Using Spin to Distinguish Models at the LHC

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This talk is based on

JHEP 10 (2005) 069	JS & Bryan Webber
JHEP 08 (2006) 055	Christiana Athanasiou, Chris Lester, JS & Bryan Webber
hep-ph/0609296	JS

Outline

- Spin Beyond the Standard Model
- Decay of a q' via a W
- Cascade Decay of a q'
- The Kullback-Leibler distance

Beyond the Standard Model

Many possible extensions to the Standard Model, known collectively as models Beyond the Standard Model (BSM).

Motivated by a number of arguments:

the hierarchy problem, dark matter, massive neutrinos, string theory,...?

Many of these models contain

1) New particles within the reach of the LHC,

2) A dark matter candidate.

Spin

- With the popularity of supersymmetric (SUSY) BSM models, it has become even more important to develop methods to deduce the spin of any new particles produced.

- If new particles with the expected charge and/or colour properties are produced, we must still have evidence that they have the correct spin before they can be declared to be SUSY-partners.

- Models such as Universal Extra Dimensions (UED) and Little Higgs with T-parity (LHT) can both have experimental signatures which could be mistaken for SUSY.

UED

- A UED model has at least 1 extra compactified spatial dimension into which all gauge fields can propagate giving Kaluza-Klein towers.

  With 1 extra dimension, y, a $\mathbb{Z}_2$ orbifold is used to get chiral fermions.

- We express 5D fields as a Fourier expansion in y:

$$F(x,y) = F_0(x) + \sum_{n=1}^{\infty} F_n(x)\cos\left(\frac{ny}{R}\right) + F_n'(x)\sin\left(\frac{ny}{R}\right)$$

  where R is the radius of the extra dimension.

- KK-parity from broken conservation of momentum in extra dimensions: $(-1)^n$.

Little Higgs

- In Little Higgs models, the Higgs is a Pseudo-Nambu-Goldstone Boson.
  This offers an alternative mechanism to break electroweak symmetry without fine-tuning.

- The minimal extension is the Littlest Higgs model:

  Global SU(5) broken to SO(5)

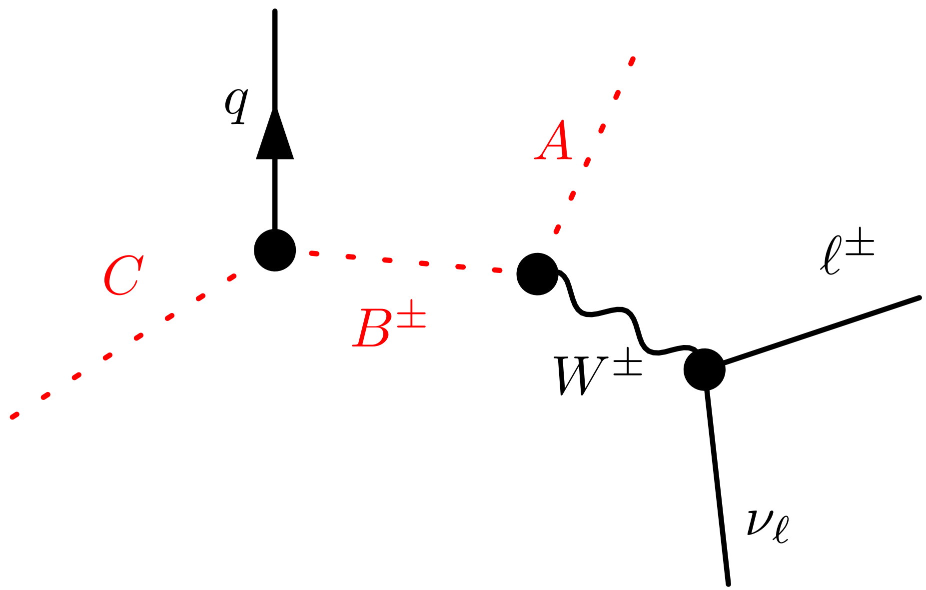
  [SU(2) $\times$ U(1)]^2 subgroup of SO(5) is then gauged.

- Collective symmetry breaking avoids one-loop quadratic divergences.

- T-parity introduced to overcome electroweak constraints.

Decays via a W

Consider the following decay of a new particle, C:

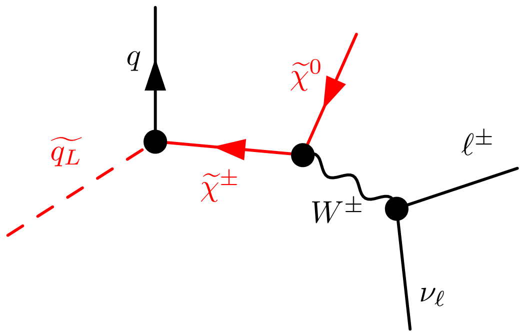


where B and A are also new particles.

These may occur in the MSSM, UED or LHT models.

Decays via a W

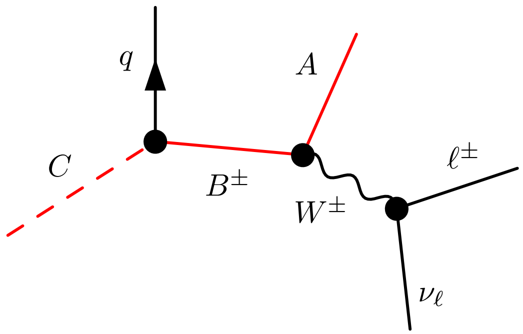
The possibilities for the spin assignments are:



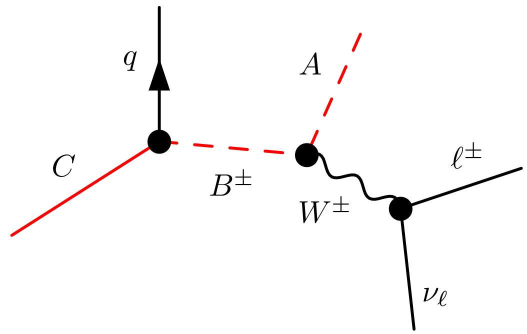
in the MSSM.

Decays via a W

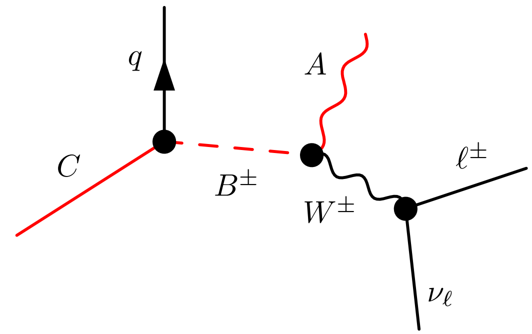
The possibilities for the spin assignments are:



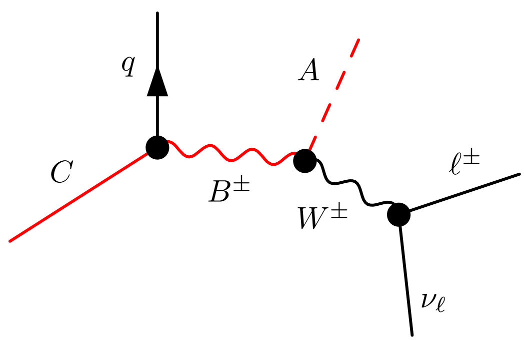
SFF



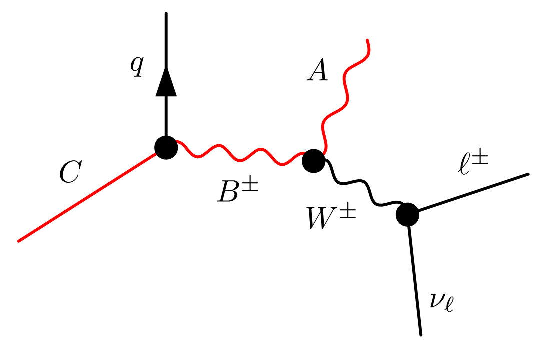
FSS



FSV



FVS



FVV

S = scalar F = fermion V = Vector

Caveat

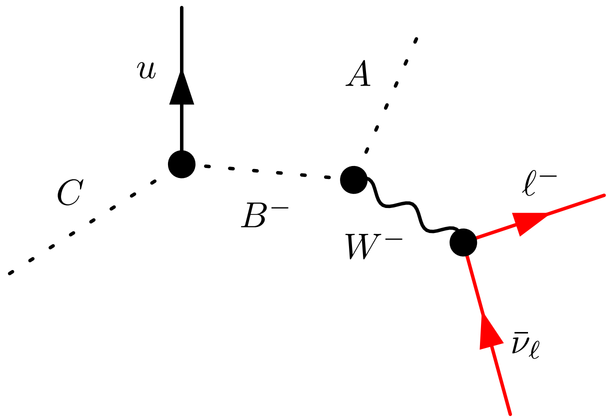
- In this initial study, we consider the simpler form of the SVV vertex: $g_{\mu\nu}$

  and of the VVV vertex:
  $g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu$.

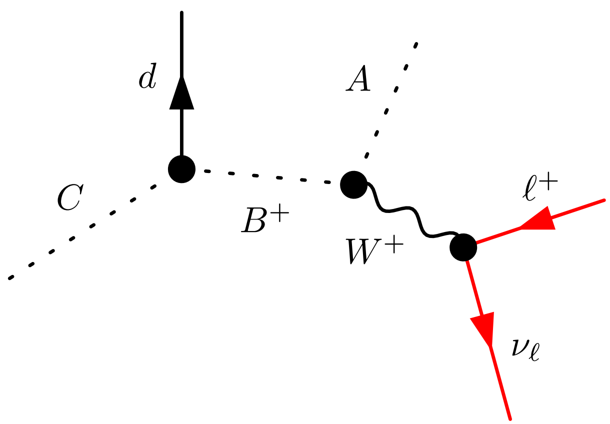
  This will be the case in a number of models, but more complicated forms are possible.

ℓ^- and ℓ^+

Process 1



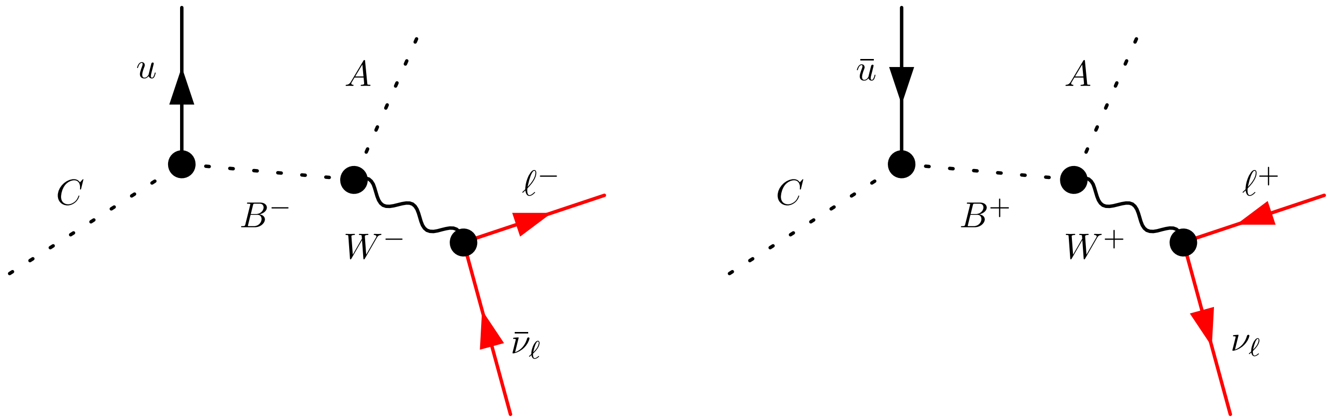
Process 2



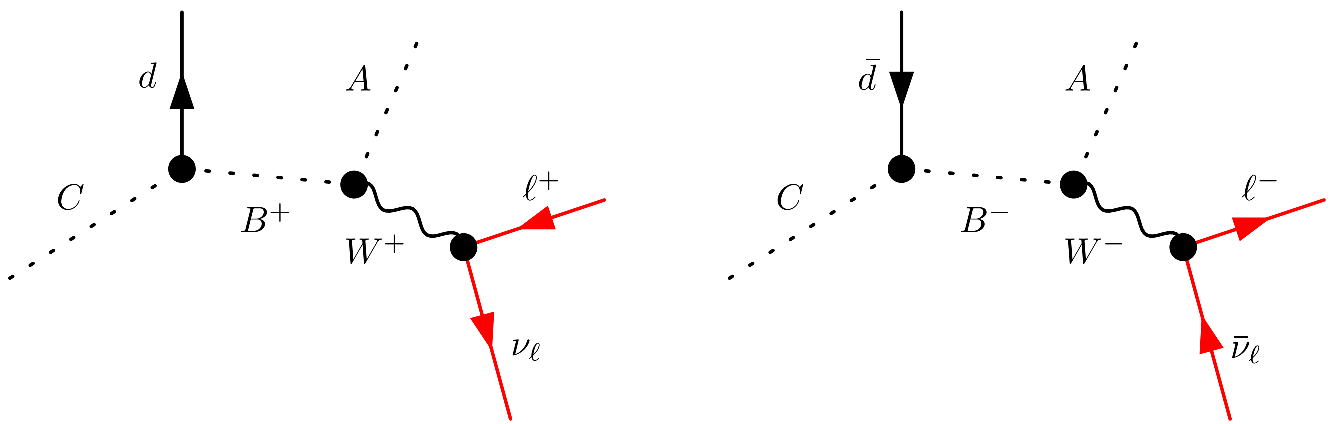
These have different spin correlations.

ℓ^- and ℓ^+

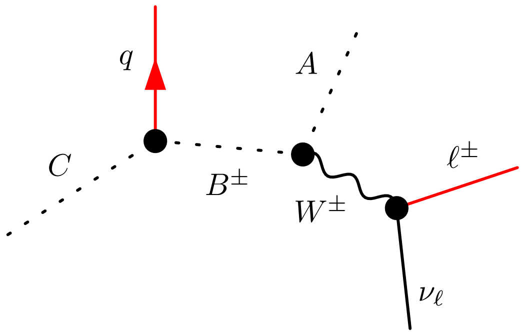
Process 1



Process 2



$m^2_{q\ell}$



In this chain, the only observable particles are the quark and the lepton.

Their invariant mass is

$$m_{q\ell} = \sqrt{(p_q + p_\ell)^2}$$

and we can treat the SM particles as massless so

$$m_{q\ell} = \sqrt{2 p_q . p_\ell}$$

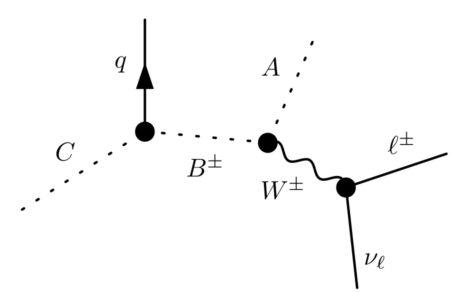
Angles

Explicitly the angular dependence of this quantity is given as

$$m_{q\ell}^2 = \frac{1}{4X} m_B^2 (1 - X) \Big(k_1 (1 - \cos\theta \cos\psi) + k_2 (\cos\theta - \cos\psi) - 2\sqrt{Y} \sin\theta \sin\psi \cos\phi \Big)$$

where
θ is angle between q and A in rest frame of B,
ψ is angle between A and ℓ in rest frame of W
and ϕ is angle between these two planes

$k_1 = 1 + Y - Z, \qquad k_2 = \sqrt{k_1^2 - 4Y}$



$X = m_B^2 / m_C^2$, $Y = m_W^2 / m_B^2$, $Z = m_A^2 / m_B^2$

Distributions

For convenience, we work with

$$\widehat{m}_{q\ell}^2 = \frac{4X}{m_B^2(1-X)} m_{q\ell}^2$$

and plot

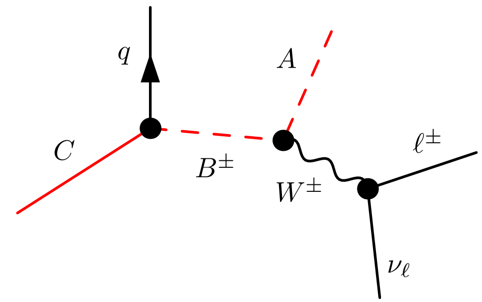
$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}\widehat{m}} = \frac{\mathrm{d}P}{\mathrm{d}\widehat{m}}$$

where Γ is the total decay rate for the chain and $\widehat{m}$ is shorthand for $\widehat{m}_{q\ell}$.

The exact analytical results are in hep-ph/0609296 (except for FVV which are too long).

Distributions

For the FSS chain,



$$\frac{\mathrm{d}P_{1,2}}{\mathrm{d}\widehat{m}} = \frac{3\widehat{m}}{2k_2^3}\begin{cases} k_1 k_2 - 2Y\log\left(\frac{k_1+k_2}{k_1-k_2}\right) & 0 \le \widehat{m}^2 \le 2k_{12}^- \\ \frac{1}{16}(6k_1 - 2k_2 - \widehat{m}^2) - 2Y\log\left(\frac{2(k_1+k_2)}{\widehat{m}^2}\right) & 2k_{12}^- \le \widehat{m}^2 \le 2k_{12}^+ \end{cases}$$

where $k_{12}^\pm = k_1 \pm k_2$.

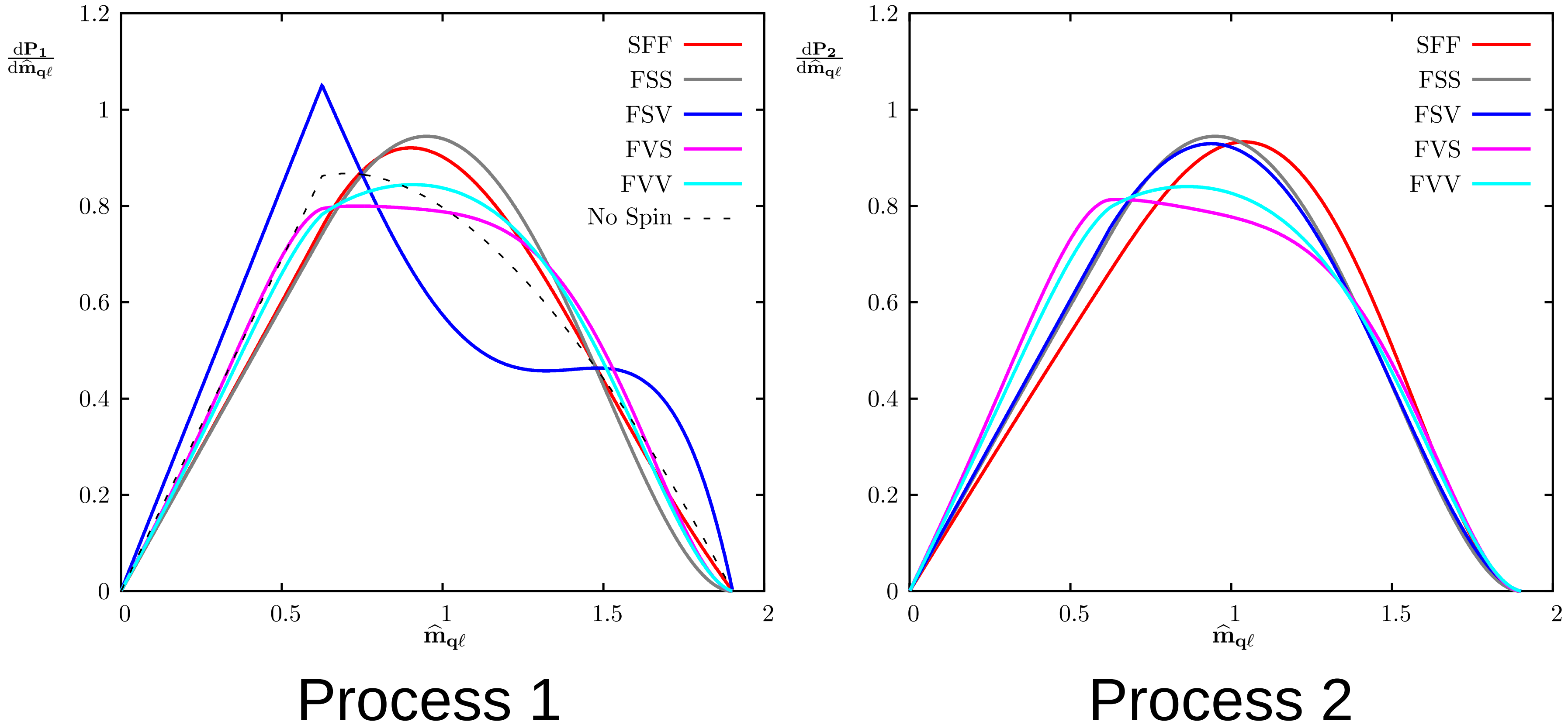
Masses

- Studied the mass spectra (in GeV/c^2) at the following Snowmass Benchmark points:

	C	B	A
SPS 1a	537	378	96
SPS 2	1533	269	79
SPS 9	1237	876	175

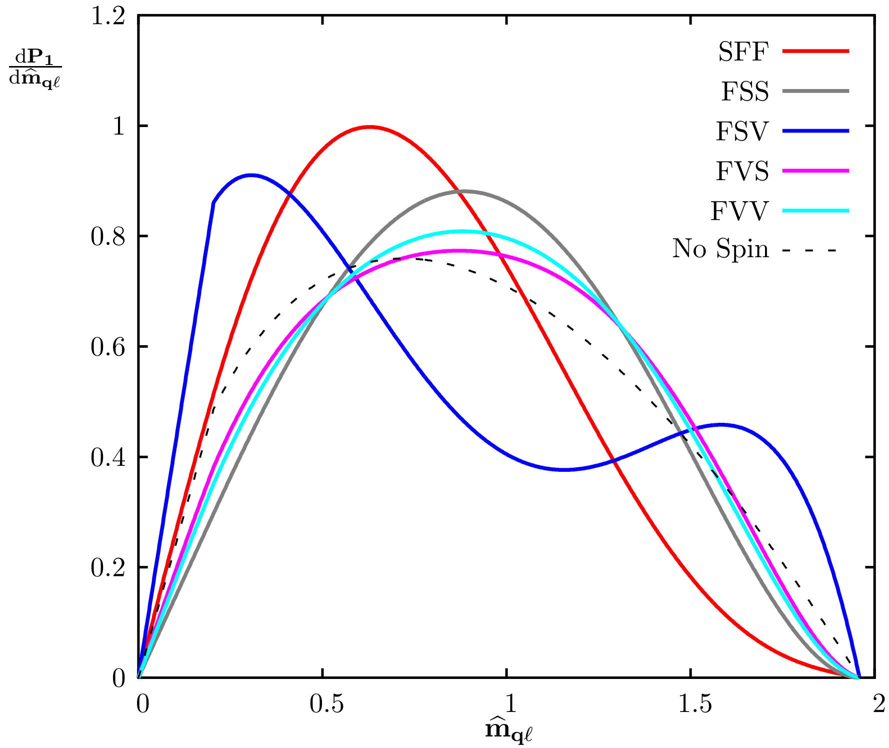
- SPS 1a and SPS 2 are mSUGRA benchmark points, while SPS 9 is an anomaly-mediated SUSY breaking point.

SPS 2 (for example)

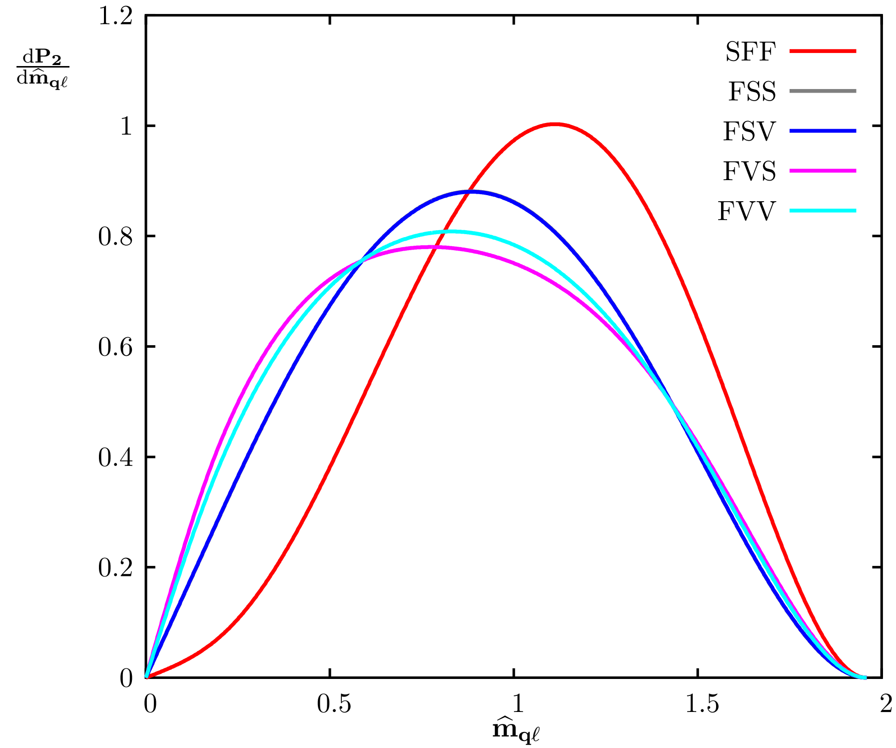


The black dotted line in the left plot shows the curve with no spin correlations.

SPS 9 (for example)



Process 1



Process 2

The black dotted line in the left plot shows the curve with no spin correlations.

P_+ and P_-

- However, in an experiment we cannot tell the difference between $\{u, \ell^-\}$ and $\{\bar{d}, \ell^-\}$. These are both

  { jet, ℓ^- }.

  Similarly for ℓ^+ events.

P_+ and P_-

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  { jet, ℓ^- }.

  Similarly for ℓ^+ events.

- The observable distributions are:

$$\frac{\mathrm{d}P_-}{\mathrm{d}\widehat{m}} = r_{d^*}\frac{\mathrm{d}P_1}{\mathrm{d}\widehat{m}} + r_{\bar{u}^*}\frac{\mathrm{d}P_2}{\mathrm{d}\widehat{m}}$$

$$\frac{\mathrm{d}P_+}{\mathrm{d}\widehat{m}} = r_{u^*}\frac{\mathrm{d}P_2}{\mathrm{d}\widehat{m}} + r_{\bar{d}^*}\frac{\mathrm{d}P_1}{\mathrm{d}\widehat{m}}.$$

  r_{d^*} and $r_{\bar{u}^*}$ add to 1 and represent the relative numbers of ℓ^- chains beginning with d^*s and $\bar{u}^*$s. Similarly for r_{u^*}, $r_{\bar{d}^*}$.

P_+ and P_-

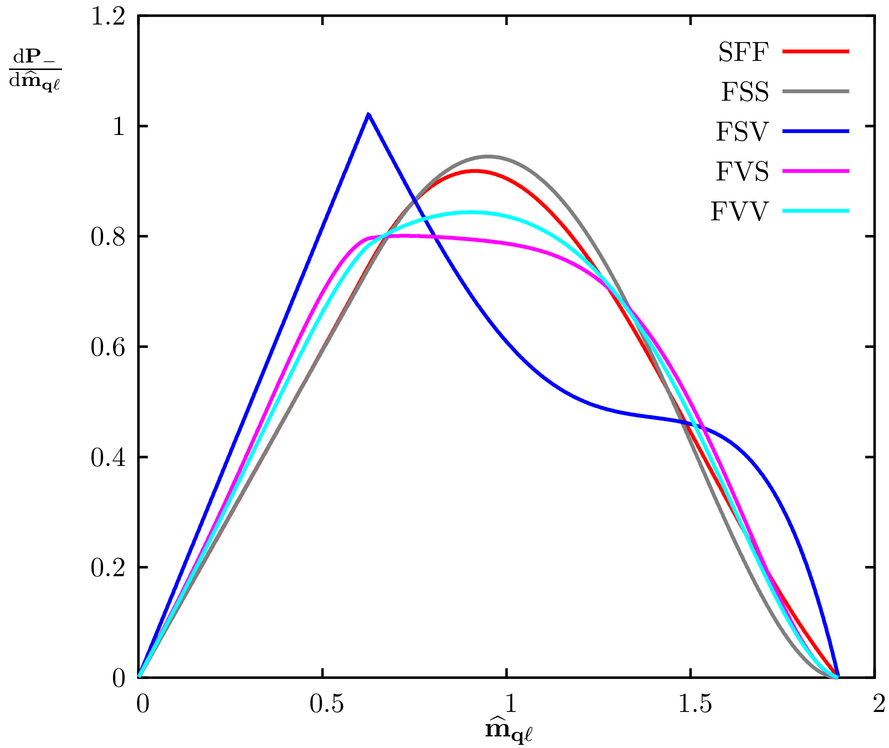
- Unfortunately, the fractions $r_{q,\bar{q}}$ reintroduce some model dependence, but the different spectra here cover a wide range of possibilities.

  HERWIG gives:

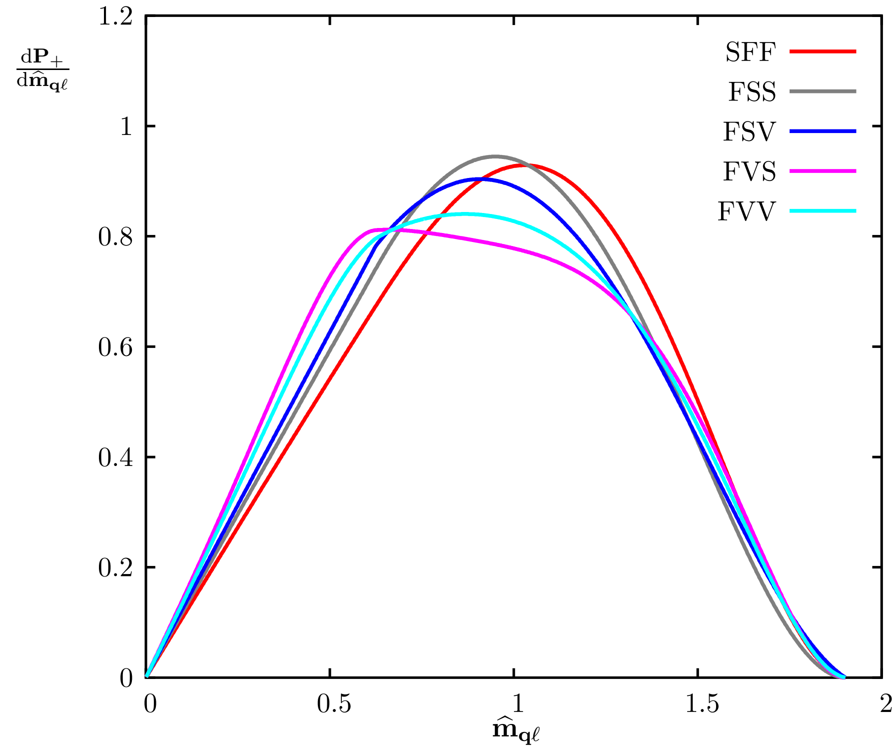
Spectrum	r_{d^*}	$r_{\bar{u}^*}$	r_{u^*}	$r_{\bar{d}^*}$
SPS 1a	0.860	0.140	0.469	0.531
SPS 2	0.900	0.100	0.911	0.089
SPS 9	0.998	0.002	0.072	0.928

- The extreme values at SPS 9 are due to large μ enhancing the effect of large $\tan\beta$.

SPS 2 (for example)

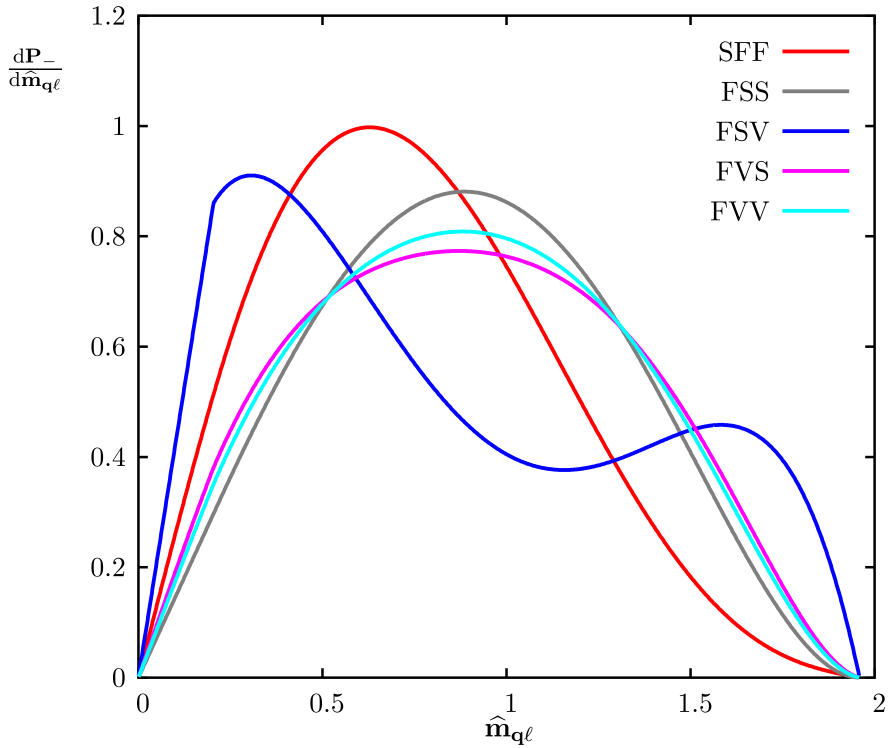


jet + ℓ^-

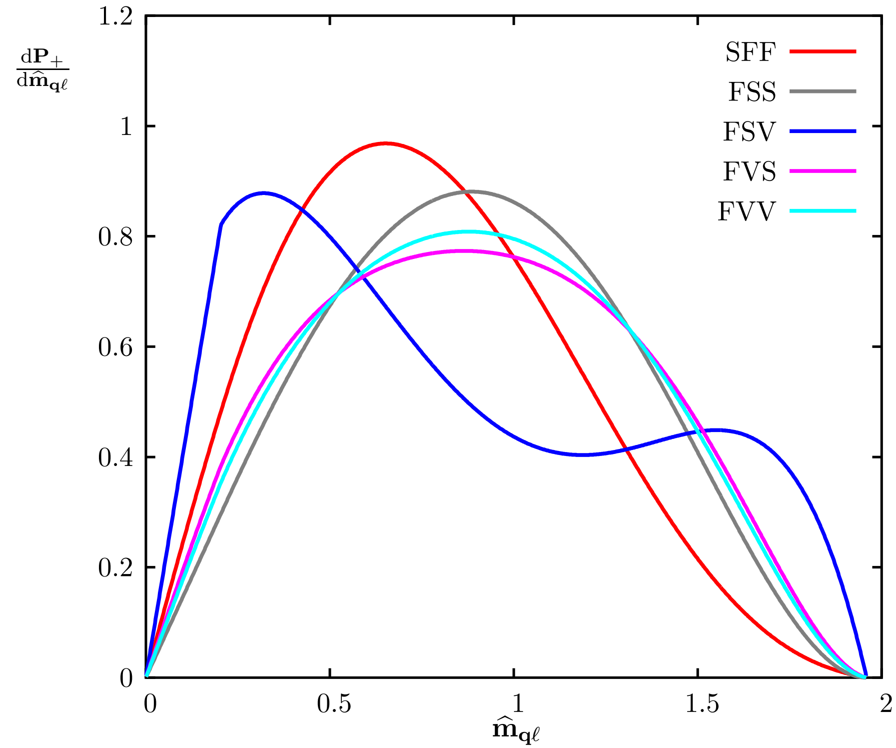


jet + ℓ^+

and SPS 9



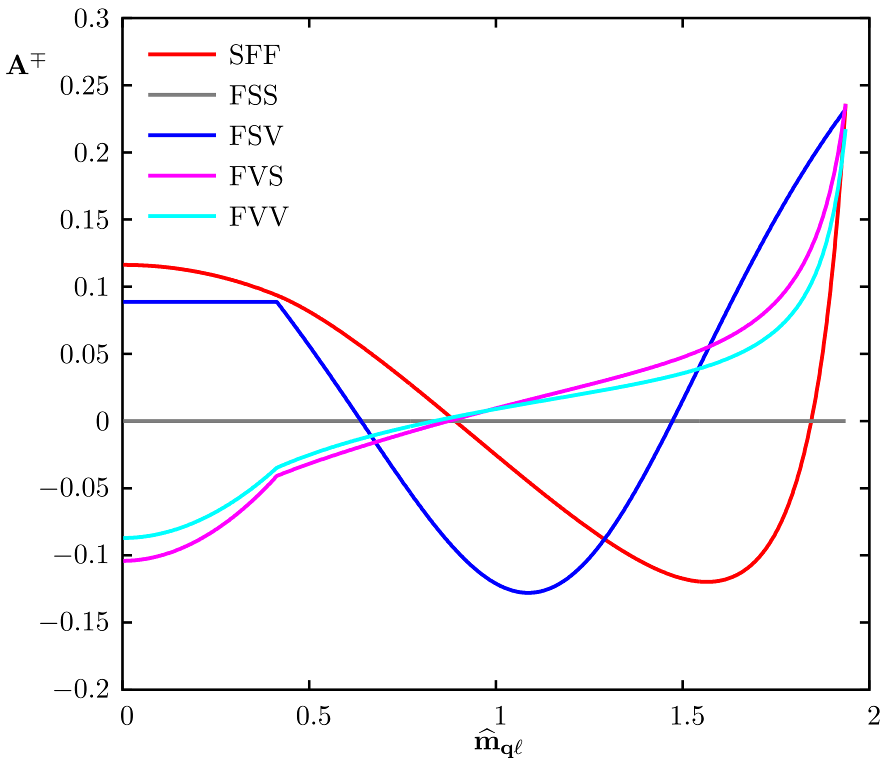
jet + ℓ^-



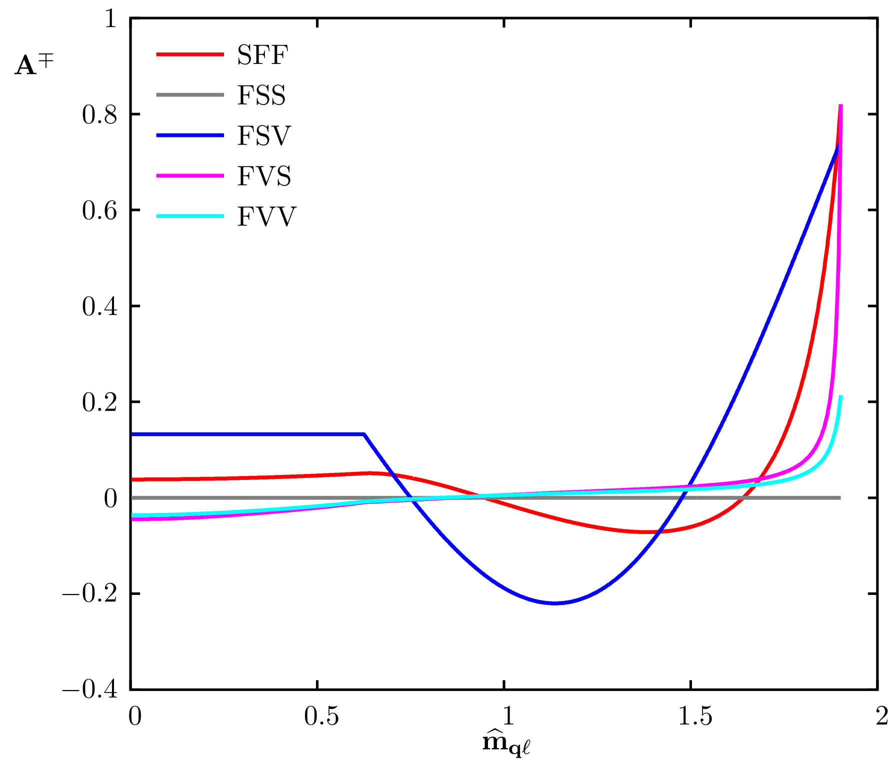
jet + ℓ^+

Asymmetry

Form asymmetry from $\mp$ distributions: $A^{\mp} = \frac{\frac{dP_-}{d\widehat{m}} - \frac{dP_+}{d\widehat{m}}}{\frac{dP_-}{d\widehat{m}} + \frac{dP_+}{d\widehat{m}}}$

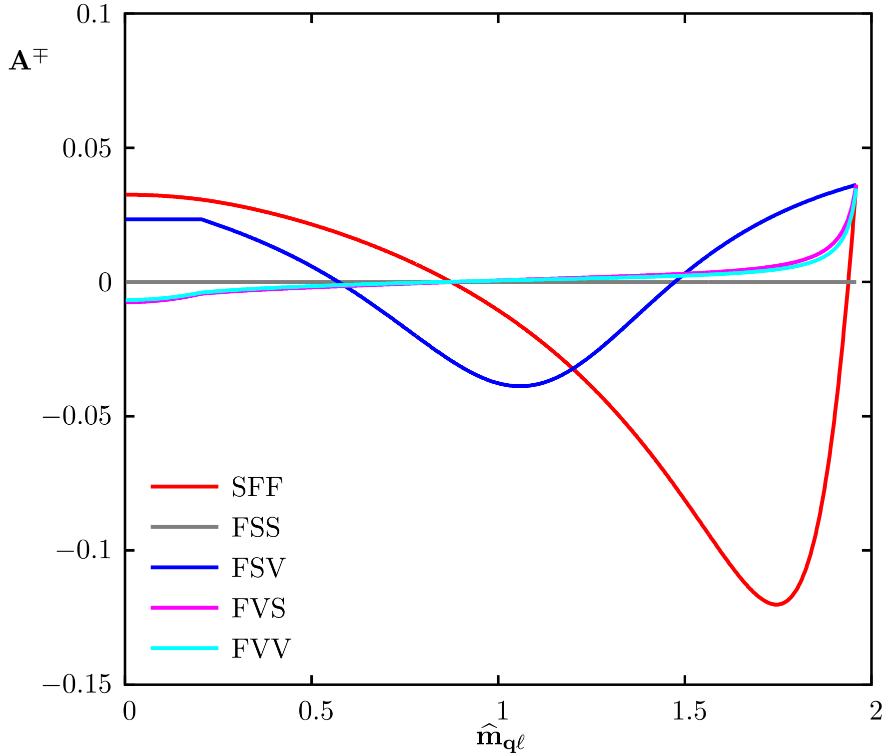


SPS 1a



SPS 2

Asymmetry



SPS 9

Smaller asymmetries here, but the original distributions were more striking at this point.

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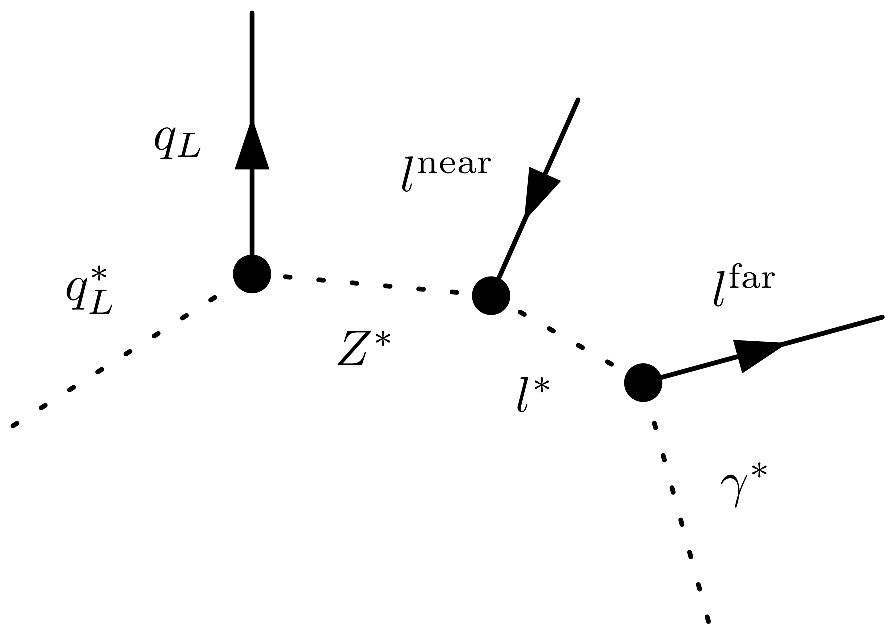
Cascade Decay

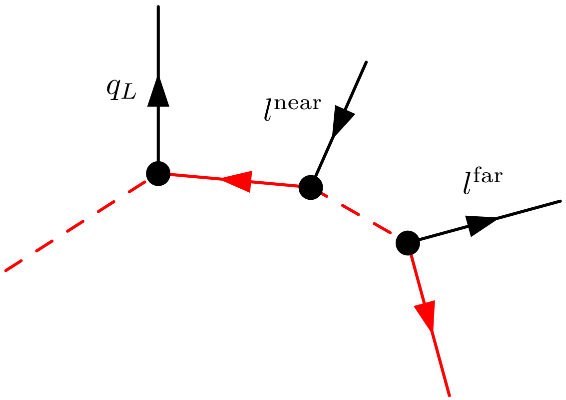
Previously this type of study was performed for the following cascade decay of a quark partner:



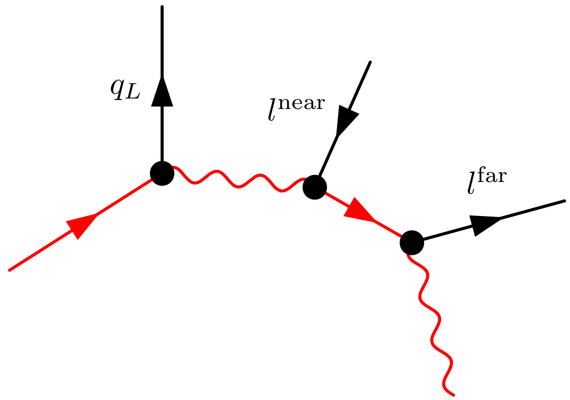
Final state is now $q\ \ell^+\ \ell^-$ and 'A'.

Cascade Decay Chains

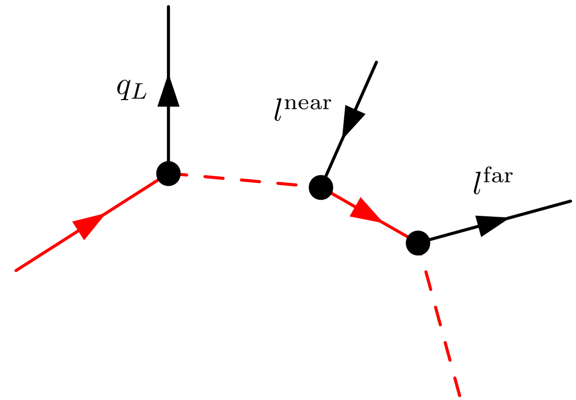
There are 6 possibilities:



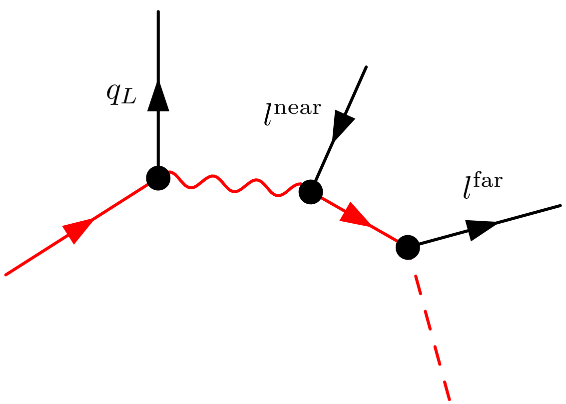
SFSF



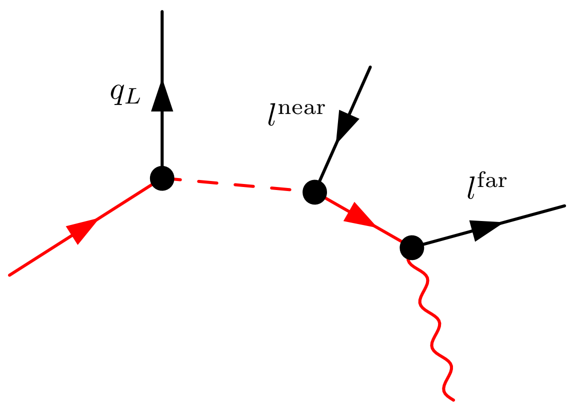
FVFV



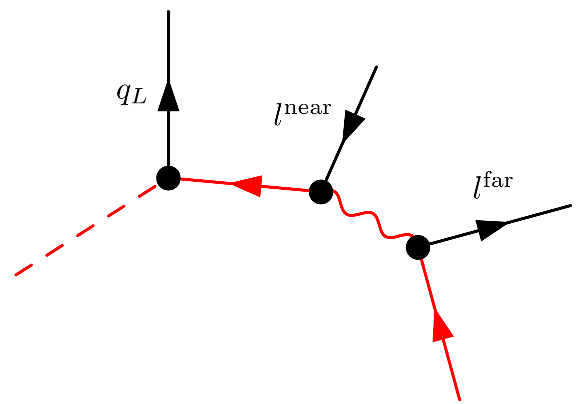
FSFS



FVFS



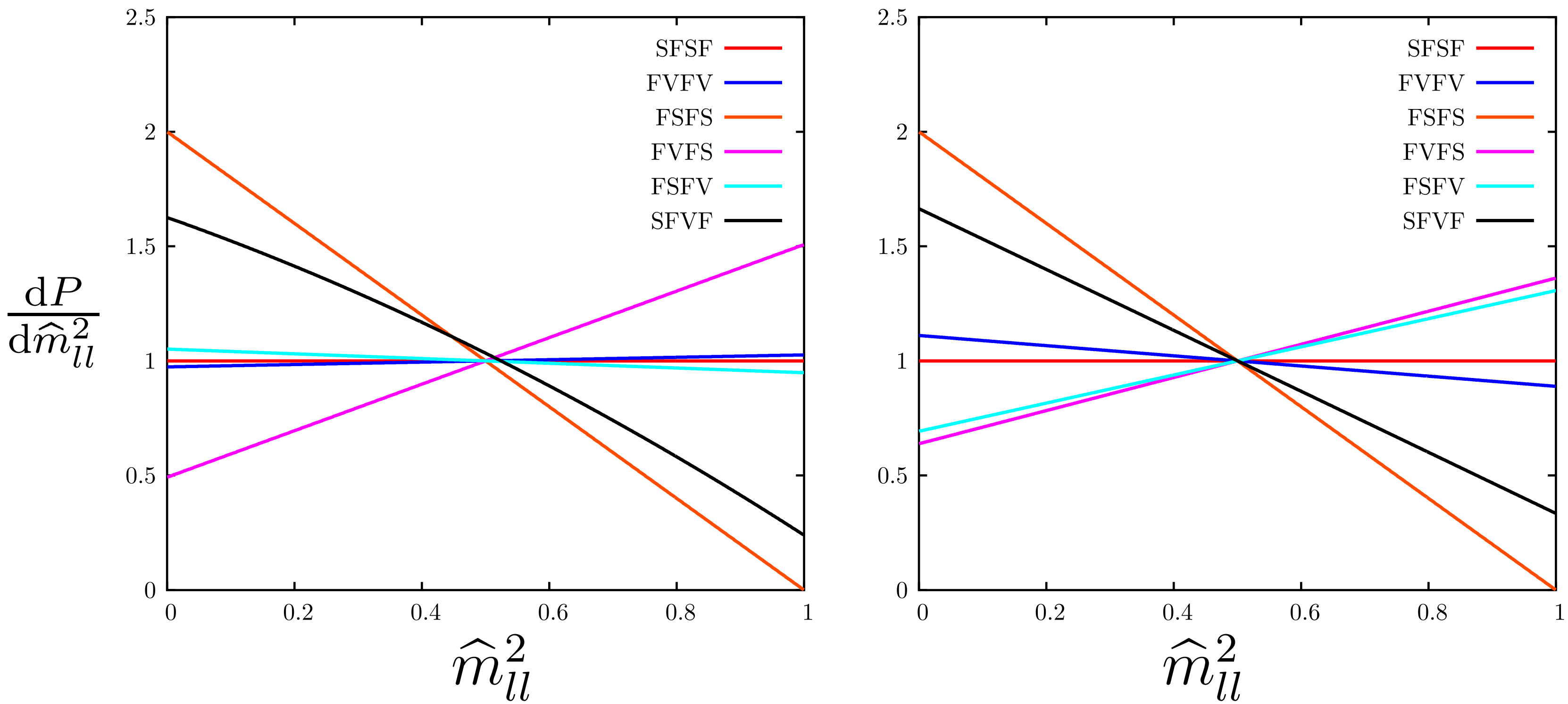
FSFV



SFVF

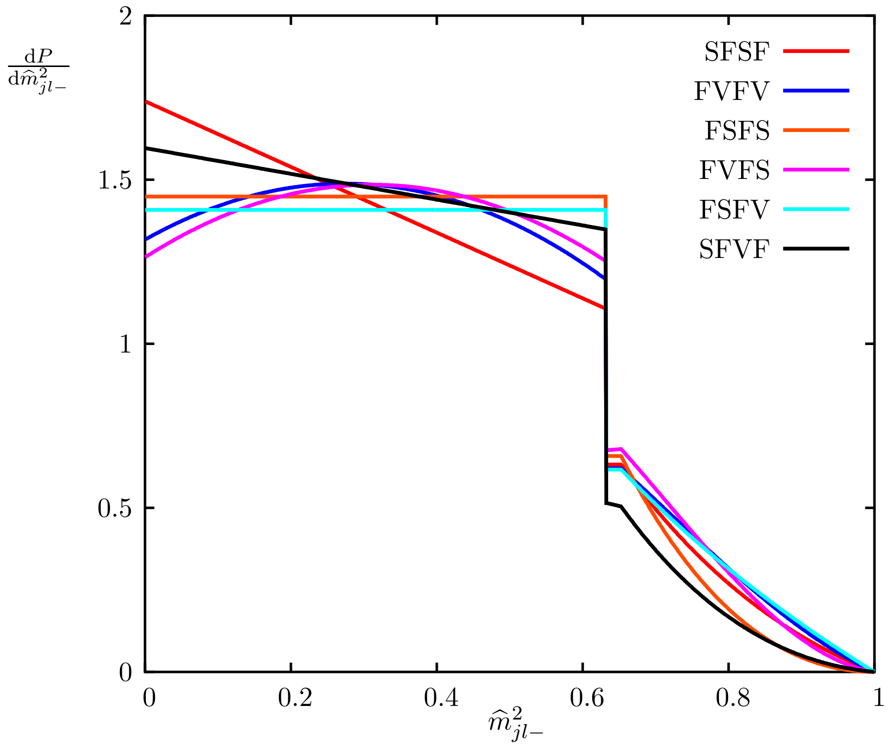
For Example, $l^{\mathrm{near}} l^{\mathrm{far}}$

We now have 3 observable particles, so 3 independent invariant mass combinations.
The m_{ll}^2 distributions for SPS 1a masses and UED masses ($R^{-1} = 800\mathrm{GeV}, \Lambda R = 20$) are:

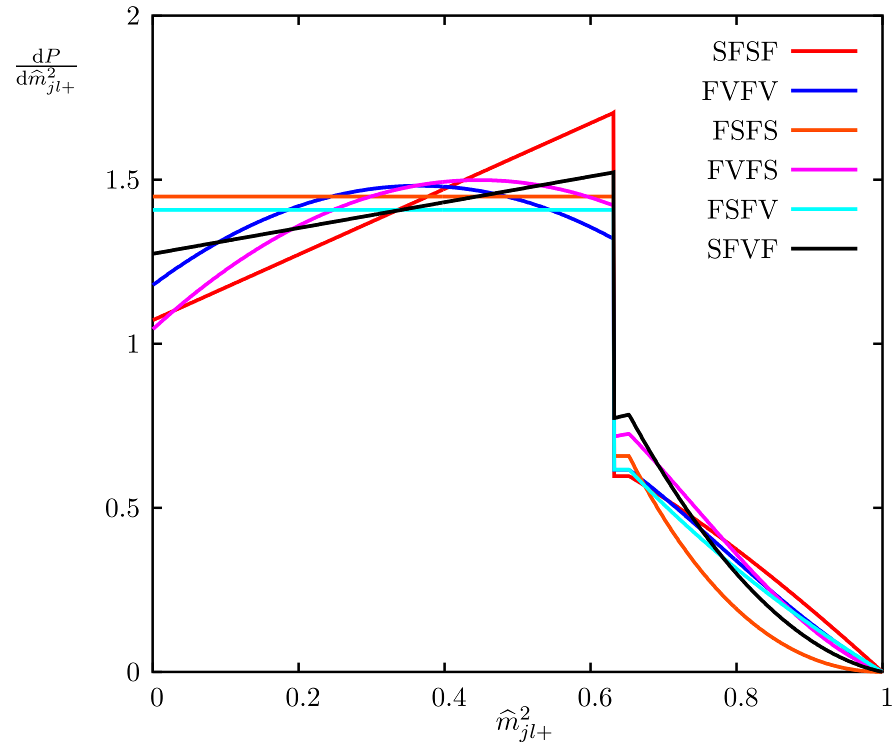


jet + $l^{\pm}$

At SPS 1a:



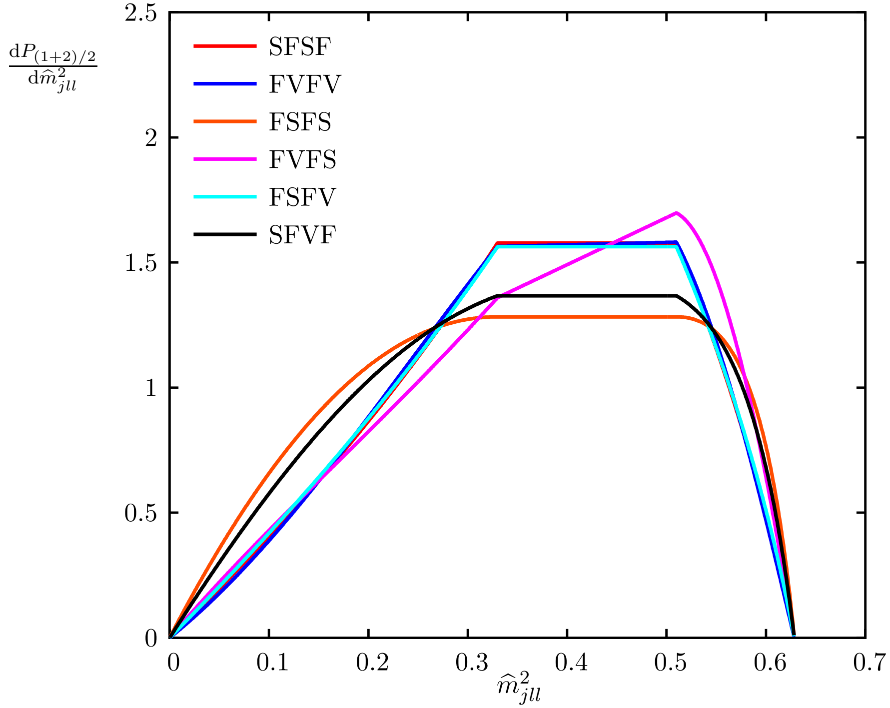
jet + ℓ^-



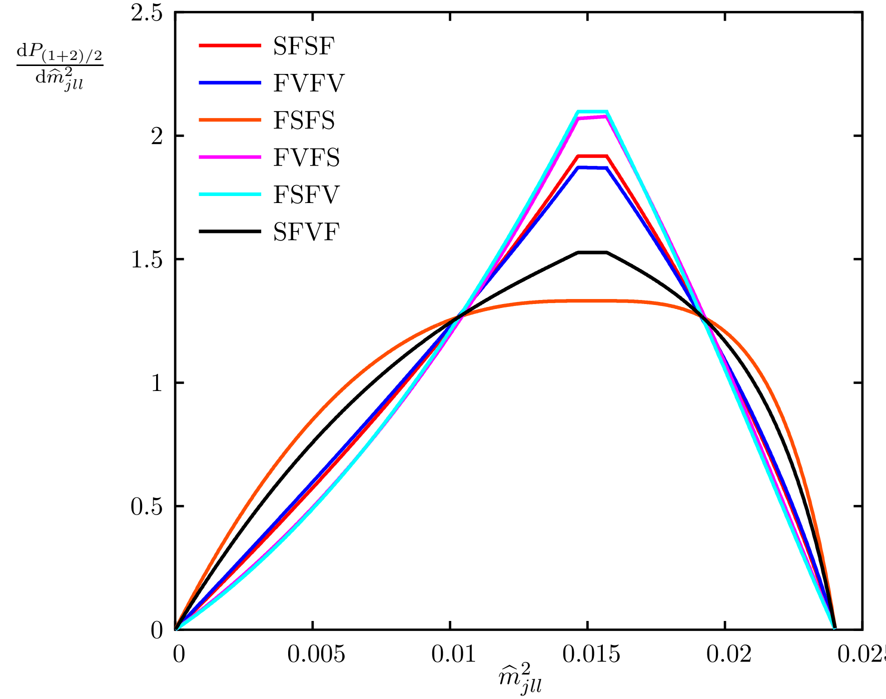
jet + ℓ^+

jet $\ell^+ \ell^-$

Also have $\widehat{m}_{jll}$ (not independent):



SPS 1a



UED-type

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Discrimination

Given a set of N invariant mass data points $\{\widehat{m}_i\}$, the statement that a model T is R times more likely than a model S can be written

$$R = \frac{p(T|\{\widehat{m}_i\})}{p(S|\{\widehat{m}_i\})}$$

Discrimination

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$$R = \frac{p(T|\{\widehat{m}_i\})}{p(S|\{\widehat{m}_i\})}$$

or equivalently by Bayes' Theorem

$$R = \frac{p(\{\widehat{m}_i\}|T)p(T)}{p(\{\widehat{m}_i\}|S)p(S)}.$$

As each event is independent, this is just

$$R\,\frac{p(S)}{p(T)} = \frac{\prod_{i=1}^{N} p(m_i|T)}{\prod_{j=1}^{N} p(m_j|S)} = \prod_{i=1}^{N} \frac{p(m_i|T)}{p(m_i|S)}.$$

Discrimination

This product can be rewritten as

$$R\,\frac{p(S)}{p(T)} = \exp\left(\sum_{i=1}^{N}\log\frac{p(m_i|T)}{p(m_i|S)}\right).$$

Discrimination

This product can be rewritten as

$$R\,\frac{p(S)}{p(T)} = \exp\left(\sum_{i=1}^{N}\log\frac{p(m_i|T)}{p(m_i|S)}\right).$$

In the limit of large N, $\sum \to \int$:

$$\sum_{i=1}^{N}\log\frac{p(\widehat{m}_i|T)}{p(\widehat{m}_i|S)} \sim N\int\,\log\left(\frac{p(\widehat{m}|T)}{p(\widehat{m}|S)}\right)\;p(\widehat{m})\;\mathrm{d}\widehat{m}$$

where $p(\widehat{m})$ is the density function for m.
Without data, we have to assume one of our models to be true. We use $p(\widehat{m}|T)$ so we are considering “if T is true, how likely are we to mistake it for S”.

Discrimination

$$\log\left(R\,\frac{p(S)}{p(T)}\right) \sim N \int \log\left(\frac{p(\widehat{m}|T)}{p(\widehat{m}|S)}\right)\, p(\widehat{m}|T)\;\mathrm{d}\widehat{m}$$

where the right hand side is N times the so-called Kullback-Leibler distance, KL(T, S).

Discrimination

$$\log\left(R\,\frac{p(S)}{p(T)}\right) \sim N \int \log\left(\frac{p(\widehat{m}|T)}{p(\widehat{m}|S)}\right)\, p(\widehat{m}|T)\,\mathrm{d}\widehat{m}$$

where the right hand side is N times the so-called Kullback-Leibler distance, KL(T, S).

In an experimental situation, it is more likely that we know the value of R we seek, and want to know how many events N this requires:

$$N \;\sim\; \frac{\log R + \log p(S)/p(T)}{\mathrm{KL}(T,S)}.$$

We will assume no prior bias for a particular model, so set $p(S) = p(T)$ for all S, T.

W-Chain

If we now substitute the P_{-} W-chain distribution at SPS 2 for example, with $R = 1000$:

S (columns), T (rows)

N	SFF	FSS	FSV	FVS	FVV
SFF	∞	1220	125	1007	2166
FSS	1608	∞	89	638	1292
FSV	121	75	∞	155	130
FVS	1027	619	177	∞	6530
FVV	2267	1240	146	6537	∞

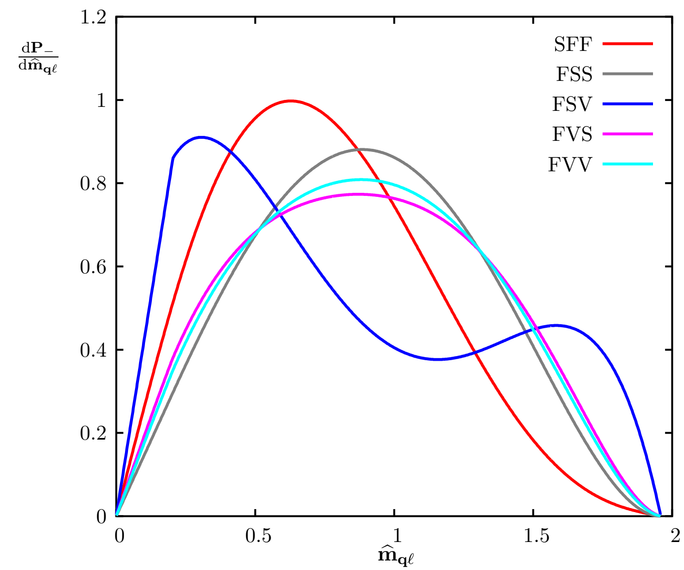
- We expect to get ∞ on the diagonal, otherwise would let a model be R times more likely than itself.
- For $R = 20$ (95% confidence) instead, multiply by $\log 20/\log 1000 \simeq 0.43$.

W-Chain

Here are the numbers for the P_{-} W-chain distribution at SPS 9 ($R = 1000$):

S (columns), T (rows)

N	SFF	FSS	FSV	FVS	FVV
SFF	∞	90	41	85	87
FSS	83	∞	36	790	1686
FSV	28	31	∞	49	42
FVS	69	742	54	∞	7451
FVV	73	1605	47	7555	∞



But this analysis only treats the P_{-} events. Can repeat for P_{+} curves separately, but even better to combine.

Combined Numbers

We consider both P_- and P_+ at once by using

$$\mathrm{KL}_{comb}(T,S) = \widehat{\mathrm{KL}}_-(T,S) + \widehat{\mathrm{KL}}_+(T,S)$$

where $\widehat{\ }$s are used as the distributions are normalised first according to the relative number of events.

If $f_\pm$ is fraction of total events with an $\ell^\pm$

$$\begin{aligned}\widehat{\mathrm{KL}}_\pm(T,S) &= \int \log\left(\frac{f_\pm\, p(\widehat{m}^\pm|T^\pm)}{f_\pm\, p(\widehat{m}^\pm|S^\pm)}\right) f_\pm\, p(\widehat{m}^\pm|T^+)\ \mathrm{d}\widehat{m} \\ &= f_\pm\ \mathrm{KL}_\pm(T,S)\end{aligned}$$

Combined Numbers

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where $\widehat{\ }$s are used as the distributions are normalised first according to the relative number of events.

The number of P_- and P_+ events in a given data sample will be known – here we estimate what it will be using HERWIG:

Spectrum	f_-	f_+
SPS 1a	0.43	0.57
SPS 2	0.32	0.68
SPS 9	0.33	0.67

Combined Numbers

At SPS 2, we get for both distributions together

N_{total}	SFF	FSS	FSV	FVS	FVV
SFF	∞	1388	312	521	837
FSS	1554	∞	261	590	1160
FSV	304	220	∞	375	375
FVS	507	577	415	∞	6416
FVV	819	1127	417	6415	∞

compared with

N_{-}	SFF	FSS	FSV	FVS	FVV
SFF	∞	1220	125	1007	2166
FSS	1608	∞	89	638	1292
FSV	121	75	∞	155	130
FVS	1027	619	177	∞	6530
FVV	2267	1240	146	6537	∞

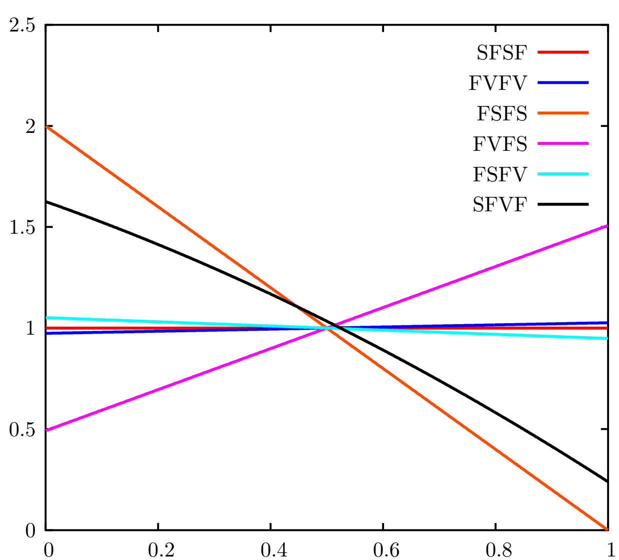
and

N_{+}	SFF	FSS	FSV	FVS	FVV
SFF	∞	1484	1064	425	649
FSS	1531	∞	2909	569	1106
FSV	1055	2549	∞	1128	3267
FVS	409	559	1131	∞	6365
FVV	630	1081	3280	6358	∞

Cascade Decays

N_{ll}	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	60486	23	148	15608	66
FVFV	60622	∞	22	164	6866	62
FSFS	36	34	∞	16	39	266
FVFS	156	173	11	∞	130	24
FSFV	15600	6864	25	122	∞	76
SFVF	78	73	187	27	90	∞

Number of events necessary for $R = 1000$ at SPS 1a.



$\uparrow \widehat{m}_{ll}$

N_{jl+}	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	1059	205	1524	758	727
FVFV	1090	∞	404	3256	4363	1746
FSFS	278	554	∞	418	741	2183
FVFS	799	6435	882	∞	2742	510
FSFV	749	4207	507	1212	∞	413
SFVF	813	1821	751	2415	1888	∞

3D Kullback-Leibler

These numbers were obtained by treating all the distributions separately.

However, we can also combine information of all 3 distributions by changing

$$m_i \;\rightarrow\; \underline{m}_i \;=\; (m^{jl+}, m^{jl-}, m^{ll})$$

Each point gives us a point in 3D phase space.

SPS 1a (for example)

N_{ll}	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	60486	23	148	15608	66
FVFV	60622	∞	22	164	6866	62
FSFS	36	34	∞	16	39	266
FVFS	156	173	11	∞	130	24
FSFV	15600	6864	25	122	∞	76
SFVF	78	73	187	27	90	∞

$\leftarrow m_{ll}$ distribution

m_{ll}, m_{jl+} and m_{jl-} $\rightarrow$

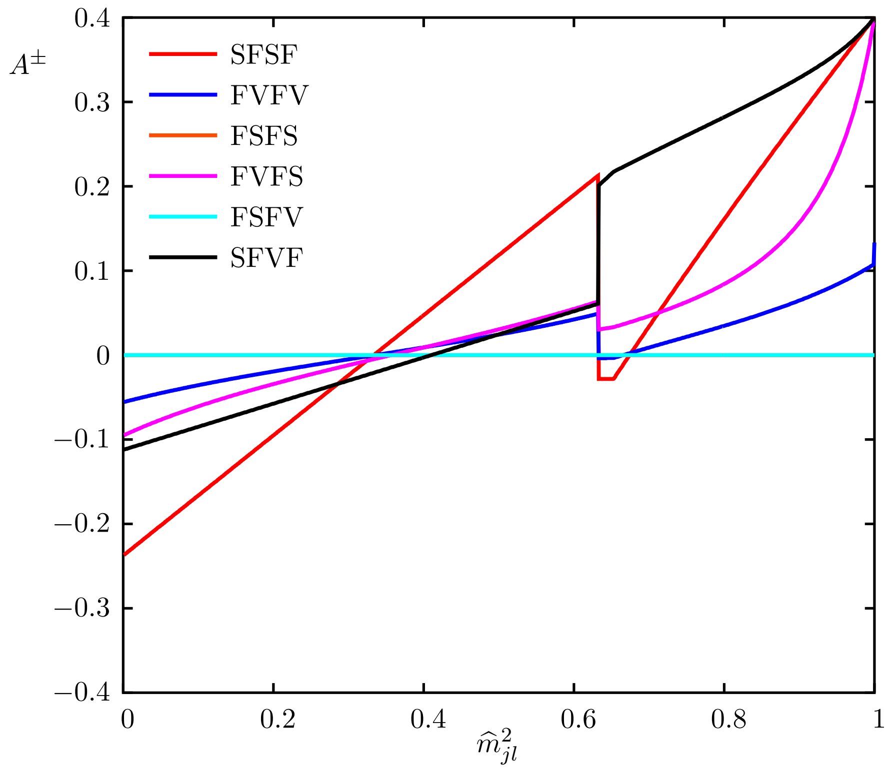
$N_{\rm all}$	SFSF	FVFV	FSFS	FVFS	FSFV	SFVF
SFSF	∞	455	21	47	348	55
FVFV	474	∞	21	54	1387	55
FSFS	33	34	∞	13	39	188
FVFS	55	67	10	∞	54	19
FSFV	341	1339	25	45	∞	66
SFVF	62	64	143	19	79	∞

Conclusions

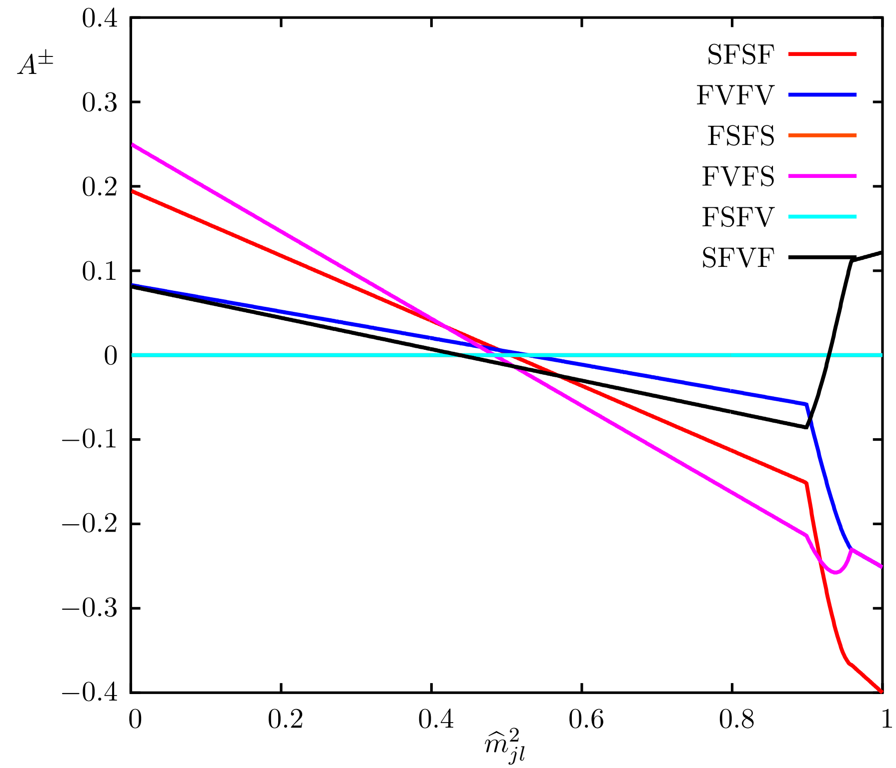
- Spin studies are very important in the LHC era.
- Decays of new particles via W bosons can be useful in spin determination.
- Cascade decays can be used to extract spin information from a number of distributions.
  - Invariant mass distributions have discriminatory power
  - Asymmetry plots provide more information.
- The Kullback-Leibler distance is an excellent tool to determine which processes are feasible for this method and which are not.

Asymmetry

And their asymmetry:



SPS 1a



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