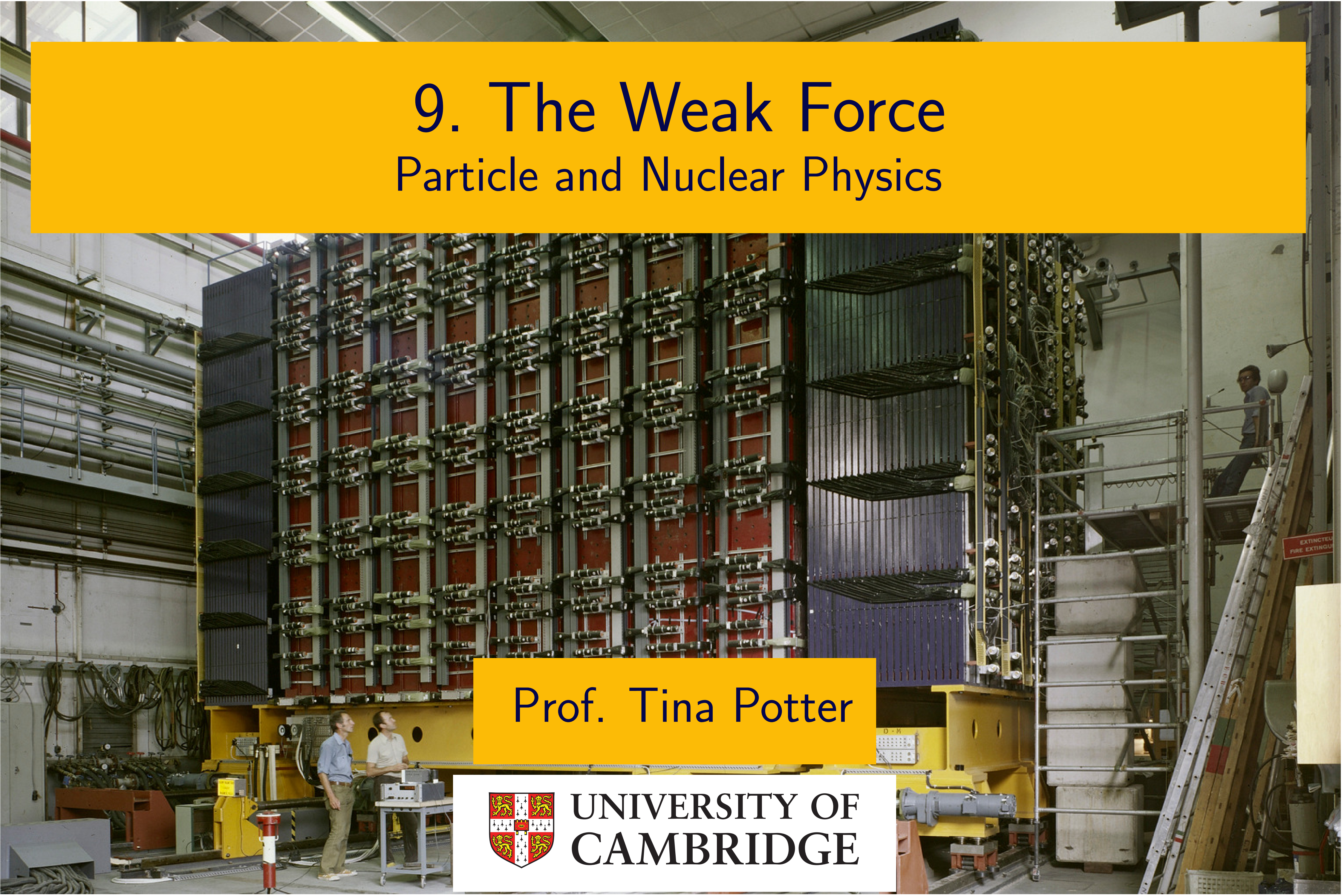
# 9. The Weak Force

## Particle and Nuclear Physics



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# In this section...

- The charged current weak interaction
- Four-fermion interactions
- Massive propagators and the strength of the weak interaction
- C-symmetry and Parity violation
- Lepton universality
- Quark interactions and the CKM

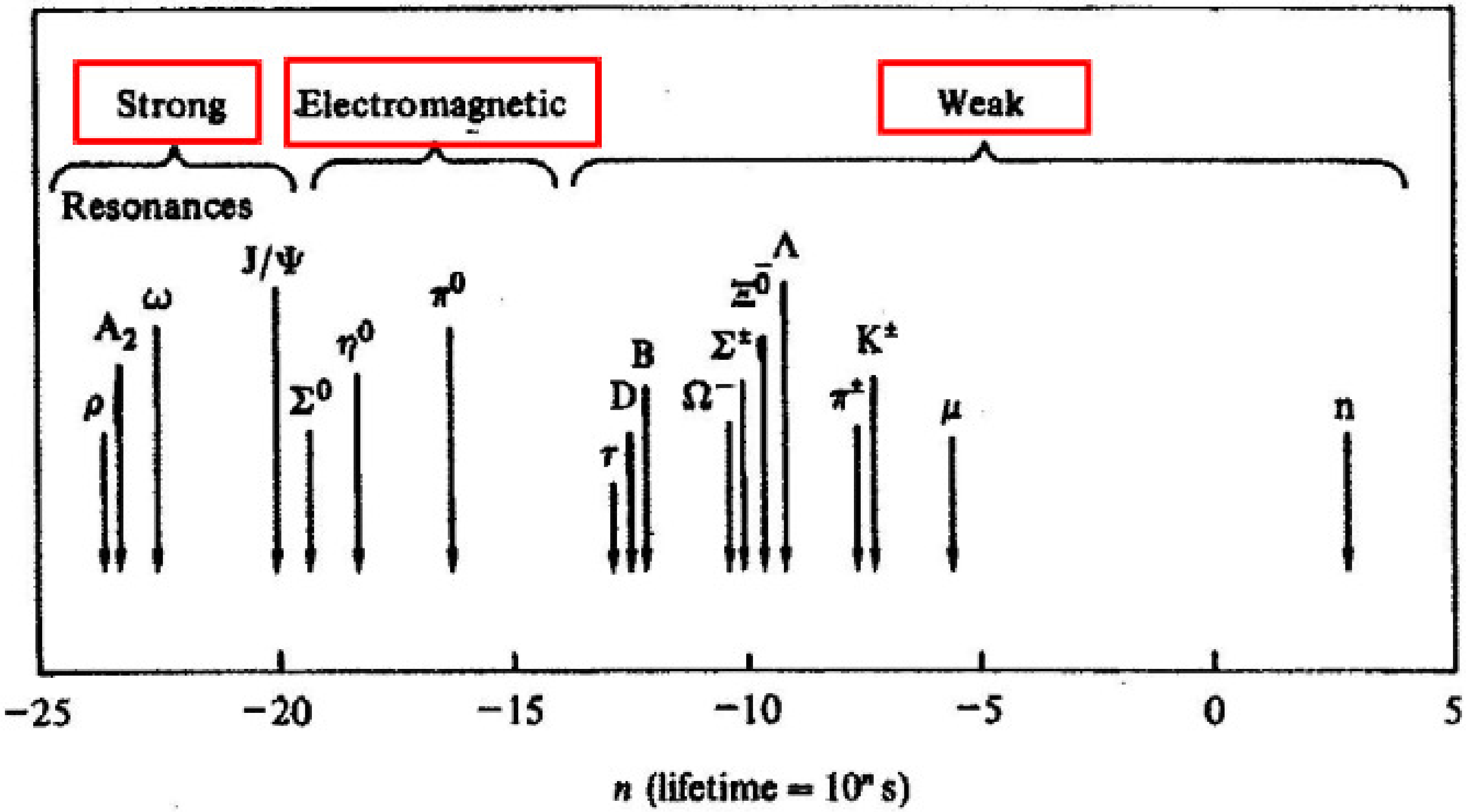
# The Weak Interaction

The weak interaction accounts for many decays in particle physics, e.g.

$$\mu^- \to e^- \bar{\nu}_e \nu_\mu \qquad \tau^- \to e^- \bar{\nu}_e \nu_\tau$$

$$\pi^+ \to \mu^- \bar{\nu}_\mu \qquad n \to p e^- \bar{\nu}_e$$

Characterised by long lifetimes and small interaction cross-sections



# The Weak Interaction

- Two types of weak interaction

  Charged current (CC): $W^\pm$ bosons

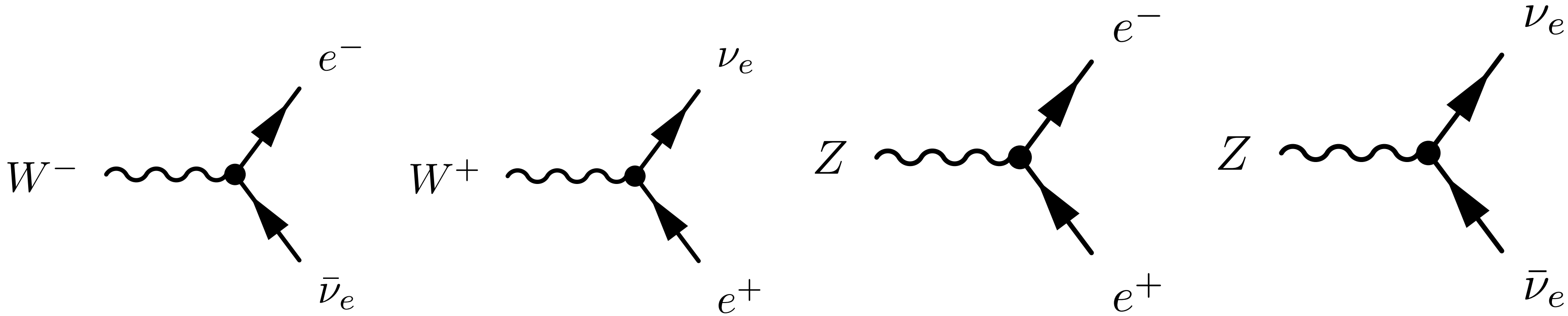
  Neutral current (NC): $Z$ bosons See Chapter 10

- The weak force is mediated by massive vector bosons:

$$m_W = 80 \text{ GeV}$$
$$m_Z = 91 \text{ GeV}$$

**Examples:** (The list below is not complete, will see more vertices later!)

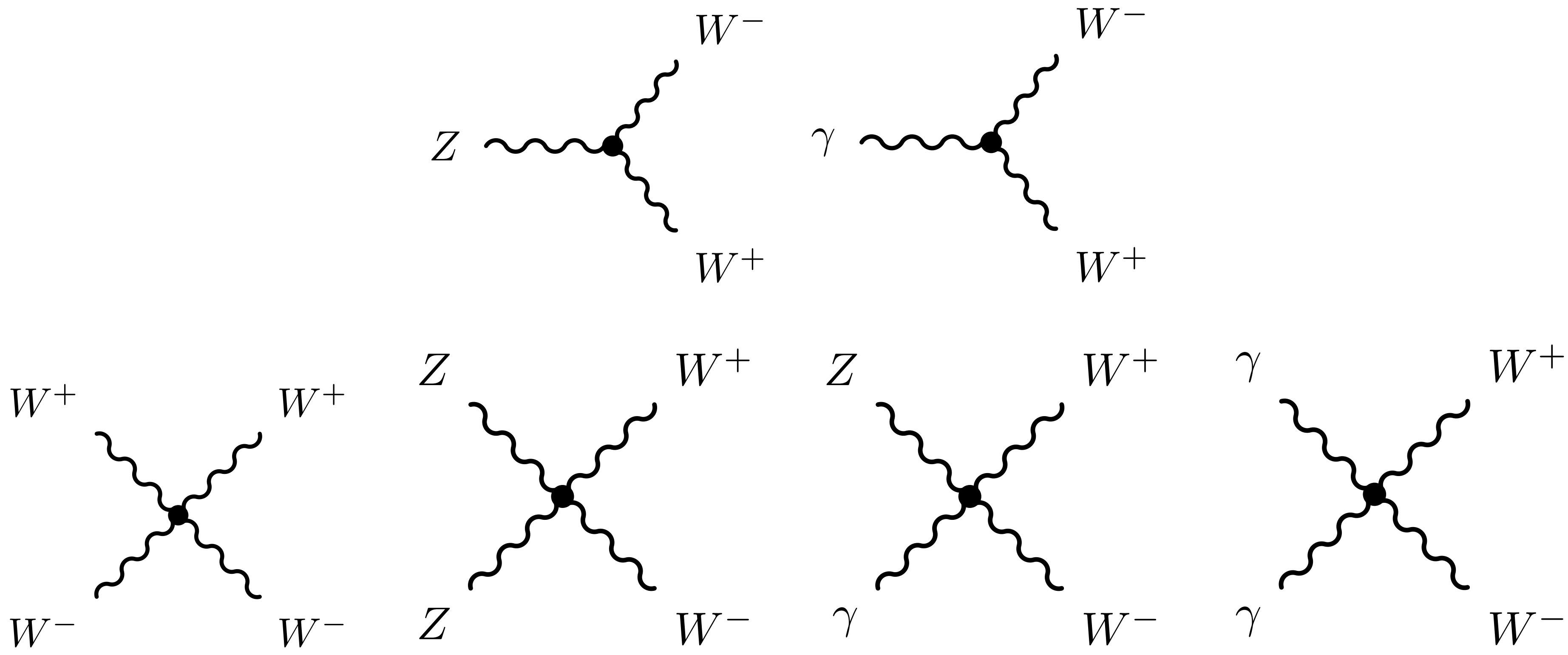
Weak interactions of electrons and neutrinos:



# Boson Self-Interactions

- In QCD the gluons carry colour charge.
- In the weak interaction the $W^{\pm}$ and $Z$ bosons carry the weak charge
- $W^{\pm}$ also carry the electric charge

$\Rightarrow$ boson self-interactions

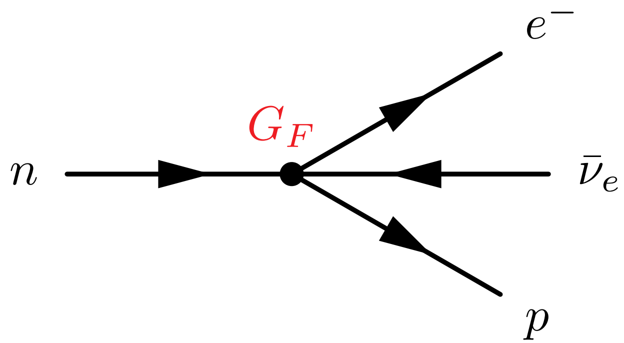


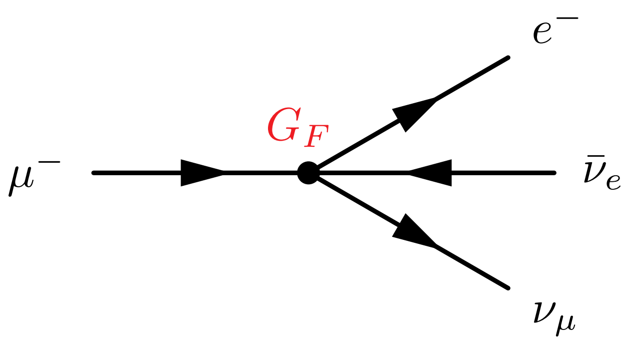

(The list above is complete as far as weak self-interactions are concerned, but we have still not seen all the weak vertices. Will see the rest later)

Weak interaction taken to be a "4-fermion contact interaction"

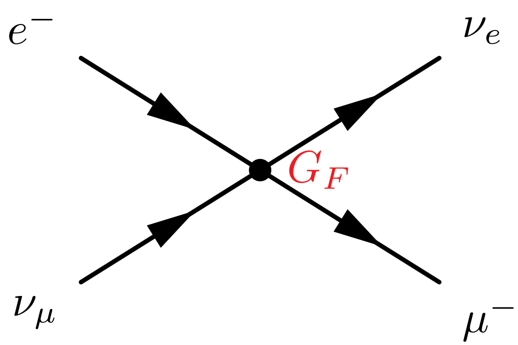
- No propagator
- Coupling strength given by the Fermi constant $G_F$
- $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

$\beta$-decay in Fermi Theory



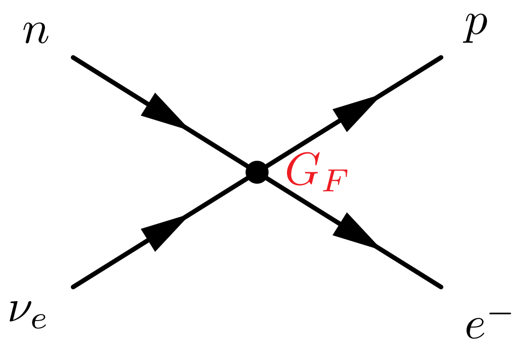




Neutrino scattering in Fermi Theory



# Why must Fermi Theory be "Wrong"?

$$\nu_e + n \rightarrow p + e^-$$




$$\mathrm{d}\sigma = 2\pi |M_{\mathrm{fi}}|^2 \frac{\mathrm{d}N}{\mathrm{d}E} = 2\pi 4 G_F^2 \frac{E_e^2}{(2\pi)^3}\,\mathrm{d}\Omega$$

$$\sigma = \frac{G_F^2 s}{\pi}$$

See Appendix F

where $E_e$ is the energy of the $e^-$ in the centre-of-mass system and $\sqrt{s}$ is the energy in the centre-of-mass system.

In the laboratory frame: $s = 2E_\nu m_n$ (fixed target collision, see Chapter 3)

$$\Rightarrow \sigma \sim (E_\nu / \text{ MeV}) \times 10^{-43}\,\mathrm{cm}^{-2}$$

- $\nu$'s only interact weakly $\therefore$ have very small interaction cross-sections.
- Here weak implies that you need approximately 50 light-years of water to stop a 1 $\mathrm{MeV}$ neutrino!

However, as $E_\nu \rightarrow \infty$ the cross-section can become very large. Violates maximum value allowed by conservation of probability at $\sqrt{s} \sim 1\ \mathrm{TeV}$ ("unitarity limit"). This is a big problem.

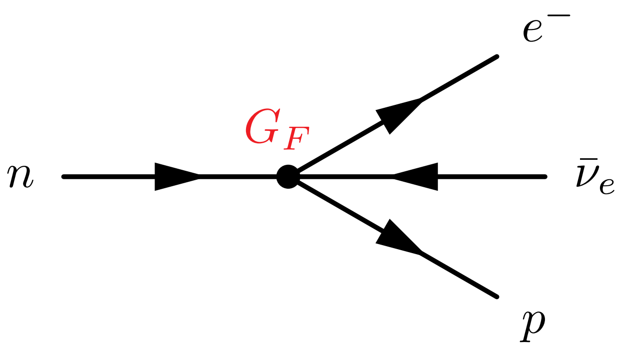
$\Rightarrow$ Fermi theory breaks down at high energies.

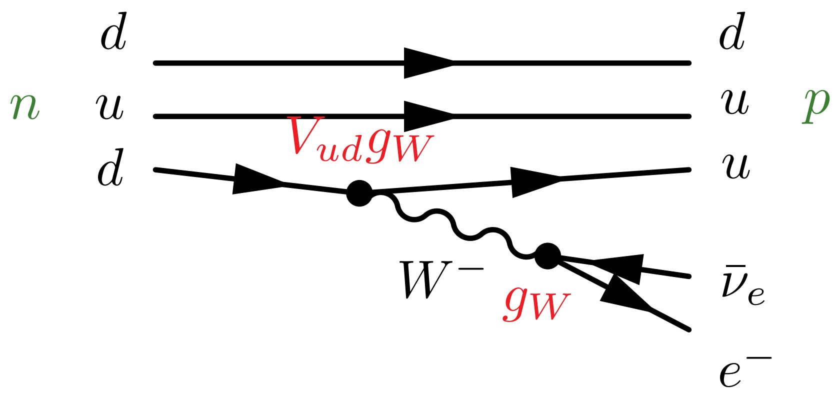
# Weak Charged Current: $W^{\pm}$ Boson

- Fermi theory breaks down at high energy
- True interaction described by exchange of charged $W^{\pm}$ bosons
- Fermi theory is the low energy ($q^2 \ll m_W^2$) effective theory of the weak interaction

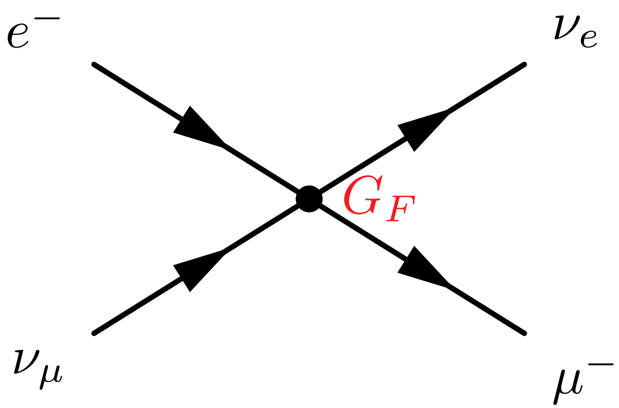
**Old Fermi Theory** | **Standard Model**

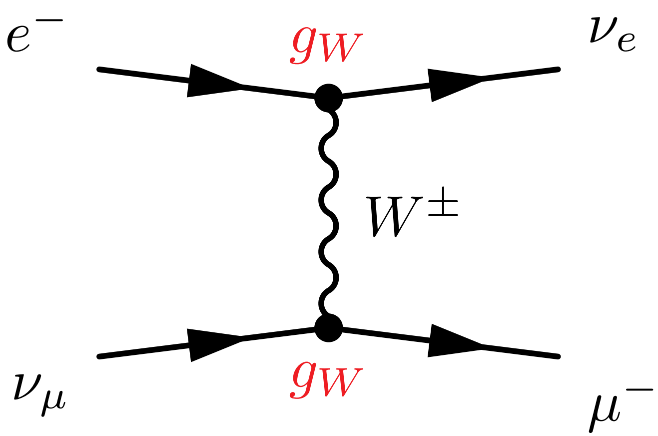
**$\beta$ decay**



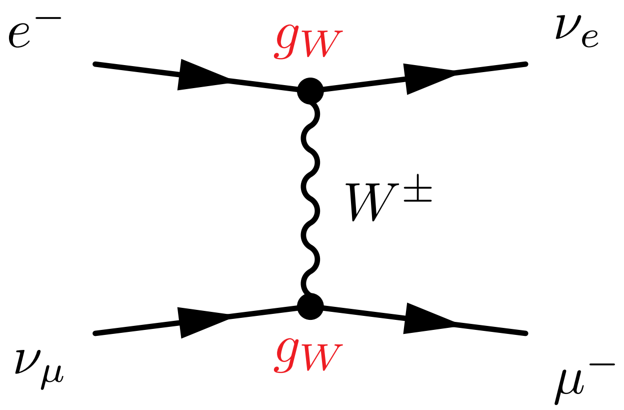

**$\nu_e e^-$ scattering**



# Weak Charged Current: $W^{\pm}$ Boson

**Weak**

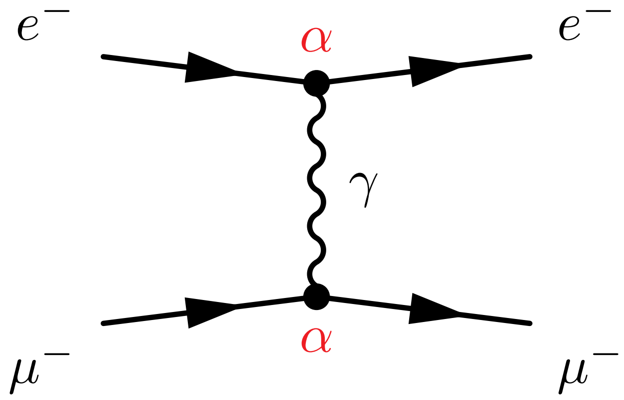



Propagator

$$\sim \frac{1}{q^2 - m_W^2}$$

$$\alpha_W = \frac{g_W^2}{4\pi}$$

**QED**




Propagator

$$\sim \frac{1}{q^2}$$

$$\alpha = \frac{e^2}{4\pi}$$

## Charged Current Weak Interaction

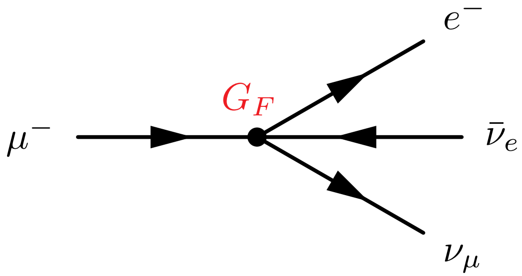
- At low energies, $q^2 \ll m_W^2$, propagator $\frac{1}{q^2 - m_W^2} \rightarrow \frac{1}{-m_W^2}$
  i.e. appears as the point-like interaction of Fermi theory.
- Massive propagator $\rightarrow$ short range

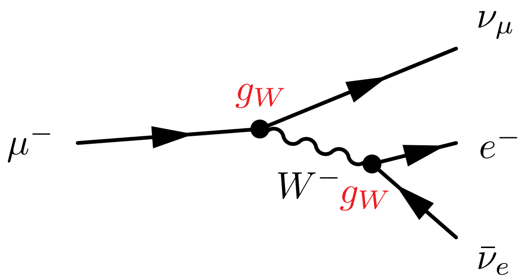
$$m_W = 80.4 \text{ GeV} \quad \Rightarrow \text{ Range} \sim \frac{1}{m_W} \sim 0.002 \text{ fm}$$

- Exchanged boson carries electromagnetic charge.
- Flavour changing - only the CC weak interaction changes flavour
- Parity violating - only the CC weak interaction can violate parity conservation

# Weak Charged Current: $W^{\pm}$ Boson

Compare Fermi theory with a massive propagator




For $q^2 \ll m_W^2$ compare matrix elements $\frac{g_W^2}{m_W^2} \to G_F$

$G_F$ is small because $m_W$ is large

The precise relationship is: $$\frac{g_W^2}{8m_W^2} \to \frac{G_F}{\sqrt{2}}$$

The numerical factors are partly of historical origin (see Perkins $4^{th}$ ed., page 210).

$m_W = 80.4\ \mathrm{GeV}$ and $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ measured in muon $\beta$ decay

$$g_W = 0.65 \quad \text{and} \quad \alpha_W = \frac{g_W^2}{4\pi} \sim \frac{1}{30}$$

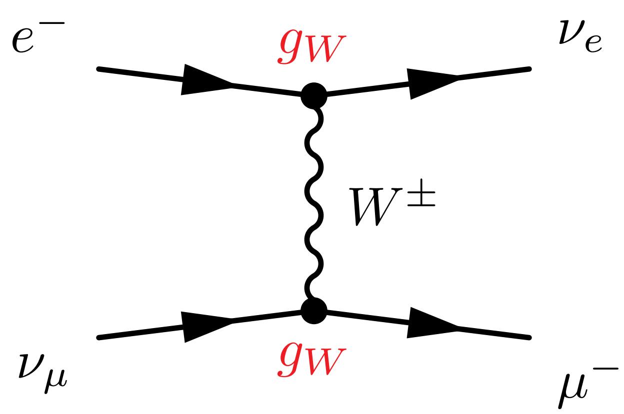
Compare to EM $\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$

The intrinsic strength of the weak interaction is actually greater than that of the electromagnetic interaction. At low energies (low $q^2$), it appears weak owing to the massive propagator.

# Weak Charged Current: $W^{\pm}$ Boson

**Neutrino Scattering with a Massive $W$ Boson**

Replace contact interaction by massive boson exchange diagram:




Fermi theory

$$\frac{d\sigma}{d\Omega} = 2\pi G_F^2 \frac{E_e^2}{(2\pi)^3}$$

Standard Model

$$\frac{d\sigma}{d\Omega} = 2\pi G_F^2 \frac{E_e^2}{(2\pi)^3} \left( \frac{m_W^2}{m_W^2 - q^2} \right)^2$$

with $|\vec{q}^2| = 4E_e^2 \sin^2\theta/2$, where $\theta$ is the scattering angle.

Integrate to give

$$\sigma = \frac{G_F^2 s}{\pi} \quad s \ll m_W^2$$

$$\sigma = \frac{G_F^2 m_W^2}{\pi} \quad s \gg m_W^2$$

see Appendix G

Cross-section is now well behaved at high energies.

# Spin and helicity

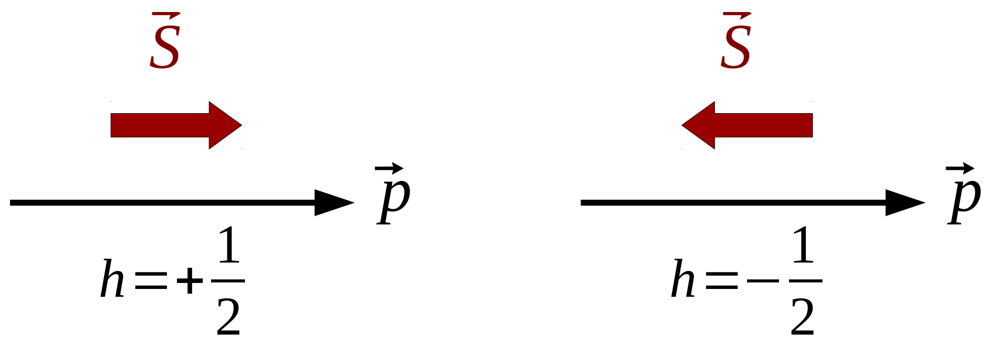
Consider a free particle of constant momentum, $\vec{p}$

- Total angular momentum, $\vec{J} = \vec{L} + \vec{S}$ is always conserved
- The orbital angular momentum, $\vec{L} = \vec{r} \times \vec{p}$ is perpendicular to $\vec{p}$
- The spin angular momentum, $\vec{S}$ can be in any direction relative to $\vec{p}$
- The value of spin $\vec{S}$ along $\vec{p}$ is always constant

The sign of the component of spin along the direction of motion is known as the "helicity",

$$h = \frac{\vec{S}.\vec{p}}{|\vec{p}|}$$

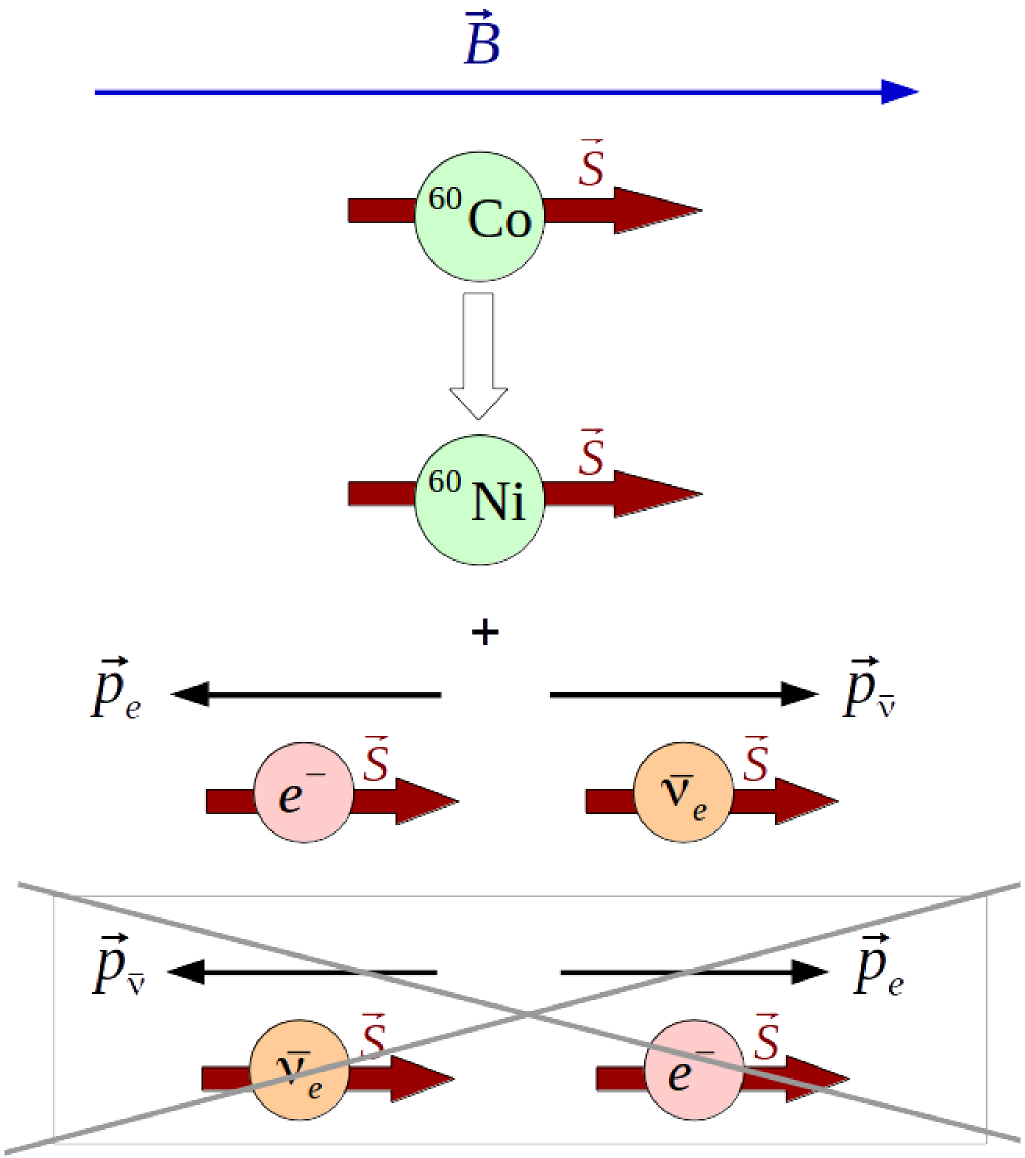
Taking spin 1/2 as an example:




"Right-handed" "Left-handed"

# The Wu Experiment

$\beta$ decay of $\quad {}^{60}\mathrm{Co} \rightarrow {}^{60}\,\mathrm{Ni} + e^- + \bar{\nu}_e$




1956
Chien-Shiung Wu

Align cooled $^{60}$Co nuclei with $\vec{B}$ field and look at direction of emission of electrons

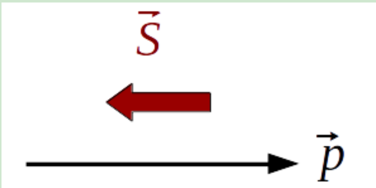
- $e^-$ always observed in direction opposite to spin – left-handed.
- $\vec{p}$ conservation: $\bar{\nu}$ must be emitted in opposite direction – right-handed.
- Right-handed $e^-$ not observed here $\Rightarrow$ Parity Violation

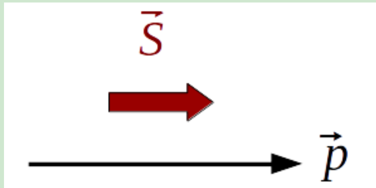
# The Weak Interaction and Helicity

The weak interaction distinguishes between left- and right-handed states. This is an experimental observation, which we need to build into the Standard Model.

The weak interaction couples preferentially to

left-handed particles and right-handed antiparticles




To be precise, the probability for weak coupling to the $\pm$ helicity state is

$\frac{1}{2}\left[1 \mp \frac{v}{c}\right]$ for a lepton $\rightarrow$ coupling to RH particles vanishes

$\frac{1}{2}\left[1 \pm \frac{v}{c}\right]$ for an antilepton $\rightarrow$ coupling to LH antiparticles vanishes

**$\Rightarrow$ right-handed $\nu$'s do not exist**

**left-handed $\bar{\nu}$'s do not exist**

Even if they did exist, they would be unobservable.

# Charge Conjugation

C-symmetry: the physics for $+Q$ should be the same as for $-Q$.

This is true for QED and QCD, but not the Weak force...

$$\begin{array}{ccc} \text{LH } e^- & \xrightarrow{\text{Charge Conjugation}} & \text{LH } e^+ \\ \textit{EM, Weak} & & \textit{EM, } \cancel{\textit{Weak}} \end{array}$$

$$\begin{array}{ccc} \text{RH } e^- & \xrightarrow{\text{Charge Conjugation}} & \text{RH } e^+ \\ \textit{EM, } \cancel{\textit{Weak}} & & \textit{EM, Weak} \end{array}$$

$$\begin{array}{ccc} \text{LH } \nu_e & \xrightarrow{\text{Charge Conjugation}} & \text{LH } \bar{\nu}_e \\ \textit{Weak} & & \cancel{\textit{Weak}} \end{array}$$

**C-symmetry is maximally violated in the weak interaction.**

# Parity Violation

**Parity is always conserved in the strong and EM interactions**

$\eta \rightarrow \pi^0\pi^0\pi^0$ $\qquad\qquad$ $\eta \rightarrow \pi^+\pi^-$

# Parity Violation

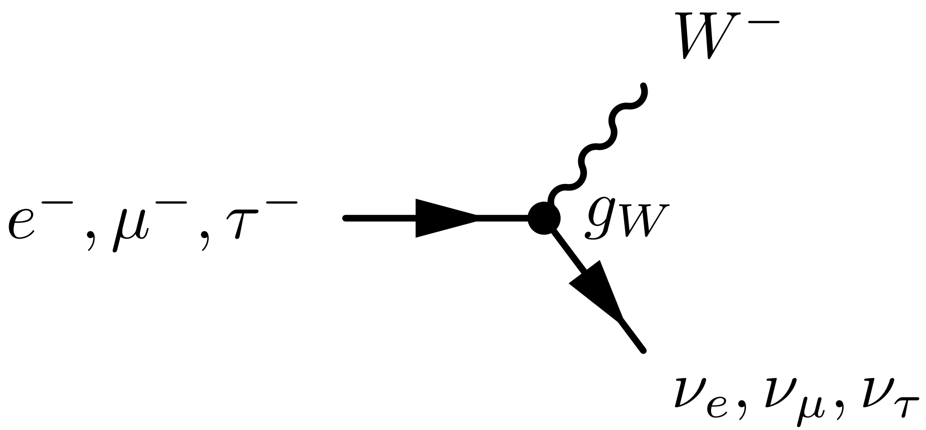
**Parity is often conserved in the weak interaction, but not always**

The weak interaction treats LH and RH states differently and therefore can violate parity (because the interaction Hamiltonian does not commute with $\hat{P}$).

$K^+ \rightarrow \pi^+\pi^-\pi^+$ $\qquad\qquad$ $K^+ \rightarrow \pi^+\pi^0$

# Weak interactions of leptons

All weak charged current lepton interactions can be described by the $W$ boson propagator and the weak vertex:




The Standard Model Weak CC Lepton Vertex

+ antiparticles

- $W$ bosons only "couple" to the (left-handed) lepton and neutrino within the same generation

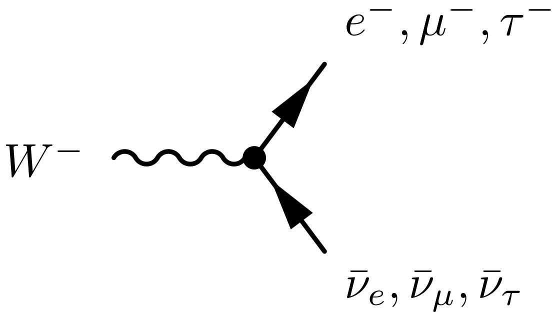
$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

e.g. no $W^{\pm} e^- \nu_\mu$ coupling

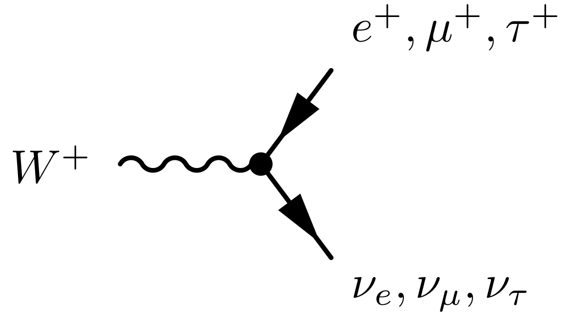
- Coupling constant $g_W$ $\quad \alpha_W = \frac{g_W^2}{4\pi}$

# Weak interactions of leptons *Examples*

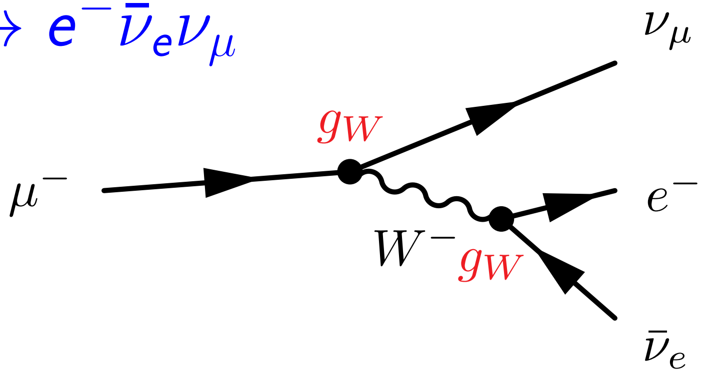
$W^- \to e^-\bar{\nu}_e,\ \mu^-\bar{\nu}_\mu,\ \tau^-\bar{\nu}_\tau$



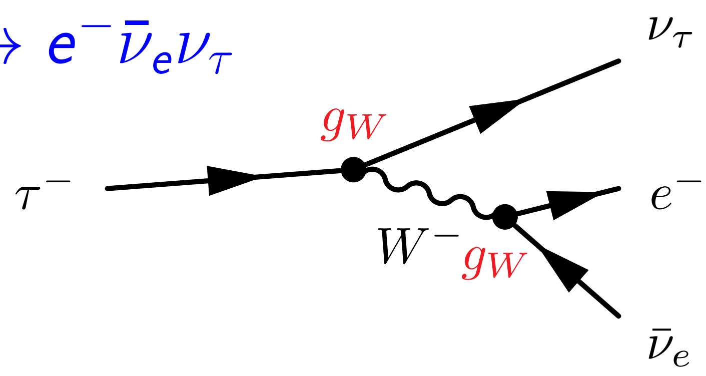

$W^+ \to e^+\nu_e,\ \mu^+\nu_\mu,\ \tau^+\nu_\tau$



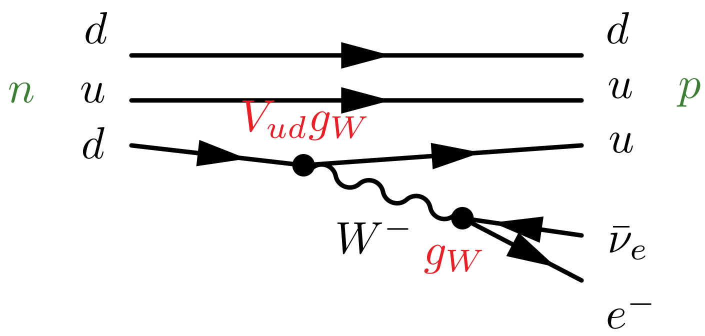

$\mu^- \to e^-\bar{\nu}_e\nu_\mu$



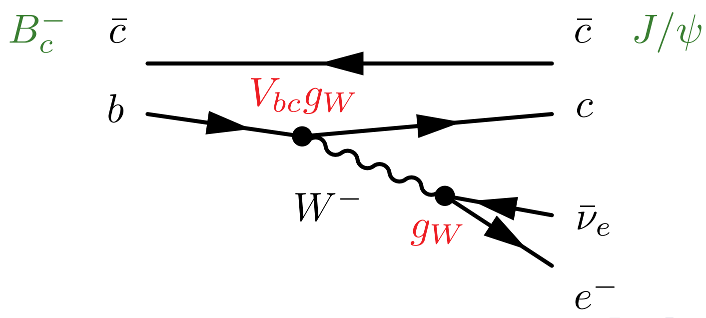

$\tau^- \to e^-\bar{\nu}_e\nu_\tau$




$n \to pe^-\bar{\nu}_e$

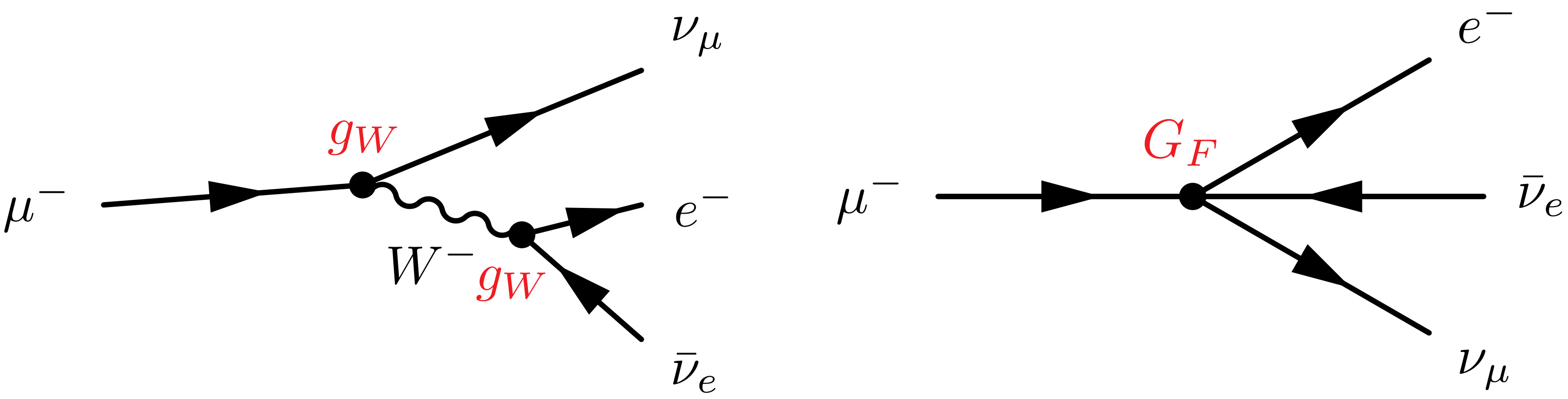



$B_c^- \to J/\psi e^-\bar{\nu}_e$



# $\mu$ Decay

- Muons are fundamental leptons ($m_\mu \sim 206 m_e$)
- Electromagnetic decay $\mu^- \to e^-\gamma$ is not observed (branching ratio $< 2.4 \times 10^{-12}$) $\Rightarrow$ the EM interaction does not change flavour.
- Only the weak CC interaction changes lepton type, but only within a generation. "Lepton number conservation" for each lepton generation.
- Muons decay weakly: $\mu^- \to e^-\bar{\nu}_e\nu_\mu$




As $m_\mu \ll m_W$ can safely use Fermi theory to calculate decay width (analogous to nuclear $\beta$ decay).

# $\mu$ Decay

Fermi theory gives decay width $\propto m_\mu^5$ (Sargent Rule)

However, more complicated phase space integration (previously neglected kinetic energy of recoiling nucleus) and taking account of helicity/spin gives different constants

$$\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F^2}{192\pi^3} m_\mu^5$$

- Muon mass and lifetime known with high precision.

$$m_\mu = 105.6583715 \pm 0.0000035 \text{ MeV}$$

$$\tau_\mu = (2.1969811 \pm 0.0000022) \times 10^{-6} \text{ s}$$

- Use muon decay to fix strength of weak interaction $G_F$

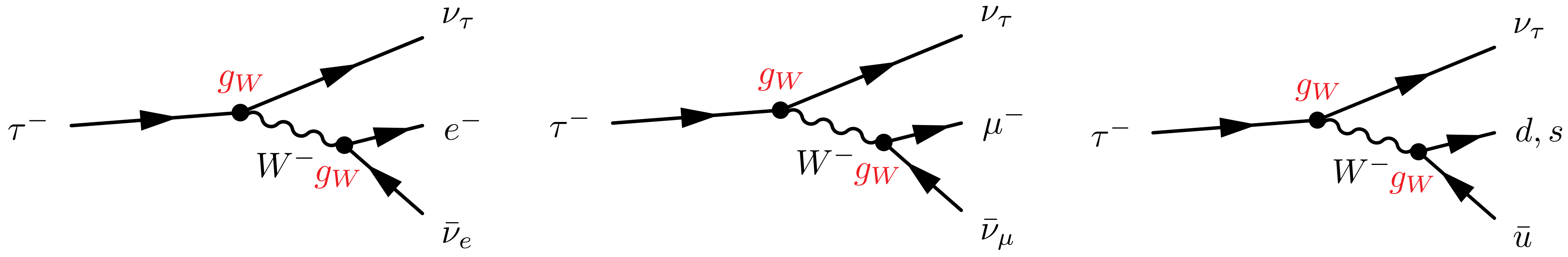
$$G_F = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

- $G_F$ is one of the best determined fundamental quantities in particle physics.

# $\tau$ Decay

The $\tau$ mass is relatively large $m_\tau = 1.77686 \pm 0.00012\ \mathrm{GeV}$

Since $m_\tau > m_\mu, m_\pi, m_p, \ldots$ there are a number of possible decay modes




Measure the $\tau$ branching fractions as:

$$\tau^- \to e^- \bar{\nu}_e \nu_\tau \quad 17.83 \pm 0.04\%$$

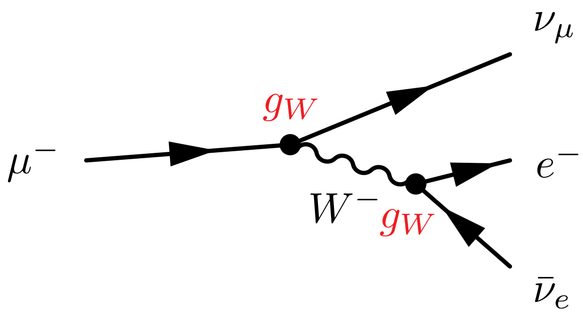
$$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \quad 17.41 \pm 0.04\%$$

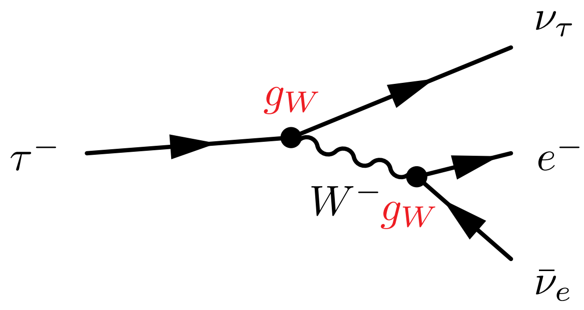
$$\tau^- \to \mathrm{hadrons} \quad 64.76 \pm 0.06\%$$

# Lepton Universality

Do all leptons have the same weak coupling?

Look at measurements of the decay rates and branching fractions.




$$\frac{1}{\tau_\mu} = \Gamma_{\mu\to e} = \frac{G_F^2}{192\pi^3} m_\mu^5$$

$$\frac{1}{\tau_\tau} = \frac{\Gamma_{\tau\to e}}{B(\tau\to e)} = \frac{1}{0.178} \frac{G_F^2}{192\pi^3} m_\tau^5$$

If weak interaction strength is universal, expect: $\frac{\tau_\tau}{\tau_\mu} = 0.178 \frac{m_\mu^5}{m_\tau^5}$

Measure $m_\mu$, $m_\tau$, $\tau_\mu$ to high precision:

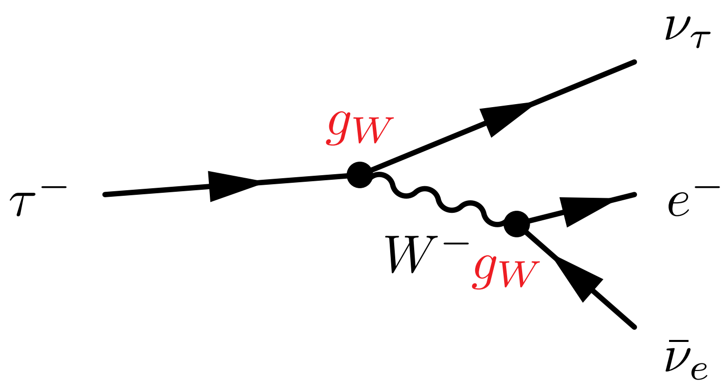
$$m_\mu = 105.6583715 \pm 0.0000035 \text{ MeV}$$
$$m_\tau = 1.77686 \pm 0.00012 \text{ GeV}$$
$$\tau_\mu = (2.1969811 \pm 0.0000022) \times 10^{-6} \text{ s}$$

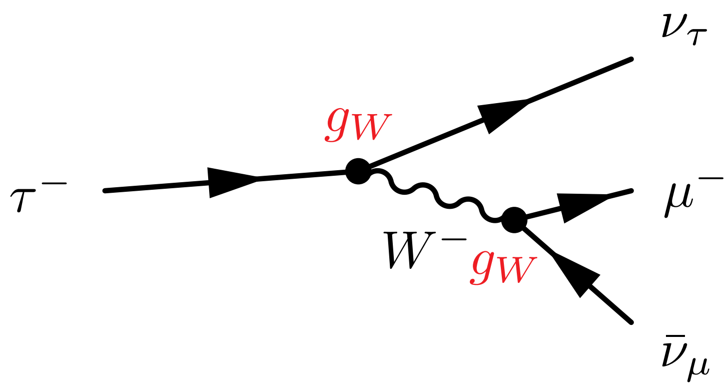
**Predict** $\tau_\tau = (2.903 \pm 0.005) \times 10^{-13}\,\text{s}$ **Measure** $\tau_\tau = (2.903 \pm 0.005) \times 10^{-13}\,\text{s}$

$\Rightarrow$ **same weak CC coupling for $\mu$ and $\tau$**

# Lepton Universality

We can also compare




If the couplings are the same, expect: $$\frac{B(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)}{B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)} = 0.9726$$

(the small difference is due to the slight reduction in phase space due to the non-negligible muon mass).

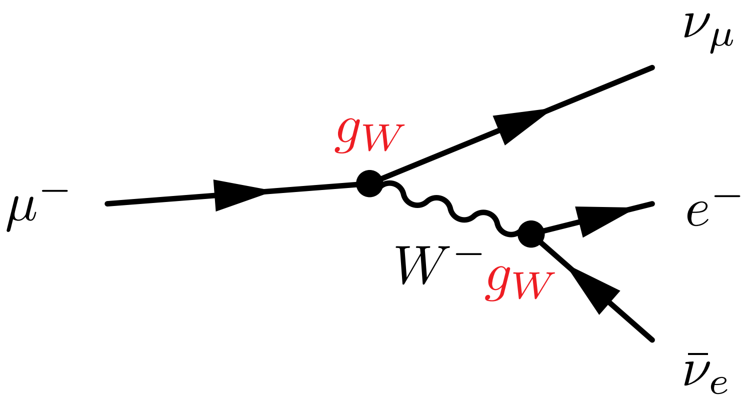
Measured $$\frac{B(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)}{B(\tau^- \to e^-\bar{\nu}_e\nu_\tau)} = 0.974 \pm 0.005$$ consistent with prediction.

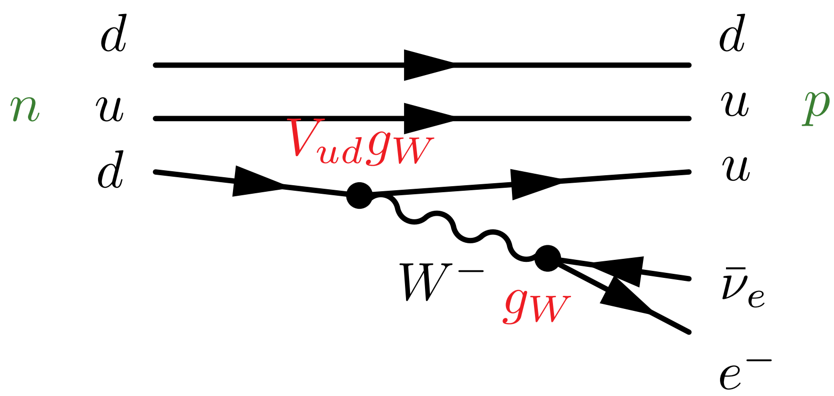
$\Rightarrow$ **same weak CC coupling for $e$, $\mu$ and $\tau$**

$\Rightarrow$ **Lepton Universality**

# Universality of Weak Coupling

Compare $G_F$ measured from $\mu^-$ decay with that from nuclear $\beta$ decay




$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$

$G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\ \mathrm{GeV}^{-2}$

Ratio $\dfrac{G_F^\beta}{G_F^\mu} = 0.974 \pm 0.003$

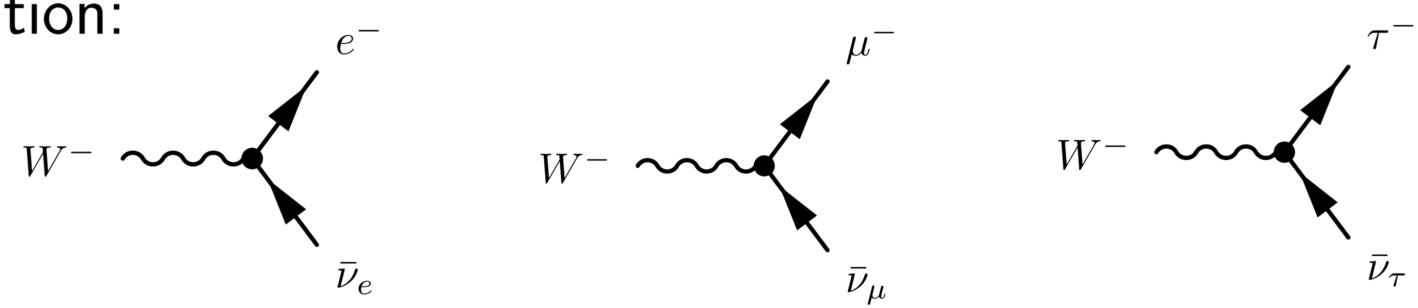
Conclude that the strength of the weak interaction is almost the same for leptons as for quarks. But the difference is significant, and has to be explained.

# Weak Interactions of Quarks

Impose a symmetry between leptons and quarks, so weak CC couplings take place within one generation:

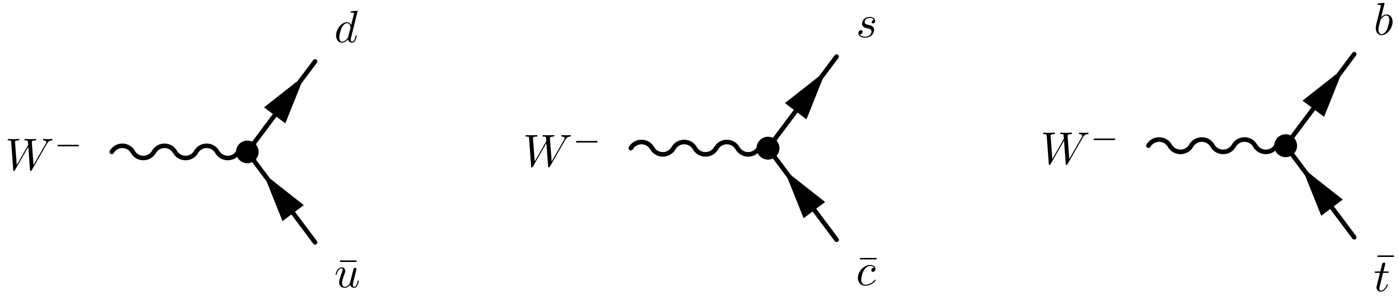
**Leptons**

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

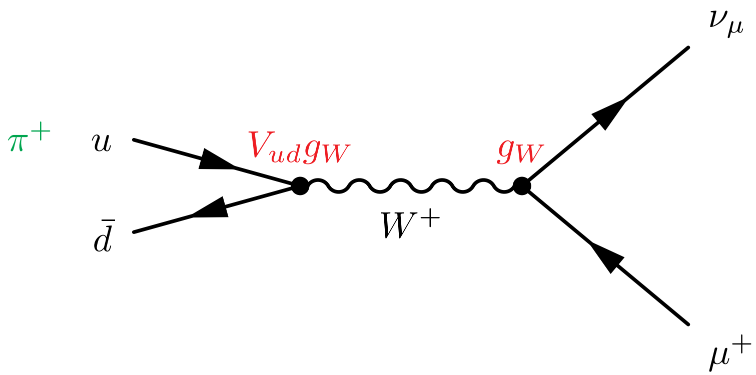



**Quarks**

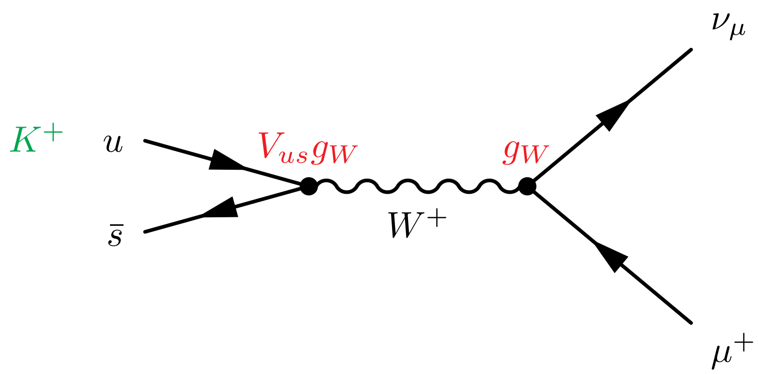
$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$




So $\pi^+ \to \mu^+ \nu_\mu$ would be allowed




but $K^+ \to \mu^+ \nu_\mu$ would not




**But we have observed $K^+ \to \mu^+ \nu_\mu$ !** (much smaller rate than $\pi^+$ decay.)

# Quark Mixing

Instead, alter the lepton-quark symmetry to: (only considering 1st and 2nd gen. here)

Leptons Quarks

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \qquad \begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix}$$

where $d' = d\cos\theta_C + s\sin\theta_C$
$s' = -d\sin\theta_C + s\cos\theta_C$

Now, the down type quarks in the weak interaction are actually linear superpositions of the down type quarks

i.e. weak eigenstates $(d', s')$ are superpositions of the mass eigenstates $(d, s)$

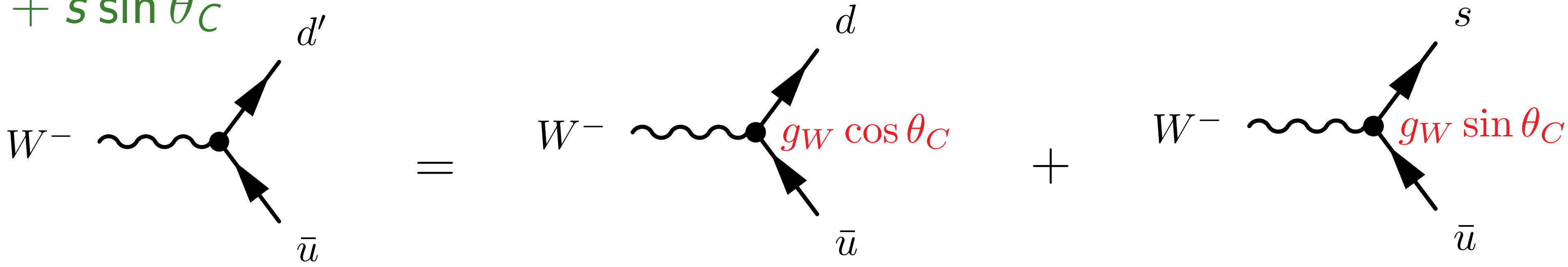
Weak Eigenstates
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$
Mass Eigenstates

$\Rightarrow$ Cabibbo angle $\theta_C \sim 13^\circ$ (from experiment)

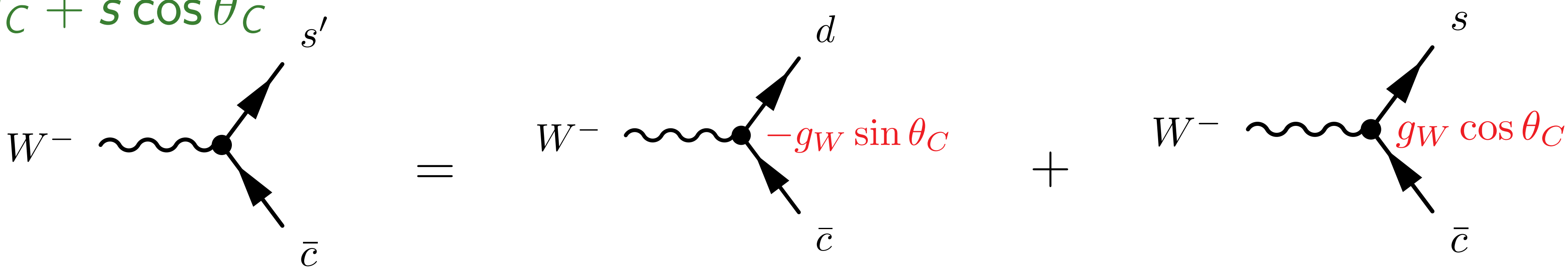
# Quark Mixing

Now, the weak coupling to quarks is:

$d \cos\theta_C + s \sin\theta_C$




$-d \sin\theta_C + s \cos\theta_C$




Quark mixing explains the lower rate of $K^+ \to \mu^+\nu_\mu$ compared to $\pi^+ \to \mu^+\nu_\mu$

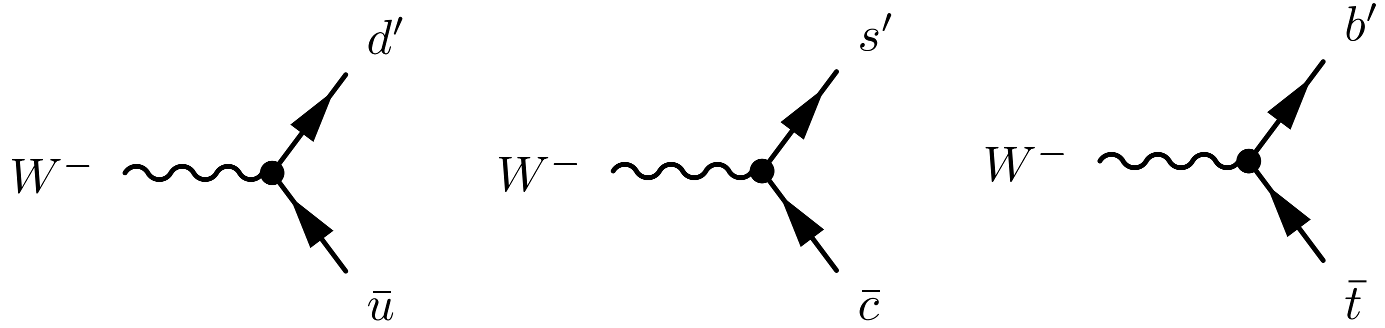
and the ratio $\frac{G_F^\beta}{G_F^\mu} = 0.974 \pm 0.003$

Difference in couplings affects $|M|^2 \propto (G_F^\beta)^2 \propto (\cos\theta_C)^2$

Now expect $\frac{G_F^\beta}{G_F^\mu} = \cos\theta_C$ which holds for $\theta_C \sim 13^\circ$

# CKM matrix *Cabibbo-Kobayashi-Maskawa Matrix*

Extend quark mixing to three generations




Weak Eigenstates
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$
Mass Eigenstates

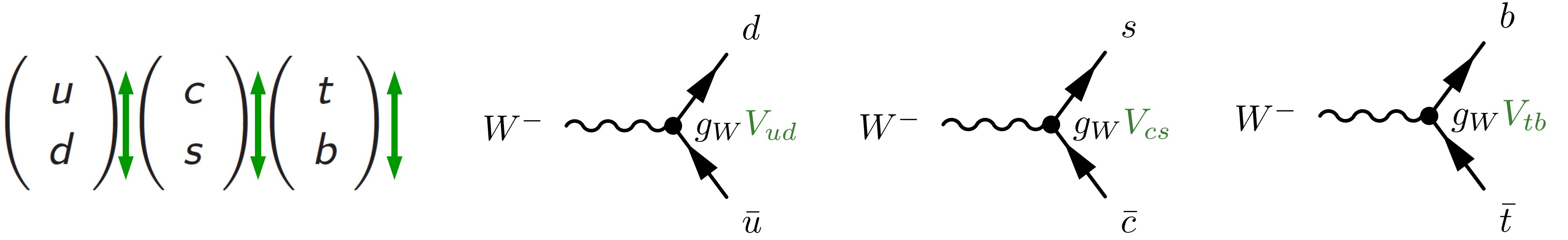
$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} \cos\theta_C & \sin\theta_C & \sin^3\theta_C e^{-i\delta} \\ -\sin\theta_C & \cos\theta_C & \sin^2\theta_C \\ \sin^3\theta_C e^{i\delta} & -\sin^2\theta_C & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 0.975 & 0.220 & 0.01 \\ -0.220 & 0.975 & 0.05 \\ 0.01 & -0.05 & 1 \end{pmatrix}$$

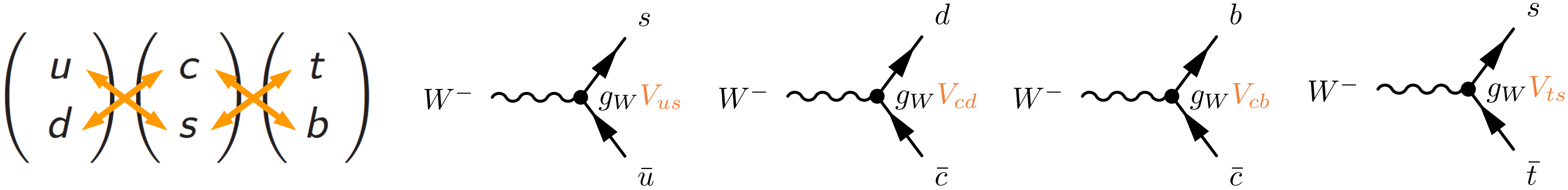
Unitary matrix.
Mixing angle $\theta_C \sim 13^\circ$
Charge-Parity violating phase $\delta \sim 69^\circ$
(full CKM matrix includes 3 mixing angles)

# Quark Mixing

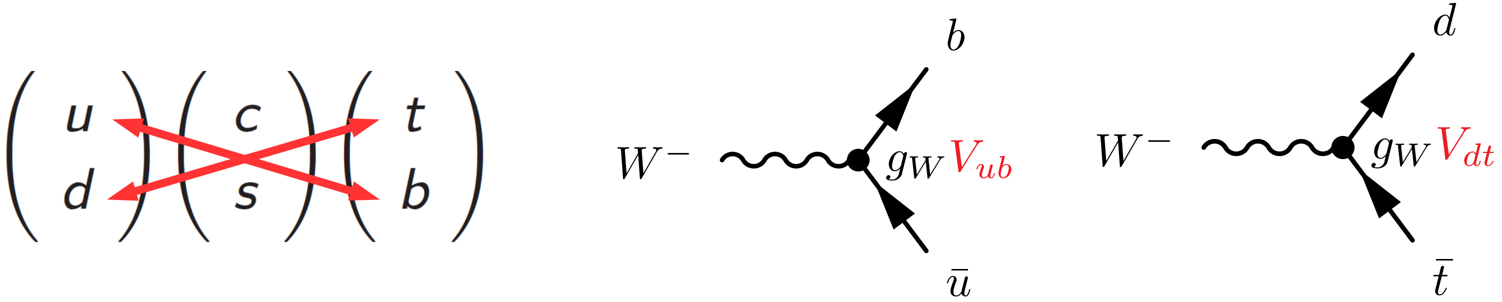
Weak interactions between quarks of the same family are "Cabibbo Allowed"




between quarks differing by one family are "Cabibbo Suppressed"

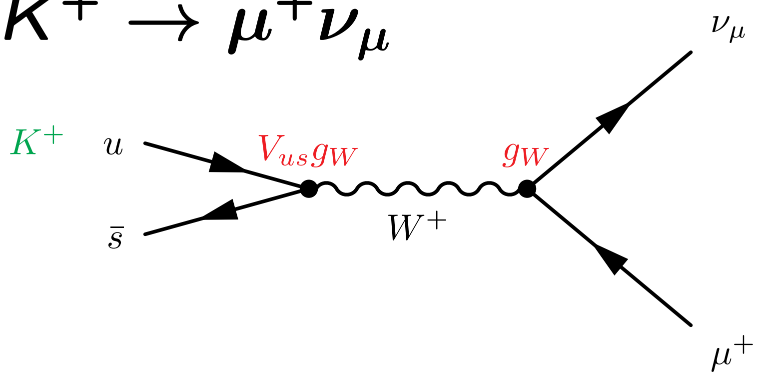



between quarks differing by two families are "Doubly Cabbibo Suppressed"



# Quark Mixing Examples

$K^+ \rightarrow \mu^+\nu_\mu$




$u\bar{s}$ coupling $\Rightarrow$ Cabbibo suppressed

$$|M|^2 \propto g_W^4 V_{us}^2 = g_W^4 \sin^2\theta_C$$

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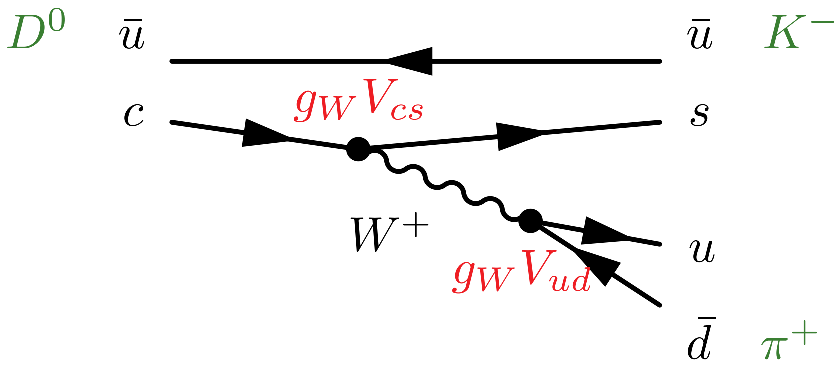
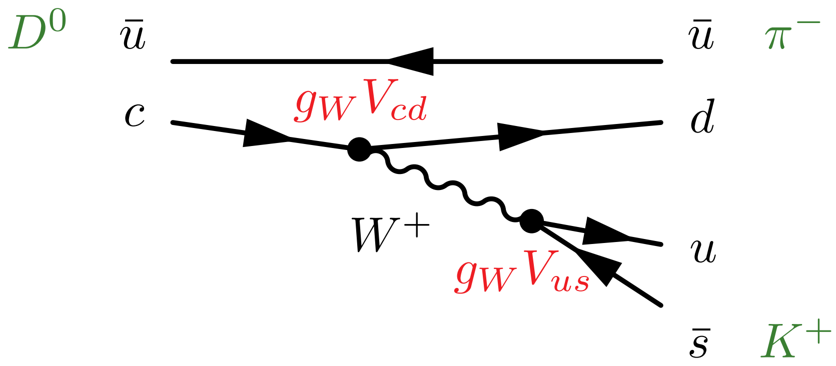
$D^0 \rightarrow K^-\pi^+$




$D^0 \rightarrow K^+\pi^-$



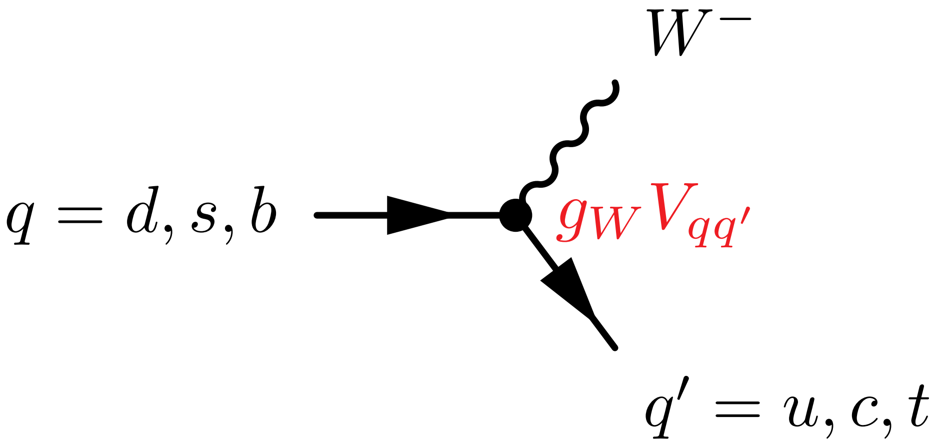

Expect $\dfrac{\Gamma(D^0 \rightarrow K^+\pi^-)}{\Gamma(D^0 \rightarrow K^-\pi^+)} \sim \dfrac{(g_W^2 V_{cd} V_{us})^2}{(g_W^2 V_{cs} V_{ud})^2} = \dfrac{\sin^4\theta_C}{\cos^4\theta_C} \sim 0.0028$

Measure $0.0038 \pm 0.0008$

$D^0 \rightarrow K^+\pi^-$ is Doubly Cabibbo suppressed (two Cabibbo suppressed vertices)

# Summary of the Weak CC Vertex

All weak charged current quark interactions can be described by the $W$ boson propagator and the weak vertex:




The Standard Model
Weak CC Quark Vertex

$+$ antiparticles

- $W^{\pm}$ bosons always change quark flavour
- $W^{\pm}$ prefers to couple to quarks in the same generation, but quark mixing means that cross-generation coupling can occur.
  Crossing two generations is less probable than one.

  $W$-lepton coupling constant $\longrightarrow g_W$
  $W$-quark coupling constant $\longrightarrow g_W V_{\mathrm{CKM}}$

# Summary

**Weak interaction (charged current)**

- Weak force mediated by massive $W$ bosons $\quad m_W = 80.385 \pm 0.015 \text{ GeV}$
- Weak force intrinsically stronger than EM interaction

$$\alpha_W \sim \frac{1}{30} \qquad \alpha_{EM} \sim \frac{1}{137}$$

- Universal coupling to quarks and leptons, but...
- Quarks take part in the interaction as mixtures of the mass eigenstates
- Parity & C-symmetry can be violated due to the helicity structure of the interaction
- Strength of the weak interaction given by

$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

from $\mu$ decay.

Problem Sheet: q.23-25

Up next... Section 10: Electroweak Unification