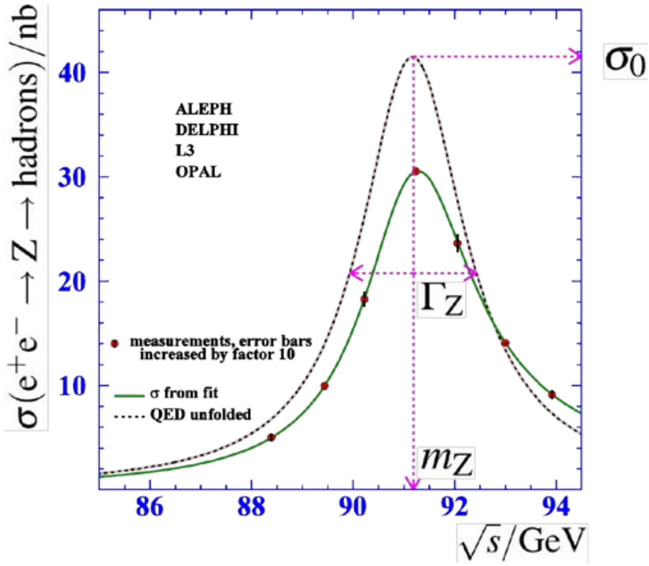
Particle Physics

Michaelmas Term 2011
Prof Mark Thomson

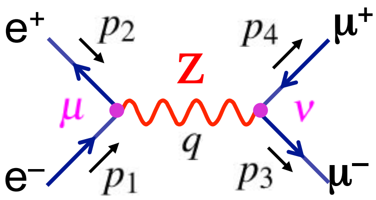


Handout 14 : Precision Tests of the Standard Model

The Z Resonance

★ Want to calculate the cross-section for $e^+e^- \to Z \to \mu^+\mu^-$

•Feynman rules for the diagram below give:



e^+e^- vertex: $\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

$\mu^+\mu^-$ vertex: $\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)$

$$\Rightarrow \quad -iM_{fi} = [\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].\frac{-ig_{\mu\nu}}{q^2-m_Z^2}.[\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]$$

$$\Rightarrow \quad \boxed{M_{fi} = -\frac{g_Z^2}{q^2-m_Z^2}g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].[\overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]}$$

★ Convenient to work in terms of helicity states by explicitly using the Z coupling to **LH and RH** chiral states (ultra-relativistic limit so helicity = chirality)

$$\tfrac{1}{2}(c_V - c_A\gamma^5) = c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

LH and RH projections operators

hence $c_V = (c_L + c_R),\;\; c_A = (c_L - c_R)$

and
$$\tfrac{1}{2}(c_V - c_A\gamma^5) = \tfrac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5) = c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

with $c_L = \tfrac{1}{2}(c_V + c_A),\;\; c_R = \tfrac{1}{2}(c_V - c_A)$

★ Rewriting the matrix element in terms of LH and RH couplings:

$$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1+\gamma^5)u(p_1)] \times [c_L^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)v(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1+\gamma^5)v(p_4)]$$

★ Apply projection operators remembering that in the ultra-relativistic limit

$$\tfrac{1}{2}(1-\gamma^5)u = u_\downarrow;\quad \tfrac{1}{2}(1+\gamma^5)u = u_\uparrow,\quad \tfrac{1}{2}(1-\gamma^5)v = v_\uparrow,\quad \tfrac{1}{2}(1+\gamma^5)v = v_\downarrow$$

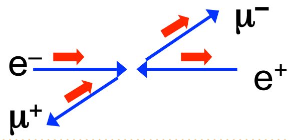
$$\Rightarrow\quad M_{fi} = -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu u_\uparrow(p_1)] \times [c_L^\mu \overline{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu v_\downarrow(p_4)]$$

★ For a combination of **V** and **A** currents, $\overline{u}_\uparrow\gamma^\mu v_\uparrow = 0$ etc, gives four orthogonal contributions

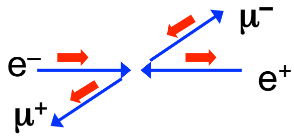
$$\Rightarrow\quad \boxed{-\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)] \times [c_L^\mu \overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]}$$

★ Sum of 4 terms

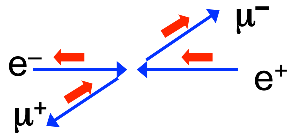
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu}[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



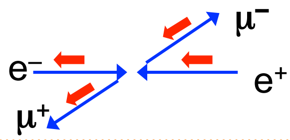
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu}[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu}[\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu}[\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state particles

★ Fortunately we have calculated these terms before when considering

$e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ giving: (pages 137-138)

$$[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] = s(1+\cos\theta)$$ etc.

★ Applying the QED results to the Z exchange with

$$\frac{e^2}{q^2} \rightarrow \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

where $q^2 = s = 4E_e^2$

gives:

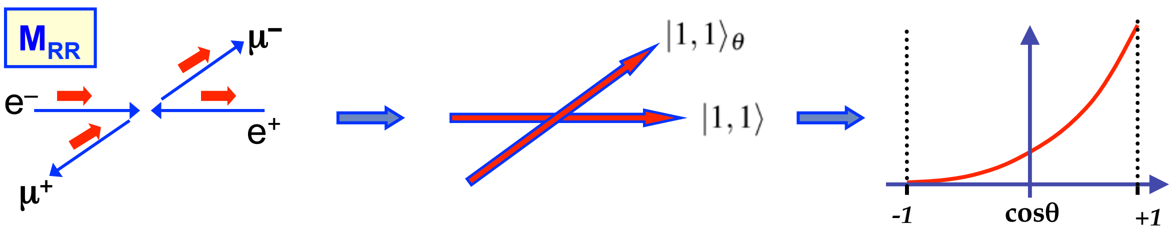
$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

★ As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.



The Breit-Wigner Resonance

★ Need to consider carefully the propagator term $1/(s - m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson

★ To do this need to account for the fact that the Z boson is an unstable particle

•For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

•For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \qquad \text{with} \qquad \tau = \frac{1}{\Gamma_Z}$$

•Equivalent to making the replacement

$$m \rightarrow m - i\Gamma/2$$

★In the Z boson propagator make the substitution:

$$m_Z \rightarrow m_Z - i\Gamma_Z/2$$

★ Which gives:

$$(s - m_Z^2) \longrightarrow [s - (m_Z - i\Gamma_Z/2)] = s - m_Z^2 + i m_Z \Gamma_Z + \tfrac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + i m_Z \Gamma_Z$$

where it has been assumed that $\Gamma_Z \ll m_Z$

★ Which gives

$$\left| \frac{1}{s - m_Z^2} \right|^2 \rightarrow \left| \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2 = \frac{1}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$

★ And the Matrix elements become

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2 \qquad \text{etc.}$$

★ In the limit where initial and final state particle mass can be neglected:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

(page 31)

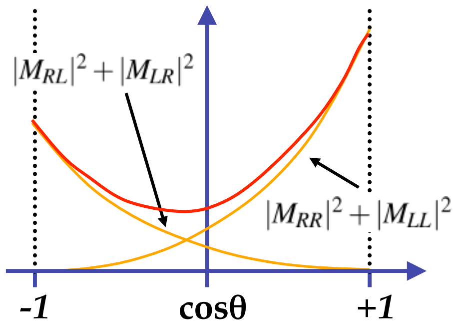
★ Giving:

$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

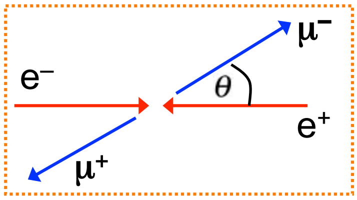
$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$



★ Because $|M_{LL}|^2+|M_{RR}|^2 \neq |M_{LR}|^2+|M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).



Cross section with unpolarized beams

★ To calculate the total cross section need to sum over all matrix elements and **average** over the **initial spin states**. Here, assuming unpolarized beams (i.e. both e⁺ and both e⁻ spin states equally likely) there a four combinations of initial electron/positron spins, so

$$\langle|M_{fi}|^2\rangle = \frac{1}{2}\cdot\frac{1}{2}\cdot(|M_{RR}|^2+|M_{LL}|^2+|M_{LR}|^2+|M_{RL}|^2)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\frac{g_Z^4 s^2}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times\{[(c_R^e)^2(c_R^\mu)^2+(c_L^e)^2(c_L^2)^2](1+\cos\theta)^2$$
$$+[(c_L^e)^2(c_R^\mu)^2+(c_R^e)^2(c_L^2)^2](1-\cos\theta)^2\}$$

★ The part of the expression {…} can be rearranged:

$$\{\ldots\} = [(c_R^e)^2+(c_L^e)^2][(c_R^\mu)^2+(c_L^\mu)^2](1+\cos^2\theta)$$
$$+2[(c_R^e)^2-(c_L^e)^2][(c_R^\mu)^2-(c_L^\mu)^2]\cos\theta \qquad (1)$$

and using $c_V^2+c_A^2 = 2(c_L^2+c_R^2)$ and $c_V c_A = c_L^2 + c_R^2$

$$\{\ldots\} = \frac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta$$

★ Hence the complete expression for the unpolarized differential cross section is:

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{1}{64\pi^2 s}\langle|M_{fi}|^2\rangle \\ &= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times \\ &\quad \left\{\tfrac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta\right\}\end{aligned}$$

★ Integrating over solid angle $d\Omega = d\phi d(\cos\theta) = 2\pi d(\cos\theta)$

$\int_{-1}^{+1}(1+\cos^2\theta)d(\cos\theta) = \int_{-1}^{+1}(1+x^2)dx = \frac{8}{3}$ and $\int_{-1}^{+1}\cos\theta d(\cos\theta) = 0$

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]}$$

★ Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions

$$(c_V^f)^2+(c_A^f)^2$$

Connection to the Breit-Wigner Formula

★ Can write the total cross section

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]}$$

in terms of the Z boson decay rates (partial widths) from page 473 (question 26)

$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^e)^2+(c_A^e)^2]$ and $\Gamma(Z\to \mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^\mu)^2+(c_A^\mu)^2]$

$$\Rightarrow \quad \sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

★ Writing the partial widths as $\Gamma_{ee} = \Gamma(Z\to e^+e^-)$ etc., the total cross section can be written

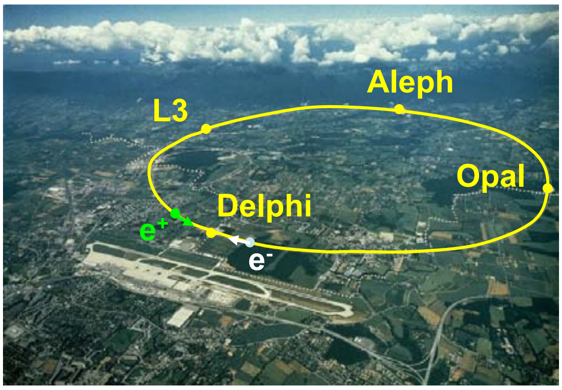
$$\boxed{\sigma(e^+e^-\to Z\to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff}} \qquad (2)$$

where f is the final state fermion flavour:

(The relation to the non-relativistic form of the part II course is given in the appendix)

Electroweak Measurements at LEP

★The **L**arge **E**lectron **P**ositron (LEP) Collider at CERN (1989-2000) was designed to make precise measurements of the properties of the Z and W bosons.



- 26 km circumference accelerator straddling French/Swiss boarder
- Electrons and positrons collided at 4 interaction points
- 4 large detector collaborations (each with 300-400 physicists):
  ALEPH,
  DELPHI,
  L3,
  OPAL

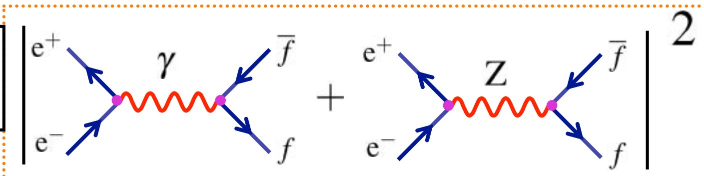
Basically a large Z and W factory:

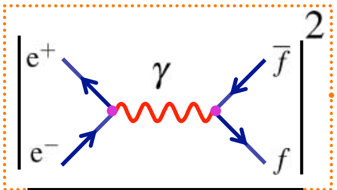
★ 1989-1995: Electron-Positron collisions at $\sqrt{s}$ = 91.2 GeV
- 17 Million Z bosons detected

★ 1996-2000: Electron-Positron collisions at $\sqrt{s}$ = 161-208 GeV
- 30000 W^+W^- events detected

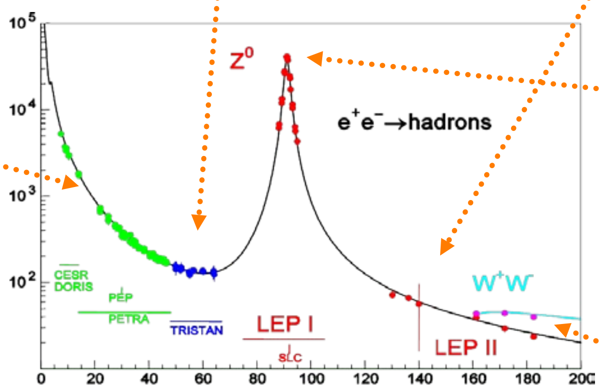
e^+e^- Annihilation in Feynman Diagrams

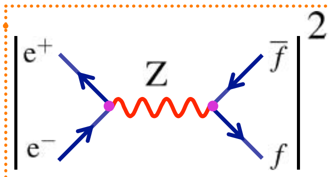
In general e^+e^- annihilation involves both photon and Z exchange : + interference





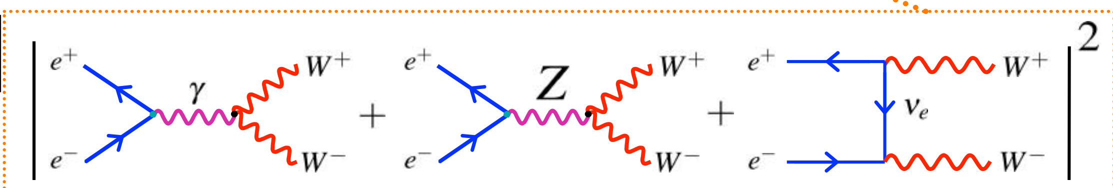
Well below Z: photon exchange dominant





At Z resonance: Z exchange dominant

High energies: WW production



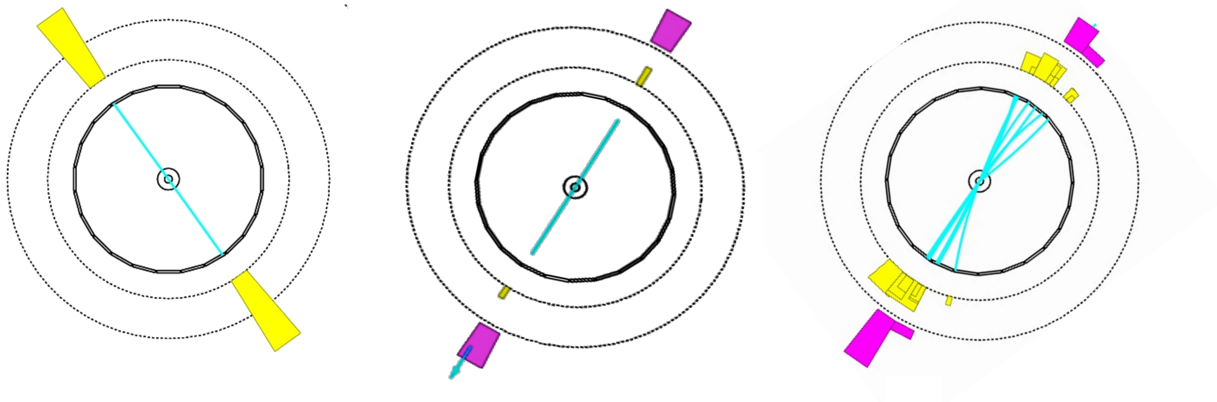
Cross Section Measurements

★ At Z resonance mainly observe four types of event:

$$e^+e^- \to Z \to e^+e^- \qquad e^+e^- \to Z \to \mu^+\mu^- \qquad e^+e^- \to Z \to \tau^+\tau^-$$
$$e^+e^- \to Z \to q\bar{q} \to \text{hadrons}$$

★ Each has a distinct topology in the detectors, e.g.

$$e^+e^- \to Z \to e^+e^- \qquad e^+e^- \to Z \to \mu^+\mu^- \qquad e^+e^- \to Z \to \text{hadrons}$$



★ To work out cross sections, first count events of each type

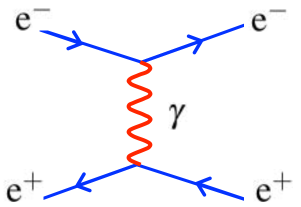
★ Then need to know "integrated luminosity" of colliding beams, i.e. the relation between cross-section and expected number of interactions

$$N_{\text{events}} = \mathscr{L}\sigma$$

★ To calculate the integrated luminosity need to know numbers of electrons and positrons in the colliding beams and the exact beam profile

- very difficult to achieve with precision of better than 10%

★ Instead "normalise" using another type of event:

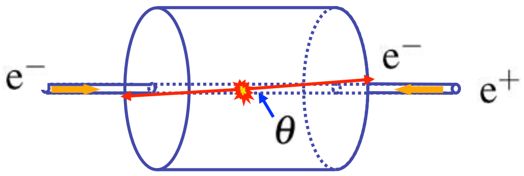


♦ Use the QED Bhabha scattering process

♦ QED, so cross section can be calculated very precisely

♦ Very large cross section – small statistical errors

♦ Reaction is very forward peaked – i.e. the electron tends not to get deflected much



$$\frac{d\sigma}{d\Omega} \propto \frac{1}{q^4} \propto \frac{1}{\sin^4\theta/2} \quad \Rightarrow \quad \boxed{\frac{d\sigma}{d\theta} \propto \frac{1}{\theta^3}}$$

Photon propagator

e.g. see handout 5

♦ Count events where the electron is scattered in the very forward direction

$$N_{\text{Bhabha}} = \mathscr{L}\sigma_{\text{Bhabha}} \quad \Rightarrow \quad \mathscr{L}$$

σ_{Bhabha} known from QED calc.

★ Hence all other cross sections can be expressed as

$$\boxed{\sigma_i = \frac{N_i}{N_{\text{Bhabha}}}\sigma_{\text{Bhabha}}} \quad \Rightarrow$$

Cross section measurements Involve just event counting !

Measurements of the Z Line-shape

★ Measurements of the Z resonance lineshape determine:

- m_Z : peak of the resonance
- Γ_Z : FWHM of resonance
- Γ_f : Partial decay widths
- N_ν : Number of light neutrino generations

★ Measure cross sections to different final states versus C.o.M. energy $\sqrt{s}$

★ Starting from

$$\sigma(e^+e^- \to Z \to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \qquad (3)$$

maximum cross section occurs at $\sqrt{s} = m_Z$ with peak cross section equal to

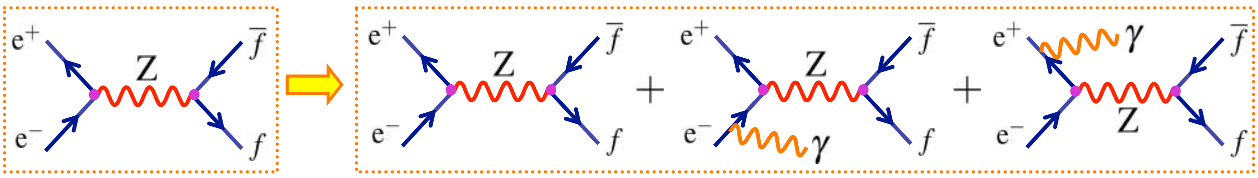
$$\boxed{\sigma^0_{f\bar{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}}$$

★ Cross section falls to half peak value at $\sqrt{s} \approx m_z \pm \frac{\Gamma_Z}{2}$ which can be seen immediately from eqn. (3)

★ Hence $\Gamma_Z = \frac{\hbar}{\tau_Z} = \text{FWHM of resonance}$

★ In practise, it is not that simple, QED corrections distort the measured line-shape

★ One particularly important correction: initial state radiation (ISR)



★ Initial state radiation reduces the centre-of-mass energy of the e^+e^- collision

$e^+ \xrightarrow{E} \;\xleftarrow{E} e^-$ $\quad \sqrt{s} = 2E$

becomes

$\xrightarrow{E} \;\xleftarrow{E-E_\gamma}$ $\quad \sqrt{s'} \approx 2E(1-\frac{E_\gamma}{2E})$

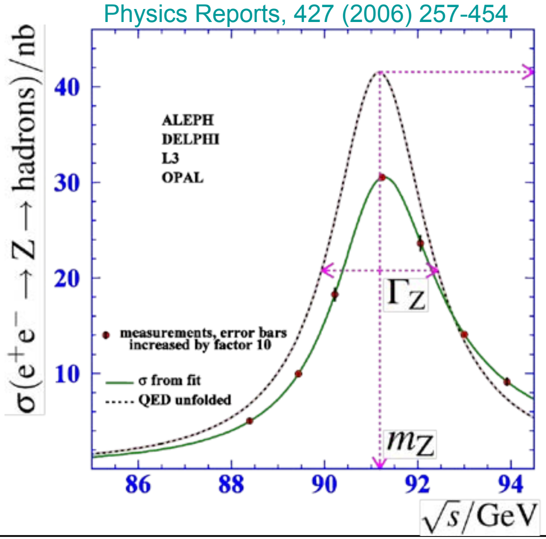
★ Measured cross section can be written:

$$\sigma_{\text{meas}}(E) = \int \sigma(E')f(E',E)\mathrm{d}E'$$

Probability of e+e- colliding with C.o.M. energy E' when C.o.M energy before radiation is E

★ Fortunately can calculate $f(E',E)$ very precisely, just QED, and can then obtain Z line-shape from measured cross section

Physics Reports, 427 (2006) 257-454



★ In principle the measurement of m_Z and Γ_Z is rather simple: run accelerator at different energies, measure cross sections, account for ISR, then find peak and FWHM

$$\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}} \qquad \boxed{\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}}$$

★ 0.002 % measurement of m_Z !

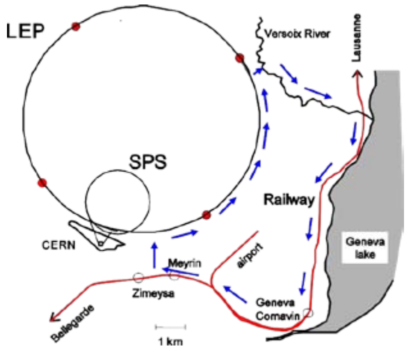
★ To achieve this level of precision – need to know energy of the colliding beams to better than 0.002 % : sensitive to unusual systematic effects…

Moon:

- As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- The nominal radius of the accelerator of 4.3 km varies by ±0.15 mm
- Changes beam energy by ~10 MeV : need to correct for tidal effects !

Trains:

- Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- Travelling via the Versoix river and using the LEP ring as a conductor.
- Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- LEP beam energy changes by ~10 MeV



Number of generations

★ Total decay width measured from Z line-shape: $\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}$

★ If there were an additional 4th generation would expect $Z \to \nu_4\overline{\nu}_4$ decays even if the charged leptons and fermions were too heavy (i.e. > $m_Z/2$)

★ Total decay width is the sum of the partial widths:

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\mathrm{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

★ Although don't observe neutrinos, $Z \to \nu\overline{\nu}$ decays affect the Z resonance shape for all final states

★ For all other final states can determine partial decay widths from peak cross sections:

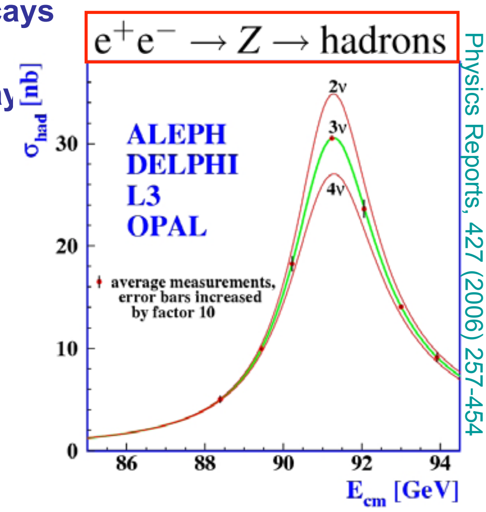
$$\sigma^0_{f\overline{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

★ Assuming lepton universality:

$$\Gamma_Z = 3\Gamma_{\ell\ell} + \Gamma_{\mathrm{hadrons}} + \boxed{N_\nu}\Gamma_{\nu\nu}$$

measured from Z lineshape | measured from peak cross sections | calculated, e.g. question 26

$$\Rightarrow \boxed{N_\nu = 2.9840 \pm 0.0082}$$



★ ONLY 3 GENERATIONS (unless a new 4th generation neutrino has very large mass)

Forward-Backward Asymmetry

★ On page 495 we obtained the expression for the differential cross section:

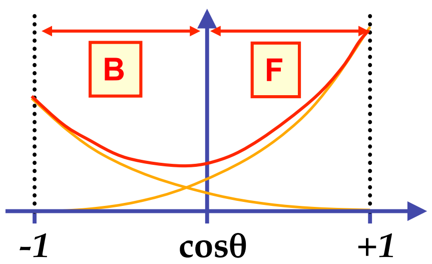
$$\langle|M_{fi}|\rangle^2 \propto [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2](1+\cos^2\theta)+[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2]\cos\theta$$

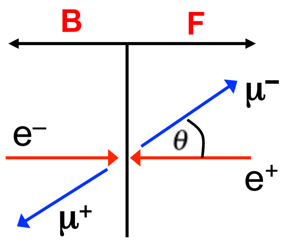
★ The differential cross sections is therefore of the form:

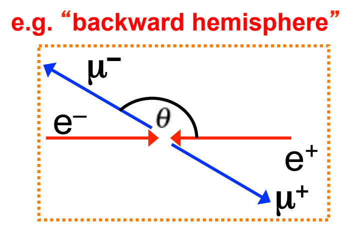
$$\frac{d\sigma}{d\Omega} = \kappa\times[A(1+\cos^2\theta)+B\cos\theta] \qquad \left\{\begin{array}{l} A = [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2] \\ B = [(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2] \end{array}\right.$$

★ Define the FORWARD and BACKWARD cross sections in terms of angle incoming electron and out-going particle

$$\boxed{\sigma_F \equiv \int_0^1 \frac{d\sigma}{d\cos\theta}d\cos\theta} \qquad \boxed{\sigma_B \equiv \int_{-1}^0 \frac{d\sigma}{d\cos\theta}d\cos\theta}$$

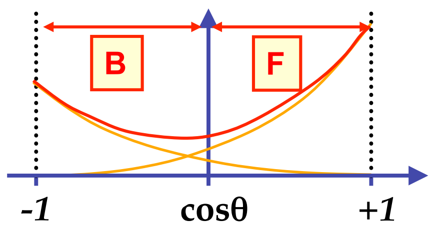






★ The level of asymmetry about **cosθ=0** is expressed in terms of the Forward-Backward Asymmetry

$$A_{FB} = \frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B}$$



• Integrating equation (1):

$$\sigma_F = \kappa\int_0^1[A(1+\cos^2\theta)+B\cos\theta]d\cos\theta = \kappa\int_0^1[A(1+x^2)+Bx]dx = \kappa\left(\frac{4}{3}A+\frac{1}{2}B\right)$$

$$\sigma_B = \kappa\int_{-1}^0[A(1+\cos^2\theta)+B\cos\theta]d\cos\theta = \kappa\int_{-1}^0[A(1+x^2)+Bx]dx = \kappa\left(\frac{4}{3}A-\frac{1}{2}B\right)$$

★ Which gives:

$$A_{FB} = \frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4}\left[\frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2}\right]\cdot\left[\frac{(c_L^\mu)^2-(c_R^\mu)^2}{(c_L^\mu)^2+(c_R^\mu)^2}\right]$$

★ This can be written as

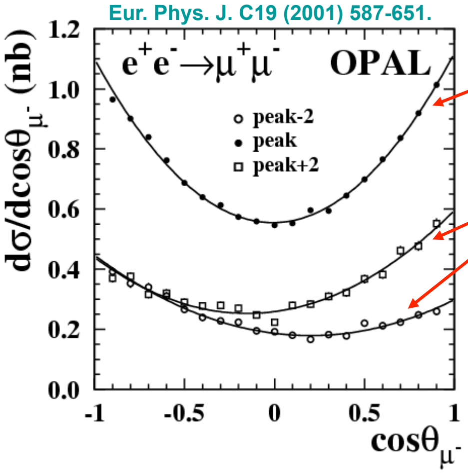
$$\boxed{A_{FB} = \frac{3}{4}A_eA_\mu} \quad \text{with} \quad \boxed{A_f \equiv \frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2} = \frac{2c_V^fc_A^f}{(c_V^f)^2+(c_A^f)^2}} \qquad (4)$$

★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

Measured Forward-Backward Asymmetries

★ Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g. $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$

OPAL Collaboration,
Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_W \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$ leads to a larger asymmetry

★LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

★To relate these measurements to the couplings uses $A_{\mathrm{FB}} = \frac{3}{4}A_e A_\mu$

★ In all cases asymmetries depend on A_e

★ To obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4}A_e^2$ (also see Appendix II for A_{LR})

Determination of the Weak Mixing Angle

★ From LEP : $A_{FB}^{0,f} = \frac{3}{4}A_e A_f$

★ From SLC : $A_{LR} = A_e$

$A_e, A_\mu, A_\tau, \ldots$

Putting everything together

$$\boxed{\begin{aligned} A_e &= 0.1514 \pm 0.0019 \\ A_\mu &= 0.1456 \pm 0.0091 \\ A_\tau &= 0.1449 \pm 0.0040 \end{aligned}}$$

includes results from other measurements

with $$A_f \equiv \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} = 2\frac{c_V/c_A}{1+(c_V/c_A)^2}$$

★ Measured asymmetries give ratio of vector to axial-vector Z coupings.

★ In SM these are related to the weak mixing angle

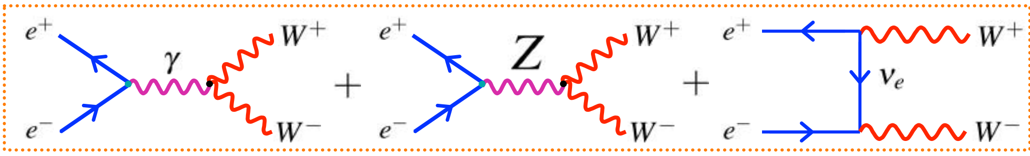
$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q\sin^2\theta_W}{I_W^3} = 1 - \frac{2Q}{I_3}\sin^2\theta_W = 1 - 4|Q|\sin^2\theta_W$$

★ Asymmetry measurements give precise determination of $\sin^2\theta_W$

$$\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$$

W^+W^- Production

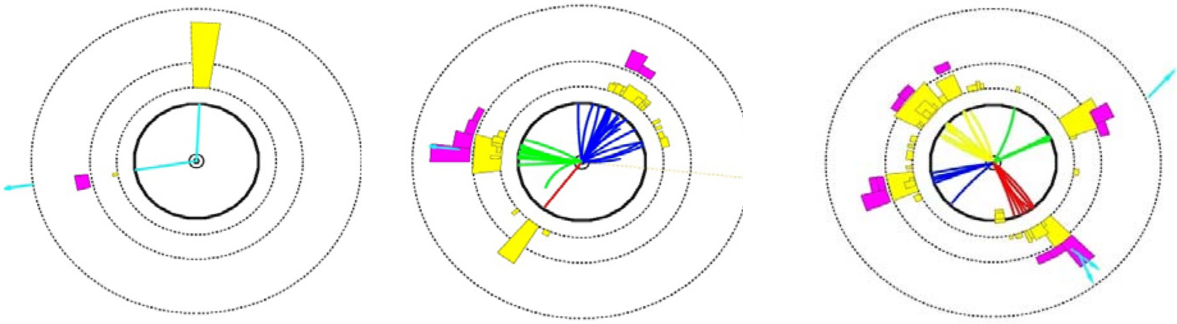
★ From 1995-2000 LEP operated above the threshold for W-pair production
★ Three diagrams "CC03" are involved



★ W bosons decay (p.459) either to leptons or hadrons with branching fractions:

$$Br(W^- \to \text{hadrons}) \approx 0.67 \qquad Br(W^- \to e^-\overline{\nu}_e) \approx 0.11$$
$$Br(W^- \to \mu^-\overline{\nu}_\mu) \approx 0.11 \qquad Br(W^- \to \tau^-\overline{\nu}_\tau) \approx 0.11$$

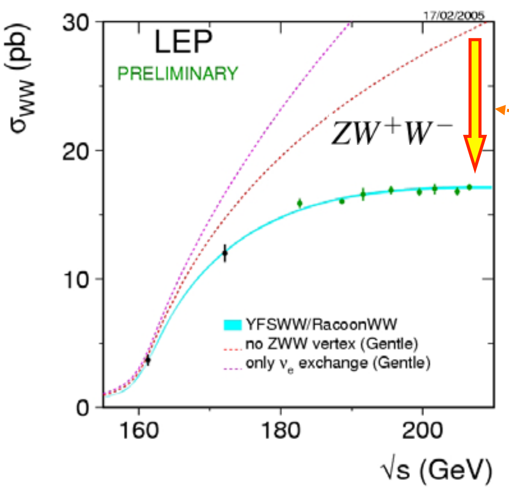
★ Gives rise to three distinct topologies



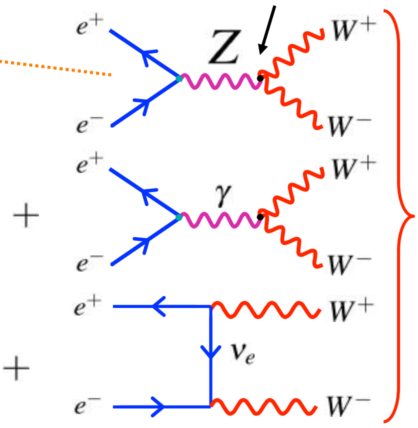
$W^+W^- \to \ell^+\nu\ell^-\overline{\nu}$ $\qquad$ $W^+W^- \to q\overline{q}\ell\nu$ $\qquad$ $W^+W^- \to q\overline{q}q\overline{q}$

$e^+e^- \to W^+W^-$ Cross Section

★ Measure cross sections by **counting events** and normalising to low angle Bhabha scattering events



★ Data consistent with SM expectation
★ Provides a direct test of ZW^+W^- vertex



★ Recall that without the Z diagram the cross section violates unitarity
★ Presence of Z fixes this problem

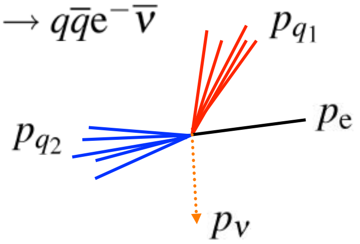
W-mass and W-width

★ Unlike $e^+e^- \to Z$, the process $e^+e^- \to W^+W^-$ is not a resonant process

⟹ Different method to measure W-boson Mass

•Measure energy and momenta of particles produced in the W boson decays, e.g.

$$W^+W^- \to q\bar{q}e^-\bar{\nu}$$

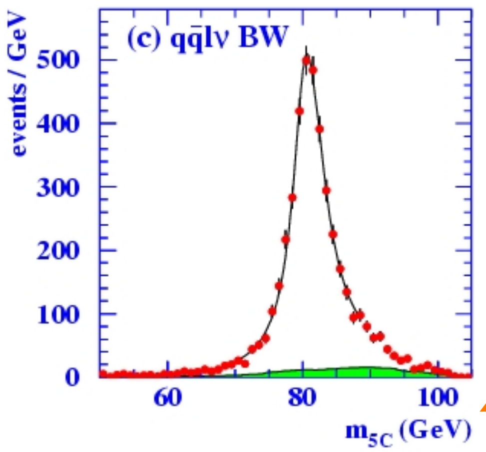


- Neutrino four-momentum from energy-momentum conservation !

$$p_{q_1} + p_{q_2} + p_e + p_\nu = (\sqrt{s}, 0)$$

- Reconstruct masses of two W bosons

$$M_+^2 = E^2 - \vec{p}^2 = (p_{q_1} + p_{q_2})^2$$
$$M_-^2 = E^2 - \vec{p}^2 = (p_e + p_\nu)^2$$



$$\boxed{\approx \tfrac{1}{2}(M_+ + M_-)}$$

★ Peak of reconstructed mass distribution gives

$$\boxed{m_W = 80.376 \pm 0.033\,\text{GeV}}$$

★ Width of reconstructed mass distribution gives:

$$\boxed{\Gamma_W = 2.196 \pm 0.083\,\text{GeV}}$$

Does not include measurements from Tevatron at Fermilab

The Higgs Mechanism

(For proper discussion of the Higgs mechanism see the Gauge Field Theory minor option)

★ In the handouts 9 and 13 introduced the ideas of gauge symmetries and EW unification. However, as it stands there is a small problem; this only works for massless gauge bosons. Introducing masses in any naïve way violates the underlying gauge symmetry.

★The Higgs mechanism provides a way of giving the gauge bosons mass

★ In this handout motivate the main idea behind the Higgs mechanism (however not possible to give a rigourous treatment outside of QFT). So resort to analogy:

Analogy:

- Consider Electromagnetic Radiation propagating through a plasma
- Because the plasma acts as a polarisable medium obtain "dispersion relation"

From IB EM:

$$n^2 = 1 - \frac{n_e e^2}{\varepsilon_0 m_e \omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

n = refractive index
ω = angular frequency
ω_p = plasma frequency

- Because of interactions with the plasma, wave-groups only propagate if they have frequency/energy greater than some minimum value

$$E > E_0 = \hbar\omega_p$$

- Above this energy waves propagate with a group velocity $v_g = \dfrac{c^2}{v_p} = nc$

- Dropping the subscript and using the previous expression for n

$$v^2 = c^2 n^2 = c^2\left(1 - \frac{\hbar^2\omega_p^2}{\hbar^2\omega^2}\right) = c^2\left(1 - \frac{E_0^2}{E^2}\right)$$

- Rearranging gives

$$\frac{E_0^2}{E^2} = 1 - \frac{v^2}{c^2} \quad \Rightarrow \quad \boxed{E = E_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \gamma m c^2}$$

with $m = E_0/c^2$

- **Massless photons propagating through a plasma behave as massive particles propagating in a vacuum !**

The Higgs Mechanism

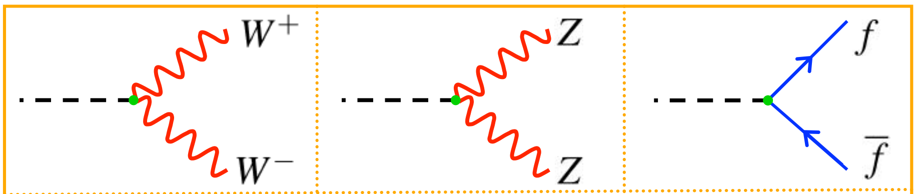
★ Propose a scalar (spin 0) field with a **non-zero vacuum expectation value (VEV)**

Massless Gauge Bosons propagating through the vacuum with a non-zero Higgs VEV correspond to massive particles.

★ The Higgs is **electrically neutral** but carries **weak hypercharge of 1/2**
★ The photon does not couple to the Higgs field and remains massless
★ The W bosons and the Z couple to weak hypercharge and become massive

★ The Higgs mechanism results in absolute **predictions for masses of gauge bosons**
★ In the SM, fermion masses are also ascribed to interactions with the Higgs field
- however, here no prediction of the masses – **just put in by hand**

Feynman Vertex factors:



$ig_W m_W g^{\mu\nu}$	$ig_Z m_Z g^{\mu\nu}$	$-i\dfrac{g_W}{2m_W}m_f$

★ Within the SM of Electroweak unification with the Higgs mechanism:

⇒ **Relations between standard model parameters**

$$m_W = \left(\frac{\pi\alpha_{em}}{\sqrt{2}G_F}\right)^{\frac{1}{2}}\frac{1}{\sin\theta_W} \qquad m_Z = \frac{m_W}{\cos\theta_W}$$

★ Hence, if you know any three of : $\alpha_{em}, G_F, m_W, m_Z, \sin\theta_W$ predict the other two.

Precision Tests of the Standard Model

★ From LEP and elsewhere have precise measurements – can test predictions of the Standard Model !

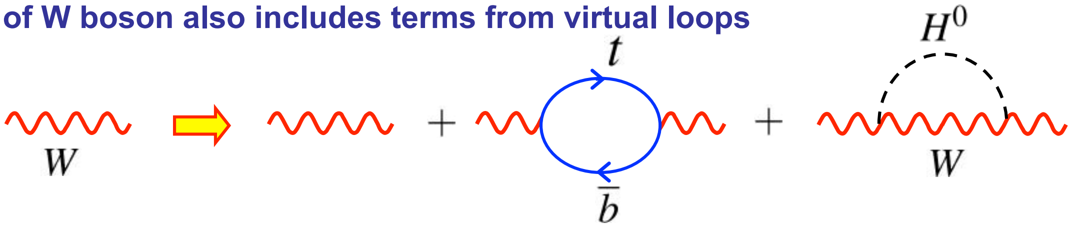
•e.g. predict: $\boxed{m_W = m_Z \cos\theta_W}$ measure $\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}}$ $\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$

•Therefore expect:

$\boxed{m_W = 79.946 \pm 0.008\,\mathrm{GeV}}$ but measure $\boxed{m_W = 80.376 \pm 0.033\,\mathrm{GeV}}$

★ Close, but not quite right – but have only considered lowest order diagrams

★ Mass of W boson also includes terms from virtual loops



$$\boxed{m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)}$$

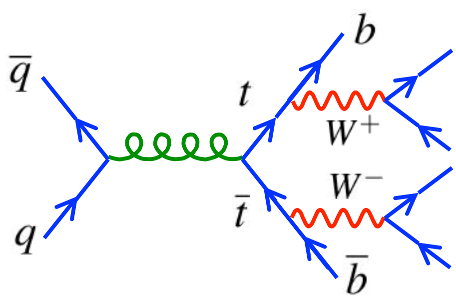
★ Above "discrepancy" due to these virtual loops, i.e. by making very high precision measurements become sensitive to the masses of particles inside the virtual loops !

The Top Quark

★ From virtual loop corrections and precise LEP data can predict the top quark mass:

$$\boxed{m_t^{\mathrm{loop}} = 173 \pm 11\,\mathrm{GeV}}$$

★ In 1994 top quark observed at the Tevatron proton anti-proton collider at Fermilab – with the predicted mass !



★ The top quark almost exclusively decays to a bottom quark since

$$|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2$$

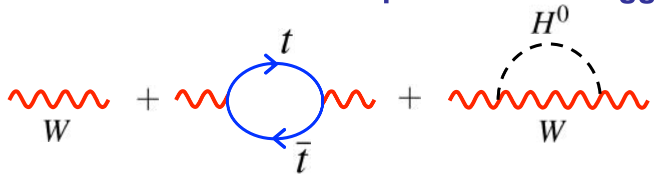
★ Complicated final state topologies:

$$t\bar{t} \to b\bar{b}q\bar{q}q\bar{q} \to 6\ \mathrm{jets}$$
$$t\bar{t} \to b\bar{b}q\bar{q}\ell\nu \to 4\ \mathrm{jets} + \ell + \nu$$
$$t\bar{t} \to b\bar{b}\ell\nu\ell\nu \to 2\ \mathrm{jets} + 2\ell + 2\nu$$

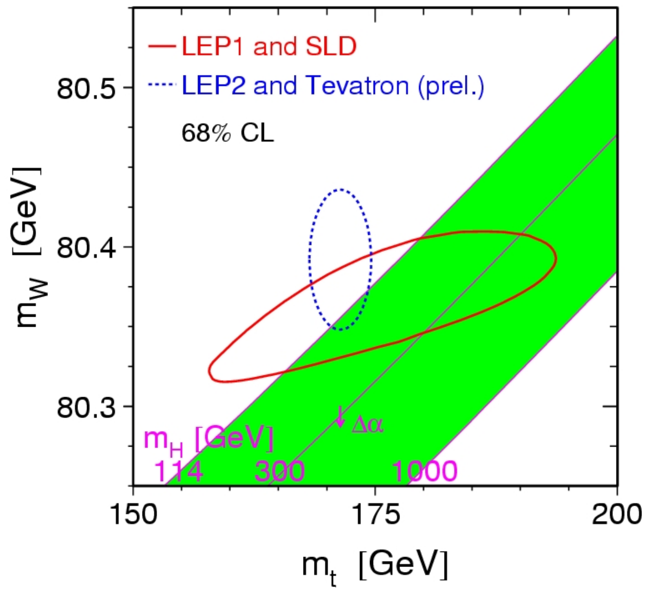
★ Mass determined by direct reconstruction (see W boson mass)

$$\boxed{m_t^{\mathrm{meas}} = 174.2 \pm 3.3\,\mathrm{GeV}}$$

★ But the W mass also depends on the Higgs mass (albeit only logarithmically)



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$



★ Measurements are sufficiently precise to have some sensitivity to the Higgs mass

★ Direct and indirect values of the top and W mass can be compared to prediction for different Higgs mass

- **Direct:** W and top masses from direct reconstruction
- **Indirect:** from SM interpretation of Z mass, θ_W etc. and

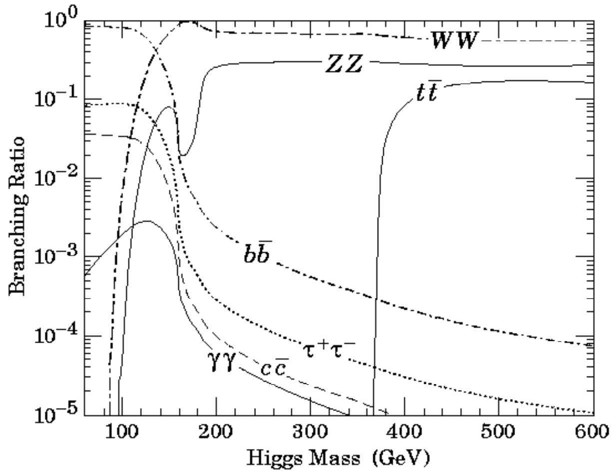
★ Data favour a light Higgs:

$$\boxed{m_H < 200\,\mathrm{GeV}}$$

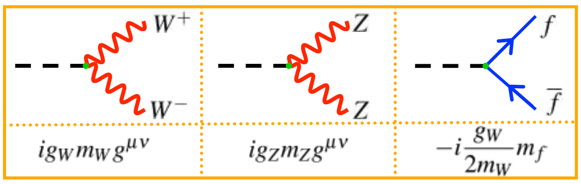
Hunting the Higgs

★ The Higgs boson is an essential part of the Standard Model – but does it exist ?

★ Consider the search at LEP. Need to know how the Higgs decays



- Higgs boson couplings proportional to mass



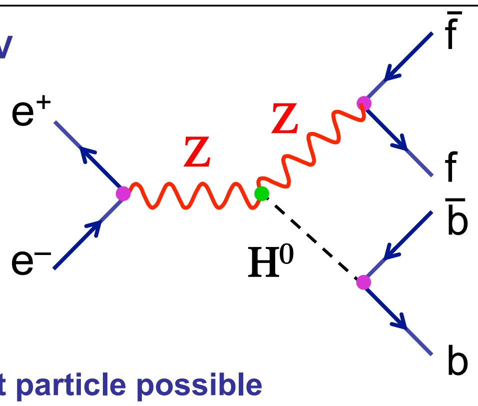
- Higgs decays predominantly to heaviest particles which are energetically allowed (Question 30)

$m_H < 2m_W$	mainly $H^0 \to b\bar{b}$	+ approx 10% $H^0 \to \tau^+\tau^-$
$2m_W < m_H < 2m_t$	almost entirely	$H^0 \to W^+W^- \;+\; H^0 \to ZZ$
$m_H > 2m_t$	either	$H^0 \to W^+W^-,\ H^0 \to ZZ,\ H^0 \to t\bar{t}$

A Hint from LEP ?

★ LEP operated with a C.o.M. energy upto 207 GeV

★ For this energy (assuming the Higgs exists) the main production mechanism would be the "Higgsstrahlung" process



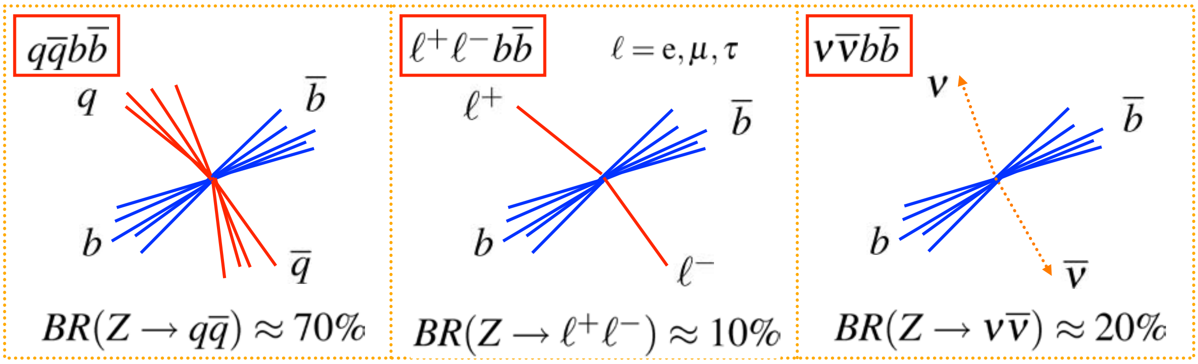
★ Need enough energy to make a Z and H; therefore could produce the Higgs boson if

$$m_H < 207\,\mathrm{GeV} - m_Z$$

i.e. if $\quad m_H < 116\,\mathrm{GeV}$

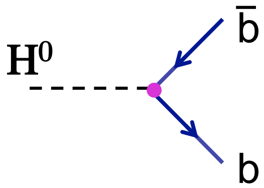
★ The Higgs predominantly decays to the heaviest particle possible

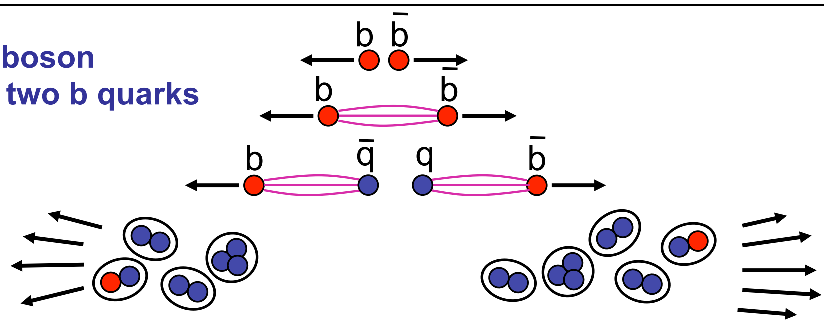
★ For $m_H < 116\,\mathrm{GeV}$ this is the b-quark (not enough mass to decay to WW/ZZ/tt)



Tagging the Higgs Boson Decays

★ One signature for a Higgs boson decay is the production of two b quarks



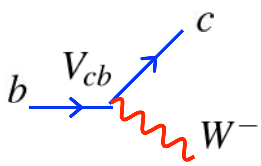


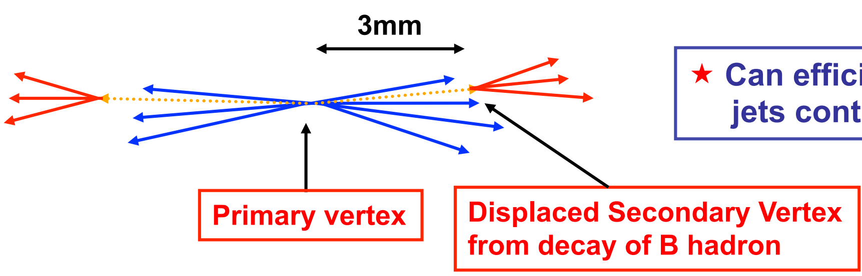
★ Each jet will contain one b-hadron which will decay weakly

★ Because V_{cb} is small $(V_{cb} \approx 0.04)$ hadrons containing b-quarks are relatively long-lived

★ Typical lifetimes of $\tau \sim 1 \times 10^{-12}\,\mathrm{s}$

★ At LEP b-hadrons travel approximately **3mm** before decaying

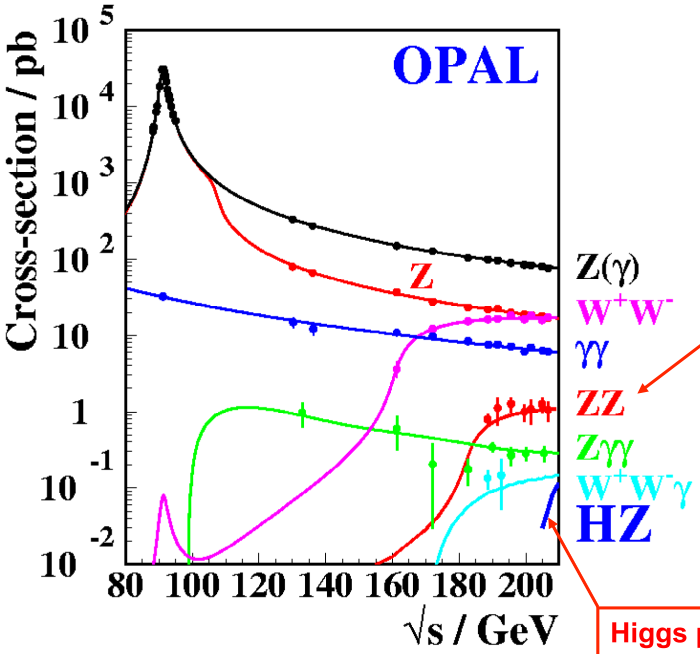




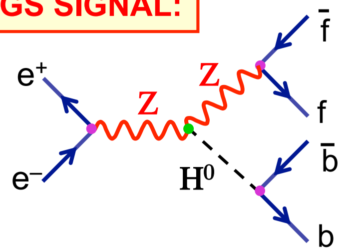
★ Can efficiently identify jets containing b quarks

★ Clear experimental signature, but small cross section, e.g. for $m_H \approx 115\,\mathrm{GeV}$ would only produce a few tens of $\mathrm{e^+e^-} \to H^0$ events at LEP

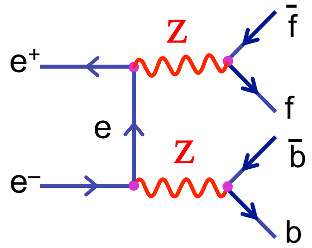
★ In addition, there are large "backgrounds"



HIGGS SIGNAL:

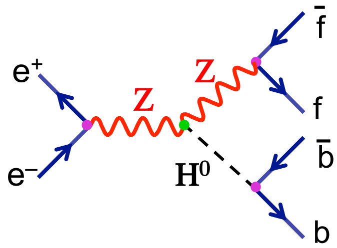


MAIN BACKGROUND:



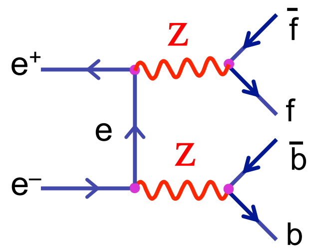
Higgs production cross section (m_H=115 GeV)

★ The only way to distinguish



$m_{b\bar{b}} = m_H$

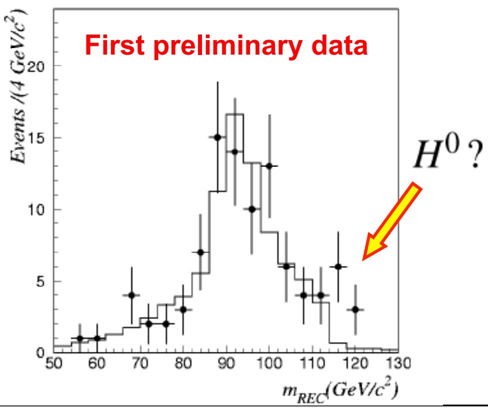
from



$m_{b\bar{b}} = m_Z$

is the from the invariant mass of the jets from the boson decays

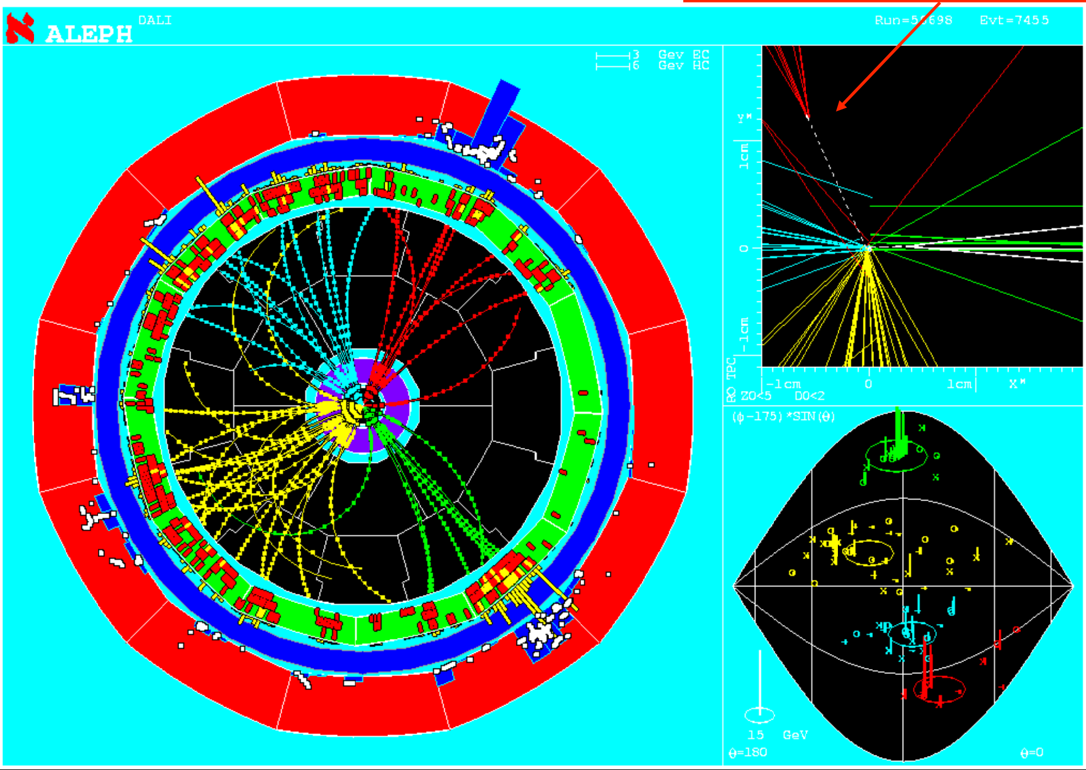
★ In 2000 (the last year of LEP running) the ALEPH experiment reported an excess of events consistent with being a Higgs boson with mass 115 GeV



- ALEPH found 3 events which were high relative probability of being signal
- L3 found 1 event with high relative probability of being signal
- OPAL and DELPHI found none

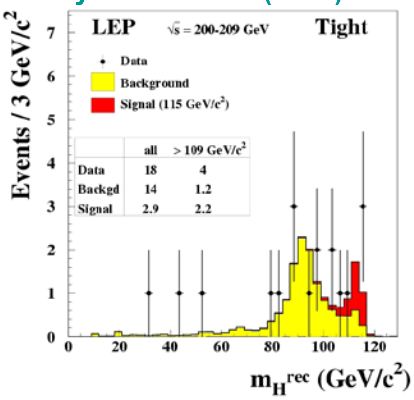
Example event:

Displaced vertex from b-decay



Combined LEP Results

Phys. Lett. B565 (2003) 61-75



★ **Final combined LEP results fairly inconclusive**

★ **A hint rather than strong evidence…**

★ **All that can be concluded:**

$$\boxed{m_H > 114\,\text{GeV}}$$

★ **Over to the LHC…**

★ **Preliminary results from early LHC data set upper limits on Higgs mass**

$$m_H < 140\,\text{GeV}$$

★ **The net is closing in….**

With the 2011 LHC data, the SM Higgs will either be found or excluded
A major discovery may be just around the corner…

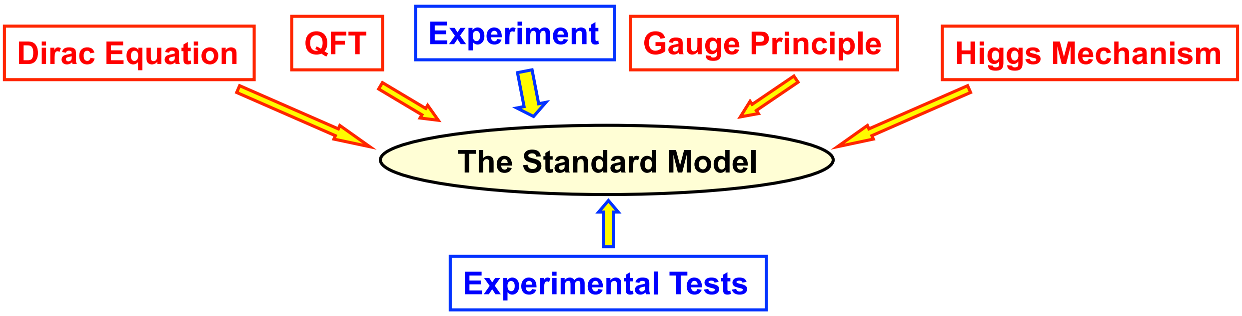
★ **The SM will then be complete…**

Concluding Remarks

★ In this course (I believe) we have covered almost all aspects of modern particle physics (and to a fairly high level)

★ The Standard Model of Particle Physics is one of the great scientific triumphs of the late 20^{th} century

★ Developed through close interplay of experiment and theory



★ Modern experimental particle physics provides many precise measurements. and the **Standard Model** successfully describes all current data !

★ Despite its great success, we should not forget that it is just a model; a collection of beautiful theoretical ideas cobbled together to fit with experimental data.

★ There are many issues / open questions…

The Standard Model : Problems/Open Questions

★ The Standard Model has too many free parameters:

$$m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, m_e, m_\mu, m_\tau, m_d, m_s, m_b, m_u, m_c, m_t$$

$$\theta_{12}, \theta_{13}, \theta_{23}, \delta \quad + \quad \lambda, A, \rho, \eta \qquad e, G_F, \theta_W, \alpha_S \qquad m_H, \theta_{CP}$$

★ Why three generations ?
★ Why $SU(3)_c \times SU(2)_L \times U(1)$?
★ Unification of the Forces
★ Origin of CP violation in early universe ?
★ What is Dark Matter ?
★ Why is the weak interaction V-A ?
★ Why are neutrinos so light ?
★ Does the Higgs exist ? + gives rise to huge cosmological constant
★ Ultimately need to include gravity

Over the last 25 years particle physics has progressed enormously.

In the next 10 years we will almost certainly have answers to some of the above questions – maybe not the ones we expect…

The End

Appendix I: Non-relativistic Breit-Wigner

★ For energies close to the peak of the resonance, can write $\sqrt{s} = m_Z + \Delta$

$$s = m_Z^2 + 2m_Z\Delta + \Delta^2 \approx m_Z^2 + 2m_Z\Delta \qquad \text{for} \quad \Delta \ll m_Z$$

so with this approximation

$$\begin{aligned}(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2 \approx (2m_Z\Delta)^2 + m_Z^2\Gamma_Z^2 &= 4m_Z^2(\Delta + \tfrac{1}{4}\Gamma_Z^2) \\ &= 4m_Z^2[(\sqrt{s}-m_Z)^2 + \tfrac{1}{4}\Gamma_Z^2]\end{aligned}$$

★ Giving: $\sigma(e^+e^- \to Z \to f\overline{f}) \approx \dfrac{3\pi}{m_Z^4}\dfrac{s}{(\sqrt{s}-m_Z)^2 + \frac{1}{4}\Gamma_Z^2}\Gamma_e\Gamma_f$

★ Which can be written:

$$\sigma(E) = \frac{g\lambda_e^2}{4\pi}\frac{\Gamma_i\Gamma_f}{(E-E_0)^2 + \frac{1}{4}\Gamma^2} \qquad (3)$$

$\Gamma_i,\ \Gamma_f$: are the partial decay widths of the initial and final states

$E,\ E_0$: are the centre-of-mass energy and the energy of the resonance

$g = \frac{(2J_Z+1)}{(2S_e+1)(2S_e+1)}$ is the spin counting factor $g = \frac{3}{2\times 2}$

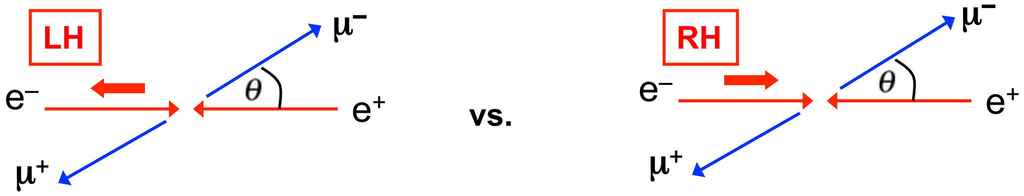
$\lambda_e = \frac{2\pi}{E}$: is the Compton wavelength (natural units) in the C.o.M of either initial particle

★ This is the **non-relativistic** form of the Breit-Wigner distribution first encountered in the part II particle and nuclear physics course.

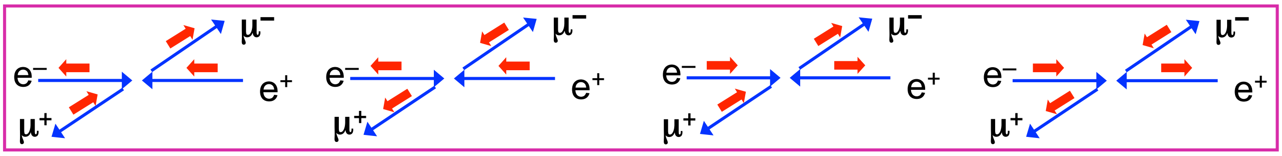
Appendix II: Left-Right Asymmetry, A_{LR}

★ At an e^+e^- linear collider it is possible to produce polarized electron beams
e.g. SLC linear collider at SLAC (California), **1989-2000**

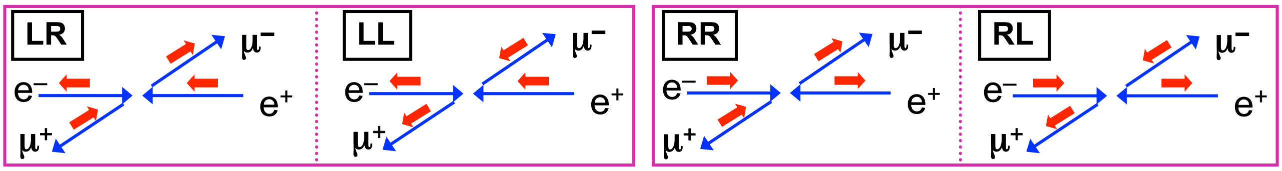
★ Measure cross section for any process for LH and RH electrons separately



▪ At LEP measure total cross section: sum of 4 helicity combinations:



▪ At SLC, by choosing the polarization of the electron beam are able to measure cross sections **separately** for LH / RH electrons



★ Averaging over the two possible polarization states of the positron for a given electron polarization:

$$\langle|M_L|\rangle^2 = \tfrac{1}{2}(|M_{LL}|^2 + |M_{LR}|^2) \qquad \langle|M_R|\rangle^2 = \tfrac{1}{2}(|M_{RL}|^2 + |M_{RR}|^2)$$

$$\Rightarrow \quad \boxed{\sigma_L = \tfrac{1}{2}(\sigma_{LR} + \sigma_{LL})} \qquad \boxed{\sigma_R = \tfrac{1}{2}(\sigma_{RR} + \sigma_{RL})}$$

★ Define cross section asymmetry:

$$\boxed{A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}}$$

★ Integrating the expressions on page 494 gives:

$$\sigma_{LL} \propto (c_L^e)^2(c_L^\mu)^2 \quad \sigma_{LR} \propto (c_L^e)^2(c_R^\mu)^2 \quad \sigma_{RL} \propto (c_R^e)^2(c_L^\mu)^2 \quad \sigma_{RR} \propto (c_R^e)^2(c_R^\mu)^2$$

$$\Rightarrow \quad \sigma_L \propto (c_L^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2] \qquad \sigma_R \propto (c_R^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2]$$

$$\boxed{A_{LR} = \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = A_e}$$

★ Hence the Left-Right asymmetry for any cross section depends only on the couplings of the electron