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The influence of the experimental methodology on the QED theoretical uncertainties on the measurement of M_{W} at LEP

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Abstract: Previous studies of the QED systematic uncertainties on the LEP measurement of the W-boson mass, M_{W}, have used idealized event selections and fitting procedures. In this paper, the Monte Carlo tandem of KoralW and YFSWW is used to investigate how the full experimental M_{W} extraction procedure affects these estimates. It is found that the kinematic fitting used in the experimental determination of M_{W} enhances the sensitivity to QED corrections involving real photon production. It is concluded that the previous estimates of the QED theoretical uncertainty on the LEP2 M_{W} measurement may be too small. A simple procedure for approximating the effect of the kinematic fit at the level of the generated four fermions is proposed. This procedure would allow previous theoretical studies to be repeated using a much closer approximation of the real experimental mass extraction method. Finally, the possibility of setting experimental limits on $\mathcal{O}(\alpha)$ theoretical uncertainties using identified $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+\mathrm{W}^-\gamma$ events is discussed.

Keywords: Standard Model, Electromagnetic Processes and Properties.

Contents

1. Introduction

It is anticipated that the final experimental error on the mass of the W boson, M_{W}, from LEP 2 will be approximately 35 MeV. A potential source of theoretical uncertainty is the treatment of QED and electroweak corrections in the Monte Carlo programs used to simulate the process $\mathrm{e^+e^-} \to 4f(\gamma)$. In recent years $\mathcal{O}(\alpha)$ electroweak corrections have been implemented in the YFSWW [1] and RacoonWW [2] programs. The current estimate of the theoretical systematic uncertainty on the LEP measurement of M_{W} due to these predominantly QED corrections is less than 5 MeV [3]. However, as pointed out by the authors, this estimate is based on an idealized M_{W} extraction procedure. Indeed, the authors suggest that the study should be repeated using the full experimental fitting procedure.

In this paper the concurrent Monte Carlo combination [4] of KoralW [5] and YFSWW [1] (referred to as KandY) is used to study the sensitivity of the full experimental analysis to $\mathcal{O}(\alpha)$ corrections. Monte Carlo events generated using KandY are passed through the OPAL detector simulation [6] and W-boson reconstruction algorithm [7]. In this way, the previous estimates of the QED systematic uncertainties can be extended to the full experimental M_{W} extraction procedure. It is found that the sensitivity to the so-called [3] $\mathcal{O}(\alpha)$ non-leading (NL) electroweak corrections is significantly enhanced by the kinematic fitting used in the experimental analyses. This enhancement is a result of the relatively large change in the cross section for production of $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ events where the photon is away from the region collinear with the $\mathrm{e^+e^-}$ beams. Consequently, measurements of the cross section for $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ in this region place constraints on the related M_{W} systematic uncertainties.

It is suggested that previous studies [3] be repeated using a closer approximation of the real experimental procedure. With this in mind, a simple procedure to approximate the effects of the kinematic fit at the level of the generated four fermions is proposed.

2. The experimental measurement of M_{W} at LEP 2

The main measurements of M_{W} at LEP are obtained from the direct reconstruction of the W-boson invariant mass distribution in $\mathrm{e^+e^-} \to \mathrm{q\overline{q}}\ell\overline{\nu}_\ell$ and $\mathrm{e^+e^-} \to \mathrm{q\overline{q}q\overline{q}}$ events. Although the exact methodologies used by the four LEP experiments differ [7, 8], there are many common features and the general principles of the analyses are summarized below.

The experimental event selections for $\mathrm{e^+e^-} \to \mathrm{W^+W^-}$ have high efficiency, typically 80 % − 85 %. Once events are selected, algorithms are applied to reconstruct the four-momenta of the fermions. In $\mathrm{e^+e^-} \to \mathrm{q\overline{q}}\ell\overline{\nu}_\ell$ events, the charged particle tracks and any clusters in the calorimeters (electromagnetic and hadronic) which are not associated with the lepton are forced into two jets using the Durham [9] or LUCLUS [10] algorithms. In $\mathrm{e^+e^-} \to \mathrm{q\overline{q}q\overline{q}}$ events, tracks and clusters are forced into four jets (although DELPHI and OPAL allow for the possibility of a fifth 'gluon' jet). The jet energy resolution is relatively poor, $\sigma_E/E > 60\ \%/\sqrt{E}$, and neutrinos from leptonic W-boson decays are unobserved. Consequently, kinematic fits are employed to improve the event-by-event mass resolution. The kinematic fits impose the constraints of energy and momentum conservation. In most analyses the additional constraint of equal masses for the two W bosons is imposed, effectively neglecting Γ_{W}, referred to as a five constraint (5C) fit. The result of the 5C kinematic fit is a single reconstructed mass for each event which approximates to the average of the invariant masses of the fermion pairs from the two W-boson decays. In general, photons from initial state radiation (ISR) within the detector acceptance are not treated as separate particles in the fit. Although the four LEP collaborations employ different procedures to extract M_{W} from the reconstructed mass distribution, each method essentially corresponds to fitting the peak of the reconstructed mass distribution and calibrating out experimental biases using Monte Carlo event samples.

The single most important aspect of the kinematic fit is the constraint that the sum of the energies of the four fermions is equal to the centre-of-mass energy of the $\mathrm{e^+e^-}$ collision, $\sqrt{s}$. Due to the poor jet energy resolution, this constraint significantly improves the invariant mass resolution. However, in the presence of ISR, this procedure introduces a bias in the reconstructed mass, since the energies of the four fermions should be constrained to the centre-of-mass energy *after* photon radiation, $\sqrt{s'}$, rather than to $\sqrt{s}$. As a result, for events with ISR, the kinematic fit will return a mass value which is too large. On average, the event-by-event mass returned by the kinematic fit approximately corresponds to

$$\overline{M} \approx \frac{1}{2}(M_{12} + M_{34}) \times \sqrt{\frac{s}{s'}}\,. \tag{2.1}$$

Hence, the reconstructed invariant mass distribution depends strongly on the $\sqrt{s'}$ distribution and as a result the peak of reconstructed W-boson mass is approximately 100 MeV higher than M_{W} (the exact value depending on $\sqrt{s}$). The bias in the fitted mass is even greater, typically a few hundred MeV. This bias is subsequently determined using Monte Carlo events and the data measurement corrected. Consequently, deficiencies in the Monte Carlo simulation may result in systematic biases and it is necessary for the Monte Carlo programs to predict accurately the $\sqrt{s'}$ distribution.

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3. QED uncertainties on the LEP M_{W} measurement

The current estimate of 5 MeV for the QED/Electroweak theoretical uncertainty on the LEP M_{W} measurement is based on the Monte Carlo generated invariant mass distribution of the $\mu^-\overline{\nu}_\mu$ system in $\mathrm{e}^+\mathrm{e}^- \to \mu^-\overline{\nu}_\mu u\overline{d}(\gamma)$ events [3]. Uncertainties are obtained from the shifts in the invariant mass distribution which result from dropping higher order theoretical terms. The simplest definition of invariant mass is obtained directly from the four-momenta of the generated fermions (termed BARE mass[1]). However, in the real experimental analyses, photons from final state radiation (FSR) are usually recombined with the nearest jet or lepton. This recombination is emulated at the level of the generated particles by recombining photons with the closest fermion if the photon has energy $E_\gamma < 1\,\mathrm{GeV}$ or if the invariant mass of the photon and fermion is less than 5 GeV (25 GeV), termed the CALO5 (CALO25) photon recombination scheme. In this way FSR photons are usually recombined with the appropriate fermion. In the CALO5/CALO25 schemes photons within 5° of the beam direction are treated as unobserved and are not recombined, reflecting the holes in experimental acceptance along the beam direction.

The main difference between the definitions of BARE and CALO masses and the experimentally reconstructed mass arises from the kinematic fit employed in the experimental analyses. As a result of the kinematic fit, $\mathcal{O}(\alpha)$ QED corrections may affect the reconstructed mass in two ways: either by modifying the invariant mass distribution of the fermion pairs from W decays or by modifying the $\sqrt{s'}$ distribution. In the first case, the theoretical estimates of mass biases obtained using either BARE or CALO schemes provide a good estimate of the true experimental bias. However, in second case, even if the invariant mass distribution of the fermion pairs is unchanged, the experimentally measured mass distribution may be biased as a result of the kinematic fit scaling the measured energies to $\sqrt{s}$ rather than $\sqrt{s'}$. Consequently, previous estimates of the impact of theoretical corrections which modify either the rate or energy spectrum of real photons produced in the process $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+\mathrm{W}^-\gamma$ may be too small.

To study possible differences between the previous methods used to estimate M_{W} systematic uncertainties and the true impact on the experimental measurement the Monte Carlo tandem of KORALW and YFSWW (KandY) is used. A total of four million $\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+\mathrm{W}^-$ events was generated over 8 separate values of $\sqrt{s}$ reflecting the LEP 2 data sample used in the W-mass determination ($183\,\mathrm{GeV} < \sqrt{s} < 209\,\mathrm{GeV}$). The events were passed through the full OPAL detector simulation [6] and W-mass reconstruction procedure including the 5C kinematic fit [12]. The following studies are based on events passing the OPAL $\mathrm{W}^+\mathrm{W}^- \to \mathrm{q}\overline{\mathrm{q}}\ell\overline{\nu}_\ell$ and $\mathrm{W}^+\mathrm{W}^- \to \mathrm{q}\overline{\mathrm{q}}\mathrm{q}\overline{\mathrm{q}}$ event selections. The KandY program produces correction weights enabling generated events to be reweighted to the corresponding theoretical prediction removing certain terms. Correction weights were produced corresponding to: the replacement of the Screened Coulomb correction [13] with the unscreened correction; degrading the treatment of ISR from $\mathcal{O}(\alpha^3)$ to $\mathcal{O}(\alpha^2)$ YFS exponentiated [14] leading logarithm (LL); and dropping the so-called $\mathcal{O}(\alpha)$ NL electroweak

[1]The nomenclature of reference [3] is used throughout this paper rather than that of [11].

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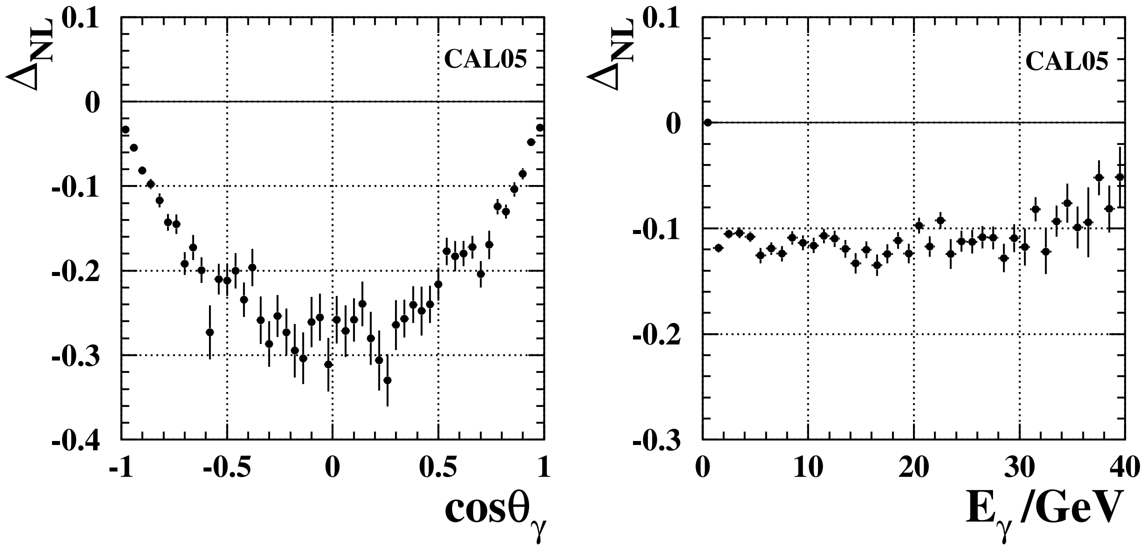

Figure 1: The effect of the $\mathcal{O}(\alpha)$ NL corrections in KandY on the distributions of the energy and angle with respect to the e^- beam of the highest energy photon in the event. Only photons which have not been recombined using CALO5 scheme are included.

corrections of KandY. It is important to note that the corrections termed '$\mathcal{O}(\alpha)$ NL' in [3] include bremsstrahlung from the W bosons (referred to as WSR) and the resulting interference with ISR.

The treatment of the Coulomb correction modifies the invariant mass distribution of the fermion pairs from the W decays. Consequently, the shift in the BARE/CALO invariant mass distribution provides a good measure of the true experimental bias. Degrading the treatment of ISR from $\mathcal{O}(\alpha^3)$ to $\mathcal{O}(\alpha^2)$ exponentiated LL has negligible impact on the experimental measurement of M_{W} [7] (the measurement is sensitive to ISR through a modification of the $\sqrt{s'}$ distribution, however the effects of the $\mathcal{O}(\alpha^3)$ terms are small). The effect of the $\mathcal{O}(\alpha)$ NL electroweak corrections, which include bremsstrahlung from the W bosons, is more interesting. To study potential changes in distributions it is convenient to define the fractional change in a differential cross section,

$$\Delta_{NL} = \frac{(d\sigma_{KY}^{NL} - d\sigma_{KY})}{d\sigma_{KY}}, \tag{3.1}$$

where $d\sigma_{KY}$ is the differential cross section in a given bin from the full KandY generation and $d\sigma_{KY}^{NL}$ is the corresponding distribution without the $\mathcal{O}(\alpha)$ NL electroweak corrections.

Figure 1 shows the effect of the $\mathcal{O}(\alpha)$ NL corrections on the photon energy and polar angle distribution. These plots are similar to those of Reference [15] showing that the experimental selections do not introduce significant biases for events with additional photons.[2] The inclusion of the $\mathcal{O}(\alpha)$ NL corrections reduces the number of real photons produced in the detector acceptance. The maximum fractional change occurs in the region

[2]The plots in Reference [15] are for $\mathrm{W^+W^-} \to \mathrm{q\bar{q}}\ell\bar{\nu}_\ell\gamma$ events generated at $\sqrt{s} = 200\,\mathrm{GeV}$ and do not include the effects of event selection.

Scheme	ΔM^{NL}
BARE	+1 MeV
CALO5	+3 MeV
CALO5*	+22 MeV
5C FIT	+21 MeV

Table 1: The change in the mean W-boson mass in the range 75 GeV−90 GeV when dropping the electroweak $\mathcal{O}(\alpha)$ NL corrections from KandY. Shifts are shown for three different theoretical definitions of the event-by-event reconstructed mass: BARE, CALO5 and the new definition proposed in this paper, CALO5*. 5C FIT refers to the shift in the average of reconstructed mass distribution for the full OPAL event reconstruction using a 5C kinematic fit.

perpendicular to the beam axis where the $W^+W^-\gamma$ cross section is decreased by 30 %. This modification of the photon rate and angular distribution is predominantly due the inclusion of radiation from the W bosons. Specifically, the reduction in the $W^+W^-\gamma$ cross section results from interference between ISR and radiation from the W bosons.[3] The photon energy spectrum is not greatly distorted by the NL corrections as can be seen from figure 1.

The potential bias in the measurement of M_W due to the $\mathcal{O}(\alpha)$ NL terms is investigated using four definitions of reconstructed mass were considered:

- $\overline{M}_{\mathrm{BARE}}$: the average of the two W-boson masses determined from the four-momenta of the four fermions.
- $\overline{M}_{\mathrm{CALO5}}$: the average of the two W-boson masses determined from the four-momenta of the four fermions after applying the CALO5 photon recombination procedure.
- $M_{5\mathrm{C}}$: the mass returned by the OPAL 5C kinematic fit. Only events for which the fit converges with a probability of greater than 0.1 % are plotted.
- $\overline{M}_{\mathrm{CALO5*}}$: $\overline{M}_{\mathrm{CALO5}}$ rescaled by $\sqrt{\frac{s}{s'}}$ where $\sqrt{s'}$ is the invariant mass of the four fermion system after recombination of photons:

$$\overline{M}_{\mathrm{CALO5*}} = \overline{M}_{\mathrm{CALO5}} \times \sqrt{\frac{s}{s'}}\,. \tag{3.2}$$

The CALO5* definition, proposed in this paper, represents an attempt to emulate the effects of the kinematic fit at the level of the four-momenta of the fermions. The distortions of the W-mass peak, plotted for above definitions of invariant mass, are shown in figure 2. To quantify the effect on the W-mass measurement, the resulting biases in the mean reconstructed W-boson mass (in the region 75 GeV−90 GeV) are listed in table 1. The inclusion of the $\mathcal{O}(\alpha)$ NL corrections has almost no effect on the BARE mass distribution and the mean value changes by just 1 MeV. A small but visible effect is observed for the

[3]This was verified using YFSWW. The reduction in the cross section for photon production shown in figure 1a was reproduced by switching from `KEYCOR=2` to `KEYCOR=3` in YFSWW, corresponding to switching from YFS exponentiation for ISR alone to the full YFS form factor including radiation from the W bosons (WSR) and interference between ISR and WSR.

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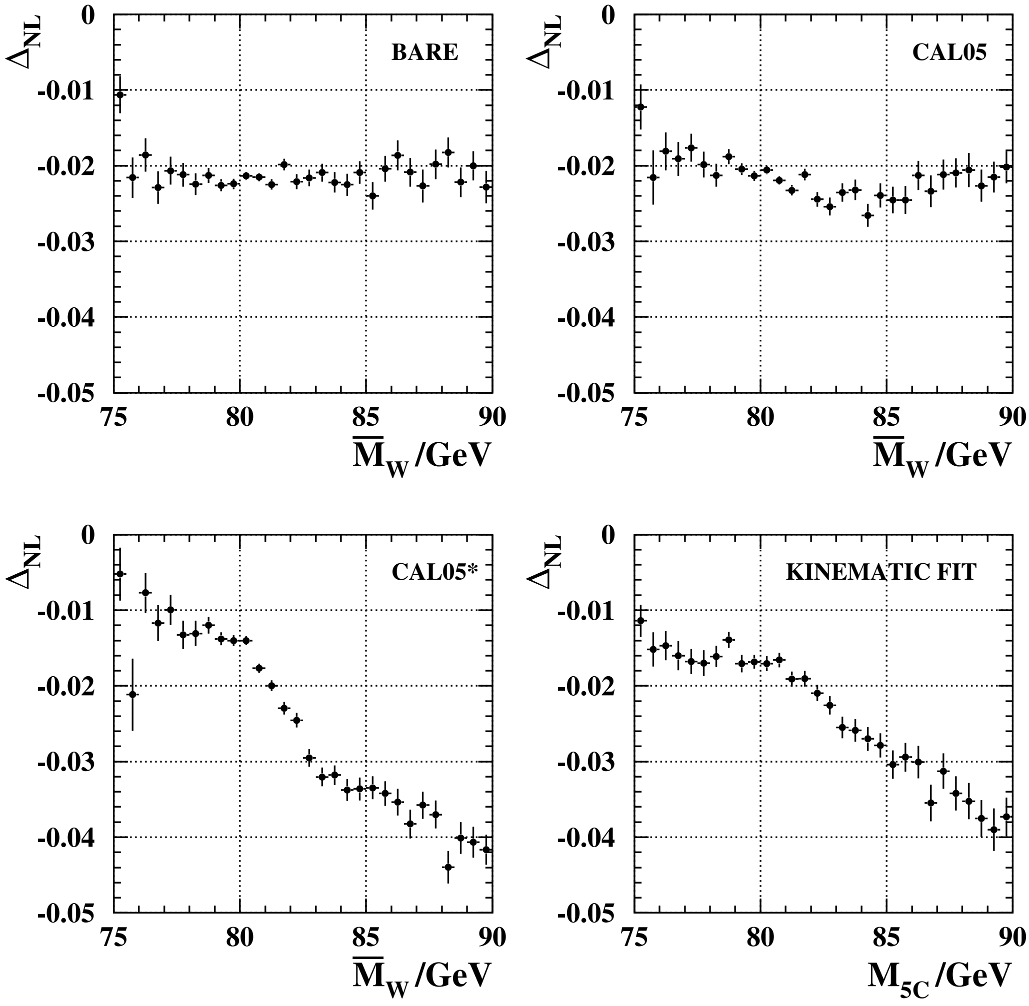

Figure 2: Fractional change in the differential cross-section as a function of the reconstructed mass using the BARE, CALO5, and CALO5* schemes compared to the experimental bias using the OPAL 5C kinematic fit.

mass in the CALO5 scheme giving a bias in the mean mass of 3 MeV. These results are consistent with those presented in [3]. However, the inclusion of the $\mathcal{O}(\alpha)$ NL corrections produces a significant shift in the experimentally reconstructed mass from the 5C kinematic fit, with the mean reconstructed mass in the range 75 GeV$-$90 GeV shifted by 21 MeV. Although the invariant mass distribution of the fermion pairs is almost unaffected the reconstructed mass distribution is significantly biased. The origin of this modification of the $\sqrt{s'}$ distribution is the reduction in the production cross section for $e^+e^- \to W^+W^-\gamma$, which results from the interference of ISR and radiation from the W bosons. It should be noted that by scaling the mean mass in the CALO5 scheme by $\sqrt{s/s'}$ (denoted CALO5*) the effect of the kinematic fit can be approximated at the level of the generated fermions.

The effect of the kinematic fit is to make the experimental analyses more sensitive to theoretical corrections which modify the rate/spectrum of photons produced in the process $e^+e^- \to W^+W^-\gamma$. The bias of 21 MeV associated with the $\mathcal{O}(\alpha)$ NL corrections in KandY

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is almost an order of magnitude larger than previously quoted. Since the origin of the 21 MeV bias is interference between ISR and radiation from the W bosons, one might expect this to be well accounted for in YFSWW, but this requires further evaluation. It should be noted that the 21 MeV bias would be reduced significantly if observed photons were treated as additional particles in the kinematic fits. In this case the energies of the fermions from W decay would be scaled correctly to $\sqrt{s'}$.

For the OPAL W-mass extraction procedure the effect of the $\mathcal{O}(\alpha)$ NL corrections on the full M_{W} reconstruction procedure is somewhat smaller than the shift in the average event-by-event reconstructed mass quoted above. Using a modified version of the OPAL M_{W} convolution fit [7] the bias associated with dropping the $\mathcal{O}(\alpha)$ NL corrections is 14 MeV.

4. Limits from the measurement of the $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ cross section

Measurements of the process $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ have been performed at LEP [16, 17, 18]. These measurements, which are restricted to photons within the experimental acceptance (typically $|\cos\theta_\gamma| < 0.95$), provide a direct probe of $\mathcal{O}(\alpha)$ QED corrections which affect real photon production away from the region collinear with the $\mathrm{e^+e^-}$ beams. Of particular interest here are the $\mathcal{O}(\alpha)$ NL electroweak corrections of KandY which decrease the calculated $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ cross section. The largest change in the differential $\mathrm{W^+W^-}\gamma$ cross section occurs close to the beam direction (as would be expected from ISR-WSR interference). However, when integrated over the photon polar angle, 88 % of the change in cross section occurs within the nominal experimental acceptance of $|\cos\theta_\gamma| < 0.95$. Hence, by measuring the $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ cross section for events where the photon is observed, it is possible to probe experimentally the $\mathcal{O}(\alpha)$ NL corrections of KandY.

When the four LEP experiments publish final results including all LEP 2 data, the combined experimental precision on the $\mathrm{W^+W^-}\gamma$ cross section should be approximately 7 %. The inclusion of the $\mathcal{O}(\alpha)$ NL electroweak corrections of KandY produces a 20 % decrease in the $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ cross section for $|\cos\theta_\gamma| < 0.95$ and $E_\gamma > 5\,\mathrm{GeV}$ (cuts chosen to match the experimental acceptances of the LEP detectors). Thus, using rate alone, the measurements are of sufficient statistical precision to provide a useful test of the KandY implementation of WSR and in particular of WSR−ISR interference. Agreement with the predictions of KandY would provide the first indirect observation (albeit only at the 3 σ level) of real photon radiation from the W bosons through interference with ISR diagrams. The statistical precision of this test would be improved slightly by including angular information in addition to the photon rate.

In the context of the M_{W} measurement at LEP the inclusion of the $\mathcal{O}(\alpha)$ NL corrections in KandY produces a 21 MeV shift in the mean reconstructed mass which results from the modification of the $\sqrt{s'}$ distribution. This is a direct consequence of the decrease in the $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ cross section. Therefore, measurements of the $\mathrm{e^+e^-} \to \mathrm{W^+W^-}\gamma$ cross section at LEP provide constraints on the $\mathcal{O}(\alpha)$ QED uncertainties related to real photon production away from the region collinear with the beams. A 20 % reduction in the $\mathrm{W^+W^-}\gamma$ cross section corresponds to a 21 MeV shift in the reconstructed value of

M_W. Assuming the bias scales linearly, a measurement of the $W^+W^-\gamma$ cross section with a statistical precision of 7 % would correspond to approximately 7 MeV on the associated systematic uncertainty on M_W. It should be noted that this limit would apply to all theoretical corrections to the process $e^+e^- \to W^+W^-$ which modify the rate of real photon production away from the collinear region; it is not specific to the $\mathcal{O}(\alpha)$ NL corrections of KandY. Given that the expected statistical uncertainty on the LEP M_W measurement is approximately 35 MeV, a LEP combined measurement of the $W^+W^-\gamma$ cross section would provide a useful experimental constraint on the $\mathcal{O}(\alpha)$ QED systematic uncertainty.

5. Conclusions

Previous studies of QED systematic uncertainties on the LEP measurement of M_W have used idealized event selections and fitting procedures. In this paper the KandY Monte Carlo program was used to study the effect of the experimental methodology on the estimated QED systematic uncertainty. It is found that the kinematic fitting used in the experimental determination of M_W enhances the sensitivity to all QED corrections involving real photon production and it is concluded that previous estimates of the QED theoretical uncertainty on the LEP 2 M_W measurement may be too small. A simple procedure for approximating the effect of the kinematic fit at the level of the generated four fermions is proposed. This procedure would allow previous theoretical studies to be extended to include a good approximation of the real experimental mass extraction method. Finally, it is shown that useful experimental limits on $\mathcal{O}(\alpha)$ theoretical uncertainties related to real photon production away from region collinear with the e^+e^- beams may be inferred from LEP measurements of the $e^+e^- \to W^+W^-\gamma$ cross section.

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