Questions for Part II Particle Physics
Lent term, 2004

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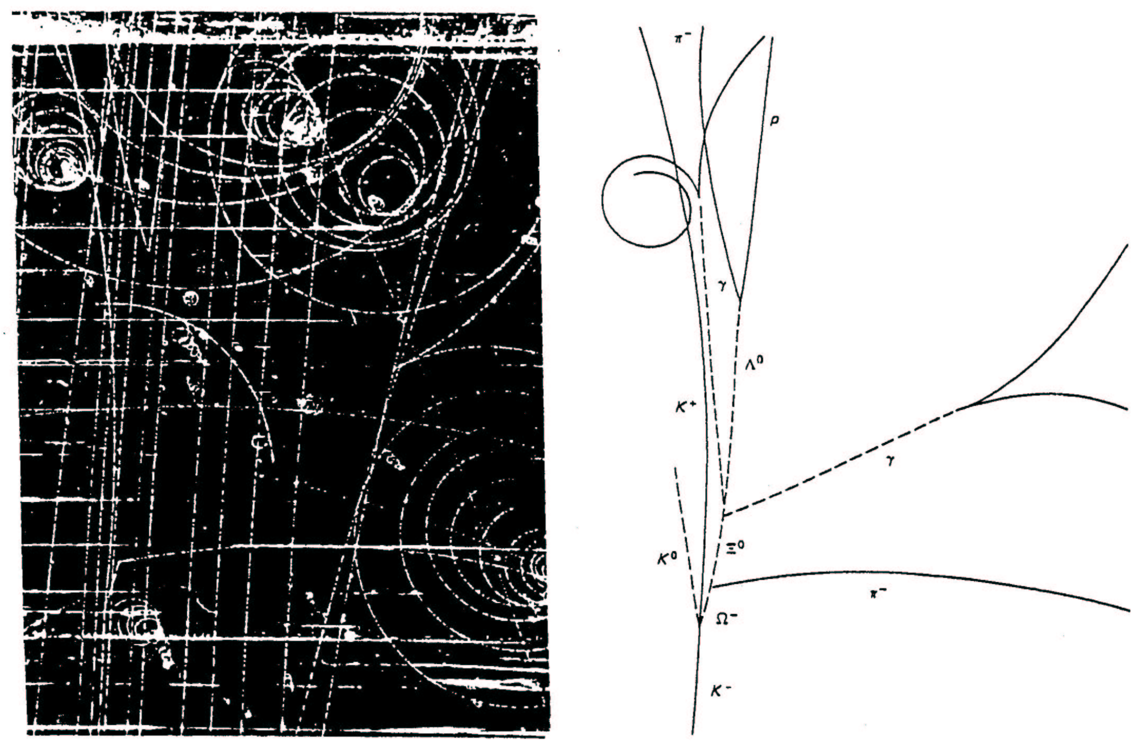
Relativistic Kinematics and QED (Handouts I & II)

1. The figure shows a photograph and line diagram of the event corresponding to the first observation of the Ω^- baryon ($\Omega^- \to \Xi^0\pi^-$, $\Xi^0 \to \Lambda^0\pi^0$) in a K^-p interaction in a liquid hydrogen bubble chamber (from Barnes *et al.*, Phys. Rev. Lett. **12** (1964) 204):



(a) The two photons from the $\pi^0 \to \gamma\gamma$ decay are both seen to convert to e^+e^- pairs. Show that the process $\gamma \to e^+e^-$ is kinematically forbidden *in vacuo*. Draw a Feynman diagram representing photon conversion in material, as seen in the figure.

(b) The π^- (track 4) and Ξ^0 from the Ω^- decay have momenta of 281 MeV/c and 1906 MeV/c respectively. Their spatial opening angle is 71^0. Estimate the mass of the Ω^- and compute its momentum.

(c) The length of the Ω^- flight path is 2.5 cm. Calculate the proper lifetime of the Ω^-.

[$m(\pi^-)$=139.6 MeV/c^2, $m(\Xi^0)$=1315 MeV/c^2]

2. At a collider the interaction rate, R, for a particular process, is given by $R = \sigma L$, where L is the luminosity of the colliding beams (units of $\mathrm{cm^{-2}s^{-1}}$). The total number of interactions is therefore related to the integrated luminosity, $\mathcal{L} = \int L\mathrm{d}t$, by $N = \mathcal{L}\sigma$.

(a) In a high energy $\mathrm{e^+e^-}$ collider there are two counter-rotating beams of particles. The particles in each beam are located in n discrete equal current bunches. Show that

$$L = fn\frac{N_{e^-}N_{e+}}{A},$$

where f is the revolution frequency, N_{e^-} and N_{e^+} are the numbers of particles in each bunch in the $\mathrm{e^+e^-}$ beams, and A is the effective area of each beam (assuming constant intensity across A).

(b) At the LEP e^+e^- collider (27 km circumference) the electron and positron beam currents are both 1.0 mA. In each beam there are four equally spaced bunches of electrons/positrons. The bunches have an effective area of $1.8 \times 10^4\,\mu\mathrm{m}^2$. An experiment is located at one of the crossing points. Calculate the luminosity of the colliding beams (as seen at the experiment).

(c) In 1994 the LEP accelerator operated for a period of 200 days with a 65 % duty cycle (*i.e.* the beams were colliding for 65% of this time). Calculate the corresponding integrated luminosity.

(d) In practice, the integrated luminosity is measured rather than calculated from the equation above. In the Opal experiment (1989-2000) the luminosity was measured by counting the number of $e^+e^- \to e^+e^-$ interactions for which the e^- scattering angle with respect to the incoming e^- beam lies between 25 mrad and 58 mrad. The $e^+e^- \to e^+e^-$ cross section for this angular acceptance is calculated (to high precision) to be 78.76 nb. During the 1994 operation of the LEP collider, 3,945,974 $e^+e^- \to e^+e^-$ events were observed (between 25 mrad and 58 mrad). Calculate the integrated luminosity.

(e) During the 1994 period of operation 67,791 interactions were classified as being $\mathrm{e^+e^-} \to \mu^+\mu^-$. The efficiency for correctly classifying a genuine $\mathrm{e^+e^-} \to \mu^+\mu^-$ event is 90.71 %. In addition, 1.0 % of events classified as $\mathrm{e^+e^-} \to \mu^+\mu^-$ are background from mis-identified $\mathrm{e^+e^-} \to \tau^+\tau^-$ interactions. Determine the experimentally measured value (in nb) of $\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)$.

(f) During the same period of operation 1,520,277 $\mathrm{e^+e^-} \to \mathrm{q\overline{q}}$ interactions were observed. Neglecting possible inefficiencies and background contributions determine $\sigma(\mathrm{e^+e^-} \to \mathrm{q\overline{q}})$. The theoretical cross section for $e^+e^- \to e^+e^-$ (between 25 mrad and 58 mrad) is calculated to a precision of 0.06 %. Comment on this in the light of the statistical error on $\sigma(\mathrm{e^+e^-} \to \mathrm{q\overline{q}})$?

3. Drell-Yan production of charged lepton pairs in hadron-hadron interactions, *e.g.* $\pi\mathrm{N} \to \mu^+\mu^- +$ *hadrons*, proceeds via quark-antiquark annihilation into a single virtual photon (where N is a neutron or proton).

Draw the lowest order Feynman diagrams that contribute to the Drell-Yan process $\pi^+\mathrm{p} \to \mu^+\mu^- +$ *hadrons*.

Show that the Drell-Yan cross sections in π^+p, π^+n, π^-p and π^-n interactions would be expected to be in the ratio 1 : 2 : 8 : 4.

What would you expect the Drell-Yan cross sections for pp and $\mathrm{\bar{p}p}$ collisions to be compared with the Drell-Yan cross section for π^+p interactions ?

4. (a) Verify the quark model predictions given in the lectures for the following meson masses:

Meson	Calculated (MeV/c^2)	Observed (MeV/c^2)
π	140	138
K	484	496
η	559	549
ρ	780	776
ω	780	783
K*	896	892
ϕ	1032	1020

What would you predict for the mass of the η' meson (measured mass 958 MeV/c^2) ?

$$\eta = \frac{1}{\sqrt{6}}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} - 2\mathrm{s\bar{s}})$$
$$\eta' = \frac{1}{\sqrt{3}}(\mathrm{u\bar{u}} + \mathrm{d\bar{d}} + \mathrm{s\bar{s}})$$

(b) What must be the total spin of any pair of quarks in the baryons in the $J^P = \frac{3}{2}^+$ decuplet? Hence predict the masses of the decuplet baryons and compare your predictions with the measured values.

[In part a) Assume m_u=m_d=310 MeV/c^2, m_s=483 MeV/c^2 and a spin-spin interaction coefficient A= 0.0615 GeV^3.]

[In part b) Assume m_u=m_d=360 MeV/c^2, m_s=540 MeV/c^2 and a spin-spin interaction coefficient A= 0.026 GeV^3.]

5. Derive the magnetic moments of the proton and neutron in the quark model as follows:

i) Assuming all the quarks are in l=0 states what must be the total spin of the two u quarks in the proton ? (Give reasons for your answer).

ii) Hence show that the wave function for a proton in the s_z=+$\frac{1}{2}$ state can be written as

$$\frac{1}{\sqrt{6}}(2u{\uparrow}u{\uparrow}d{\downarrow} - u{\uparrow}u{\downarrow}d{\uparrow} - u{\downarrow}u{\uparrow}d{\uparrow})$$

and derive a similar expression for the neutron.

iii) Assuming u and d quarks have equal mass write their magnetic moments in terms of this quark mass.

iv) Hence predict the ratio of the proton and neutron magnetic moments. Compare with the observed values: $\mu_p = 2.79\mu_N$, $\mu_n = -1.91\mu_N$. What value for the quark mass is needed to give these values? Is this sensible?

6. The figure shown over the page (from Boyarski *et al.*, Phys. Rev. Lett. **34** (1975) 1357) shows the original e^+e^- annihilation cross section measurements from the Mark II Collaboration which contributed to the discovery of the J/ψ meson. The measurements were made during a fine scan of the $e^\pm$ beam energies at the SPEAR storage ring at SLAC which consisted of oppositely circulating e^+ and e^- beams of equal energy.

The observed width (a few MeV) of the J/ψ resonance is due dominantly to the energy spread inherent in the e^+, e^- beams at each scan point. The *relative* centre of mass energy between scan points is known very precisely however (to about 1 part in 10^4). The actual J/ψ width is much smaller than the observed width, but can be extracted from the data as follows:

(a) The Breit-Wigner formula for the scattering of two particles of spin s_1 and s_2 in the region of a resonance of spin J is:

$$\sigma(E) = \frac{\lambda^2}{4\pi}\frac{(2J+1)}{(2s_1+1)(2s_2+1)}\frac{\Gamma_i\Gamma_f}{[(E-E_R)^2+\Gamma^2/4]}$$

where λ is the de Broglie wavelength of the incoming particles in the centre of mass, E is the centre of mass energy, E_R is the resonance energy, Γ is the total width of the resonance and Γ_i (Γ_f) is the partial width for decay into the initial (final) state. Show that, for the production of the J/ψ resonance in e^+e^- collisions, the integrated elastic cross section under the resonance peak is given by

$$\sigma' \equiv \int \sigma_{el}(E)dE = \frac{3}{8}\lambda^2 B^2 \Gamma$$

where B is the branching ratio for the decay $J/\psi \to e^+e^-$.

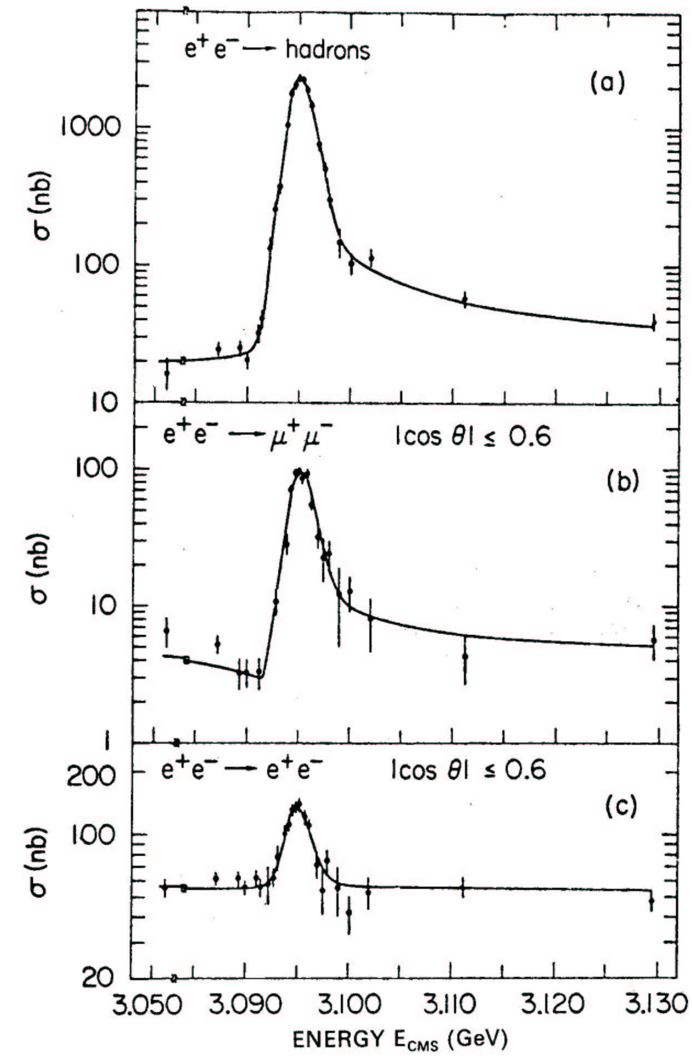
(b) Assume that, at each scan point, the beam energy spread produces a spread of centre of mass energies E' distributed about the average centre of mass energy E according to a probability distribution $f(E'-E)$. Show that the measured area under the resonance peak, $\int \sigma_{meas}(E)dE$, is the same as the true area under the peak, $\int \sigma(E)dE$.

(c) Given that the differential cross section $d\sigma/d\Omega$ for the process $e^+e^- \to J/\psi \to e^+e^-$ is proportional to $1+\cos^2\theta$ where θ is the angle between the final e^- and the beam direction, calculate the fraction of J/ψ decays contained within the acceptance region $|\cos\theta| < 0.6$ imposed for the e^+e^- and $\mu^+\mu^-$ channels.

(d) Use the data in the figure to estimate the quantities σ' and B defined above [1]. Hence estimate Γ and Γ_{ee} for the J/ψ.

(e) The corresponding widths for the ϕ meson are Γ=4.4 MeV and Γ_{ee}=1.37 keV. Explain why the J/ψ and ϕ mesons have similar leptonic widths Γ_{ee} but very different total widths Γ.

[1] Note that the measured cross sections contain a significant non-resonant contribution which must be subtracted. Optionally: draw the leading order Feynman diagrams representing these non-resonant processes and suggest what might be causing the dip structure seen in the $\mu^+\mu^-$ channel. Also optionally: explain how radiative effects might cause the high tail seen in the hadronic channel.



7. Write brief notes on :

   a) The evidence for the existence of quarks.

   b) The evidence for colour.

   c) Charmonium.

Weak Interactions (Handout VI)

8. The Ω^- baryon (sss), produced in the event shown in example 1, is seen to decay weakly through the decay chain $\Omega^- \to \Xi^0\pi^-$, $\Xi^0 \to \Lambda^0\pi^0$ and $\Lambda^0 \to \mathrm{p}\pi^-$. Draw the quark line diagrams for the decays of the Ω^-, the Ξ^0 and the Λ^0.

   Explain, with the aid of quark line diagrams, why the Ω^- is not observed to decay via the strong decay $\Omega^- \to \Xi^-\overline{\mathrm{K}^0}$ and that the weak decay $\Omega^- \to \Lambda^0\pi^-$ is strongly suppressed.

   [$\Xi^0(uss)$, $\Xi^-(dss)$, $\overline{\mathrm{K}^0}(s\bar{\mathrm{d}})$, $\Lambda^0(uds)$]

9. Show that the τ lepton decay branching ratios should be approximately in the ratios

$$\tau \to e : \tau \to \mu : \tau \to hadrons = 1 : 1 : 3.0$$

Estimate the mean lifetime of the τ lepton, assuming the branching ratio for $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ is 18%.

[$m_\tau = 1.777$ GeV/c^2, $m_\mu = 0.106$ GeV/c^2 and the mean μ lifetime is 2.197×10^{-6} s.]

10. Draw the Feynman for the weak decays ($b \to q\, e^- \,\overline{\nu}_e$) and $\tau^- \to e^- \,\nu_\tau \,\overline{\nu}_e$).

In the limit that all final state fermions are massless explain why

$$\frac{\Gamma(b \to q\, e^- \,\overline{\nu}_e)}{\Gamma(\tau^- \to e^- \,\nu_\tau \,\overline{\nu}_e)} = |V_{qb}|^2 \frac{m_b^5}{m_\tau^5}.$$

When the final quark mass is taken into account the above result becomes

$$\frac{\Gamma(b \to q\, e^- \,\overline{\nu}_e)}{\Gamma(\tau^- \to e^- \,\nu_\tau \,\overline{\nu}_e)} = |V_{qb}|^2 \frac{m_b^5}{m_\tau^5} f(m_q/m_b),$$

where

$$f(x) = 1 - 8x^2 - 24x^4 \ln x + 8x^6 - x^8.$$

To a reasonable approximation, the inclusive decay rate of the B$^-$($b\overline{u}$) meson into all final states containing leptons, $B^- \to e^- \,\overline{\nu}_e + hadrons$, may be calculated by considering just the decay of the b-quark *i.e.* neglecting the fact that the b-quark exists in a bound meson state. The lifetime of the B$^-$ meson is measured to be $(1.65 \pm 0.03) \times 10^{-12}$ s. The B$^-$ branching fraction to $e^- \overline{\nu}_e + hadrons$ is measured to be

$$Br(\mathrm{B}^- \to e^- \,\overline{\nu}_e + hadrons) = 10.2 \pm 0.9\,\%.$$

From these data obtain upper limits on the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$.

Comment on the inferred value of $|V_{tb}|$

[$m_u = 0.35$ GeV, $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, $m_\tau = 1.777$ GeV, $\tau_{\tau^-} = (2.906 \pm 0.011) \times 10^{-13}$ s and $Br(\tau^- \to e^- \nu_\tau \overline{\nu}_e) = 17.8\,\%$]

11. From a measurement of the width of the Z^0 particle it is possible to deduce that it is very likely that there are only three generations of quarks and leptons. Explain why this is so, indicating in your answer what other measurements are required to make this deduction.

The Breit-Wigner formula for a reaction cross-section is given by

$$\sigma = \frac{g\pi}{k^2} \frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \Gamma^2/4}.$$

Explain the meaning of the symbols. Derive an expression that gives the probability of finding a state with a finite lifetime, τ, at an at an energy E and explain why you would expect the cross-section for a process that occurs via a resonance to have the form shown above.

In the OPAL experiment at LEP the cross-section for $e^+e^- \to \tau^+\tau^-$ was measured at various centre-of-mass energies. Some of the results are shown below. Plot these data and make estimates of the Z^0 boson mass, M_{Z^0}, the total width of the Z^0 boson, Γ_Z, and the partial decay width Γ_τ (assuming lepton universality of the Neutral Current).

E_{cm}/GeV	$\sigma(e^+e^- \to \tau^+\tau^-)$/nb
88.481	0.2769 ± 0.0235
89.442	0.4892 ± 0.0091
90.223	0.8331 ± 0.0368
91.283	1.4988 ± 0.0213
91.969	1.1892 ± 0.0235
92.971	0.7089 ± 0.0105
93.717	0.4989 ± 0.0276

Why is the measured resonance curve asymmetric and what else needs to be taken into account when determining accurately M_{Z^0}, Γ_Z and Γ_τ ?

12. Estimate the total decay width, Γ_Z, and the lifetime of the Z^0 boson using the resonant cross-section ratio,

$$\frac{\sigma(e^+e^- \to Z^0 \to \text{hadrons})}{\sigma(e^+e^- \to Z^0 \to \mu^+\mu^-)} = 20.7,$$

and the Z^0 partial decay widths, $\Gamma(Z^0 \to \mu^+\mu^-) = 83.3$ MeV and $\Gamma(Z^0 \to \nu_\mu\bar{\nu}_\mu) = 166.5$ MeV. Make clear any assumptions you make.

13. The number of neutrino species, in principle, can be determined from the total width of the W boson. Using the Standard Model prediction of the partial width for $W \to e\bar{\nu}_e$ decays,

$$\Gamma(W \to e\bar{\nu}_e) = \frac{G_F}{\sqrt{2}} \frac{M_W^3}{6\pi},$$

the mass of the W boson, $M_W = 80$ GeV/c^2 and the total width, $\Gamma_W = 2.1$ GeV, estimate the number of light neutrino species. Make clear any assumptions you make.

[$G_F = 1.2 \times 10^{-5}$ GeV^{-2}.]

14. Draw all possible lowest order Feynman diagrams for the neutrino scattering processes:

i) $\nu_e \, e^- \to \nu_e \, e^-$
ii) $\overline{\nu}_e \, e^- \to \overline{\nu}_e \, e^-$
iii) $\nu_\mu \, e^- \to \nu_\mu \, e^-$
iv) $\overline{\nu}_\mu \, e^- \to \overline{\nu}_\mu \, e^-$
v) $\nu_e \, n \to e^- \, p$

(a) Using Fermi theory of weak interactions and assuming spinless particles, calculate an approximate expression for the cross section for $\nu_e \, e^- \to \nu_e \, e^-$ for centre-of-mass energy $\sqrt{s}$. Comment on its behavior at high energies.

(b) Derive an expression for $\sigma(\nu_e \, e^-)$ in terms of the energy of the neutrino in the laboratory frame.

(c) The majority of high energy neutrinos produced in the sun are produced in the reaction:

$$^{8}\mathrm{B} \;\to\; {}^{7}\mathrm{Be} + e^+ + \nu_e.$$

Assuming these neutrinos have a mean energy of approximately 7.5 MeV and the flux at the earth is $5.2 \times 10^6 \, \mathrm{cm}^{-2}\mathrm{s}^{-1}$, estimate the neutrino interaction rate in the 50 kton Super-Kamiokande water Čerenkov detector.

Optional extra: By considering the helicities in the process $\nu_e \, e^- \to \nu_e \, e^-$ discuss qualitatively:
a) how the cross section depends on the scattering angle between the neutrino in the initial and final state,
b) how the magnitude of the cross section is different from that obtained from Fermi theory.

15. Consider each of the sets of processes given below. In each case, with the aid of line diagrams using the Standard Model vertices, determine which processes are allowed and which are forbidden. By considering the strength of the forces involved, list the processes in each set in order of expected rate.

a) $\pi^0 \to \gamma\gamma$, $\quad \pi^0 \to \pi^- e^+ \nu_e \quad$ and $\quad \pi^0 \to \nu\bar{\nu}$;
b) $\rho^0 \to \pi^0\gamma$, $\quad \rho^0 \to \pi^+\pi^- \quad$ and $\quad \rho^0 \to \pi^0\pi^0$;
c) $e^+e^- \to \tau^+\tau^-$, $\quad \bar{\nu}_\mu + \tau^- \to \tau^- + \bar{\nu}_\mu \quad$ and $\quad \nu_\tau + \mathrm{p} \to \tau^+ + \mathrm{n}$;
d) $\mathrm{B}^0(\bar{b}d) \to \mathrm{D}^-(\bar{c}d)\pi^+$, $\quad \mathrm{B}^0 \to \pi^+\pi^- \quad$ and $\quad \mathrm{B}^0 \to J/\psi \mathrm{K}^0$.

16. See `http://www.hep.phy.cam.ac.uk/~thomson/particles/questions/Q16.html`

17. Show that if there are two neutrino mass eigenstates ν_2 and ν_3 with masses m_2 and m_3 and energies E_2 and E_3, mixed so that

$$\nu_\mu = \nu_2 \cos\theta + \nu_3 \sin\theta$$
$$\nu_\tau = -\nu_2 \sin\theta + \nu_3 \cos\theta$$

then the number of muon neutrinos observed at a distance L from the muon source is

$$|\nu_\mu(L)|^2 = |\nu_\mu(L=0)|^2 \times \left[1 - \sin^2(2\theta)\,\sin^2\left\{\left(\frac{E_3 - E_2}{2\hbar}\right)\frac{L}{c}\right\}\right].$$

If m_2 and m_3 are very much less than the neutrino momentum, p, show that

$$|\nu_\mu(L)|^2 \approx |\nu_\mu(L=0)|^2 \times \left[1 - \sin^2(2\theta)\,\sin^2\left\{A\left(\frac{(m_2^2 - m_3^2)L}{p}\right)\right\}\right]$$

where A is a constant.

In 2005 the MINOS experiment will start to study neutrino oscillations by pointing a beam of 1-5 GeV/c muon neutrinos from Fermilab, Illinois, at the 5400 ton MINOS far dectector in the SOUDAN mine in Minnesota, 730 km away. The experiment aims to make a precise measurement of $m_3^2 - m_2^2$.

Sketch the expected energy spectrum of muon neutrinos at the MINOS far detector if $\sin^2(2\theta) = 0.90$ and $m_3^2 - m_2^2 = 2.5\times10^{-3}$ (eV/c^2)^2. Assume that the energy spectum of neutrinos produced by the beam at Fermilab is of uniform intensity in the range 1-5 GeV and zero elsewhere (i.e. a top-hat function).

If muon neutrinos oscillate into tau neutrinos, will any τ leptons (produced by charged current interactions) be observed in the MINOS far detector ?

[$A = 1.27$ s^{-1} if m_2 and m_3 are measured in eV/c^2, p in GeV/c and L in km. The mass of the τ^- is $1.777\,\mathrm{GeV}/c^2$.]

Answers

1. (b) 1689 MeV/c^2, 2015 MeV/c

   (c) 70 ps

2. (b) $4.9 \times 10^{30}\,\mathrm{cm^{-2}s^{-1}}$

   (c) $55.0\,\mathrm{pb}^{-1}$

   (d) $50.1\,\mathrm{pb}^{-1}$

   (e) $1.477 \pm 0.006\,\mathrm{nb}$ (the Standard Model predicts $1.476\,\mathrm{nb}$)

   (f) $30.32 \pm 0.02\,\mathrm{nb}$

3. $\sigma(\mathrm{p\bar{p}}) = 17\,\sigma(\pi^+\mathrm{p})$

4. (a) $M(\eta') = 349$ MeV/c^2

   (b) S=1, $M(\Delta) = 1.230$ GeV/c^2, $M(\Sigma^*) = 1.383$ GeV/c^2, $M(\Xi) = 1.535$ GeV/c^2, $M(\Omega^-) =$ 1.687 GeV/c^2.

5. (i) S=1

   (iv) $\mu_\mathrm{p}/\mu_\mathrm{n} = -1.5$, Mass of quark = 330 MeV/c^2.

6. (c) 50.4 %

   (d) $\sigma' \approx 800$ nb MeV, $B \approx 0.055$, $\Gamma \approx 75$ keV, $\Gamma_{ee} \approx 5.3$ keV.

9. 0.3 ps

10. $|V_{cb}| < 0.05$ and $|V_{ub}| < 0.03$

12. 2.47 GeV, 2.66×10^{-25} s.

13. 3

14. a few hundred interactions per day