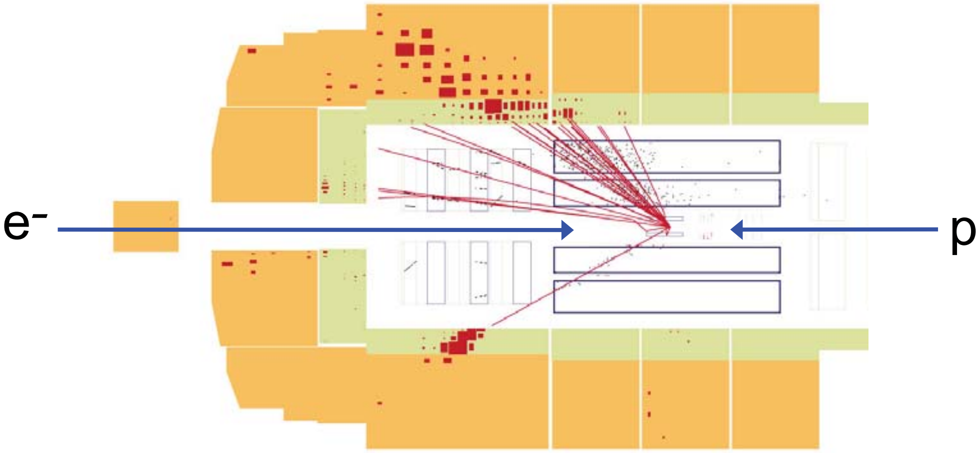
Particle Physics

Michaelmas Term 2011
Prof Mark Thomson



Handout 6 : Deep Inelastic Scattering

e⁻ p Elastic Scattering at Very High q^2

★ **At high q^2 the Rosenbluth expression for elastic scattering becomes**

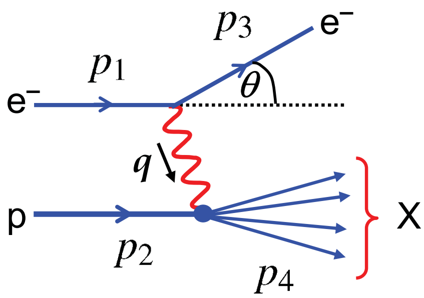
$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{elastic} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2}\frac{E_3}{E_1}\left(\frac{q^2}{2M^2}G_M^2\sin^2\frac{\theta}{2}\right) \qquad \tau = -\frac{q^2}{4M^2} \gg 1$$

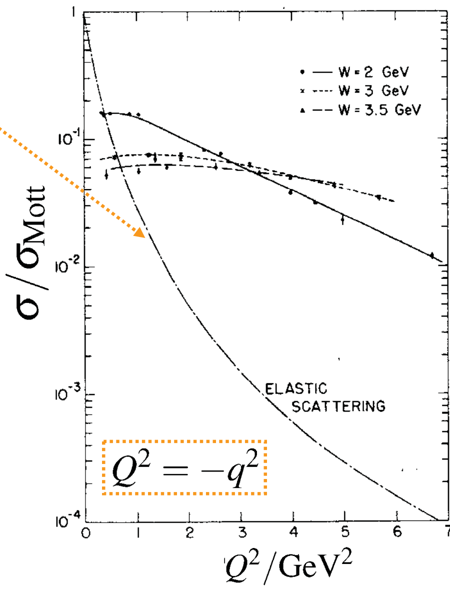
• **From e⁻ p elastic scattering, the proton magnetic form factor is**

$$G_M(q^2) \approx \frac{1}{(1+q^2/0.71\mathrm{GeV}^2)^2} \Rightarrow G_M(q^2) \propto q^{-4} \quad \text{at high } q^2$$

$$\Rightarrow \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{elastic} \propto q^{-6}$$

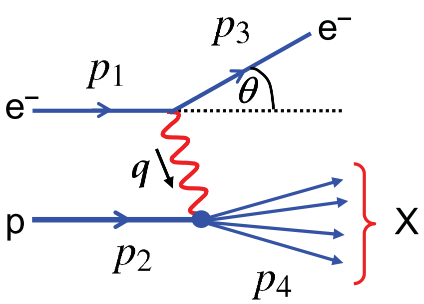
• **Due to the finite proton size, elastic scattering at high q^2 is unlikely and inelastic reactions where the proton breaks up dominate.**





M.Breidenbach et al., Phys. Rev. Lett. 23 (1969) 935

Kinematics of Inelastic Scattering



- For inelastic scattering the mass of the final state hadronic system is no longer the proton mass, M
- The final state hadronic system must contain at least one **baryon** which implies the final state invariant mass $M_X > M$

$$M_X^2 = p_4^2 = (E_4^2 - |\vec{p}_4|^2)$$

★ For inelastic scattering introduce four new kinematic variables: $\boxed{x, y, \nu, Q^2}$

★ **Define:** $\boxed{x \equiv \frac{Q^2}{2p_2.q}}$ **Bjorken x** (Lorentz Invariant)

where $\boxed{Q^2 \equiv -q^2}$ $\quad Q^2 > 0$

- Here $M_X^2 = p_4^2 = (q+p_2)^2 = -Q^2 + 2p_2.q + M^2$

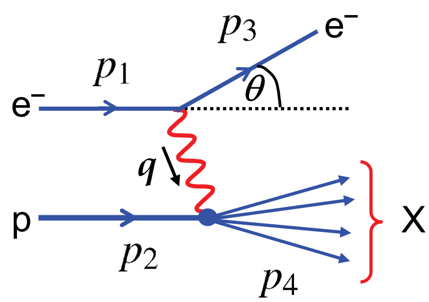
$\Rightarrow \quad Q^2 = 2p_2.q + M^2 - M_X^2 \quad \Rightarrow \quad Q^2 \leq 2p_2.q$

Note: in many text books W is often used in place of M_X

hence $\boxed{0 < x < 1 \text{ inelastic}}$ $\quad \boxed{x = 1 \quad \text{elastic}}$

Proton intact $M_X = M$

★ **Define:** $\boxed{y \equiv \frac{p_2.q}{p_2.p_1}}$ (Lorentz Invariant)



- In the Lab. Frame:

$p_1 = (E_1, 0, 0, E_1) \quad p_2 = (M, 0, 0, 0)$

$q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$

$$\rightarrow \quad y = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$

So y is the fractional energy loss of the incoming particle

$$\boxed{0 < y < 1}$$

- In the C.o.M. Frame (neglecting the electron and proton masses):

$$p_1 = (E, 0, 0, E); \quad p_2 = (E, 0, 0, -E); \quad p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$

$$\rightarrow \quad y = \tfrac{1}{2}(1 - \cos\theta^*) \qquad \text{for } E \gg M$$

★ **Finally Define:** $\boxed{\nu \equiv \frac{p_2.q}{M}}$ (Lorentz Invariant)

- In the Lab. Frame: $\nu = E_1 - E_3$

  ν is the energy lost by the incoming particle

Relationships between Kinematic Variables

- Can rewrite the new kinematic variables in terms of the squared centre-of-mass energy, s, for the electron-proton collision

$$e^- \xrightarrow{p_1} \qquad \xleftarrow{p_2} p$$

$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1.p_2 = 2p_1.p_2 + M^2 + \cancel{m_e^2}$$

Neglect mass of electron

$$2p_1.p_2 = s - M^2$$

- For a fixed centre-of-mass energy, it can then be shown that the four kinematic variables

$$Q^2 \equiv -q^2 \qquad x \equiv \frac{Q^2}{2p_2.q} \qquad y \equiv \frac{p_2.q}{p_2.p_1} \qquad \nu \equiv \frac{p_2.q}{M}$$

are not independent.

- i.e. the scaling variables x and y can be expressed as

$$x = \frac{Q^2}{2M\nu} \qquad y = \frac{2M}{s - M^2}\nu$$

Note the simple relationship between y and ν

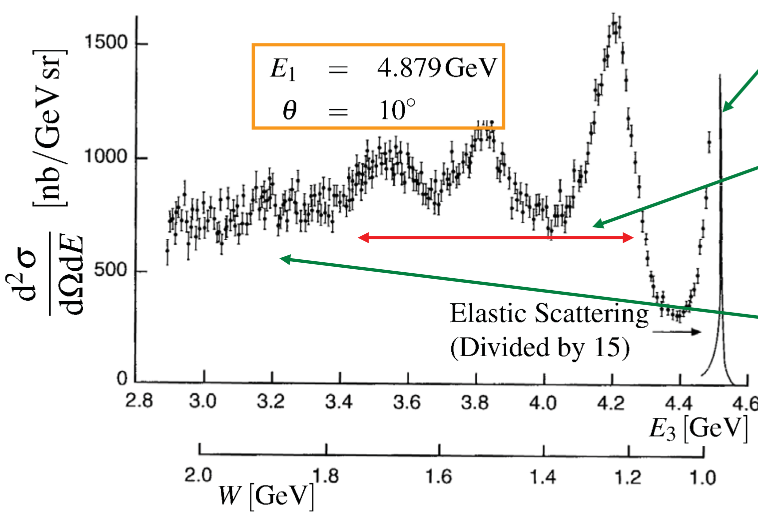
and $$xy = \frac{Q^2}{s - M^2} \quad \Rightarrow \quad Q^2 = (s - M^2)xy$$

- For a fixed centre of mass energy, the interaction kinematics are completely defined by any **two** of the above kinematic variables (except y and ν)
- For elastic scattering $(x = 1)$ there is only one independent variable. As we saw previously if you measure electron scattering angle know everything else.

Inelastic Scattering

Example: Scattering of 4.879 GeV electrons from protons at rest

- Place detector at **10°** to beam and measure the **energies** of scattered **e⁻**
- Kinematics fully determined from the **electron energy** and **angle** !
- e.g. for this energy and angle : the invariant mass of the final state hadronic system $W^2 = M_X^2 = 10.06 - 2.03E_3$ (try and show this)

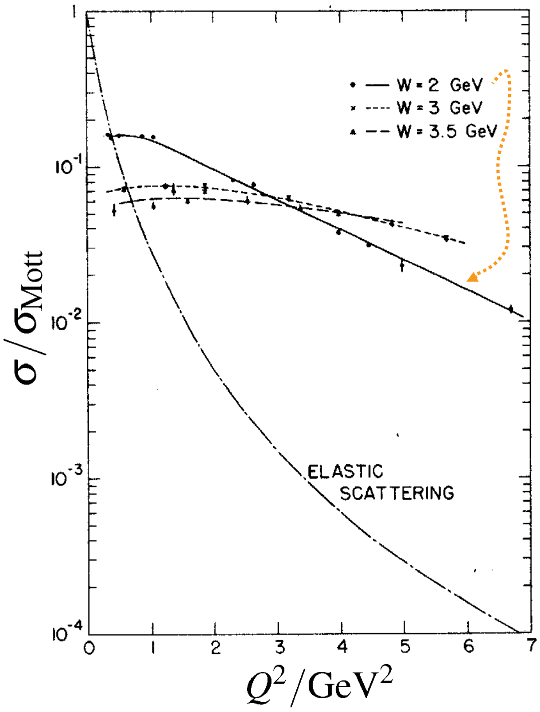


- **Elastic Scattering**
  proton remains intact
  $W = M$
- **Inelastic Scattering**
  produce "excited states" of proton e.g. $\Delta^+(1232)$
  $W = M_\Delta$
- **Deep Inelastic Scattering**
  proton breaks up resulting in a many particle final state

DIS = large W

Inelastic Cross Sections

- Repeat experiments at different angles/beam energies and determine q^2 dependence of elastic and inelastic cross-sections



M.Breidenbach et al., Phys. Rev. Lett. 23 (1969) 935

- Elastic scattering falls of rapidly with q^2 due to the proton not being point-like (i.e. form factors)
- Inelastic scattering cross sections only weakly dependent on q^2
- Deep Inelastic scattering cross sections almost independent of q^2 !

i.e. "Form factor" → 1

➡ **Scattering from point-like objects within the proton !**

Elastic → Inelastic Scattering

★ **Recall:** Elastic scattering (Handout 5)

- Only one independent variable. In Lab. frame express differential cross section in terms of the electron scattering angle (Rosenbluth formula)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2}\frac{E_3}{E_1}\left(\frac{G_E^2+\tau G_M^2}{(1+\tau)}\cos^2\frac{\theta}{2}+2\tau G_M^2\sin^2\frac{\theta}{2}\right) \qquad \tau = \frac{Q^2}{4M^2}$$

Note: here the energy of the scattered electron is determined by the angle.

- In terms of the Lorentz invariant kinematic variables can express this differential cross section in terms of Q^2 (Q13 on examples sheet)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[\frac{G_E^2+\tau G_M^2}{(1+\tau)}\left(1-y-\frac{M^2y^2}{Q^2}\right)+\frac{1}{2}y^2G_M^2\right]$$

which can be written as:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[f_2(Q^2)\left(1-y-\frac{M^2y^2}{Q^2}\right)+\frac{1}{2}y^2f_1(Q^2)\right]$$

★ **Inelastic scattering**

- For Deep Inelastic Scattering have two independent variables. Therefore need a double differential cross section

Deep Inelastic Scattering

★ It can be shown that **the most general Lorentz Invariant expression** for $e^-p \to e^-X$ inelastic scattering (via a single exchanged photon is):

$$\boxed{\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\frac{M^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]} \quad (1)$$

INELASTIC SCATTERING

c.f.
$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1-y-\frac{M^2y^2}{Q^2}\right)f_2(Q^2) + \frac{1}{2}y^2f_1(Q^2)\right]$$

ELASTIC SCATTERING

We will soon see how this connects to the quark model of the proton

- **NOTE:** The form factors have been replaced by the **STRUCTURE FUNCTIONS**

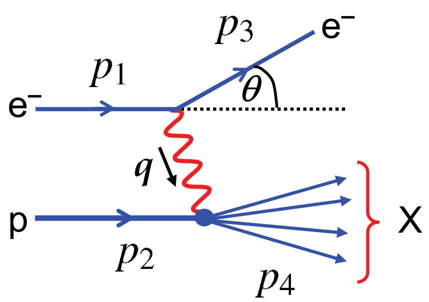
$$F_1(x,Q^2) \quad \text{and} \quad F_2(x,Q^2)$$

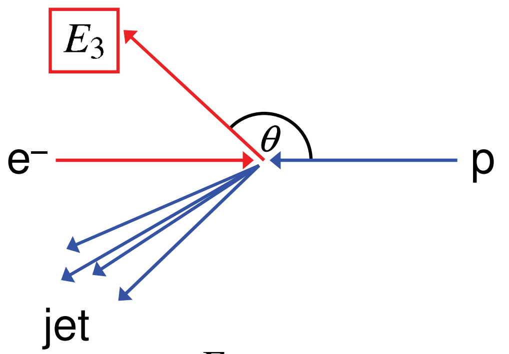
which are a function of x and Q^2: can not be interpreted as the Fourier transforms of the charge and magnetic moment distributions. We shall soon see that they describe the **momentum distribution** of the quarks within the proton

★ In the limit of high energy (or more correctly $Q^2 \gg M^2y^2$) eqn. (1) becomes:

$$\boxed{\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]} \quad (2)$$

- In the Lab. frame it is convenient to express the cross section in terms of the angle, θ , and energy, E_3, of the scattered electron – **experimentally well measured.**





$$Q^2 = 4E_1E_3\sin^2\theta/2; \quad x = \frac{Q^2}{2M(E_1-E_3)}; \quad y = 1-\frac{E_3}{E_1}; \quad \nu = E_1 - E_3$$

- In the Lab. frame, Equation (2) becomes: (see examples sheet Q13)

$$\boxed{\frac{d^2\sigma}{dE_3\,d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2}\left[\frac{1}{\nu}F_2(x,Q^2)\cos^2\frac{\theta}{2} + \frac{2}{M}F_1(x,Q^2)\sin^2\frac{\theta}{2}\right]} \quad (3)$$

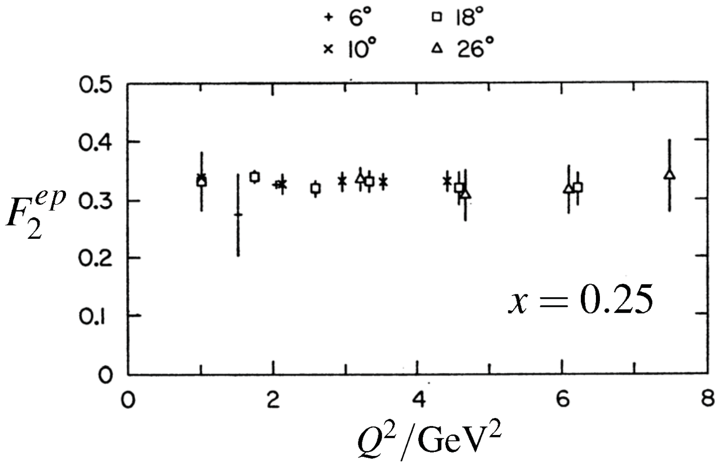
Electromagnetic Structure Function

Pure Magnetic Structure Function

Measuring the Structure Functions

★ To determine $F_1(x,Q^2)$ and $F_2(x,Q^2)$ for a given x and Q^2 need measurements of the differential cross section at several different scattering angles and incoming electron beam energies (see Q13 on examples sheet)

Example: electron-proton scattering F_2 vs. Q^2 at fixed x



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203

♦ Experimentally it is observed that both F_1 and F_2 are (almost) independent of Q^2

Bjorken Scaling and the Callan-Gross Relation

★ The near (see later) independence of the structure functions on Q^2 is known as **Bjorken Scaling**, i.e.

$$\boxed{F_1(x,Q^2) \rightarrow F_1(x) \qquad F_2(x,Q^2) \rightarrow F_2(x)}$$

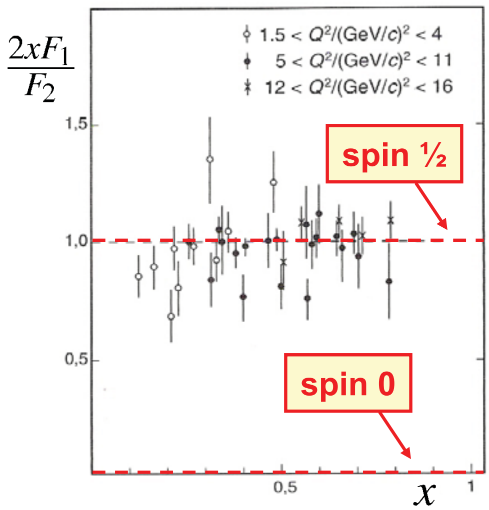
• It is strongly suggestive of scattering from **point-like** constituents within the proton

★ It is also observed that $F_1(x)$ and $F_2(x)$ are not independent but satisfy the **Callan-Gross relation**

$$\boxed{F_2(x) = 2xF_1(x)}$$

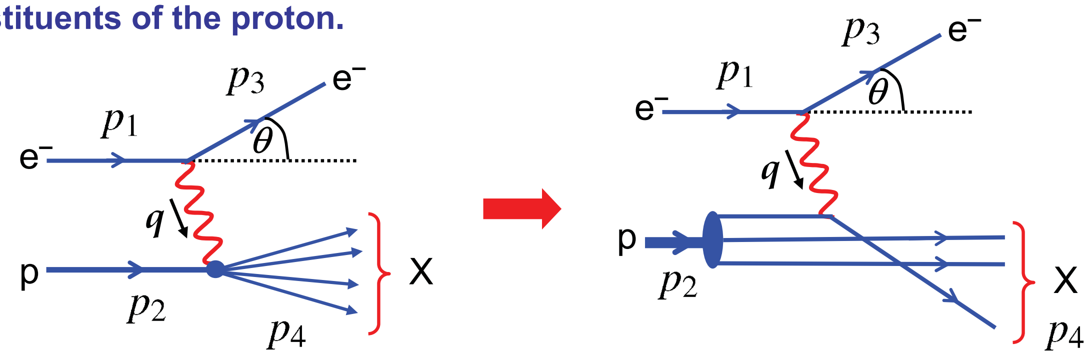
• As we shall soon see this is exactly what is expected for scattering from **spin-half** quarks.

• **Note** if quarks were spin zero particles we would expect the purely magnetic structure function to be zero, i.e. $F_1(x) = 0$



The Quark-Parton Model

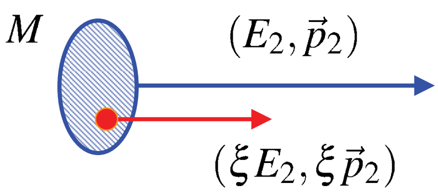
- Before quarks and gluons were generally accepted Feynman proposed that the proton was made up of point-like constituents "partons"
- Both Bjorken Scaling and the Callan-Gross relationship can be explained by assuming that Deep Inelastic Scattering is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton.

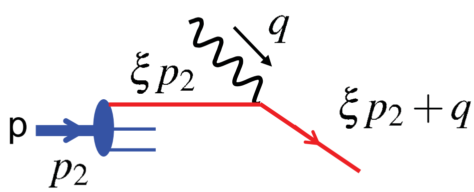


Scattering from a proton with structure functions → Scattering from a point-like quark within the proton

★ How do these two pictures of the interaction relate to each other?

- In the parton model the basic interaction is ELASTIC scattering from a "quasi-free" spin-½ quark in the proton, i.e. treat the quark as a free particle!
- The parton model is most easily formulated in a frame where the proton has very high energy, often referred to as the "infinite momentum frame", where we can neglect the proton mass and $p_2 = (E_2, 0, 0, E_2)$
- In this frame can also neglect the mass of the quark and any momentum transverse to the direction of the proton.
- Let the quark carry a fraction ξ of the proton's four-momentum.





- After the interaction the struck quark's four-momentum is $\xi p_2 + q$

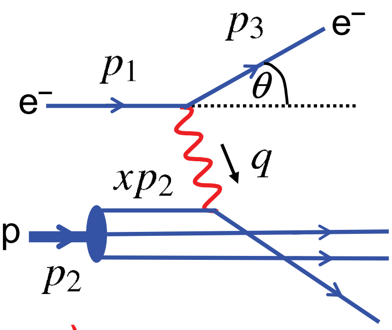
$$(\xi p_2 + q)^2 = m_q^2 \approx 0 \quad \rightarrow \quad \cancel{\xi^2 p_2^2} + q^2 + 2\xi p_2.q = 0 \qquad (\xi^2 p_2^2 = m_q^2 \approx 0)$$

$$\rightarrow \quad \xi = \frac{Q^2}{2p_2.q} = x$$

Bjorken x can be identified as the fraction of the proton momentum carried by the struck quark (in a frame where the proton has very high energy)

- In terms of the proton momentum

$$s=(p_1+p_2)^2 \simeq 2p_1.p_2 \qquad y=\frac{p_2.q}{p_2.p_1} \qquad x=\frac{Q^2}{2p_2.q}$$



- But for the underlying quark interaction

$$s^q=(p_1+xp_2)^2=2xp_1.p_2=xs$$

$$y_q=\frac{p_q.q}{p_q.p_1}=\frac{xp_2.q}{xp_2.p_1}=y$$

$$x_q=1$$ (elastic, i.e. assume quark does not break up)

- Previously derived the Lorentz Invariant cross section for $e^-\mu^- \rightarrow e^-\mu^-$ elastic scattering in the ultra-relativistic limit (handout 4 + Q10 on examples sheet).

Now apply this to $e^-q \rightarrow e^-q$

$$\frac{d\sigma}{dq^2}=\frac{2\pi\alpha^2 e_q^2}{q^4}\left[1+\left(1+\frac{q^2}{s_q}\right)^2\right]$$

e_q is quark charge, i.e. $e_u=+2/3; \quad e_d=-1/3$

- Using $-q^2=Q^2=(s_q-m^2)x_q y_q$ ➡ $\dfrac{q^2}{s_q}=-y_q=-y$

$$\frac{d\sigma}{dQ^2}=\frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1+(1-y)^2\right]$$

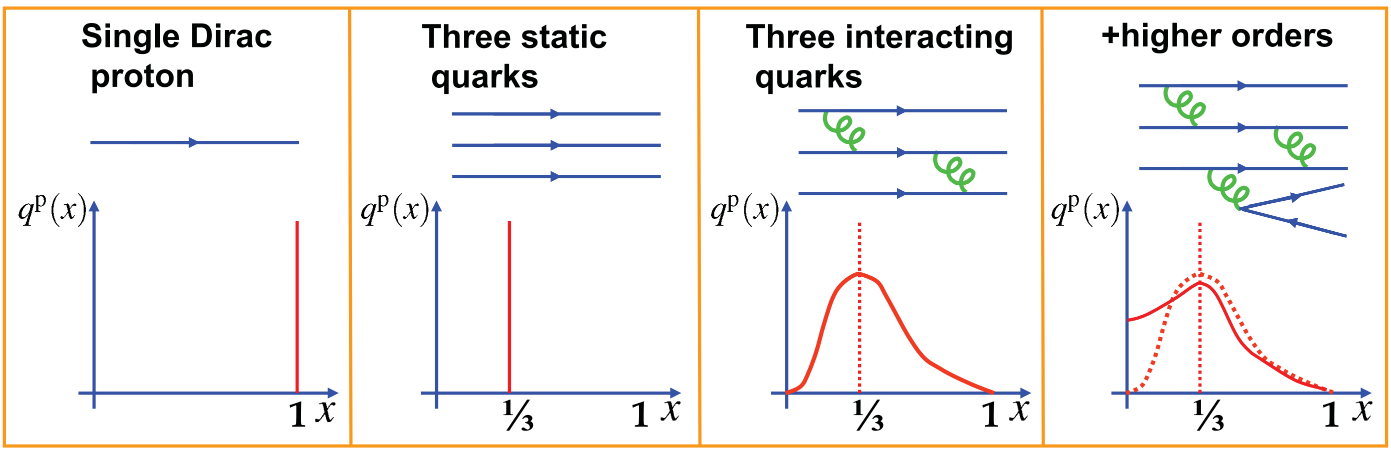
$$\boxed{\frac{d\sigma}{dQ^2}=\frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]} \qquad (3)$$

★ This is the expression for the differential cross-section for elastic e^-q scattering from a quark carrying a fraction x of the proton momentum.

- Now need to account for distribution of quark momenta within proton

★ Introduce parton distribution functions such that $q^p(x)dx$ is the number of quarks of type q within a proton with momenta between $x \rightarrow x+dx$

- Expected form of the parton distribution function ?



★ The cross section for scattering from a **particular quark type** within the proton which in the range $x \rightarrow x + \mathrm{d}x$ is

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right] \times e_q^2 q^{\mathrm{p}}(x)\mathrm{d}x$$

★ Summing over all types of quark within the proton gives the expression for the electron-proton scattering cross section

$$\boxed{\frac{\mathrm{d}^2\sigma^{\mathrm{ep}}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\sum_q e_q^2 q^{\mathrm{p}}(x)} \qquad (5)$$

★ Compare with the electron-proton scattering cross section in terms of structure functions (equation (2)):

$$\boxed{\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]} \qquad (6)$$

★ By comparing (5) and (6) obtain the parton model prediction for the structure functions in the general L.I. form for the differential cross section

$$\boxed{F_2^{\mathrm{p}}(x,Q^2) = 2xF_1^{\mathrm{p}}(x,Q^2) = x\sum_q e_q^2 q^{\mathrm{p}}(x)} \Rightarrow$$

Can relate measured structure functions to the underlying quark distributions

<u>**The parton model predicts:**</u>

- **Bjorken Scaling** $F_1(x,Q^2) \rightarrow F_1(x) \qquad F_2(x,Q^2) \rightarrow F_2(x)$
  - ✶ Due to scattering from point-like particles within the proton
- **Callan-Gross Relation** $F_2(x) = 2xF_1(x)$
  - ✶ Due to scattering from spin half Dirac particles where the magnetic moment is directly related to the charge; hence the "electro-magnetic" and "pure magnetic" terms are fixed with respect to each other.

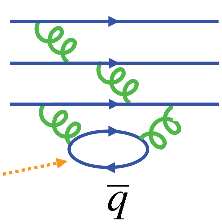
★ At present parton distributions cannot be calculated from QCD

- Can't use perturbation theory due to large coupling constant

★ Measurements of the structure functions enable us to determine the parton distribution functions !

★ For electron-proton scattering we have:

$$F_2^{\mathrm{p}}(x) = x\sum_q e_q^2 q^{\mathrm{p}}(x)$$



- Due to higher orders, the proton contains not only up and down quarks but also anti-up and anti-down quarks (will neglect the small contributions from heavier quarks)

• For electron-proton scattering have:

$$F_2^{\mathrm{ep}}(x) = x\sum_q e_q^2 q^{\mathrm{p}}(x) = x\left(\frac{4}{9}u^{\mathrm{p}}(x) + \frac{1}{9}d^{\mathrm{p}}(x) + \frac{4}{9}\overline{u}^{\mathrm{p}}(x) + \frac{1}{9}\overline{d}^{\mathrm{p}}(x)\right)$$

• For electron-neutron scattering have:

$$F_2^{\mathrm{en}}(x) = x\sum_q e_q^2 q^{\mathrm{n}}(x) = x\left(\frac{4}{9}u^{\mathrm{n}}(x) + \frac{1}{9}d^{\mathrm{n}}(x) + \frac{4}{9}\overline{u}^{\mathrm{n}}(x) + \frac{1}{9}\overline{d}^{\mathrm{n}}(x)\right)$$

★ Now assume "isospin symmetry", i.e. that the neutron (ddu) is the same as a proton (uud) with up and down quarks interchanged, i.e.

$$d^{\mathrm{n}}(x) = u^{\mathrm{p}}(x); \qquad u^{\mathrm{n}}(x) = d^{\mathrm{p}}(x)$$

and define the neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^{\mathrm{p}}(x) = d^{\mathrm{n}}(x); \qquad d(x) \equiv d^{\mathrm{p}}(x) = u^{\mathrm{n}}(x)$$

$$\overline{u}(x) \equiv \overline{u}^{\mathrm{p}}(x) = \overline{d}^{\mathrm{n}}(x); \qquad \overline{d}(x) \equiv \overline{d}^{\mathrm{p}}(x) = \overline{u}^{\mathrm{n}}(x)$$

giving:

$$\boxed{F_2^{\mathrm{ep}}(x) = 2xF_1^{\mathrm{ep}}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right)} \quad (7)$$

$$\boxed{F_2^{\mathrm{en}}(x) = 2xF_1^{\mathrm{en}}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right)} \quad (8)$$

• Integrating (7) and (8) :

$$\int_0^1 F_2^{\mathrm{ep}}(x)\mathrm{d}x = \int_0^1 x\left(\frac{4}{9}[u(x)+\overline{u}(x)] + \frac{1}{9}[d(x)+\overline{d}(x)]\right)\mathrm{d}x = \frac{4}{9}f_u + \frac{1}{9}f_d$$

$$\int_0^1 F_2^{\mathrm{en}}(x)\mathrm{d}x = \int_0^1 x\left(\frac{4}{9}[d(x)+\overline{d}(x)] + \frac{1}{9}[u(x)+\overline{u}(x)]\right)\mathrm{d}x = \frac{4}{9}f_d + \frac{1}{9}f_u$$

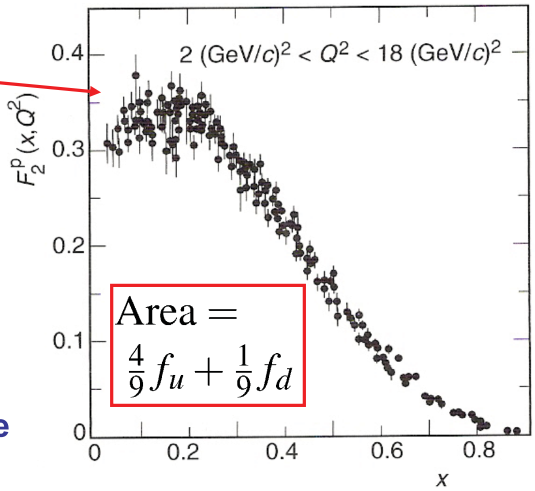
★ $f_u = \int_0^1 [xu(x) + x\overline{u}(x)]\mathrm{d}x$ is the fraction of the proton momentum carried by the up and anti-up quarks

Experimentally

$$\int F_2^{\mathrm{ep}}(x)\mathrm{d}x \approx 0.18$$

$$\int F_2^{\mathrm{en}}(x)\mathrm{d}x \approx 0.12$$

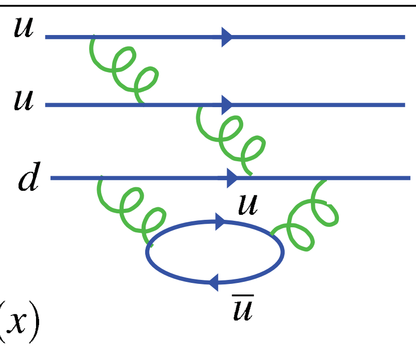
$$\Rightarrow \boxed{f_u \approx 0.36 \qquad f_d \approx 0.18}$$



✶ In the proton, as expected, the up quarks carry twice the momentum of the down quarks

✶ The quarks carry just over 50 % of the total proton momentum. The rest is carried by gluons (which being neutral doesn't contribute to electron-nucleon scattering).

Valence and Sea Quarks



- As we are beginning to see the proton is complex…
- The parton distribution function $u^{\mathrm{p}}(x) = u(x)$ includes contributions from the "valence" quarks and the virtual quarks produced by gluons: the "sea"
- Resolving into valence and sea contributions:

$$u(x) = u_{\mathrm{V}}(x) + u_{\mathrm{S}}(x) \qquad d(x) = d_{\mathrm{V}}(x) + d_{\mathrm{S}}(x)$$
$$\overline{u}(x) = \overline{u}_{\mathrm{S}}(x) \qquad \overline{d}(x) = \overline{d}_{\mathrm{S}}(x)$$

- The proton contains two valence up quarks and one valence down quark and would expect:

$$\int_0^1 u_{\mathrm{V}}(x)\mathrm{d}x = 2 \qquad \int_0^1 d_{\mathrm{V}}(x)\mathrm{d}x = 1$$

- But no ***a priori*** expectation for the total number of sea quarks !
- But sea quarks arise from gluon quark/anti-quark pair production and with $m_u = m_d$ it is reasonable to expect

$$u_{\mathrm{S}}(x) = d_{\mathrm{S}}(x) = \overline{u}_{\mathrm{S}}(x) = \overline{d}_{\mathrm{S}}(x) = S(x)$$

- With these relations (7) and (8) become

$$F_2^{\mathrm{ep}}(x) = x\left(\frac{4}{9}u_{\mathrm{V}}(x) + \frac{1}{9}d_{\mathrm{V}}(x) + \frac{10}{9}S(x)\right) \qquad F_2^{\mathrm{en}}(x) = x\left(\frac{4}{9}d_{\mathrm{V}}(x) + \frac{1}{9}u_{\mathrm{V}}(x) + \frac{10}{9}S(x)\right)$$

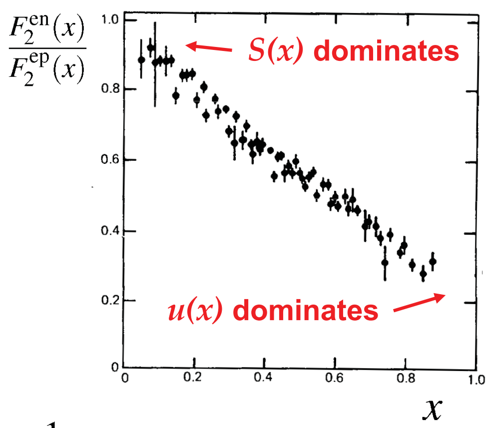
Giving the ratio

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} = \frac{4d_{\mathrm{V}}(x) + u_{\mathrm{V}}(x) + 10S(x)}{4u_{\mathrm{V}}(x) + d_{\mathrm{V}}(x) + 10S(x)}$$

- The sea component arises from processes such as $g \to \overline{u}u$. Due to the $1/q^2$ dependence of the gluon propagator, much more likely to produce low energy gluons. Expect the sea to comprise of low energy $q/\overline{q}$
- Therefore at low x expect the sea to dominate:

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} \to 1 \quad \text{as} \quad x \to 0$$

Observed experimentally



- At high x expect the sea contribution to be small

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} \to \frac{4d_{\mathrm{V}}(x) + u_{\mathrm{V}}(x)}{4u_{\mathrm{V}}(x) + d_{\mathrm{V}}(x)} \quad \text{as} \quad x \to 1$$

Note: $u_{\mathrm{V}} = 2d_{\mathrm{V}}$ would give ratio 2/3 as $x \to 1$

Experimentally $F_2^{\mathrm{en}}(x)/F_2^{\mathrm{ep}}(x) \to 1/4 \quad \text{as} \quad x \to 1$

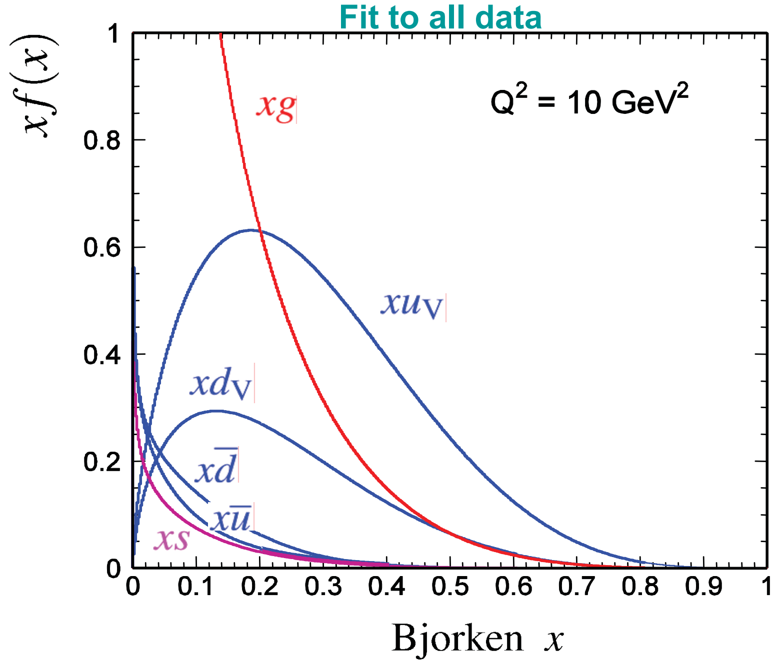
$$\Rightarrow \quad d(x)/u(x) \to 0 \quad \text{as} \quad x \to 1$$

This behaviour is not understood.

Parton Distribution Functions

- Ultimately the parton distribution functions are obtained from a fit to all experimental data including neutrino scattering (see handout 10)
  - Hadron-hadron collisions give information on gluon pdf $g(x)$

Fit to all data

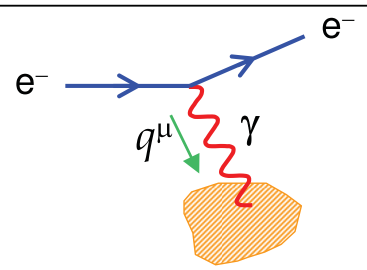


Note:
- Apart from at large x
  $u_V(x) \approx 2d_V(x)$
- For $x < 0.2$ gluons dominate
- In fits to data assume $u_s(x) = \bar{u}(x)$
- $\bar{d}(x) > \bar{u}(x)$ not understood – exclusion principle?
- Small strange quark component $s(x)$

(Try Question 12)

Scaling Violations

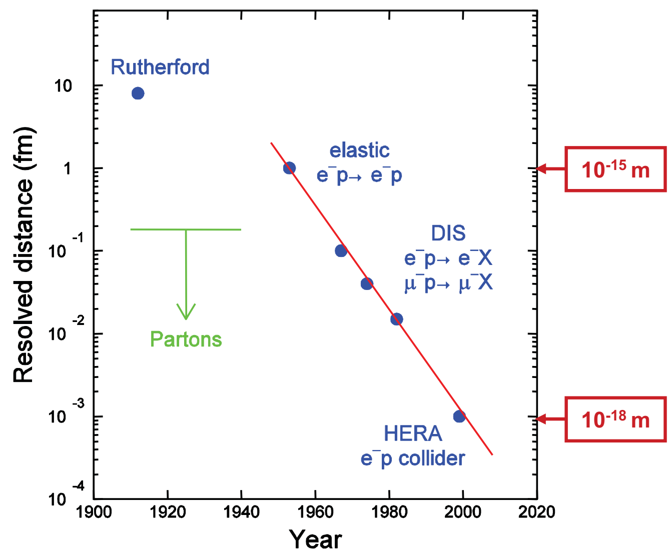
- In last 40 years, experiments have probed the proton with virtual photons of ever increasing energy
- Non-point like nature of the scattering becomes apparent when λ_γ ~ size of scattering centre



$$\lambda_\gamma = \frac{h}{|\vec{q}|} \sim \frac{1\,\mathrm{GeV\,fm}}{|\vec{q}|(\mathrm{GeV})}$$

- Scattering from point-like quarks gives rise to Bjorken scaling: no q^2 cross section dependence
- IF quarks were not point-like, at high q^2 (when the wavelength of the virtual photon ~ size of quark) would observe rapid decrease in cross section with increasing q^2.
- To search for quark sub-structure want to go to highest q^2

HERA

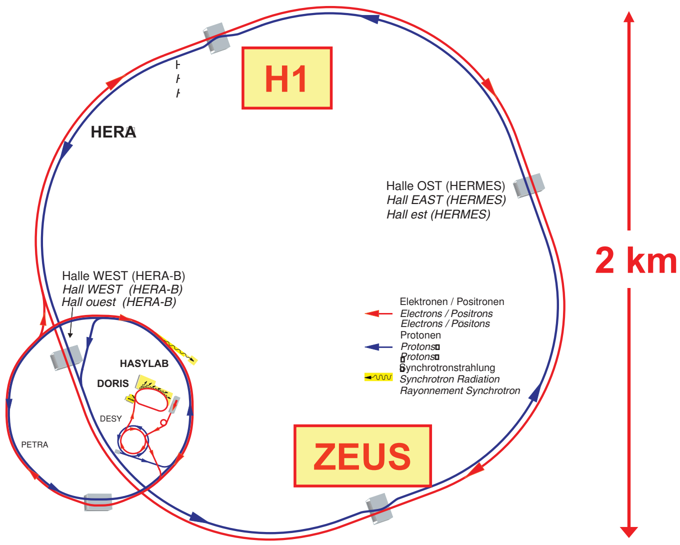


HERA $e^{\pm}p$ Collider : 1991-2007

★ DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany

$e^{\pm}$ 27.5 GeV → ← 820 GeV p

$\sqrt{s}$ = 300 GeV

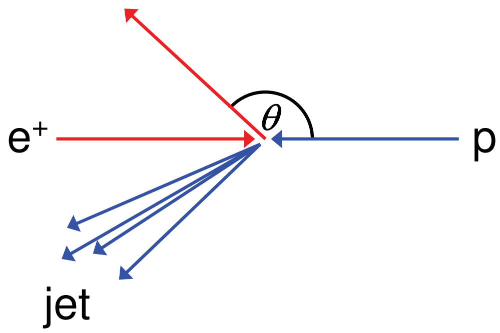


★ Two large experiments : H1 and ZEUS

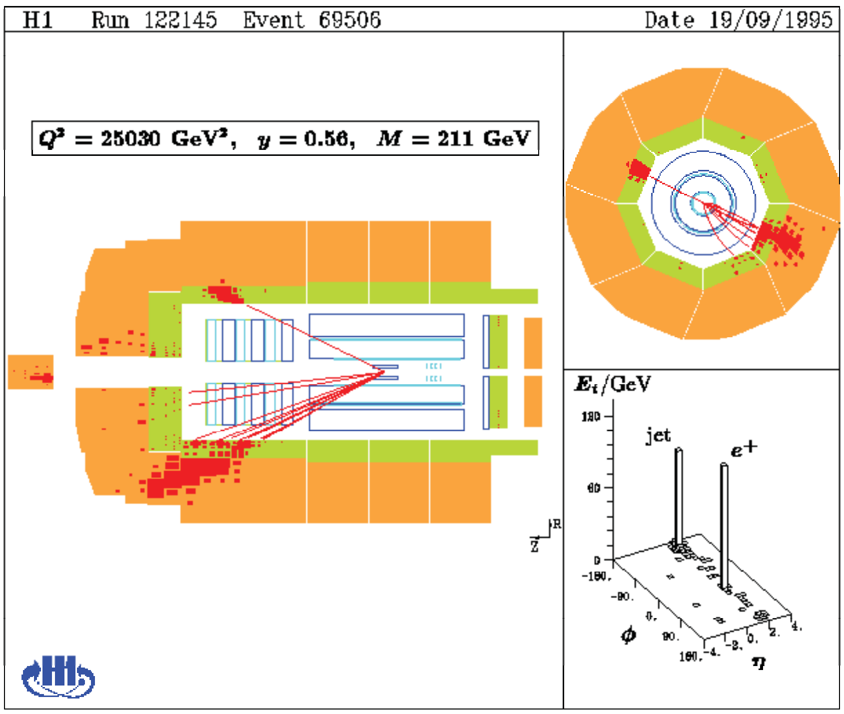
★ Probe proton at very high Q^2 and very low x

Example of a High Q^2 Event in H1

★ Event kinematics determined from electron angle and energy



★ Also measure hadronic system (although not as precisely) – gives some redundancy



$F_2(x,Q^2)$ Results

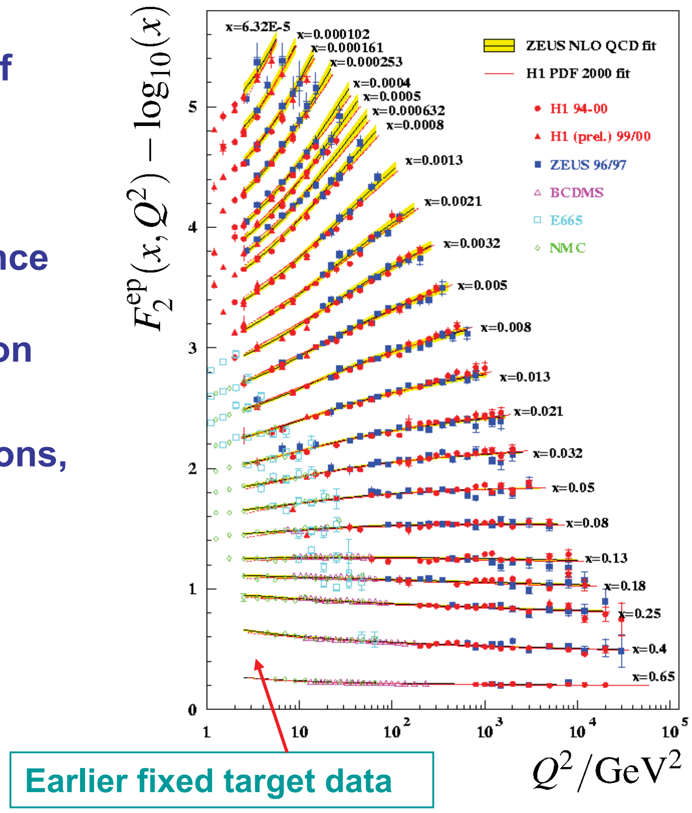
★ No evidence of rapid decrease of cross section at highest Q^2

➡ $\boxed{R_{\text{quark}} < 10^{-18}\,\text{m}}$

★ For $x > 0.05$, only weak dependence of F_2 on Q^2 : consistent with the expectation from the quark-parton model

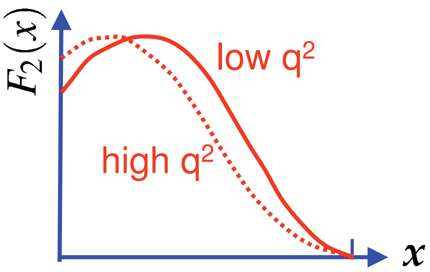
★ But observe clear scaling violations, particularly at low x

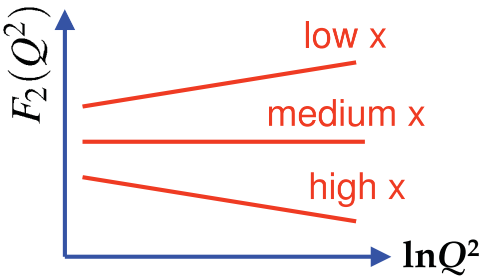
$$F_2(x,Q^2) \neq F_2(x)$$



Origin of Scaling Violations

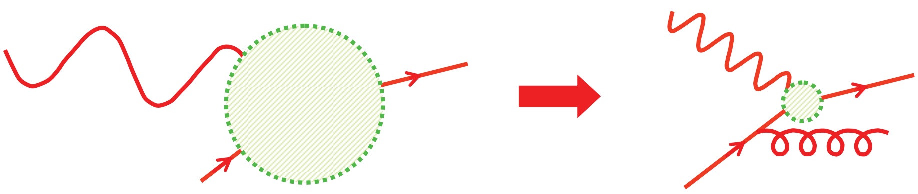
★ Observe "small" deviations from exact Bjorken scaling $F_2(x) \rightarrow F_2(x,Q^2)$





★ At high Q^2 observe more low x quarks

★ **"Explanation"**: at high Q^2 (shorter wave-length) resolve finer structure: i.e. reveal quark is sharing momentum with gluons. At higher Q^2 expect to "see" more low x quarks



★ QCD cannot predict the x dependence of $F_2(x,Q^2)$

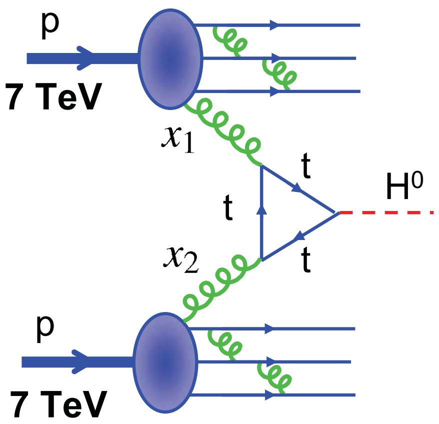
★ But QCD can predict the Q^2 dependence of $F_2(x,Q^2)$

Proton-Proton Collisions at the LHC

★ Measurements of structure functions not only provide a powerful test of QCD, the parton distribution functions are essential for the calculation of cross sections at pp and p$\bar{\text{p}}$ colliders.

• **Example:** Higgs production at the Large Hadron Collider LHC (2009-)

- The LHC will collide 7 TeV protons on 7 TeV protons
- However underlying collisions are between partons
- Higgs production the LHC dominated by "gluon-gluon fusion"

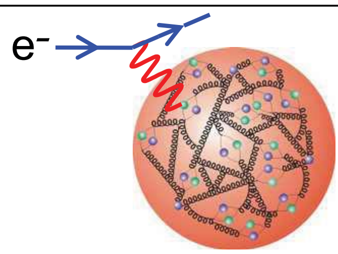


- Cross section depends on gluon PDFs

$$\sigma(pp \to HX) \sim \int_0^1 \int_0^1 g(x_1)g(x_2)\sigma(gg \to H)\mathrm{d}x_1\mathrm{d}x_2$$

- Uncertainty in gluon PDFs lead to a ±5 % uncertainty in Higgs production cross section
- Prior to HERA data uncertainty was ±25 %

Summary



- At very high electron energies $\lambda \ll r_p$: the proton appears to be a sea of quarks and gluons.
- Deep Inelastic Scattering = Elastic scattering from the quasi-free constituent quarks

  ➡ Bjorken Scaling $F_1(x,Q^2) \to F_1(x)$ point-like scattering

  ➡ Callan-Gross $F_2(x) = 2xF_1(x)$ Scattering from spin-1/2

- Describe scattering in terms of parton distribution functions $u(x), d(x), \ldots$ which describe momentum distribution inside a nucleon
- The proton is much more complex than just uud - sea of anti-quarks/gluons
- Quarks carry only 50 % of the protons momentum – the rest is due to low energy gluons
- We will come back to this topic when we discuss neutrino scattering...