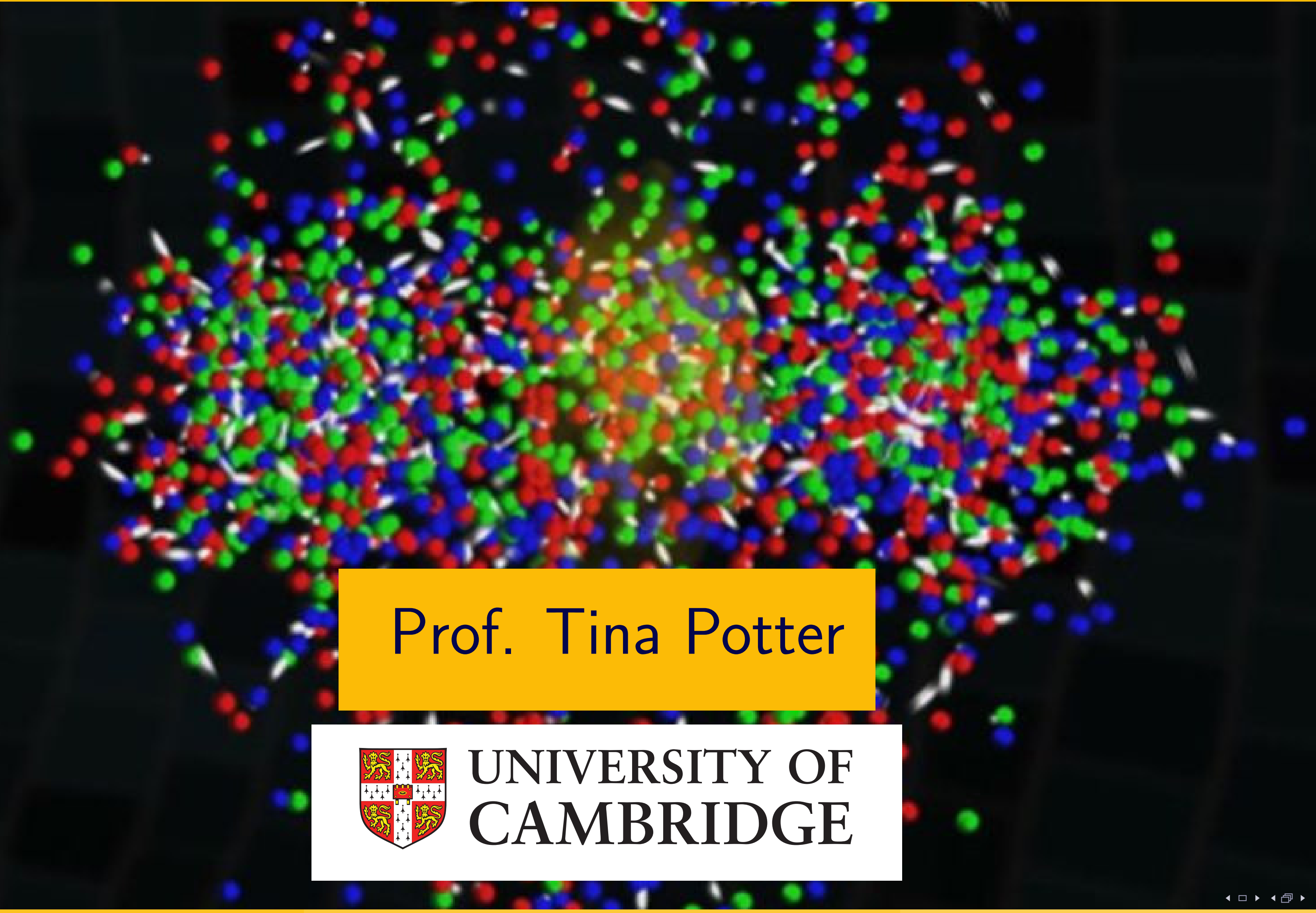
7. QCD

Particle and Nuclear Physics



Prof. Tina Potter

UNIVERSITY OF CAMBRIDGE

In this section…

- The strong vertex
- Colour, gluons and self-interactions
- QCD potential, confinement
- Hadronisation, jets
- Running of α_s
- Experimental tests of QCD

QCD

Quantum Electrodynamics is the quantum theory of the electromagnetic interaction.

- mediated by massless photons
- photon couples to electric charge
- strength of interaction: $\langle \psi_{\rm f} | \hat{H} | \psi_{\rm i} \rangle \propto \sqrt{\alpha}$ $\qquad \alpha = \frac{e^2}{4\pi} = \frac{1}{137}$

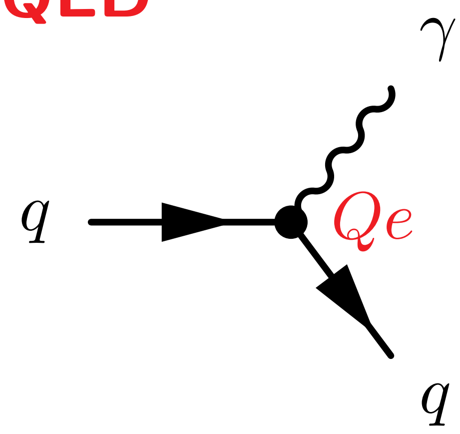
Quantum Chromodynamics is the quantum theory of the strong interaction.

- mediated by massless gluons
- gluon couples to "strong" charge
- only quarks have non-zero "strong" charge, therefore only quarks feel the strong interaction.
- strength of interaction: $\langle \psi_{\rm f} | \hat{H} | \psi_{\rm i} \rangle \propto \sqrt{\alpha_s}$ $\qquad \alpha_s = \frac{g_s^2}{4\pi} \sim 1$

The Strong Vertex

Basic QCD interaction looks like a stronger version of QED:

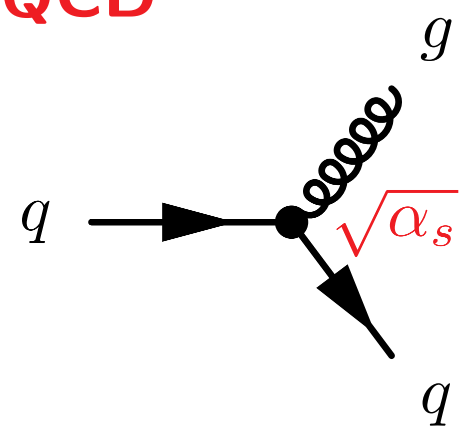
QED



$+$ antiquarks

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

QCD



$+$ antiquarks

$$\alpha_s = \frac{g_s^2}{4\pi} \sim 1$$

- The coupling of the gluon, g_s, is to the "strong" charge.
- Energy, momentum, angular momentum and charge always conserved.
- QCD vertex never changes quark flavour
- QCD vertex always conserves parity

Colour

QED:

- Charge of QED is electric charge, a conserved quantum number

QCD:

- Charge of QCD is called “ colour ”
- colour is a conserved quantum number with 3 values labelled red, green and blue.

Quarks carry	colour	r	b	g
Antiquarks carry	anti- colour	$\bar{r}$	$\bar{b}$	$\bar{g}$

- Colorless particles either have
    - no colour at all e.g. leptons, γ, W, Z and do not interact via the strong interaction
    - or equal parts r, b, g e.g. meson $q\bar{q}$ with $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$, baryon qqq with rgb
- gluons do not have equal parts r, b, g, so carry colour (e.g. $r\bar{r}$, see later)

QCD as a gauge theory

- Recall QED was invariant under gauge symmetry

$$\psi \rightarrow \psi' = \mathrm{e}^{\mathrm{i}q\alpha(\vec{r},t)}\psi$$

- The equivalent symmetry for QCD is invariance under *(non-examinable)*

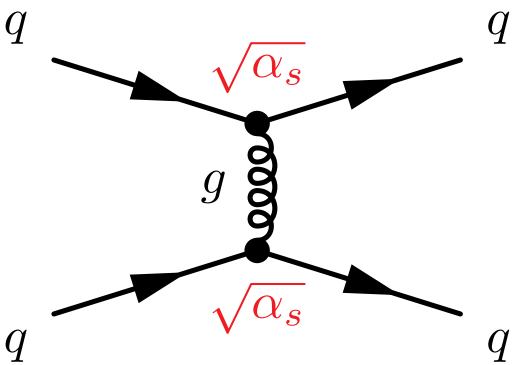
$$\psi \rightarrow \psi' = \mathrm{e}^{\mathrm{i}g\vec{\lambda}.\vec{\Lambda}(\vec{r},t)}\psi$$

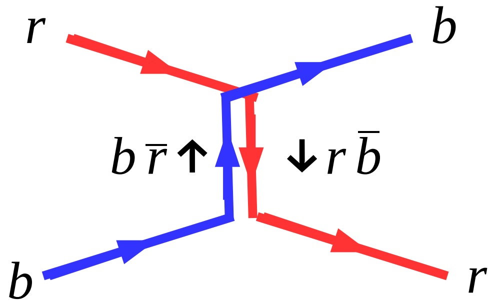
  an "SU(3)" transformation (λ are eight 3x3 matrices).
- Operates on the colour state of the quark field – a "rotation" of the colour state which can be different at each point of space and time.
- Invariance under SU(3) transformations $\rightarrow$ eight massless gauge bosons, gluons (eight in this case). Gluon couplings are well specified.
- Gluons also have self-couplings, i.e. they carry colour themselves...

Gluons

Gluons are massless spin-1 bosons, which carry the colour quantum number (unlike γ in QED which is charge neutral).

Consider a red quark scattering off a blue quark. Colour is exchanged, but always conserved (overall and at each vertex).





Expect 9 gluons (3x3): $r\bar{b}$ $r\bar{g}$ $g\bar{r}$ $g\bar{b}$ $b\bar{g}$ $b\bar{r}$ $r\bar{r}$ $b\bar{b}$ $g\bar{g}$

However: Real gluons are orthogonal linear combinations of the above states. The combination $\frac{1}{\sqrt{3}}(r\bar{r}+b\bar{b}+g\bar{g})$ is colourless and does not participate in the strong interaction. $\Rightarrow$ 8 coloured gluons

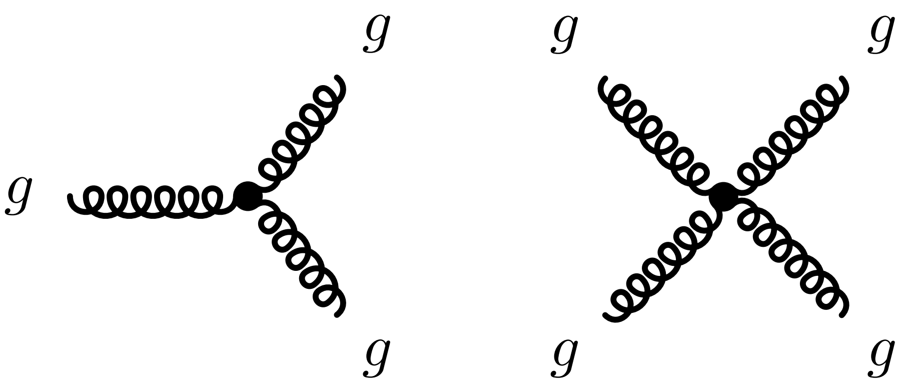
Conventionally chosen to be (all orthogonal):

$$r\bar{b} \quad r\bar{g} \quad g\bar{r} \quad g\bar{b} \quad b\bar{g} \quad b\bar{r} \quad \frac{1}{\sqrt{2}}(r\bar{r}-b\bar{b}) \quad \frac{1}{\sqrt{6}}(r\bar{r}+b\bar{b}-2g\bar{g})$$

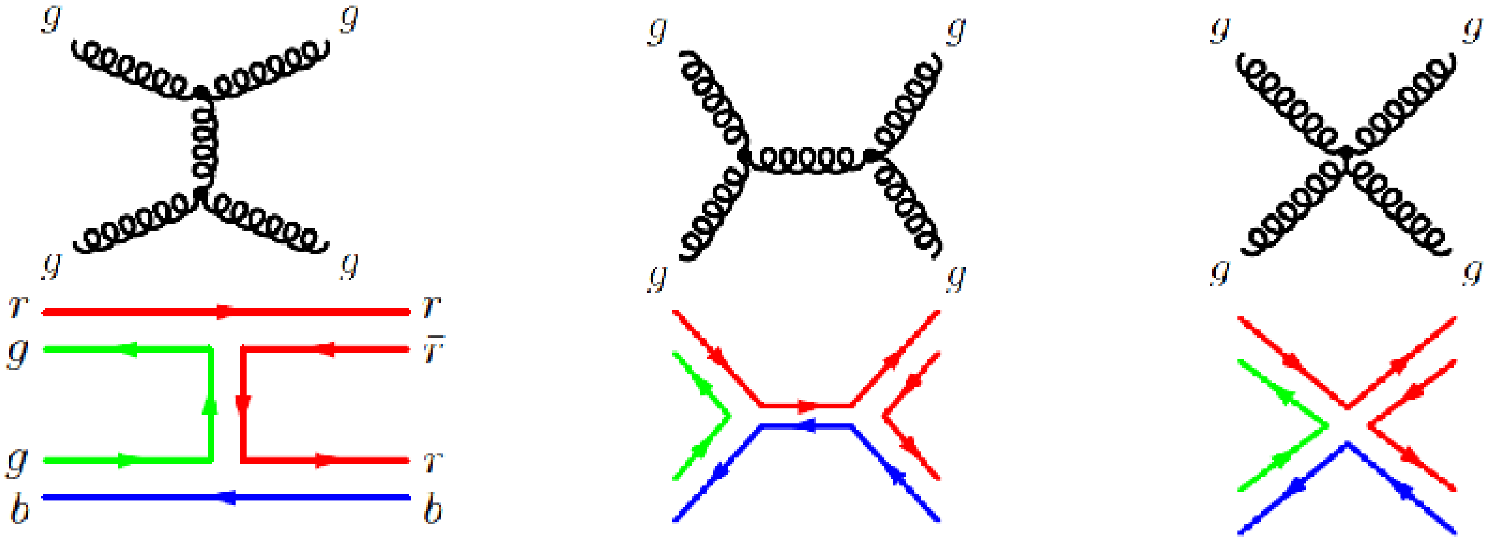
Gluon Self-Interactions

QCD looks like a stronger version of QED. However, there is one big difference and that is gluons carry colour charge.

⇒ Gluons can interact with other gluons



Example: Gluon-gluon scattering $gg \to gg$



Same colour flow in each case: $r\bar{g} + g\bar{b} \to r\bar{r} + r\bar{b}$

QCD Potential

QED Potential:

$$V_{\mathrm{QED}} = -\frac{\alpha}{r}$$

QCD Potential:

$$V_{\mathrm{QCD}} = -C\frac{\alpha_s}{r}$$

At short distances, QCD potential looks similar, apart from the "colour factor" C.

For $q\bar{q}$ in a colourless state in a meson, $C = 4/3$
For qq in a colourless state in baryon, $C = 2/3$

Note: the colour factor C arises because more than one gluon can participate in the process $q \to qg$. Obtain colour factor from averaging over initial colour states and summing over final/intermediate colour states.

Confinement

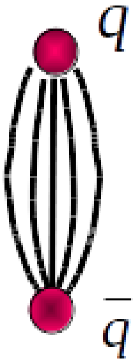
Never observe single free quarks or gluons

- Quarks are always confined within hadrons
- This is a consequence of the strong interaction of gluons.

Qualitatively, compare QCD with QED:

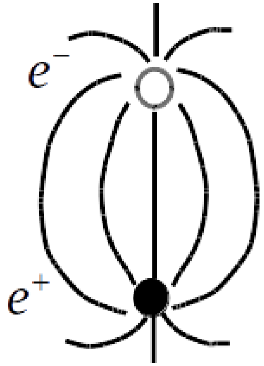
QCD

Colour field



QED

Electric field



Self interactions of the gluons squeezes the lines of force into a narrow tube or string. The string has a "tension" and as the quarks separate the string stores potential energy.

Energy stored per unit length in field $\sim$ constant $V(r) \propto r$

Energy required to separate two quarks is infinite. Quarks always come in combinations with zero net colour charge $\Rightarrow$ confinement.

How Strong is Strong?

QCD potential between quark and antiquark has two components:

- Short range, Coulomb-like term: $-\frac{4}{3}\frac{\alpha_s}{r}$
- Long range, linear term: $+kr$

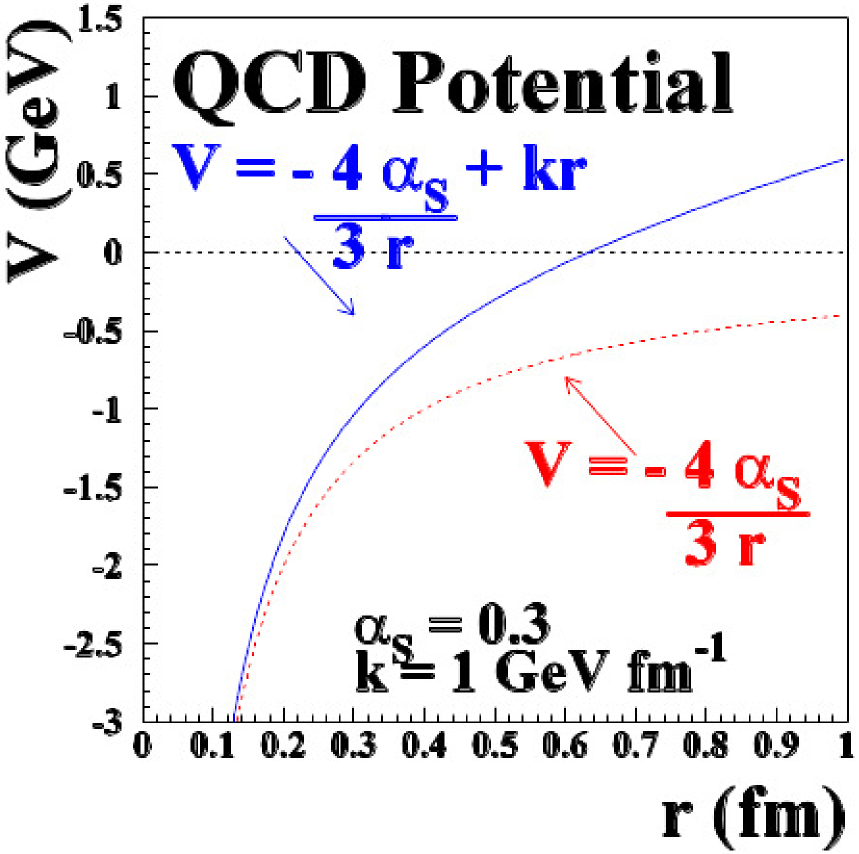
$$V_{\mathrm{QCD}} = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

with $k \sim 1\ \mathrm{GeV/fm}$

$$F = -\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{4}{3}\frac{\alpha_s}{r^2} + k$$

at large r

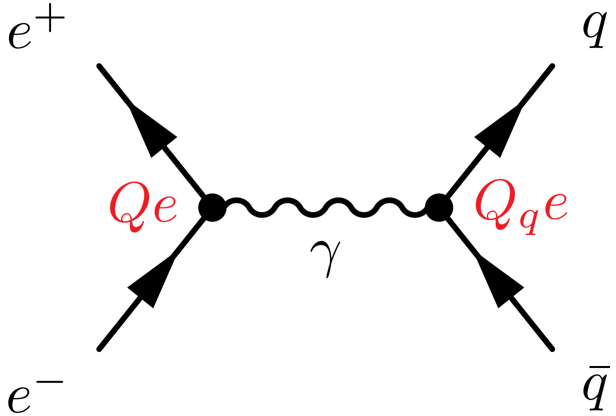
$$F = k \sim \frac{1.6 \times 10^{-10}}{10^{-15}}\mathrm{N} \; = \; 160,000\,\mathrm{N}$$

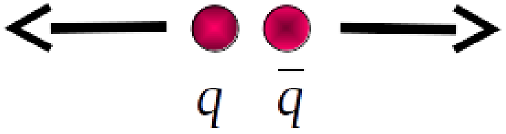


Equivalent to weight of ~150 people

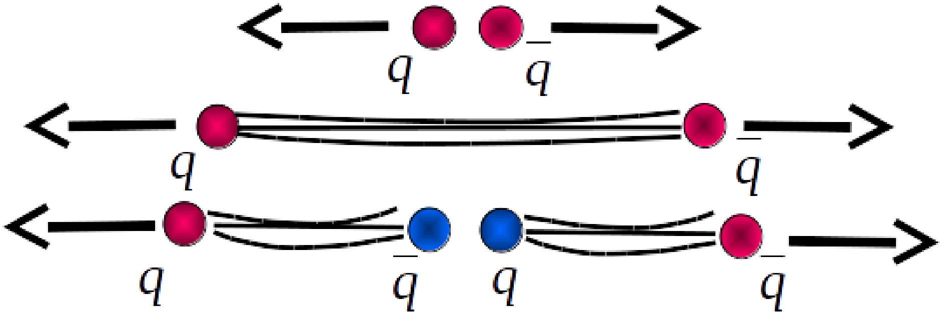
Jets

Consider the $q\bar{q}$ pair produced in $e^+e^- \to q\bar{q}$

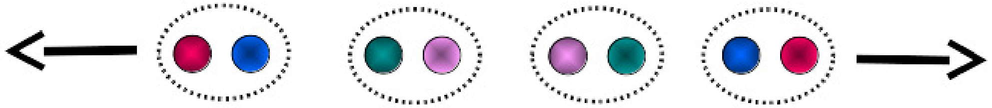




As the quarks separate, the potential energy in the colour field ("string") starts to increase linearly with separation. When the energy stored exceeds $2m_q$, new $q\bar{q}$ pairs can be created.

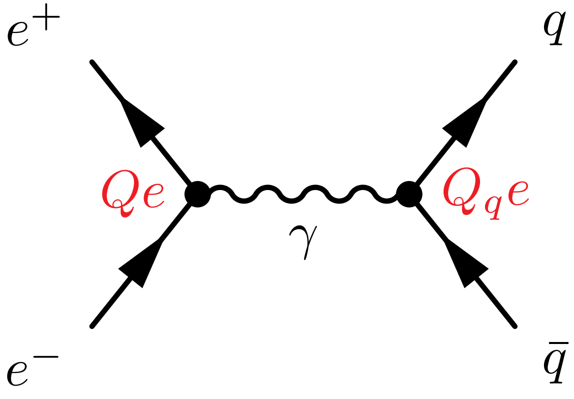


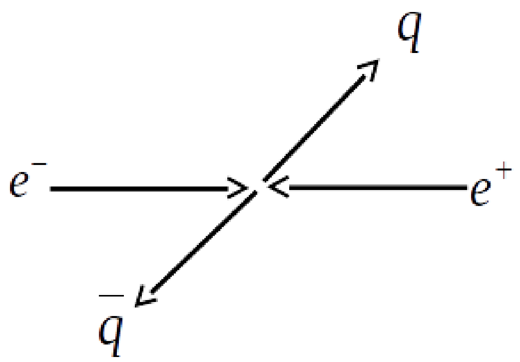
As energy decreases, hadrons (mainly mesons) freeze out

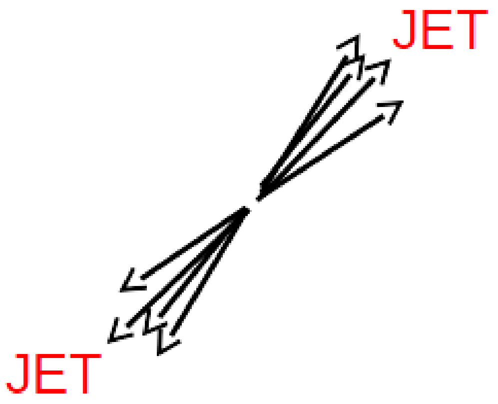


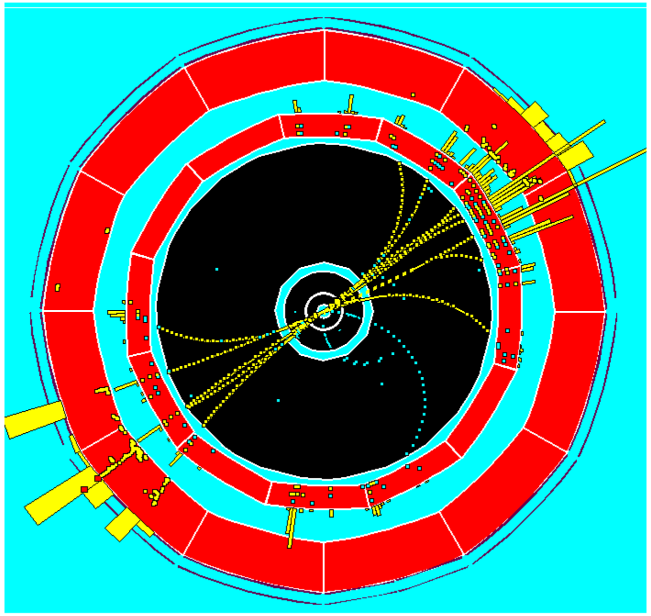
Jets

As quarks separate, more $q\bar{q}$ pairs are produced. This process is called hadronisation. Start out with quarks and end up with narrowly collimated jets of hadrons.









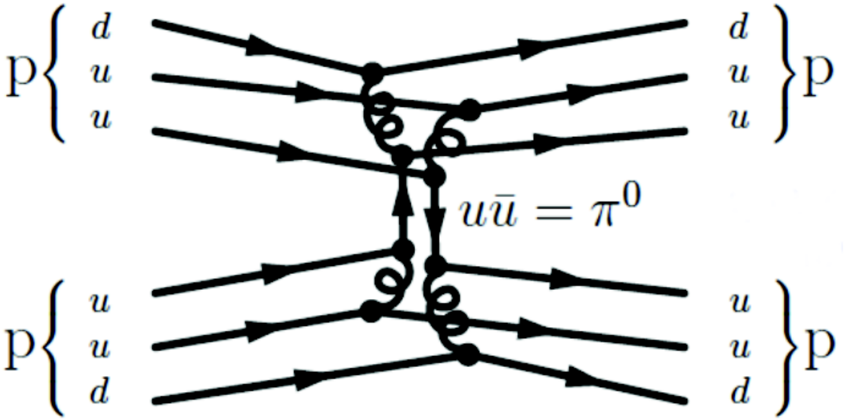
Typical $e^+e^- \rightarrow q\bar{q}$ event

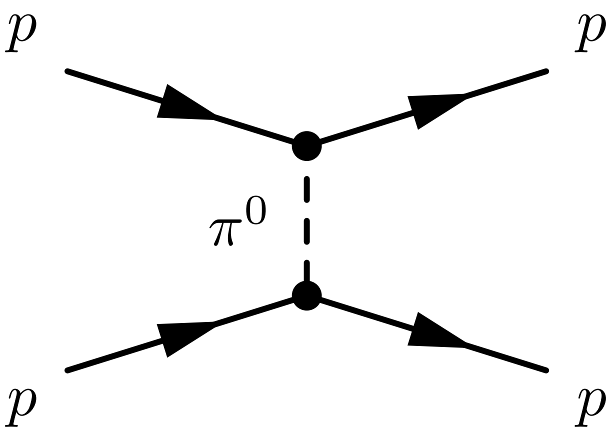
The hadrons in a quark(antiquark) jet follow the direction of the original quark(antiquark). Consequently, $e^+e^- \rightarrow q\bar{q}$ is observed as a pair of back-to-back jets.

Nucleon-Nucleon Interactions

- Bound qqq states (e.g. protons and neutrons) are colourless (colour singlets)
- They can only emit and absorb another colour singlet state, i.e. not single gluons (conservation of colour charge).
- Interact by exchange of pions.

  Example: pp scattering (One possible diagram)



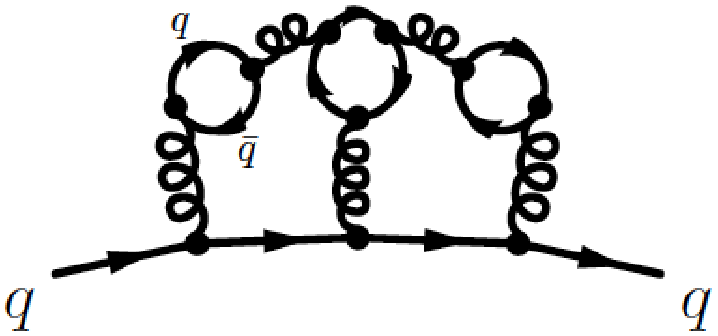


- Nuclear potential is Yukawa potential with $V(r) = -\frac{g^2}{4\pi}\frac{e^{-m_\pi r}}{r}$
- Short range force:

$$\mathrm{Range} = \frac{1}{m_\pi} = (0.140\ \mathrm{GeV})^{-1} = 7\ \mathrm{GeV}^{-1} = 7 \times (\hbar c)\,\mathrm{fm} = 1.4\,\mathrm{fm}$$

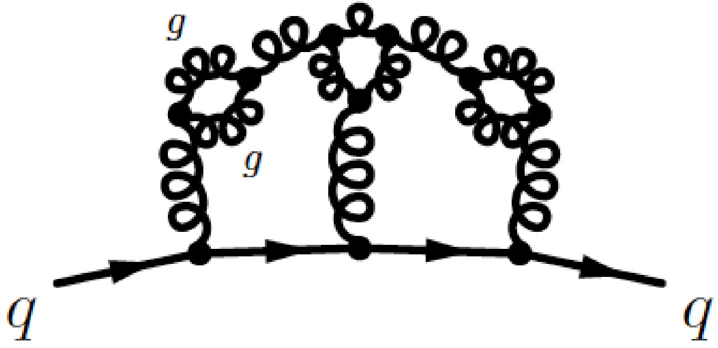
Running of α_s

- α_s specifies the strength of the strong interaction.
- But, just as in QED, α_s is not a constant. It "runs" (i.e. depends on energy).
- In QED, the bare electron charge is screened by a cloud of virtual electron-positron pairs.
- In QCD, a similar "colour screening" effect occurs.



In QCD, quantum fluctuations lead to a cloud of virtual $q\bar{q}$ pairs.

One of many (an infinite set) of such diagrams analogous to those for QED.



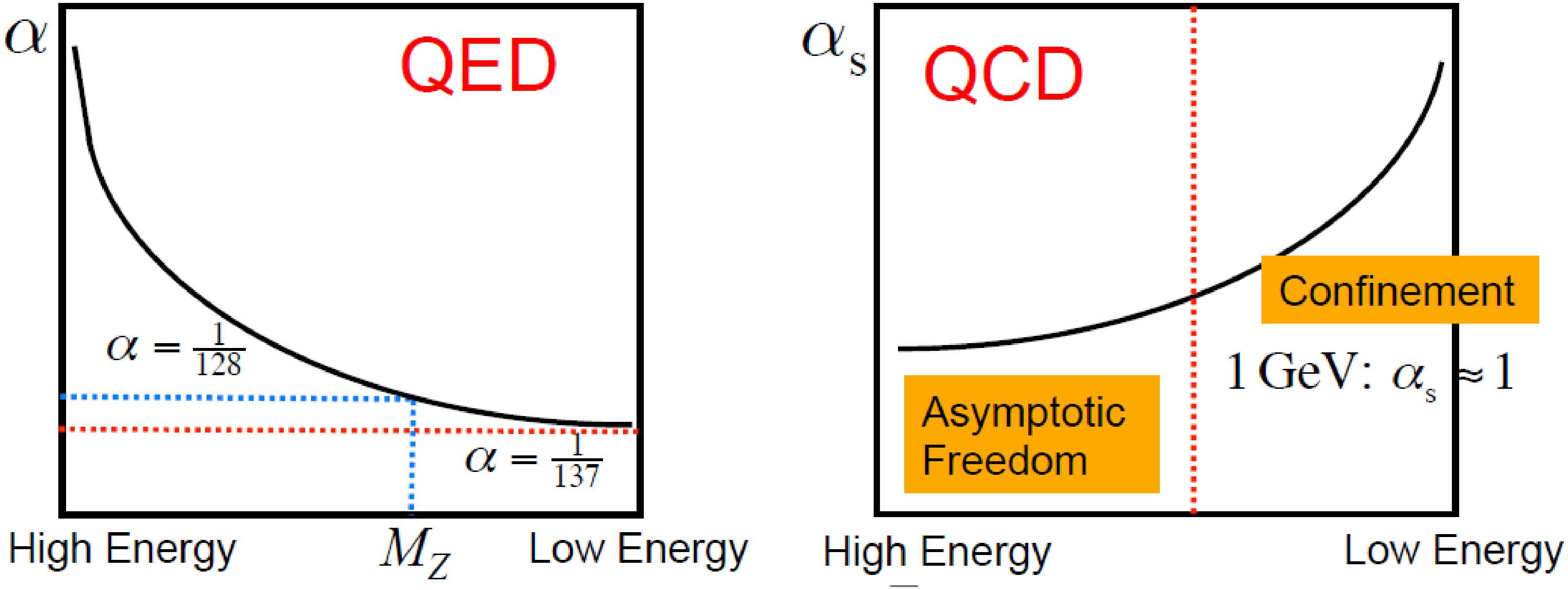
In QCD, the gluon self-interactions also lead to a cloud of virtual gluons.

One of many (an infinite set) of such diagrams.

No analogy in QED, photons do not carry the charge of the interaction.

Colour Anti-Screening

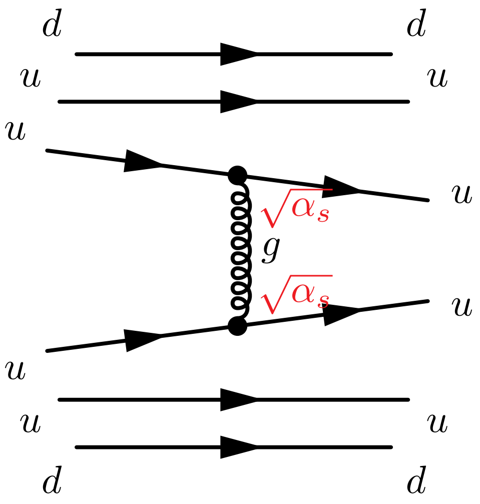
- Due to gluon self-interactions bare colour charge is screened by both virtual quarks and gluons.
- The cloud of virtual gluons carries colour charge and the effective colour charge decreases at smaller distances (high energy)!
- Hence, at low energies, α_s is large → cannot use perturbation theory.
- But at high energies, α_s is small. In this regime, can treat quarks as free particles and use perturbation theory → Asymptotic Freedom.

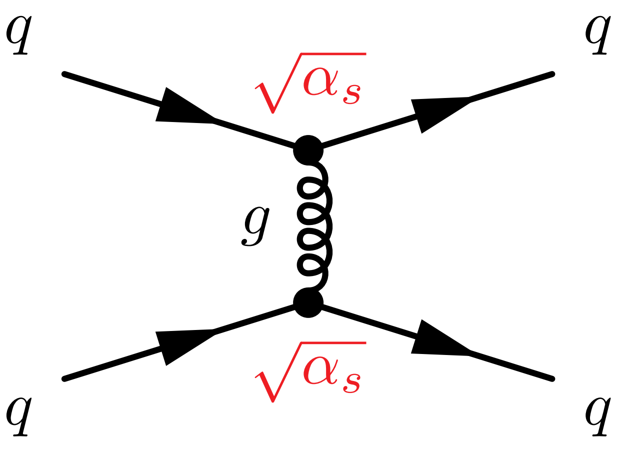


$$\sqrt{s} = 100\ \mathrm{GeV}, \quad \alpha_s = 0.12$$

Scattering in QCD

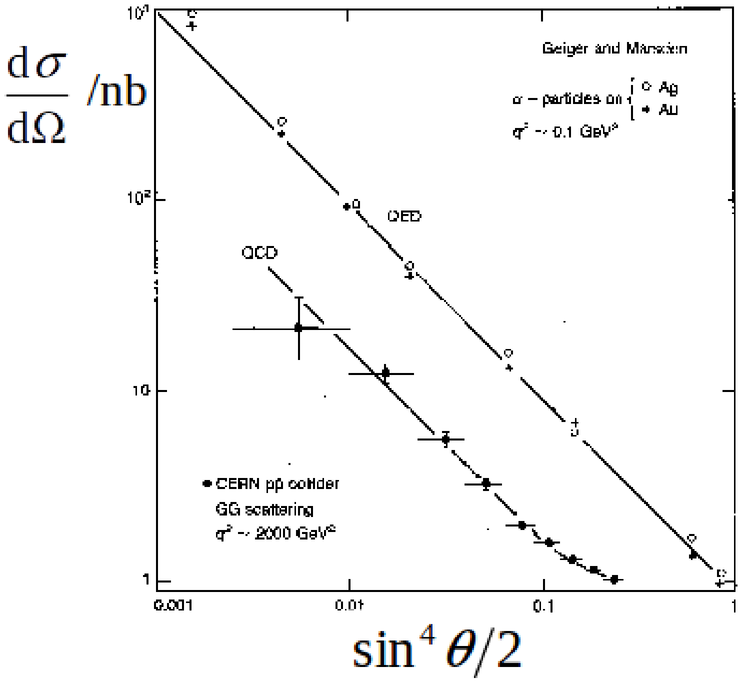
Example: High energy proton-proton scattering.





$$M \sim \frac{1}{q^2}\sqrt{\alpha_s}\sqrt{\alpha_s}$$

$$\Rightarrow \frac{d\sigma}{d\Omega} \sim \frac{(\alpha_s)^2}{\sin^4\theta/2}$$



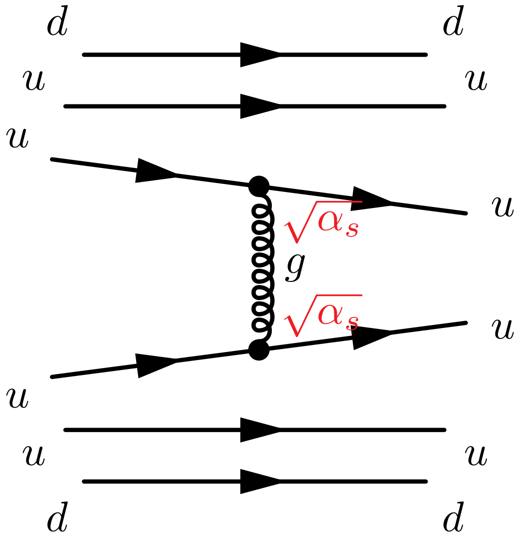
Upper points: Geiger and Marsden data (1911) for the elastic scattering of a particles from gold and silver foils.

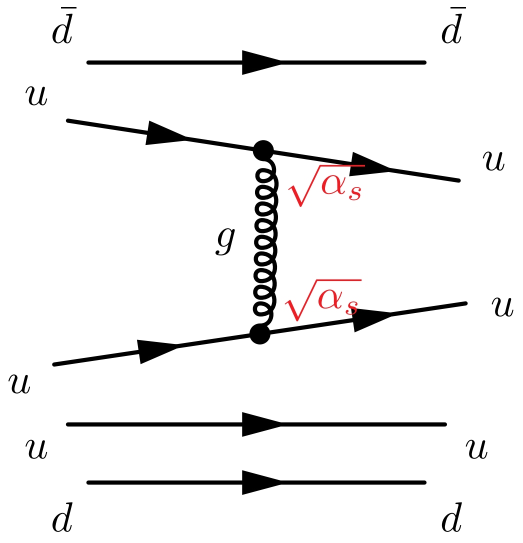
Lower points: angular distribution of quark jets observed in pp scattering at $q^2 = 2000\ \mathrm{GeV}^2$.

Both follow the Rutherford formula for elastic scattering.

Scattering in QCD

Example: *pp* vs $\pi^+ p$ scattering





Calculate ratio of $\sigma(pp)_{\rm total}$ to $\sigma(\pi^+ p)_{\rm total}$

QCD does not distinguish between quark flavours, only colour charge of quarks matters.

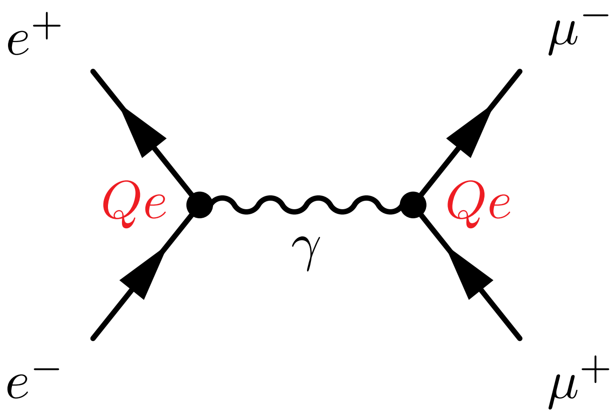
At high energy ($E \gg$ binding energy of quarks within hadrons), ratio of $\sigma(pp)_{\rm total}$ and $\sigma(\pi^+ p)_{\rm total}$ depends on number of possible quark-quark combinations.

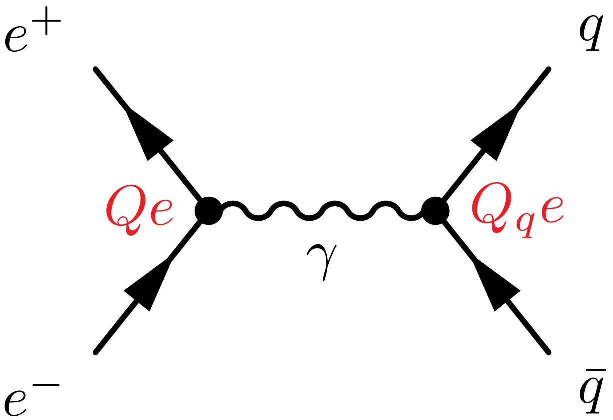
Predict: $\dfrac{\sigma(\pi p)}{\sigma(pp)} = \dfrac{2 \times 3}{3 \times 3} = \dfrac{2}{3}$ Experiment: $\dfrac{\sigma(\pi p)}{\sigma(pp)} = \dfrac{24\,\mathrm{mb}}{38\,\mathrm{mb}} \sim \dfrac{2}{3}$

QCD in e^+e^- Annihilation

e^+e^- annihilation at high energies provides direct experimental evidence for colour and for gluons.

Start by comparing the cross-sections for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to q\bar{q}$





$$M \sim \frac{1}{q^2}\sqrt{\alpha}\sqrt{\alpha}$$

$$M \sim \frac{1}{q^2}Q_q\sqrt{\alpha}\sqrt{\alpha}$$

$$\Rightarrow \sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

If we neglect the mass of the final state quarks/muons then the only difference is the charge of the final state particles:

$$Q_\mu = -1 \qquad Q_q = +\frac{2}{3}, \quad -\frac{1}{3}$$

Evidence for Colour

Consider the ratio $$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

For a single quark of a given colour $R = Q_q^2$

However, we measure $\sigma(e^+e^- \to \text{ hadrons})$ not just $\sigma(e^+e^- \to u\bar{u})$.
A jet from a u-quark looks just like a jet from a d-quark etc.
Thus, we need to sum over all available flavours (u, d, c, s, t, b) and colours (r, g, b):

$$R = 3\sum_i Q_i^2 \qquad \text{(3 colours)}$$

where the sum is over all quark flavours (i) that are kinematically accessible at centre-of-mass energy, $\sqrt{s}$, of the collider.

Evidence for Colour

Expect to see steps in R as energy is increased.

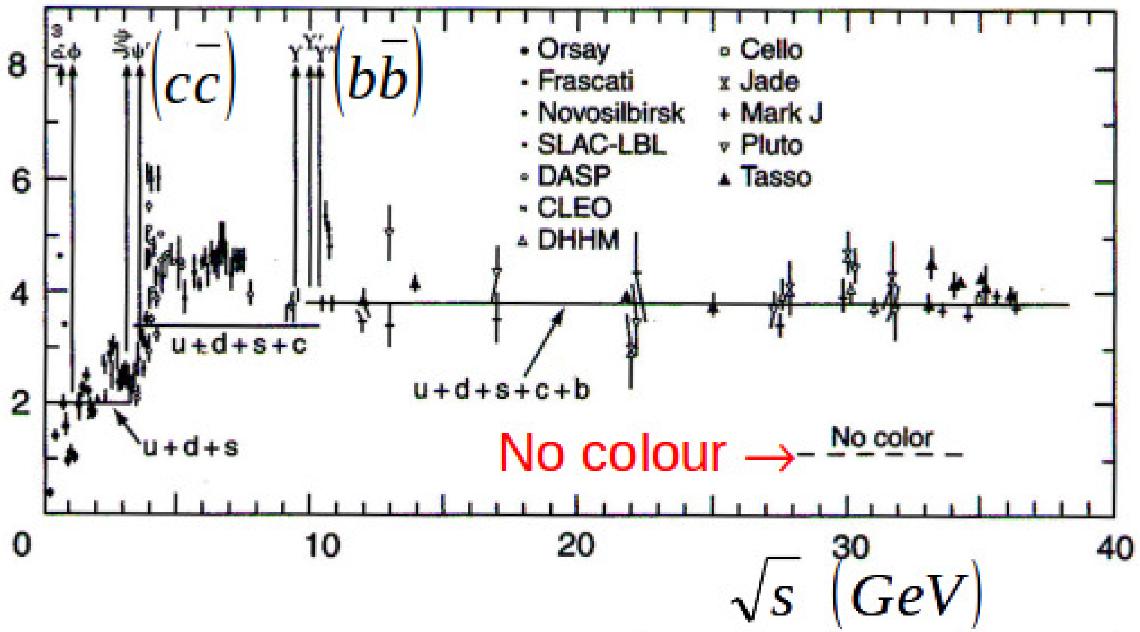
$$R = 3\sum_i Q_i^2$$

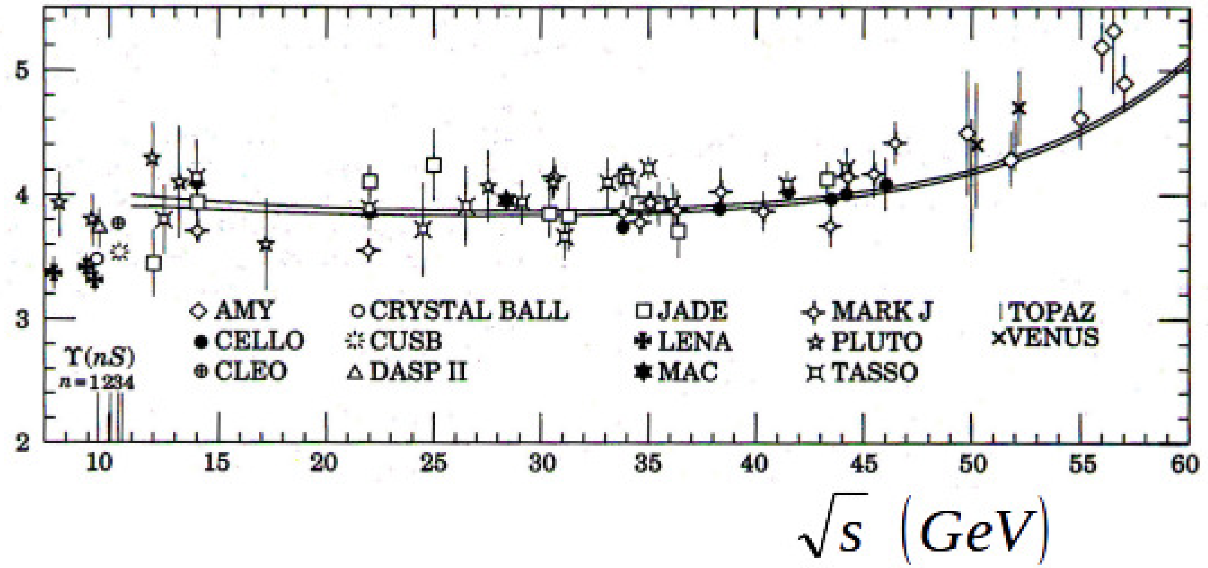
Energy	Expected ratio R
$\sqrt{s} > 2m_s, \quad \sim 1\ \mathrm{GeV}$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}\right) = 2$ uds
$\sqrt{s} > 2m_c, \quad \sim 4\ \mathrm{GeV}$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}\right) = 3\frac{1}{3}$ $udsc$
$\sqrt{s} > 2m_b, \quad \sim 10\ \mathrm{GeV}$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}\right) = 3\frac{2}{3}$ $udscb$
$\sqrt{s} > 2m_t, \quad \sim 350\ \mathrm{GeV}$	$3\left(\frac{4}{9}+\frac{1}{9}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}+\frac{4}{9}\right) = 5$ $udscbt$

Evidence for Colour

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- R increases in steps with $\sqrt{s}$
  Strong evidence for colour
- $\sqrt{s} < 11\ \mathrm{GeV}$ region observe bound state resonances: charmonium $(c\bar{c})$ and bottomonium $(b\bar{b})$
- $\sqrt{s} > 50\ \mathrm{GeV}$ region observe low edge of Z resonance $\Gamma \sim 2.5\ \mathrm{GeV}$.





Experimental Evidence for Colour

- $$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- **The existence of Ω^- (sss)**

  The Ω^- (sss) is a ($L = 0$) spin-3/2 baryon consisting of three s-quarks.

  The wavefunction: $\psi = s\uparrow s\uparrow s\uparrow$

  is symmetric under particle interchange. However, quarks are fermions, therefore require an anti-symmetric wave-function, i.e. need another degree of freedom, namely colour , whose wavefunction must be antisymmetric.

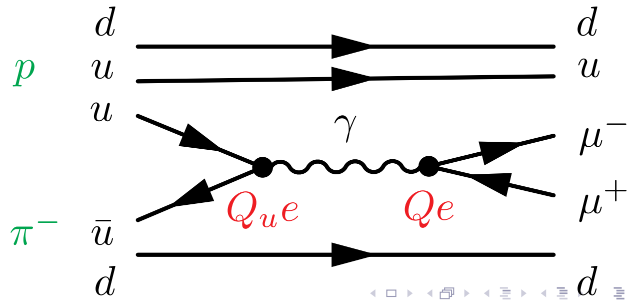
  $$\psi = (s\uparrow s\uparrow s\uparrow)\psi_{\text{colour}}$$

  $$\psi_{\text{colour}} = \frac{1}{\sqrt{6}}(rgb + gbr + brg - grb - rbg - bgr)$$

  i.e. need to introduce a new quantum number (colour) to distinguish the three quarks in Ω^- – avoids violation of Pauli's Exclusion Principle.

- **Drell-Yan process**

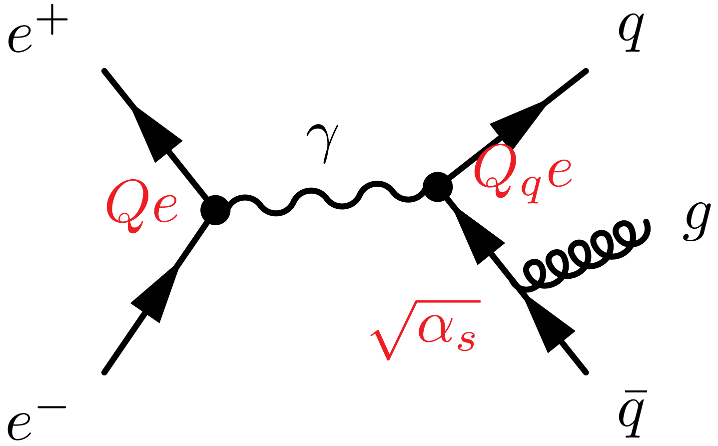
Need colour to explain cross-section; colours of the annihilating quarks must match to form a virtual photon. Cross-section suppressed by a factor $1/N_{\text{colour}}$.



Evidence for Gluons

In QED, electrons can radiate photons. In QCD, quarks can radiate gluons.

Example: $e^-e^+ \to q\bar{q}g$



$$M \sim \frac{Q_q}{q^2}\sqrt{\alpha}\sqrt{\alpha}\sqrt{\alpha_s}$$

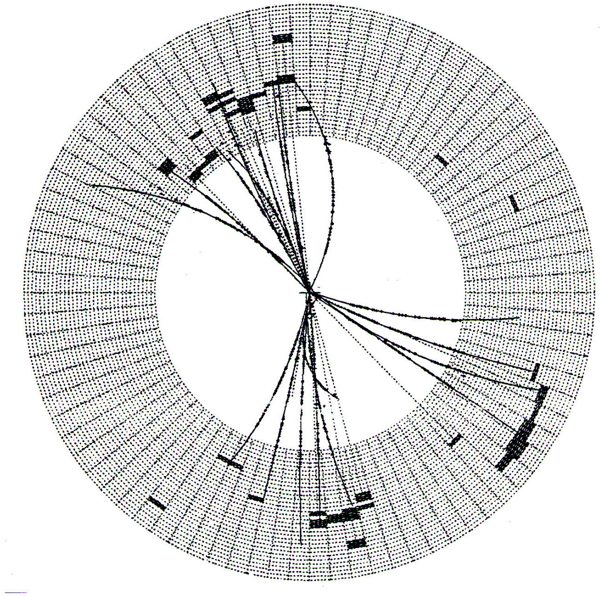
Giving an extra factor of $\sqrt{\alpha_s}$ in the matrix element, i.e. an extra factor of α_s in the cross-section.

In QED we can detect the photons. In QCD, we never see free gluons due to confinement.

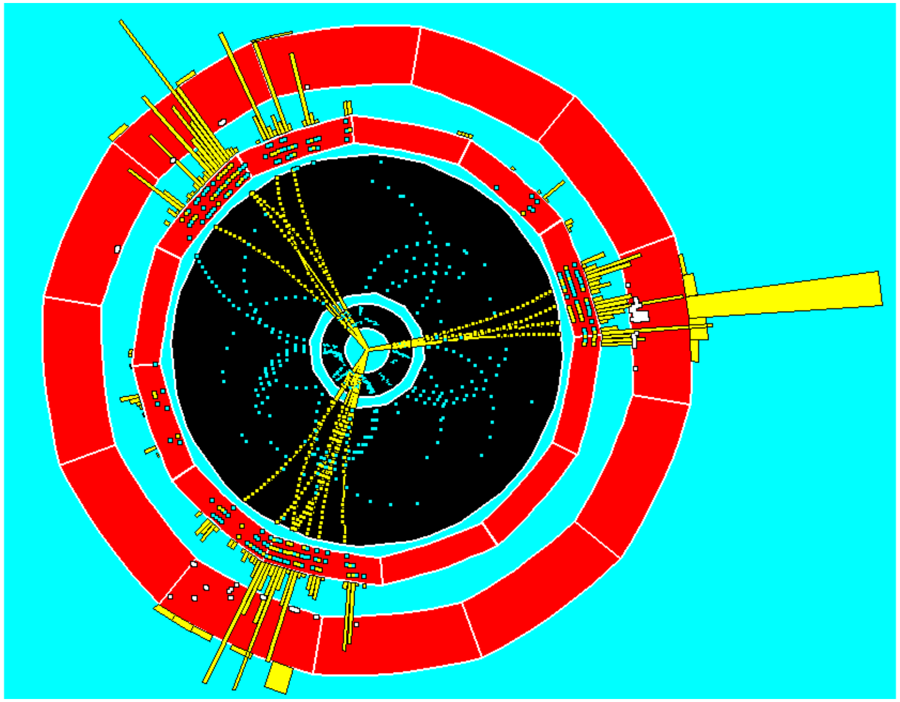
Experimentally, detect gluons as an additional jet: 3-jet events.
– Angular distribution of gluon jet depends on gluon spin.

Evidence for Gluons

JADE event $\sqrt{s} = 31\ \mathrm{GeV}$
First direct evidence of gluons (1978)

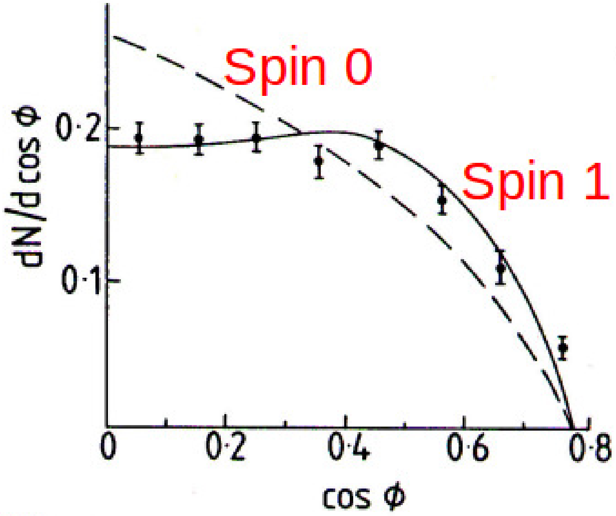


ALEPH event $\sqrt{s} = 91\ \mathrm{GeV}$ (1990)



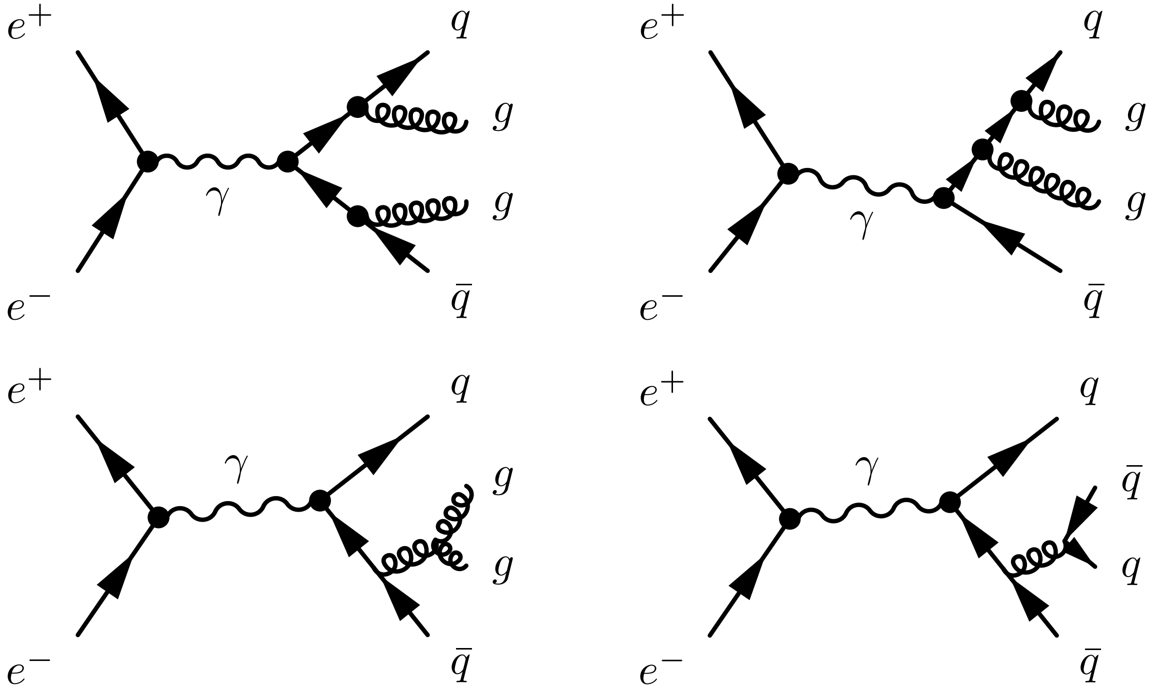
Distribution of the angle, ϕ , between the highest energy jet (assumed to be one of the quarks) relative to the flight direction of the other two (in their cm frame). ϕ distribution depends on the spin of the gluon.

$\Rightarrow$ Gluon is spin 1



Evidence for Gluon Self-Interactions

Direct evidence for the existence of the gluon self-interactions comes from 4-jet events:

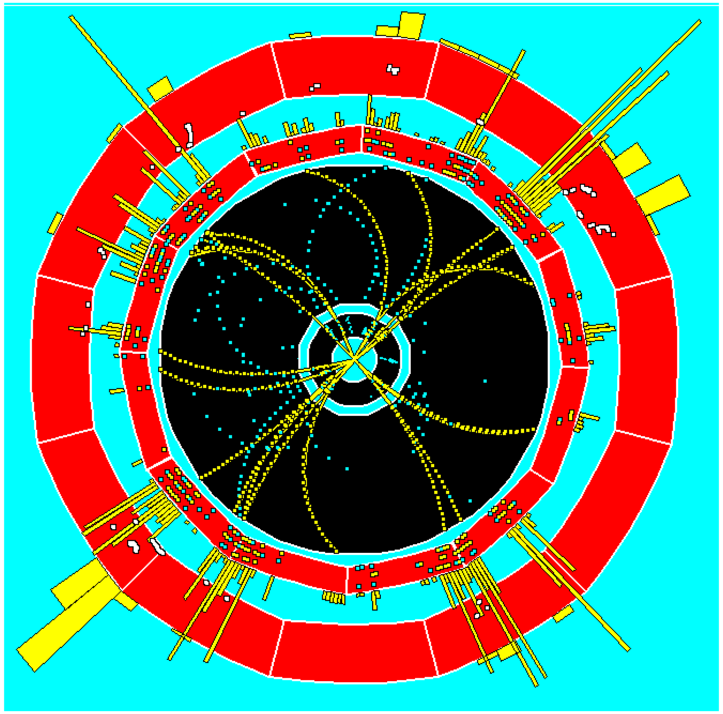


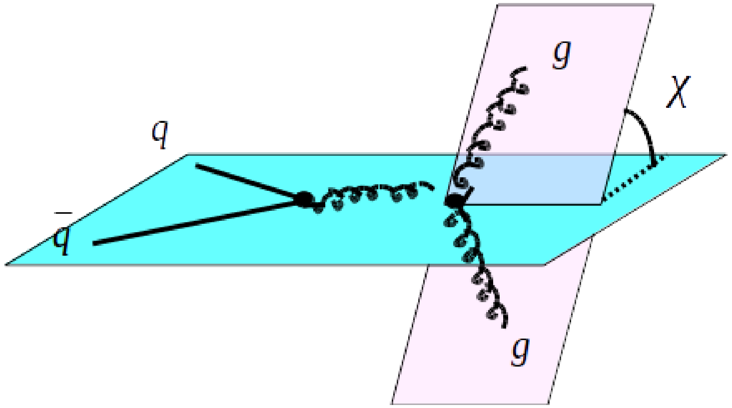
The angular distribution of jets is sensitive to existence of triple gluon vertex (lower left diagram)

- qqg vertex consists of two spin $1/2$ quarks and one spin 1 gluon
- ggg vertex consists of three spin-1 gluons
  - $\Rightarrow$ Different angular distribution.

Evidence for Gluon Self-Interactions

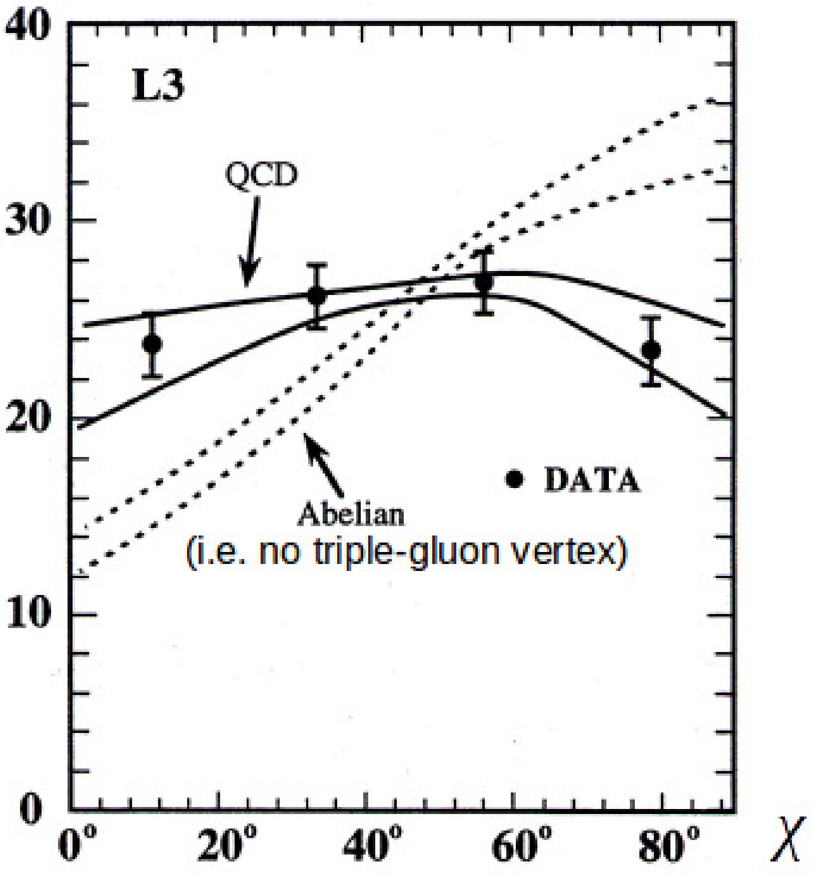
ALEPH 4-jet event





Experimental method:

- Define the two lowest energy jets as the gluons. (Gluon jets are more likely to be lower energy than quark jets).
- Measure angle χ between the plane containing the “quark” jets and the plane containing the “gluon” jets.



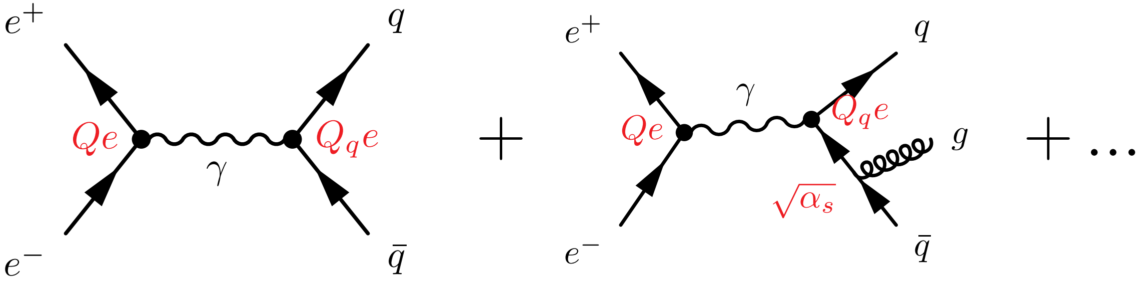
Gluon self-interactions are required to describe the experimental data.

Measurements of α_s

α_s can be measured in many ways.
The cleanest is from the ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

In practise, measure



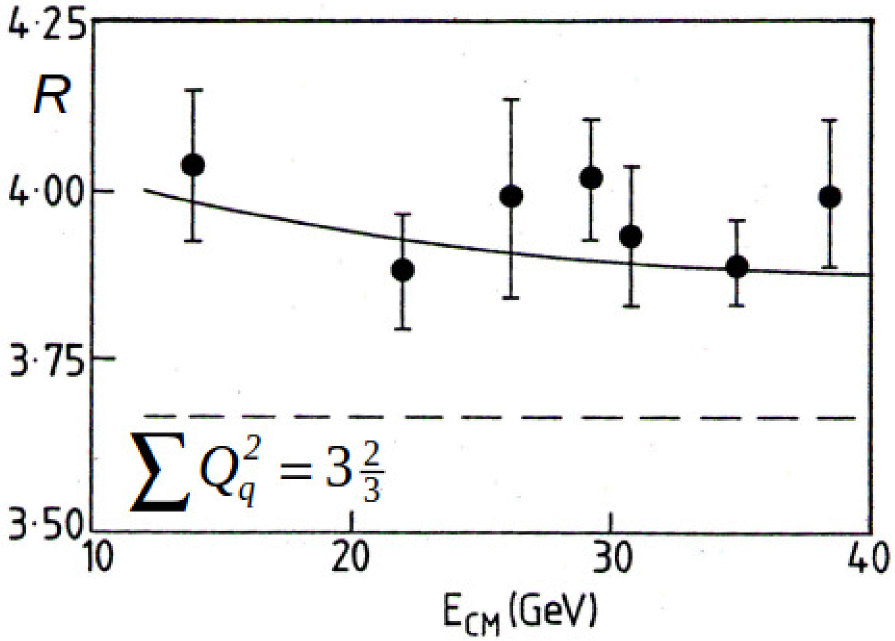
i.e. don't distinguish between 2 and 3 jets

When gluon radiation is included:

$$R = 3 \sum Q_q^2 \left(1 + \frac{\alpha_s}{\pi}\right)$$

Therefore, $$\left(1 + \frac{\alpha_s}{\pi}\right) \sim \frac{3.9}{3.66}$$

$$\alpha_s(q^2 = 25^2) \sim 0.2$$

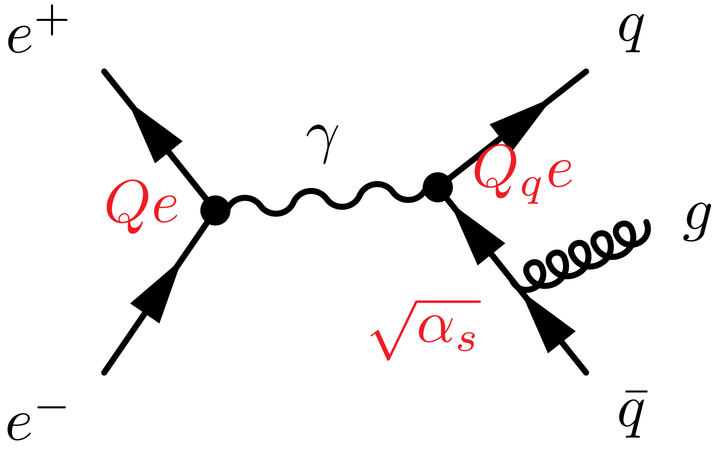


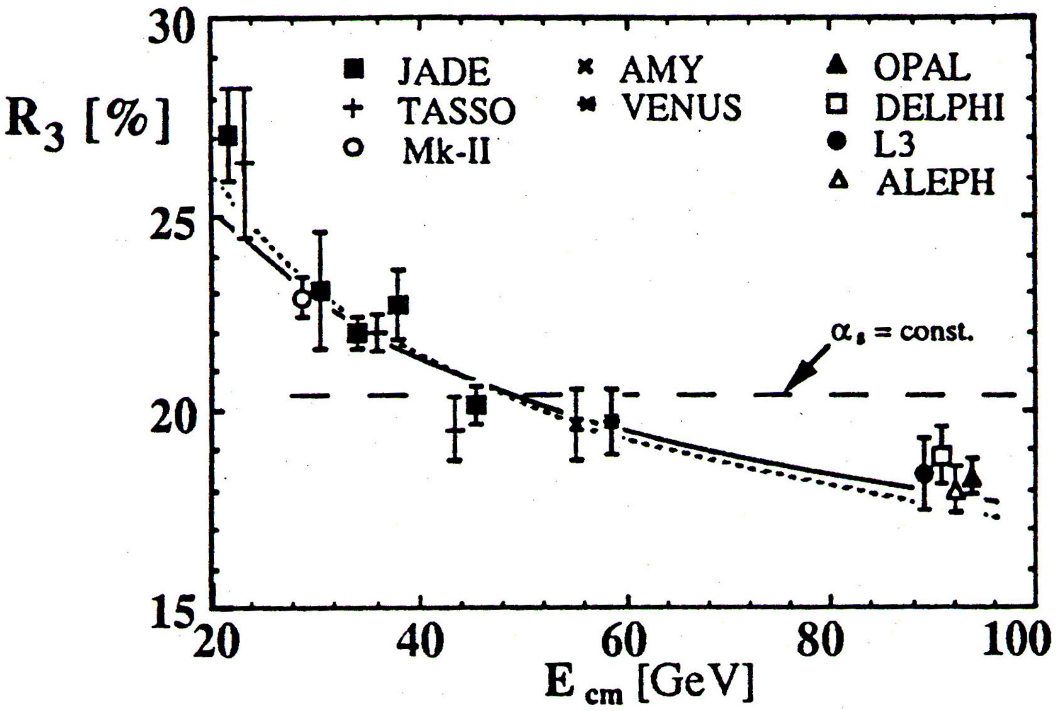
Measurements of α_s

Many other ways to measure α_s

Example: 3-jet rate $e^+e^- \to q\bar{q}g$

$$R_3 = \frac{\sigma(e^+e^- \to 3 \text{ jets})}{\sigma(e^+e^- \to 2 \text{ jets})} \propto \alpha_s$$



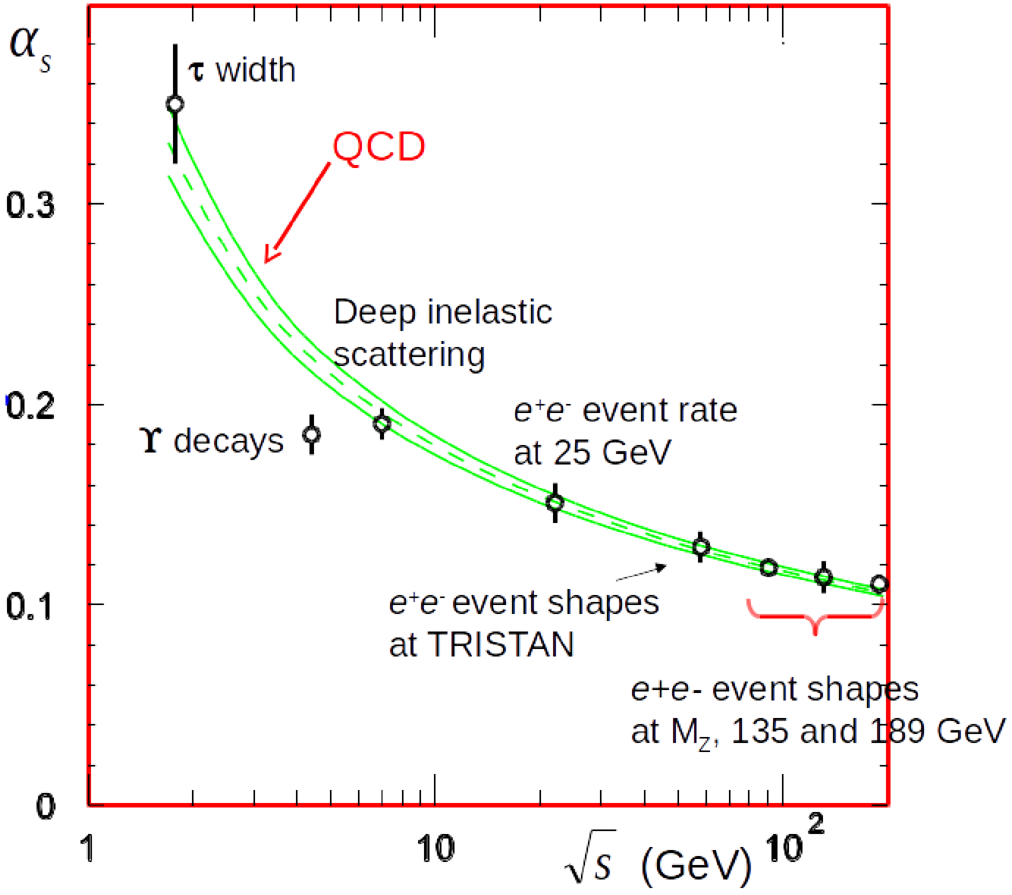


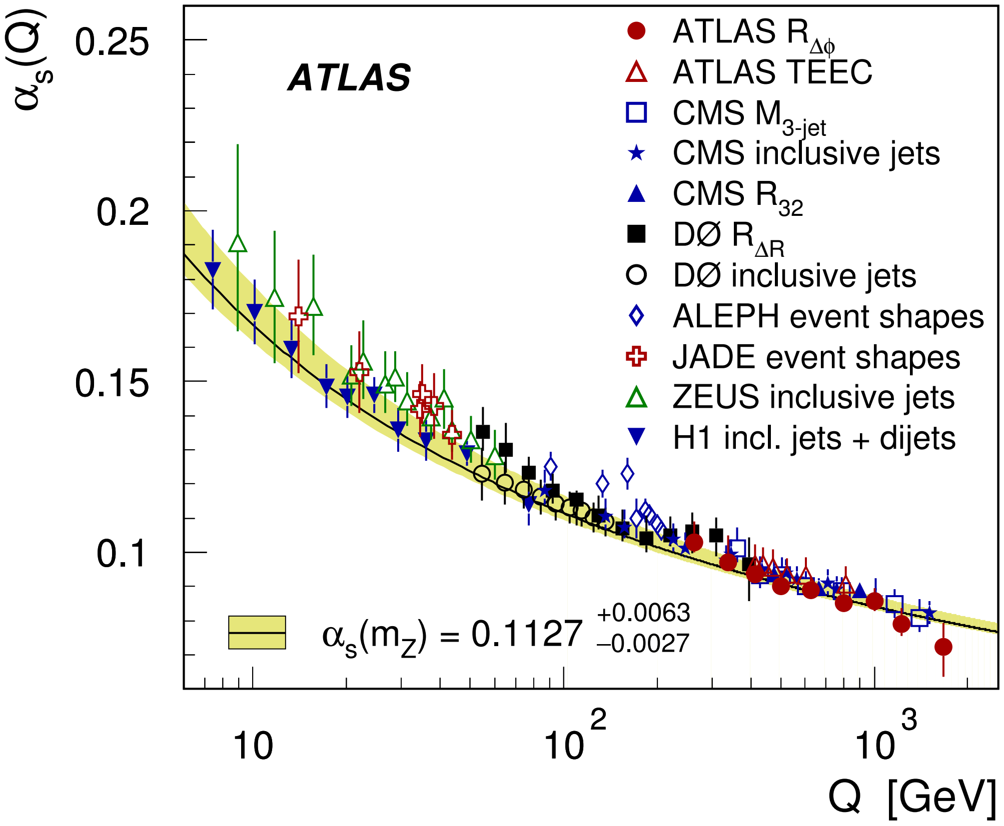
α_s decreases with energy

α_s runs!

in accordance with QCD

Observed running of α_s





Summary

- QCD is a gauge theory, similar to QED, based on SU(3) symmetry
- Gluons are vector gauge bosons, which couple to (three types of) colour charge (r, b, g)
- Gluons themselves carry colour charge – hence they have self-interactions (unlike QED).
- Leads to running of α_s, in the opposite sense to QED. Force is weaker at high energies ("asymptotic freedom") and very strong at low energies.
- Quarks and gluons are confined. Seen as hadrons and jets of hadrons.
- Tests of QCD
  - Evidence for colour
  - Existence of gluons, test of their spin and self-interactions
  - Measurement of α_s and observation that it runs.

Problem Sheet: q.15-16

Up next... Section 8: Quark Model of Hadrons