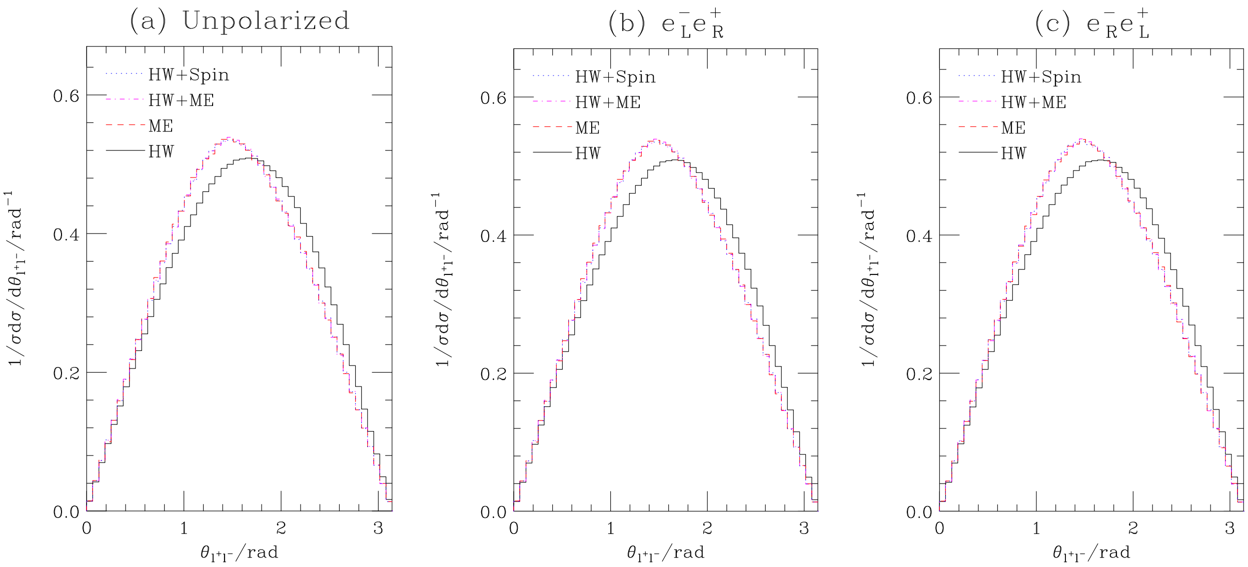
- We can also consider the three body decay $\tilde{\chi}^0_2 \to \ell^+\ell^-\tilde{\chi}^0_1$ using the SUSY parameters $M_0 = 210$ GeV, $A_0 = 0$ GeV, $\tan\beta = 10$, $M_1 = 450$ GeV, $M_2 = 350$ GeV, $M_3 = 350$ GeV.

- Study the angle between the lepton and the beam,



Neutralino Production in e^+e^- collisions

- and between the lepton and antilepton.

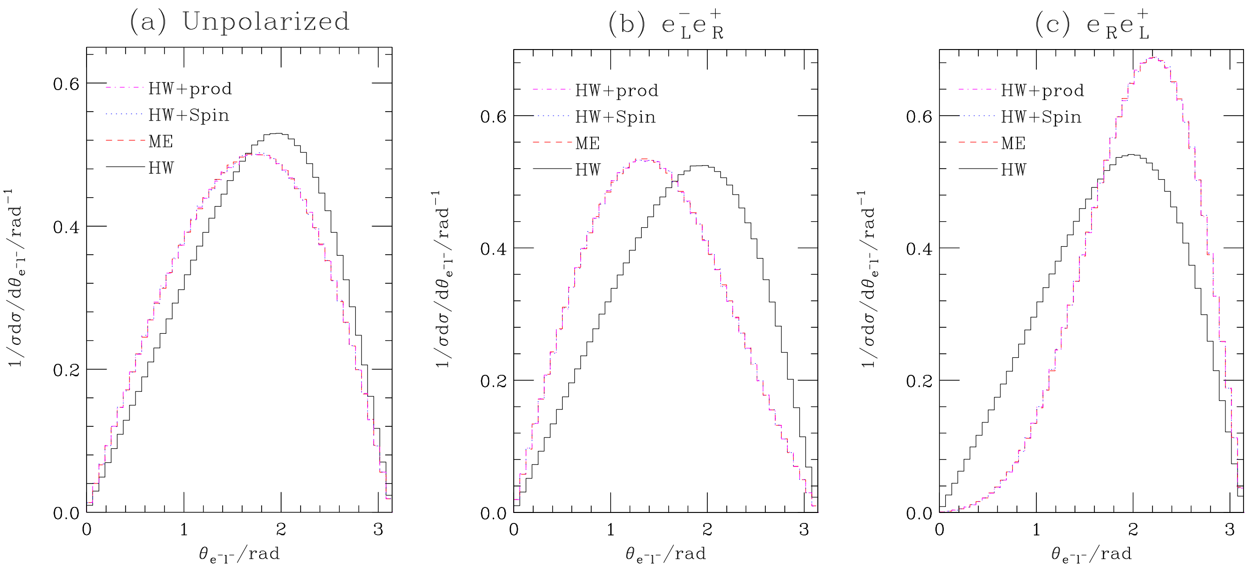


Top pair production in e^+e^- collisions

- In this example in addition to the calculation of the spin density matrix for the production we also need to calculate the decay matrix for the first decay and use this in performing the second decay.

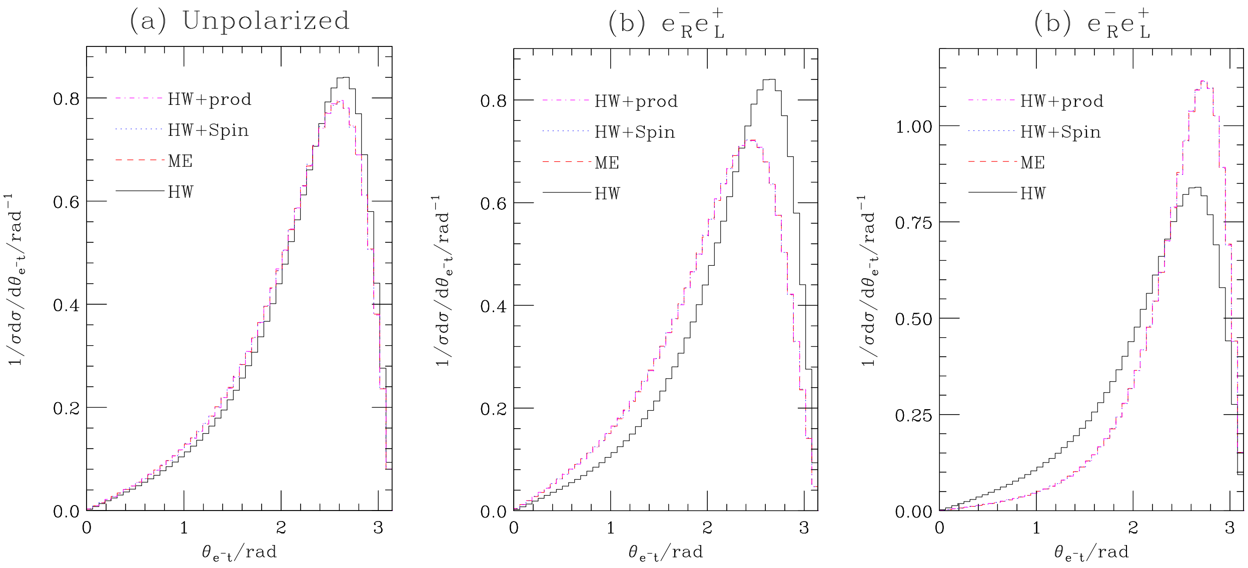
- Study the semi-leptonic decays of both top quarks and look at the correlations.

- Between the lepton produced in the antitop decay and the beam.

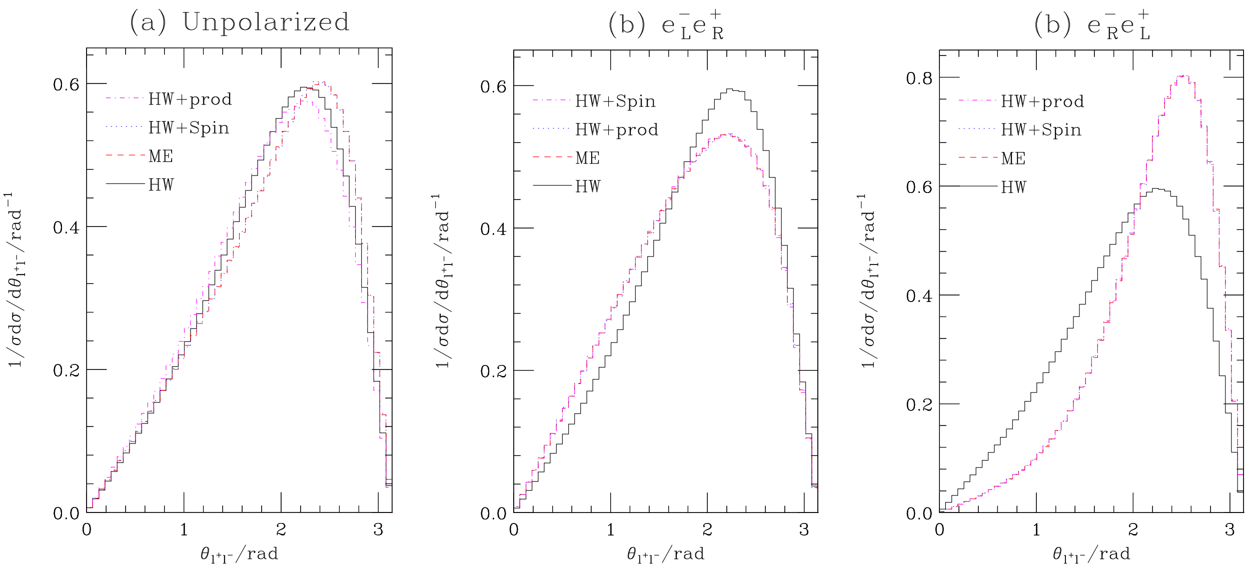


Top pair production in e^+e^- collisions

- Between the lepton and the top quark.



- Between the lepton and antilepton.



Top pair production in $\mathrm{e}^{+}\mathrm{e}^{-}$ collisions

- The correlation between the lepton and the beam or top quark is only affected by the production mechanism.

- The correlation between the lepton and antilepton is also effected by the decay of the first quark.

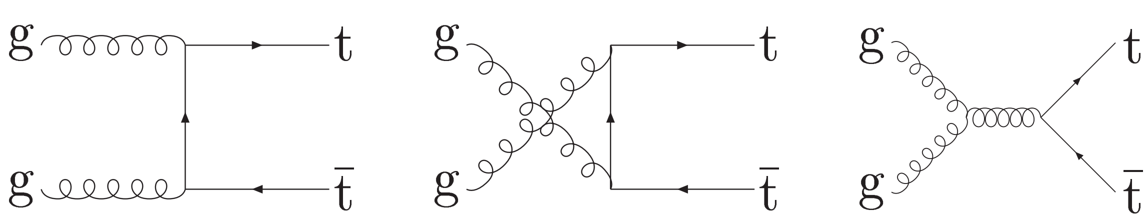
Top quark pair production in hadron collisions

- At leading order there are two processes which contribute

$$\begin{aligned} \mathrm{q\bar{q}} &\longrightarrow \mathrm{t\bar{t}}, \\ \mathrm{gg} &\longrightarrow \mathrm{t\bar{t}}. \end{aligned}$$

- The quark initiated process has the same helicity structure as $\mathrm{e^+e^-} \to \mathrm{t\bar{t}}$.

- The gluon initiated process presents complications due to the different colour flows.



- The colour matrices are

$$\begin{aligned} C_{t\,c_1c_2}^{\;ab} &= t^a_{c_1c'}t^b_{c'c_2} && t\mathrm{-channel}, \\ C_{u\,c_1c_2}^{\;ab} &= t^b_{c_1c'}t^a_{c'c_2} && u\mathrm{-channel}, \\ C_{s\,c_1c_2}^{\;ab} &= if^{abc}t^c_{c_1c_2} && s\mathrm{-channel}. \end{aligned}$$

Top quark pair production in hadron collisions

- the s-channel diagram can be written

$$C_{s\,c_1c_2}^{\;ab} = C_{t\,c_1c_2}^{\;ab} - C_{u\,c_1c_2}^{\;ab},$$

  using the definition of f^{abc}.

- This could be a problem if there is more than one colour flow in the decays as well.

- The number of colour flows would then grow exponentially with the number of particles.

- Doesn't happen for the decays we have been considering in the SM or MSSM.

- Therefore we can use

$$\sum_{a,b=1,C_N} C_{ab}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\prod_{i=1,n} D^i_{\lambda_i\lambda'_i},$$
$$\mathcal{M}^a_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{b*}_{\kappa'_1\kappa'_2;\lambda'_1\ldots\lambda'_n}$$

  to generate the momenta of the particles, etc.

Top quark pair production in hadron collisions

- In some models, e.g RPV SUSY, the decays can have more than one colour flow.

- Then we must select a colour flow in the hard process using

$$C_{aa}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}^a_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{a*}_{\kappa'_1\kappa'_2;\lambda_1\ldots\lambda_n}A_a,$$

where

$$A_a = \frac{\sum_{b,c=1,C_N} C_{bc}\rho^1_{\kappa_1\kappa'_1}\rho^2_{\kappa_2\kappa'_2}\mathcal{M}^b_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{c*}_{\kappa'_1\kappa'_2;\lambda_1\ldots\lambda_n}}{\sum_{b=1,C_N} C_{bb}\mathcal{M}^b_{\kappa_1\kappa_2;\lambda_1\ldots\lambda_n}\mathcal{M}^{b*}_{\kappa'_1\kappa'_2;\lambda_1\ldots\lambda_n}}.$$

- or the decay using

$$\rho_{\lambda_0\lambda'_0}C_{aa}\mathcal{M}^a_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^{a*}_{\lambda'_0;\lambda_1\ldots\lambda_n}E_a,$$

where

$$E_a = \frac{\sum_{b,c=1,C_N} C_{bc}\mathcal{M}^b_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^{c*}_{\lambda'_0;\lambda_1\ldots\lambda_n}}{\sum_{b=1,C_N} C_{bb}\mathcal{M}^b_{\lambda_0;\lambda_1\ldots\lambda_n}\mathcal{M}^{b*}_{\lambda'_0;\lambda_1\ldots\lambda_n}}.$$

- this stops the exponential growth but neglects some correlations.

Top quark pair production in hadron collisions

- We looked at the correlations using

$$\begin{aligned}
\mathcal{O}_1 &= \hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{q}}_{\ell^-}; \\
\mathcal{O}_2 &= \left(\hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{p}}_p\right)\left(\hat{\mathbf{q}}_{\ell^-} \cdot \hat{\mathbf{p}}_p\right); \\
\mathcal{O}_3 &= \left(\hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{k}}_t\right)\left(\hat{\mathbf{q}}_{\ell^-} \cdot \hat{\mathbf{k}}_t\right); \\
\mathcal{O}_4 &= \frac{1}{2}\left[\left(\hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{p}}_p\right)\left(\hat{\mathbf{q}}_{\ell^-} \cdot \hat{\mathbf{k}}_t\right) + \left(\hat{\mathbf{q}}_b^* \cdot \hat{\mathbf{k}}_t\right)\left(\hat{\mathbf{q}}_{\ell^-} \cdot \hat{\mathbf{p}}_p\right)\right]; \\
\mathcal{O}_5 &= \mathrm{sgn}\, y_t\ \mathcal{O}_4; \\
\mathcal{O}_6 &= \left(\hat{\mathbf{p}}_p \times \hat{\mathbf{q}}_b^*\right) \cdot \left(\hat{\mathbf{p}}_p \times \hat{\mathbf{q}}_{\ell^-}\right) = \mathcal{O}_1 - \mathcal{O}_2.
\end{aligned}$$

- for the decays $\mathrm{t} \to \mathrm{bW}^+ \to \mathrm{bq\bar{q}}$ and $\bar{\mathrm{t}} \to \bar{\mathrm{b}}\mathrm{W}^- \to \bar{\mathrm{b}}\ell^-\bar{\nu}$

- where
  $\hat{\mathbf{p}}_p$ is the beam direction
  $\hat{\mathbf{q}}_{\ell^-}$ is the momentum of the lepton
  $\hat{\mathbf{q}}_b^*$ is the momentum of the b in the t rest frame
  $\hat{\mathbf{k}}_t$ is the top quark momentum

- and looked at the correlations at the Tevatron and LHC.

Top quark pair production in hadron collisions

- Observables at the Tevatron.

	$\mathcal{O}$/%		
	Full	Spin 1	Spin 2
$\mathcal{O}_1$	2.46 ± 0.04	2.45 ± 0.02	2.41 ± 0.02
$\mathcal{O}_2$	2.94 ± 0.02	2.92 ± 0.01	2.90 ± 0.01
$\mathcal{O}_3$	1.68 ± 0.02	1.67 ± 0.01	1.64 ± 0.01
$\mathcal{O}_4$	0.03 ± 0.02	0.01 ± 0.01	0.01 ± 0.01
$\mathcal{O}_5$	1.95 ± 0.02	1.94 ± 0.01	1.92 ± 0.01
$\mathcal{O}_6$	-0.48 ± 0.03	-0.47 ± 0.02	-0.46 ± 0.02

- Observables at the LHC.

	$\mathcal{O}$/%		
	Full	Spin 1	Spin 2
$\mathcal{O}_1$	-2.01 ± 0.02	-2.04 ± 0.02	-2.04 ± 0.02
$\mathcal{O}_2$	-0.16 ± 0.02	-0.16 ± 0.02	-0.18 ± 0.02
$\mathcal{O}_3$	-0.55 ± 0.02	-0.56 ± 0.02	-0.57 ± 0.01
$\mathcal{O}_4$	0.00 ± 0.02	-0.01 ± 0.01	-0.02 ± 0.01
$\mathcal{O}_5$	-0.23 ± 0.02	-0.25 ± 0.01	-0.25 ± 0.01
$\mathcal{O}_6$	-1.85 ± 0.02	-1.87 ± 0.02	-1.86 ± 0.01

- Here the default result of HERWIG is consistent with zero.

Chargino production in $\mathrm{e}^+\mathrm{e}^-$ collisions

- In a Monte Carlo simulation we often want to include the widths of the particles.

- Can't factorized the matrix element as before.

- We have two options

  – Use off-shell momenta and on-shell mass

$$\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) = \not{p} + m + \frac{\not{l}}{2p\cdot l}(m^2 - p^2)$$

  – Use off-shell momenta and off-shell mass

$$\sum_\lambda u_\lambda(p)\bar{u}_\lambda(p) = \not{p} + m_{\mathrm{off}},$$

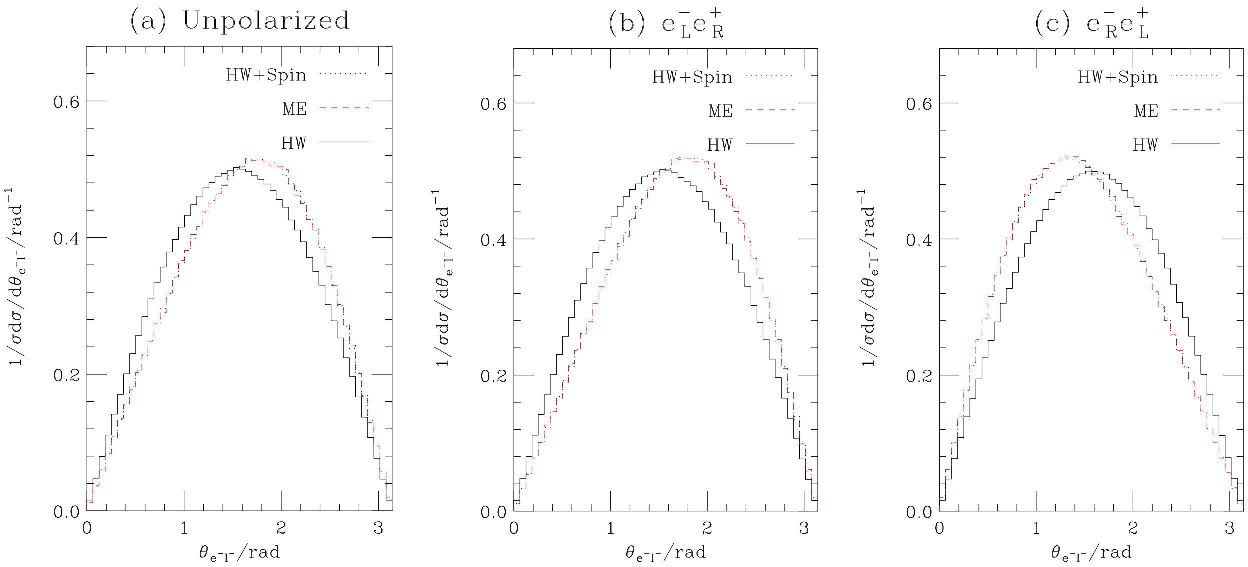
- easiest to study effects in $\tilde{\chi}^\pm_1\tilde{\chi}^\mp_2$ production.

- At point we studied the $\tilde{\chi}^\pm_1$ width is 73 MeV and the $\tilde{\chi}^\pm_2$ width is 2.9 GeV. We also considered ten times the $\tilde{\chi}^\pm_2$ width. The $\tilde{\chi}^\pm_1$ mass is 127.8 GeV and the $\tilde{\chi}^\pm_2$ mass is 357.6 GeV.

Chargino production in $\mathrm{e}^+\mathrm{e}^-$ collisions

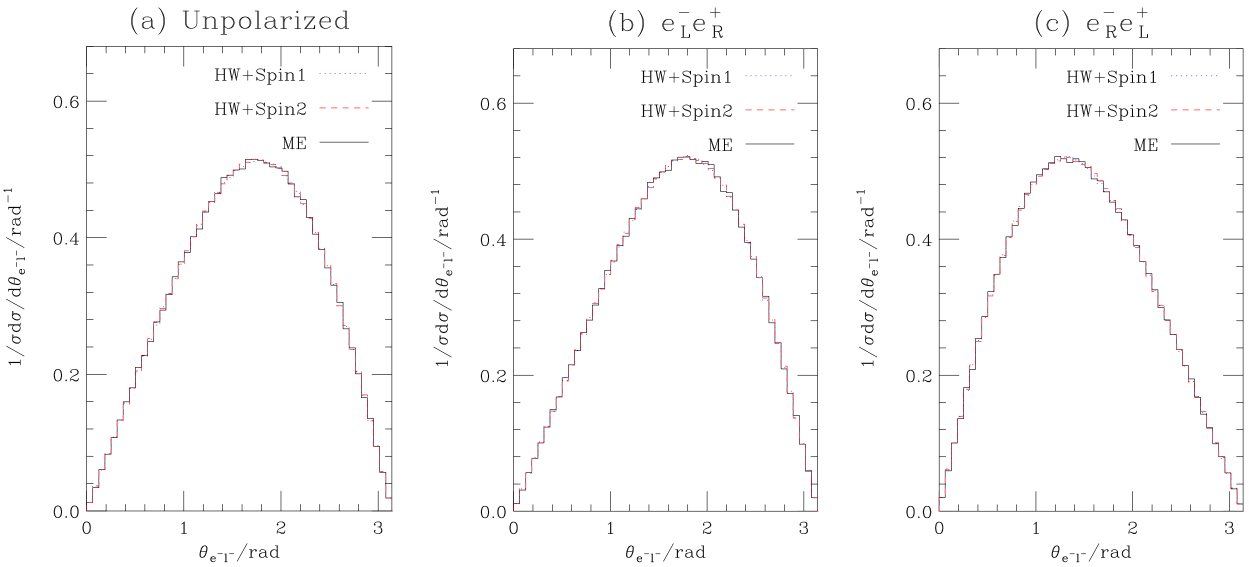
- Look at the angle between the beam and the lepton produced in decay of the $\tilde{\chi}_2^+$ via W exchange.

- No width

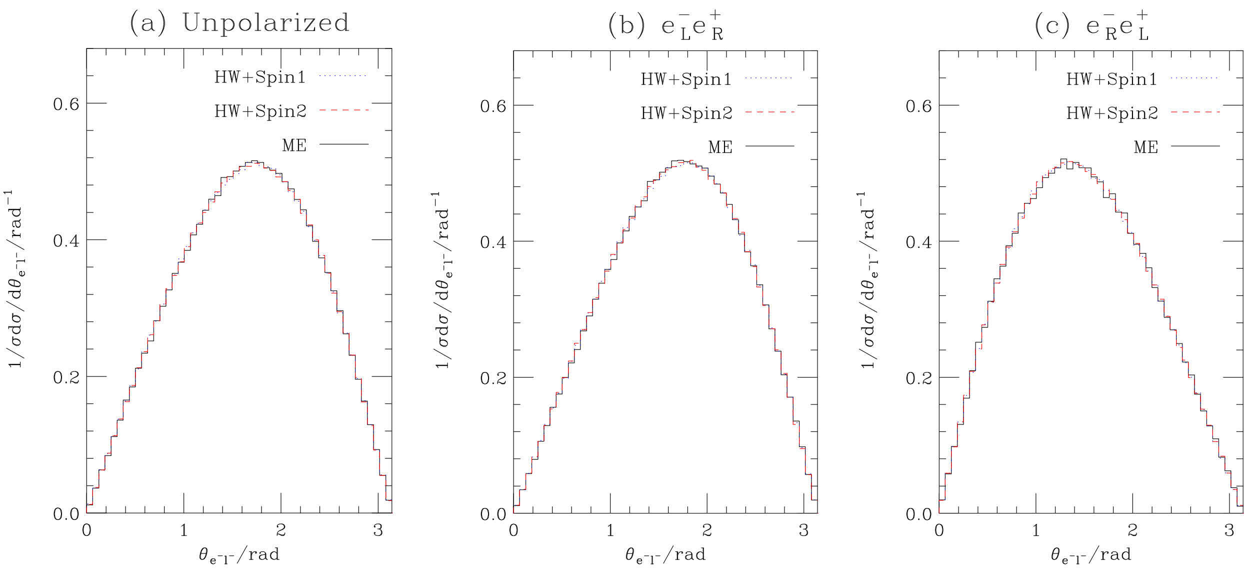


Chargino production in e^+e^- collisions

- Physical width



- Ten times physical width



Correlations with the parton shower

- For a full simulation we should also include correlations with the parton shower.
- This should be possible given the similarities of the algorithms.
- The implementation of such an algorithm is a major project and will have to wait for a C++ event generator.
- Here we will merely propose two options.

First Algorithm

- One choice is to perform the QCD algorithm as before.
- When a particle reaches the cut-off scale the decay algorithm is run.
- The QCD algorithm is run on the decay products and so on.
- The matrices can be passed up and the chain and the correlations included.
- There are problems with this algorithm:
  - The scale of α_S depends on the energy of the hard collision or mass of the decaying particle;
  - In the angular-ordered shower the maximum angle of emission depends on the direction of the colour partner which is defined by the hard process or decay;
  - The parton-shower algorithm doesn't conserve energy of momentum. The momenta of the jets must be adjusted after the parton shower to ensure 4-momentum conservation.

Second Algorithm

- Perform the QCD algorithm.
- Select an unstable particle at random.
- Use the information from the QCD algorithm to calculate a spin density matrix for the decay.
- Use the decay algorithm.
- Use the QCD algorithm with the result of the decay algorithm as input.
- Repeat this procedure.
- There are problems with this algorithm:
  - More spin density and decay matrices have to be calculated and stored;
  - The algorithm is more complicated.
- However this is probably the easier algorithm to implement.

Conclusions

- We have shown that it's possible to include spin correlations in a Monte Carlo event generator
- The algorithm:
  - allows production and decay to be generated as separate steps;
  - has a complexity which only grows linearly with the number of particles.
- The results are in good agreement with matrix element calculations.
- This will be available in the next version of HERWIG.
- We have also proposed a new algorithm for correlations with the parton shower which can hopefully be implemented in the next generation of C++ event generators.