NATURAL SCIENCES TRIPOS Part III

Tuesday 18 January 2000 2.00 to 3.30

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
Particle Physics

Answer **two** *questions only. The approximate number of marks allotted to each part of a question is indicated in the right margin where appropriate. The paper contains THREE sides and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

1 If the charged weak current were scalar in structure, the matrix element for the decay $\pi^- \to \mu^- \overline{\nu}_\mu$ would be of the form

$$M_{\rm fi} = i \frac{G_{\rm F}}{\sqrt{2}} f_\pi \bar{u}(p_2) v(p_3)$$

where f_π is the pion decay constant, p_2 is the 4-momentum of the μ^-, and p_3 is the 4-momentum of the $\overline{\nu}_\mu$.

In the rest frame of the π^-, with the μ^- travelling in the $+z$ direction, show that $\bar{u}_i(p_2) v_j(p_3)$ $(i, j = 1, 2)$ is non-zero for only two of the four possible combinations and interpret this result. Show also that the two non-zero combinations are of magnitude $\sqrt{m_\pi^2 - m_\mu^2}$ where m_π and m_μ are the masses of the π^- and μ^-, respectively. [10]

Given that the decay rate is

$$\Gamma = \frac{p^*}{8\pi m_\pi^2} \langle |M_{\rm fi}|^2 \rangle$$

where p^* is the centre of mass momentum of either final state particle, show that

$$\Gamma = G_{\rm F}^2 f_\pi^2 \frac{(m_\pi^2 - m_\mu^2)^2}{16\pi m_\pi^3} \ .$$

Hence predict the ratio $R = \Gamma(\pi^- \to {\rm e}^- \overline{\nu}_e)/\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)$ for a scalar weak charged current. [6]

Specify the *correct* structure of the charged weak current and indicate the corresponding modification which must be made to the above expression for the matrix element. Given that this modification results in an extra factor of $2m_\mu^2$ in the expression for the decay rate, obtain a new prediction for the ratio R and explain qualitatively why R is so small. [6]

The π^- and K^- mesons have lifetimes of 2.60×10^{-8} s and 1.24×10^{-8} s respectively. The $K^- \to \mu^- \overline{\nu}_\mu$ decay has a branching ratio of 63.5%. The pion and kaon decay constants are $f_\pi = 0.132\,\mathrm{GeV}$ and $f_K = 0.160\,\mathrm{GeV}$, respectively. Calculate the observed $\pi^- \to \mu^- \overline{\nu}_\mu$ and $K^- \to \mu^- \overline{\nu}_\mu$ decay rates and compare with the predicted rates. Draw Feynman diagrams for these two decays and explain what additional factors are needed to obtain agreement between the measured and predicted rates. Explain the physical origin of such factors, and describe briefly their form for interactions involving the three generations of quark flavours. [8]

You may require the following information:

The π^- has spin zero and mass 139.6 MeV. The μ^- and e^- masses are 105.7 MeV and 0.51 MeV, respectively. The K^- meson has spin zero, mass 493.7 MeV and quark content $s\bar{u}$.

$G_\mathrm{F} = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$.

$\hbar = 6.582 \times 10^{-25}$ GeV.s.

The Dirac spinors are

$$u_1 = N\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}, \quad u_2 = N\begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}, \quad v_1 = N\begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, \quad v_2 = N\begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix}$$

where $N = \sqrt{E+m}$.

You *may* also need the Dirac matrices:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

2 Summarise the essential ingredients of QCD and its theoretical formulation. [5]
The eight allowed gluon states are

$$r\bar{b}, r\bar{g}, b\bar{g}, b\bar{r}, g\bar{b}, g\bar{r}, \frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b}), \frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$$

where r, g, b denote the three colour quantum numbers. Show that the coupling $\langle\psi|G|\psi\rangle$ between a quark-antiquark pair is negative when the quark-antiquark pair is in the colour singlet state

$$\psi = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

but positive for any non-singlet colour state. Interpret this result and comment on the overall form of the quark-antiquark potential. [8]

Define the operations of *parity* and *charge conjugation*. Show that all mesons are parity eigenstates, and that certain mesons are charge conjugation eigenstates; obtain expressions for the corresponding eigenvalues. Describe the observed pattern of J^{PC} and flavour quantum numbers for mesons consisting of u, d or s quarks and $\bar{u}, \bar{d}$ or $\bar{s}$ antiquarks. [7]

The K^0 and $\overline{K}^0$ mesons have quark content $d\bar{s}$ and $s\bar{d}$, respectively. Explain how CP eigenstates can be constructed in the $K^0 - \overline{K}^0$ system, and show that, neglecting the effects of CP violation, one of the CP eigenstates can decay to a two-pion final state and the other to a three-pion final state. [10]

3 Write brief accounts of **three** of the following:

(a) electromagnetic and hadronic showers and their detection; [10]

(b) deep-inelastic $e^{\pm}p$ scattering; [10]

(c) non-calorimetric particle identification techniques; [10]

(d) experimental tests of the electroweak Standard Model; [10]

(e) the evidence for a finite neutrino mass. [10]