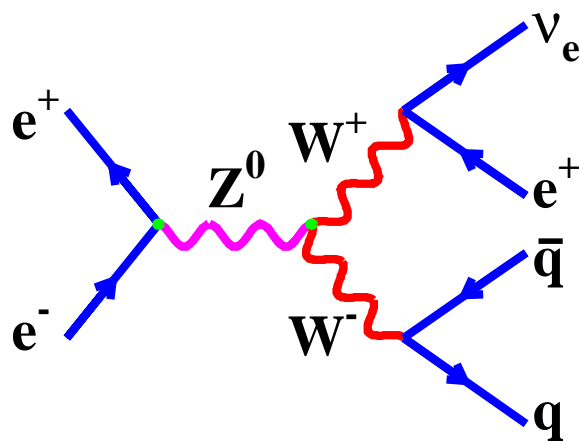
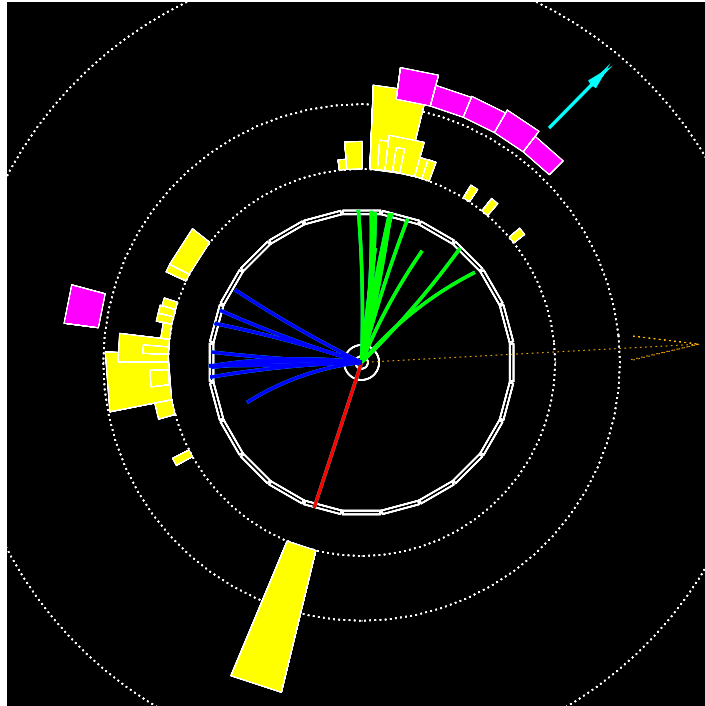


# Particle Physics

Dr M.A. Thomson



## Part II, Lent Term 2004 HANDOUT VI

# The Weak Interaction

- ★ The WEAK interaction accounts for many decays in particle physics e.g.

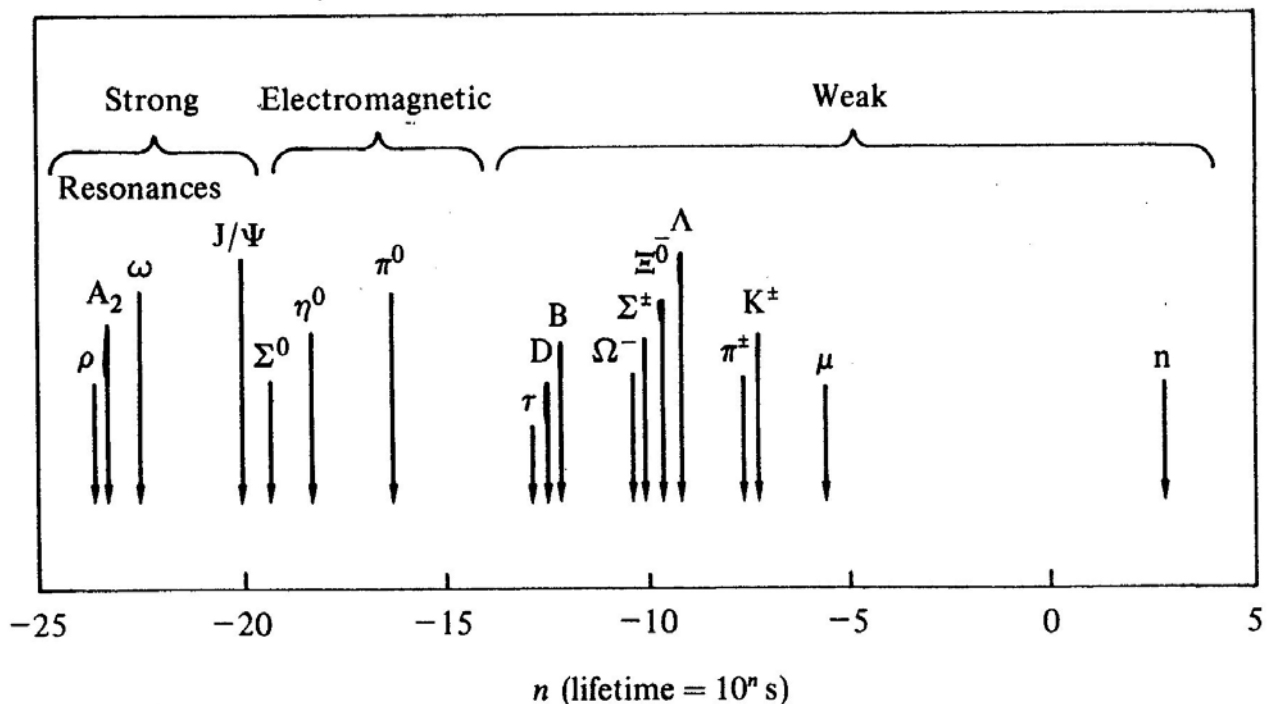
$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu,$$

$$\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau,$$

$$\pi^+ \rightarrow \mu^- \bar{\nu}_\mu,$$

$$n \rightarrow p e^- \bar{\nu}_e, \dots$$

- ★ Characterized by long lifetimes, small cross sections



★ Two types of WEAK interaction

CHARGED CURRENT (CC) -  $W^{\pm}$  Bosons

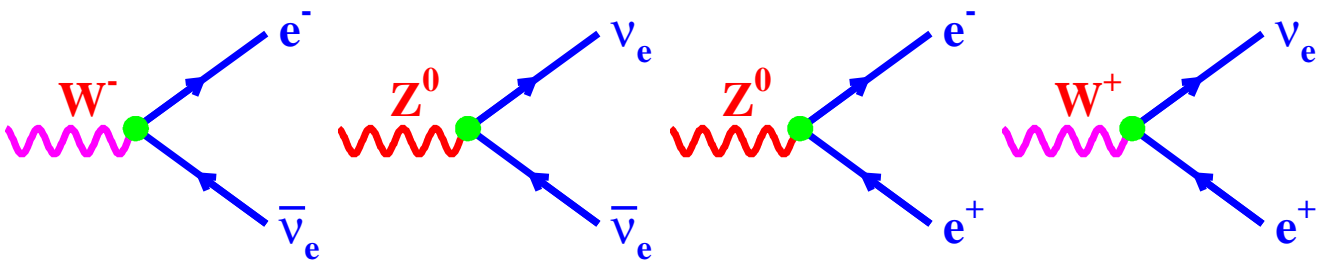
NEUTRAL CURRENT (NC) -  $Z^0$  Boson

★ WEAK force mediated by MASSIVE VECTOR BOSONS:

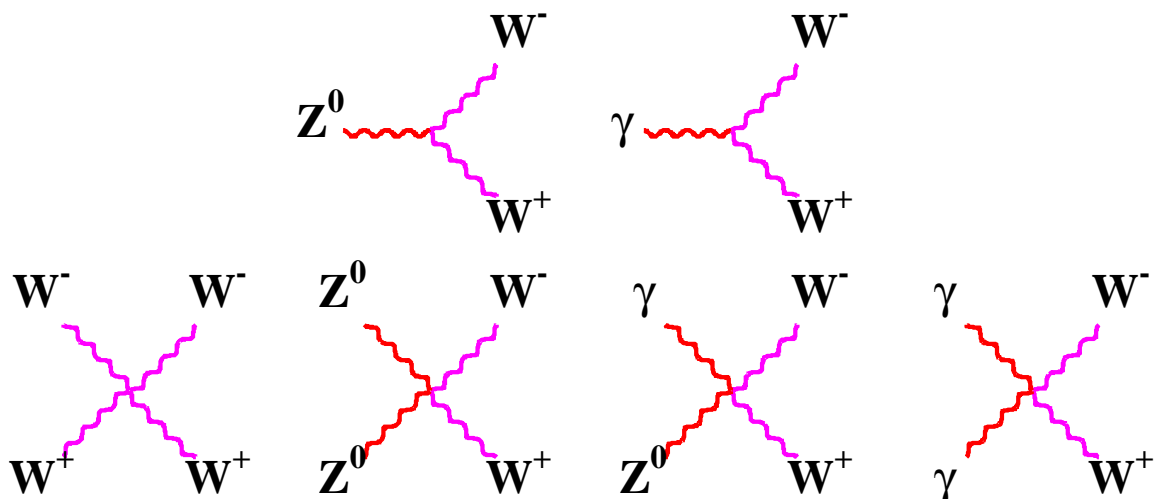
$$M_W \sim 80 \text{ GeV}$$

$$M_{Z^0} \sim 90 \text{ GeV}$$

★ e.g. the WEAK interactions of electrons and electron neutrinos:

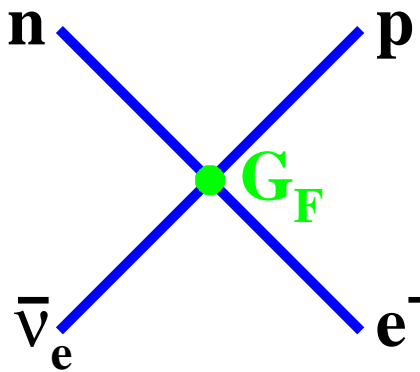


BOSON SELF-INTERACTIONS



★ also interactions with PHOTONS (W-bosons are charged)

# Fermi Theory



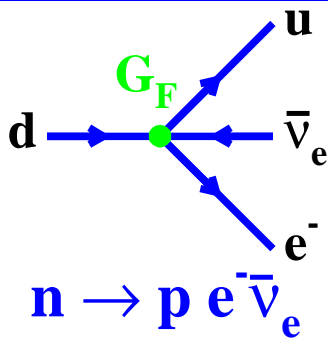
WEAK interaction taken to be a 4-fermion contact interaction

★ *i.e* no propagator

★ coupling strength  $G_F$

★  $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$

## Beta Decay in Fermi Theory



Golden Rule :

$$1/\tau = \Gamma = 2\pi |M_{fi}|^2 \rho(E)$$

$$\text{with } \rho = \frac{dN}{dE}$$

## Phase Space : 2-body vs. 3-body

★ **TWO BODY FINAL STATE:**

$$dN = \frac{E^2}{(2\pi)^3} d\Omega dE$$

(neglecting final state masses). Only consider one of the particles since the other fixed by  $(E, \tilde{p})$  conservation

★ **THREE BODY FINAL STATE (e.g  $\beta$ -decay):**

$$d^2 N = \frac{E_\nu^2}{(2\pi)^3} d\Omega_\nu dE_\nu \frac{E_e^2}{(2\pi)^3} d\Omega_e dE_e$$

now necessary to consider phase space of two of the particles - the third is then given by  $(E, \tilde{p})$  conservation

In Nuclear  $\beta$ -decay the energy released in the nuclear transition,  $E_0$ , is shared between the electron, neutrino and the recoil kinetic energy of the nucleus:

$$E_0 = E_e + E_\nu + T_{\text{recoil}}$$

Since the nucleus is much more massive than the electron/neutrino:

$$E_0 \approx E_e + E_\nu$$

and the nuclear recoil ensures momentum conservation.

For a given electron energy  $E_e$  :

$$\begin{aligned} dE_\nu &= dE_0 \\ \frac{dN}{dE_0} &= \frac{dN}{dE_\nu} \\ &= \frac{E_\nu^2}{(2\pi)^3} d\Omega_\nu \frac{E_e^2}{(2\pi)^3} d\Omega_e dE_e \end{aligned}$$

Assuming isotropic decay distributions and integrating over  $d\Omega_e d\Omega_\nu$  gives:

$$\begin{aligned} \frac{dN}{dE_0} &= (4\pi)^2 \frac{E_\nu^2}{(2\pi)^3} \frac{E_e^2}{(2\pi)^3} dE_e \\ &= \frac{E_\nu^2 E_e^2}{4\pi^4} dE_e \\ &= \frac{(E_0 - E_e)^2 E_e^2}{4\pi^4} dE_e \\ d\Gamma &= 2\pi |M_{fi}|^2 \frac{(E_0 - E_e)^2 E_e^2}{4\pi^4} dE_e \\ \frac{d\Gamma}{dE_e} &= |M_{fi}|^2 \frac{(E_0 - E_e)^2 E_e^2}{2\pi^3} \end{aligned}$$

In **FERMI theory** take:

$$|M_{fi}|^2 = G_F^2 \times f |M_{\text{nuclear}}|^2$$

where the nuclear matrix element  $|M_{\text{nuclear}}|^2$  accounts for the overlap of the nuclear wave-functions, and  $f$  is the Coulomb correction.

Here assume  $|M_{\text{nuclear}}|^2 = 1$  (super-allowed transition) and neglect  $f$ .

$$\begin{aligned} \Rightarrow \frac{d\Gamma}{dE_e} &= \frac{G_F^2}{2\pi^3} (E_0 - E_e)^2 E_e^2 \\ \Gamma &= \frac{G_F^2}{2\pi^3} \int_0^{E_0} (E_0 - E_e)^2 E_e^2 dE_e \\ \Gamma &= \frac{G_F^2}{2\pi^3} \left[ \frac{E_0^5}{3} - 2\frac{E_0^5}{4} + \frac{E_0^5}{5} \right] \\ \Gamma &= \frac{G_F^2 E_0^5}{60\pi^3} \end{aligned}$$

**SARGENT RULE:**

$$\tau \propto E^{-5}$$

★ e.g. see  $\mu^-$  and  $\tau^-$  decay

By studying lifetimes for nuclear beta decay (and applying necessary corrections, determine strength of weak coupling in FERMI theory:

$$G^\beta = 1.136 \pm 0.003 \times 10^{-5} \text{ GeV}^{-2}$$

## Beta-Decay Spectrum

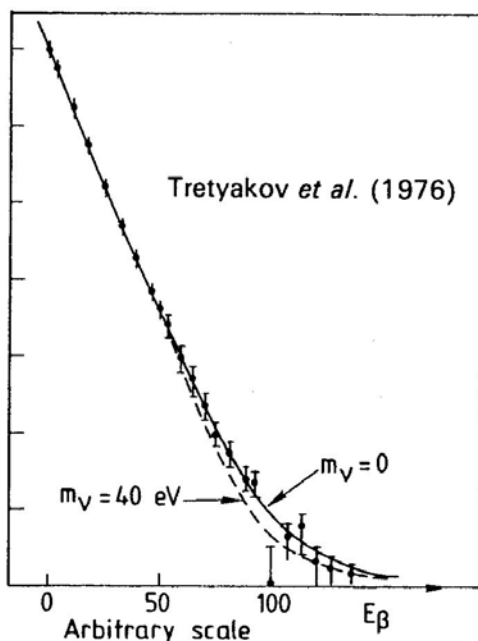
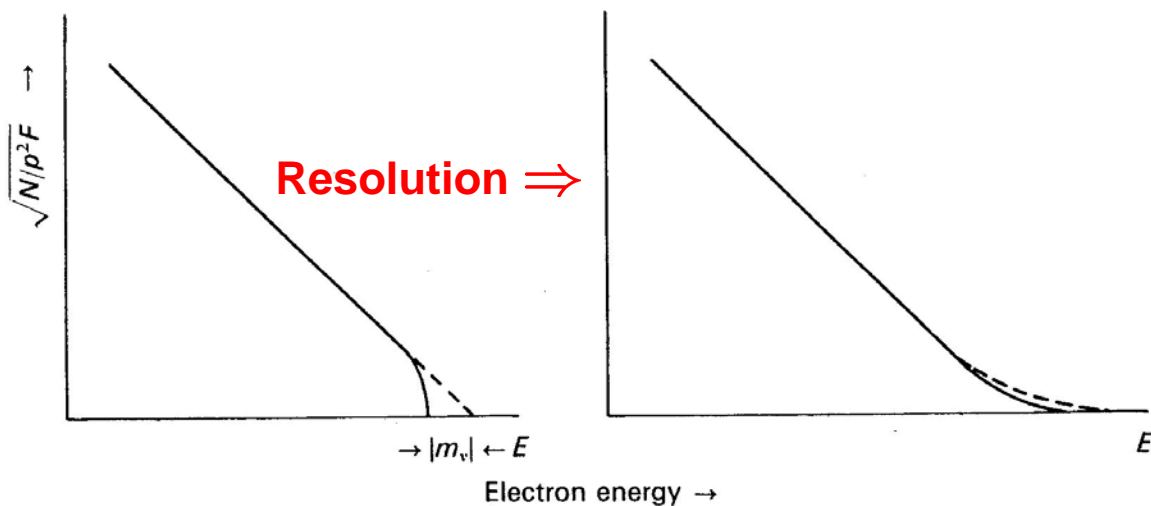
$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3} (E_0 - E_e)^2 E_e^2$$

Plot of  $\sqrt{\frac{d\Gamma}{dE_e} \frac{1}{E_e^2}}$  versus  $(E_0 - E_e)$  (Kurie plot) is linear

$$\sqrt{\frac{d\Gamma}{dE_e} \frac{1}{E_e^2}} \propto (E_0 - E_e)$$

For a non-zero neutrino mass this is modified to

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3} (E_0 - E_e)^2 E_e^2 \sqrt{1 - \left(\frac{m_\nu^2}{E_0 - E_e}\right)^2}$$



Most recent results (1999)

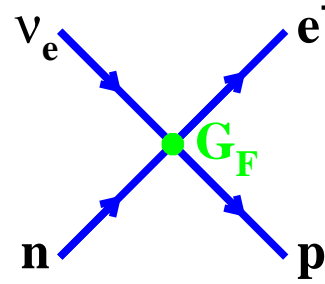
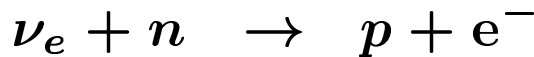
Tritium  $\beta$ -decay:

$$m(\nu_e) < 3 \text{ eV}$$

If neutrinos have mass  
 $m(\nu_e) \ll m(e)$ .

Why so small ?

## Neutrino Scattering in Fermi Theory (inverse $\beta$ -decay)



$$\begin{aligned} d\sigma &= 2\pi |M_{fi}|^2 \frac{dN}{dE} \\ &\sim 2\pi G_F^2 \frac{E_e^2}{(2\pi)^3} d\Omega \\ \sigma &\sim G_F^2 s \end{aligned}$$

where  $E_e$  is the energy of the  $e^-$  in the centre-of-mass system and  $\sqrt{s}$  is the energy in the centre-of-mass system.

In the Laboratory frame:  $s = 2E_\nu m_n$  ( see Handout I )

$$\sigma(\nu_e n) \sim (E_\nu \text{ in MeV}) \times 10^{-43} \text{ cm}^2$$

- ★ Neutrinos only interact **WEAKLY**  $\therefore$  have very small interaction cross-sections.
- ★ Here **WEAK** implies that you need approximately 50 light-years of water to stop a 1 MeV neutrino.
- ★ Communication via neutrino beams (á la Star Trek) non-trivial !

However, as  $E_\nu \rightarrow \infty$  the cross-section  $\sigma(\nu_\mu e^-)$  can become large. Violates maximum value allowed by conservation of probability at  $\sqrt{s} = 740 \text{ GeV}$  (**UNITARITY LIMIT**)

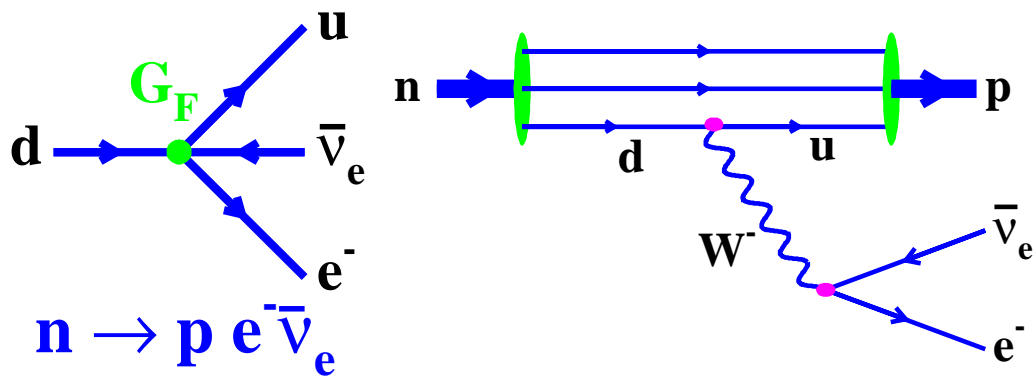
- ★ **FERMI** Theory breaks down at high energies



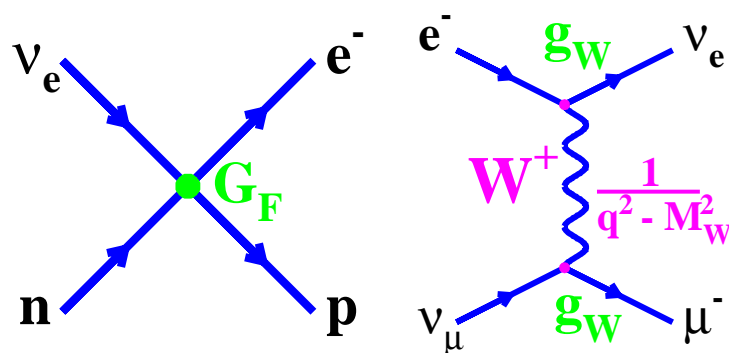
# Weak Charged Current - $W^\pm$ Boson

- ★ Fermi theory breaks down at high energy.
- ★ True interaction described by exchange of charged W-bosons ( $W^\pm$ )
- ★ Fermi theory is the low energy ( $q^2 \ll M_W^2$ ) **EFFECTIVE** theory of the WEAK interaction

## Beta-Decay:



## $\nu_\mu e^-$ Scattering:

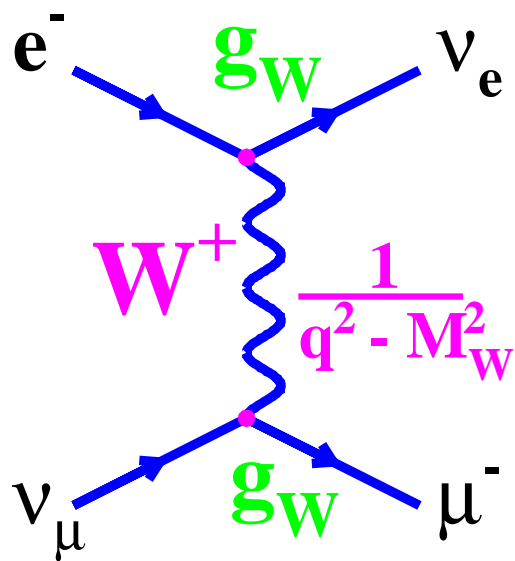


At low "energies"  $q^2 \ll M_W^2$ :

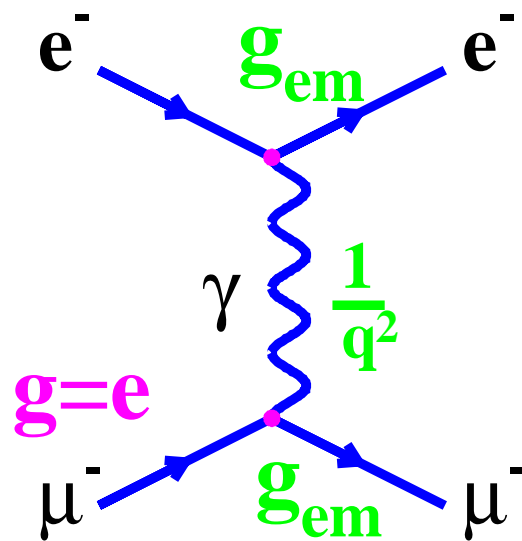
W-Boson propagator  $\frac{1}{q^2 - M_W^2} \rightarrow \frac{1}{-M_W^2}$

## Compare WEAK and QED interactions

### WEAK INTERACTION



### QED

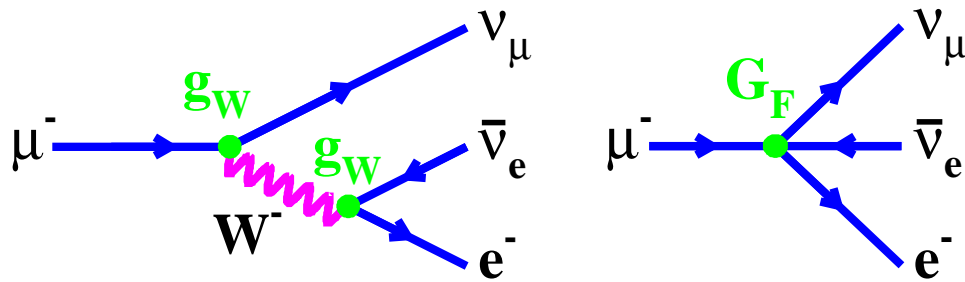


### CHARGED CURRENT WEAK INTERACTION

- ★ For  $q^2 \ll M_W^2$  propagator becomes  $\frac{1}{M_W^2}$  - i.e. appears as the **POINT-LIKE** interaction of FERMI theory.
- ★ Massive Propagator  $\rightarrow$  short range
 
$$M_W = 80.4 \pm 0.1 \text{ GeV}$$

$$\text{Range} \approx \frac{1}{M_W} \sim 0.002 \text{ fm}$$
- ★ Exchanged Boson carries electro-magnetic charge
- ★ **FLAVOUR CHANGING !**
  - ONLY WEAK interaction changes flavour**
- ★ **Parity Violating !**
  - ONLY WEAK interaction can violate parity conservation**

## COMPARE Fermi theory c.f. massive propagator



For  $q^2 \ll M_W^2$  compare matrix elements:

$$\frac{g_W^2}{M_W^2} \rightarrow G_F$$

★  $G_F$  is small because  $M_W$  is large.

The precise relationship is:

$$\frac{g_W^2}{8M_W^2} \rightarrow \frac{G_F}{\sqrt{2}}$$

The numerical factors are partly of historical origin (e.g. see Perkins 4<sup>th</sup> Edition, page 210).

$$M_W = 80.4 \text{ GeV} \text{ and } G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$$

$$\Rightarrow g_W = 0.65$$

$$\therefore \alpha_W = \frac{g_W^2}{4\pi} \approx \frac{1}{30}$$

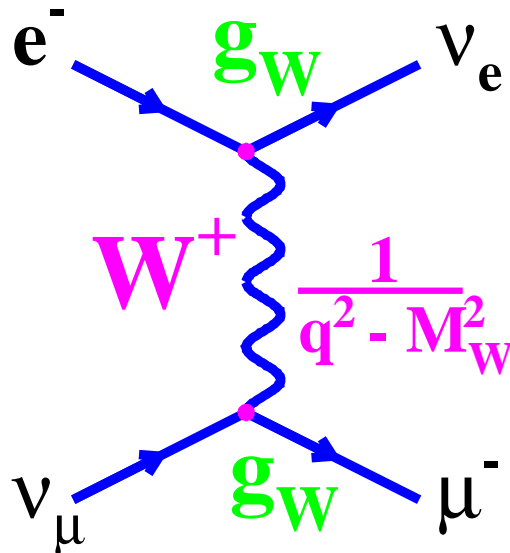
The intrinsic strength of the **WEAK** interaction is greater than that of the electro-magnetic interaction. At low energies (low  $q^2$ ) it appears weak due to the massive propagator.

★  $\alpha_S \approx 0.2$ ,  $\alpha_W \approx 0.03$ ,  $\alpha_{EM} \approx 0.01$

★ suggestive of **UNIFICATION** of the forces

## Neutrino Scattering with a Massive W Boson

Replace contact interaction by massive boson exchange diagram:



$$\frac{d\sigma}{dq^2} = \frac{1}{32\pi} \frac{g_w^4}{(q^2 - M_W^2)^2}$$

$$\text{with } |q| = 2E \sin \frac{\theta}{2}$$

where  $\theta$  is the scattering angle.

(e.g. similar to Handout I p.36)

Integrate to give:

$$\sigma = \frac{G_F^2 s}{\pi} \quad s \ll M_W^2$$

$$\sigma = \frac{G_F^2 M_W^2}{\pi} \quad s \gg M_W^2$$

Total cross section now well behaved at high energies.

# Parity Violation in Beta Decay

## Revision : Nuclear Physics

Under Parity  $\hat{P}$ :

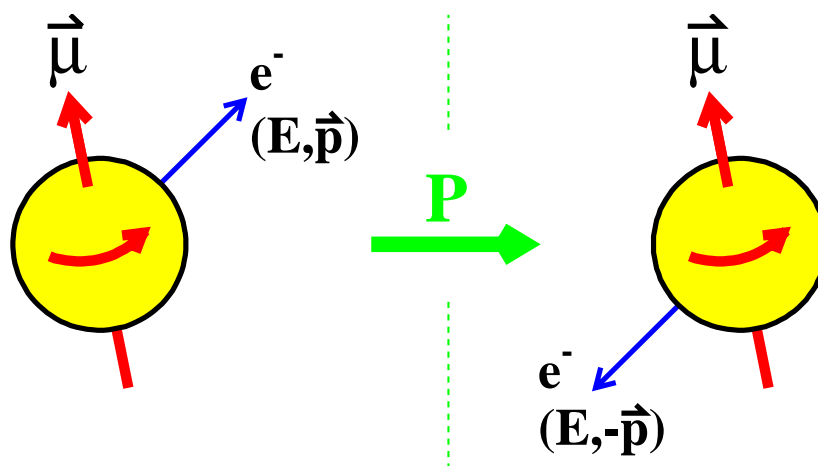
$$\begin{aligned}\tilde{\mathbf{r}} &\rightarrow -\tilde{\mathbf{r}} \\ \tilde{\mathbf{p}} \propto \nabla &\rightarrow -\tilde{\mathbf{p}} \\ \tilde{\mathbf{L}} = \tilde{\mathbf{r}} \times \tilde{\mathbf{p}} &\rightarrow \tilde{\mathbf{L}} \\ \tilde{\boldsymbol{\mu}} &\rightarrow \tilde{\boldsymbol{\mu}}\end{aligned}$$

$\hat{P}$ : Axial vectors e.g.  $\tilde{\mathbf{L}}$ ,  $\tilde{\boldsymbol{\mu}}$  do not change sign

EXPERIMENT: Align  $^{60}\text{Co}$  nuclei at low temperatures with  $\tilde{\mathbf{B}}$  field



Observe angular distribution of  $e^-$  relative to  $\tilde{\mathbf{B}}$ .



If parity conserved expect equal numbers of  $e^-$  parallel and anti-parallel to  $\tilde{\mathbf{B}}$ .

Experiment (C.S. Wu 1956) showed clear asymmetry  $\Rightarrow$  **PARITY VIOLATION** in WEAK interactions

# Origin of Parity Violation

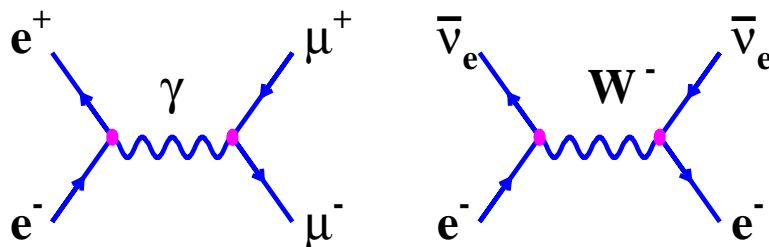
In the ultra-relativistic (massless) limit **only**

- ★ LEFT-HANDED PARTICLES and
- ★ RIGHT-HANDED ANTI-PARTICLES.

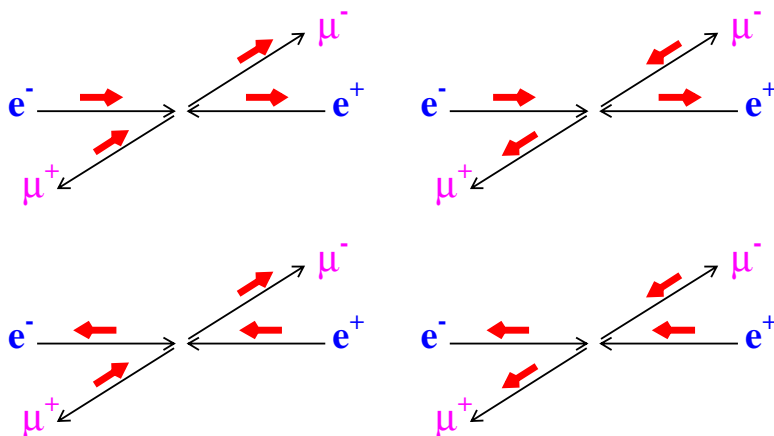
participate in the **WEAK** (charged current) interaction.

For massive fermions the weak interaction couples preferentially to **LEFT-HANDED** particles and **RIGHT-HANDED** anti-particles.

Compare QED and WEAK interaction.

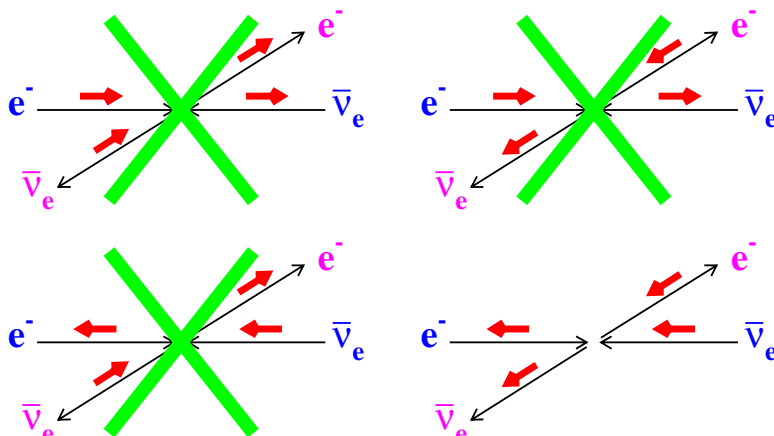


QED :  $e^+ e^- \rightarrow \mu^+ \mu^-$



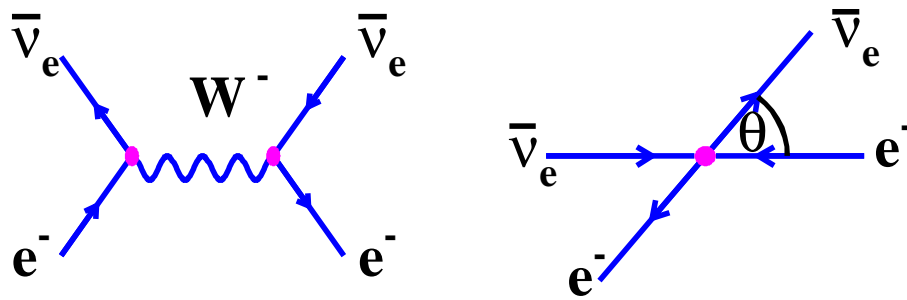
From 16 possible SPIN assignments only 4 give non-zero contributions to cross section

WEAK INTERACTION :  $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$

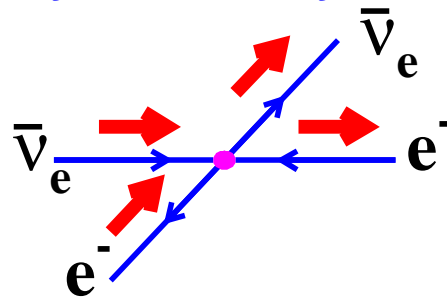


From 16 possible SPIN assignments only 1 gives a non-zero contribution to cross section.  $L\bar{R} \rightarrow L\bar{R}$

**EXAMPLE**  $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$  scattering in the centre-of-mass frame ( $s = E_e + E_\nu = 2E_e$ )



In massless limit - only one Helicity state contributes:



$$\begin{aligned} \frac{d\sigma}{d\Omega} &= 2\pi |M_{fi}|^2 \frac{E_e^2}{(2\pi)^3} \\ &= \frac{1}{16\pi^3} |M_{fi}|^2 s \end{aligned}$$

where  $s$  is centre-of-mass energy

$$M_{fi} = \left( \frac{g_W}{\sqrt{2}} \right)^2 \frac{1}{q^2 - M_W^2} \cos^2 \frac{\theta}{2}$$

For  $q^2 \ll M_W^2$

$$|M_{fi}| = \frac{2G_F}{\sqrt{2}} (1 + \cos \theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{8\pi^2} G_F^2 s (1 + \cos \theta)^2$$

$$\sigma = \frac{G_F^2 s}{3\pi}$$

## Parity Violation

The WEAK interaction treats LH and RH states differently and therefore can violate PARITY (*i.e.* the interaction Hamiltonian does not commute with  $\hat{P}$ )

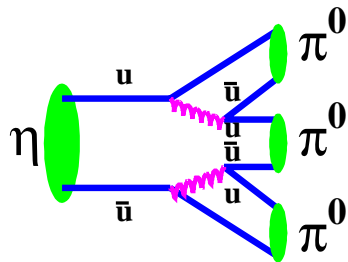
Parity ALWAYS conserved in STRONG/EM interactions

$$P = \prod_i P_i \prod_{i>j} (-1)^{L_{ij}}$$

where  $P_i$  is the intrinsic parity of the particle  $i$  and  $L_{ij}$  is the orbital angular momentum between particles  $i$  and  $j$ .

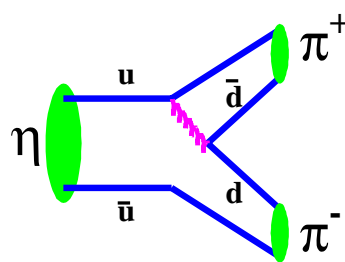
Taking  $L_{ij} = 0$

$$\begin{array}{l} \eta \rightarrow \pi^0 \pi^0 \pi^0 \\ \mathbf{J}^P \quad 0^- \quad 0^- \quad 0^- \quad 0^- \\ \mathbf{P} \quad -1 \quad -1 \end{array}$$



BR = 32 %

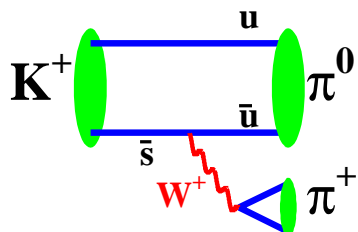
$$\begin{array}{l} \eta \rightarrow \pi^+ \pi^- \\ \mathbf{J}^P \quad 0^- \quad 0^- \quad 0^- \\ \mathbf{P} \quad -1 \quad +1 \end{array}$$



BR < 0.1 %

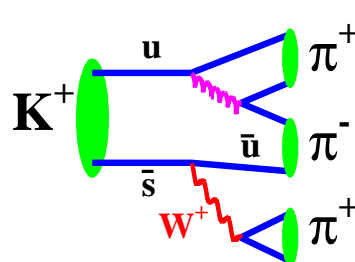
Parity is usually violated in WEAK interactions

$$\begin{array}{l} \mathbf{K}^+ \rightarrow \pi^+ \pi^0 \\ \mathbf{J}^P \quad 0^- \quad 0^- \quad 0^- \\ \mathbf{P} \quad -1 \quad +1 \end{array}$$



BR = 21 %

$$\begin{array}{l} \mathbf{K}^+ \rightarrow \pi^+ \pi^- \pi^+ \\ \mathbf{J}^P \quad 0^- \quad 0^- \quad 0^- \quad 0^- \\ \mathbf{P} \quad -1 \quad -1 \end{array}$$



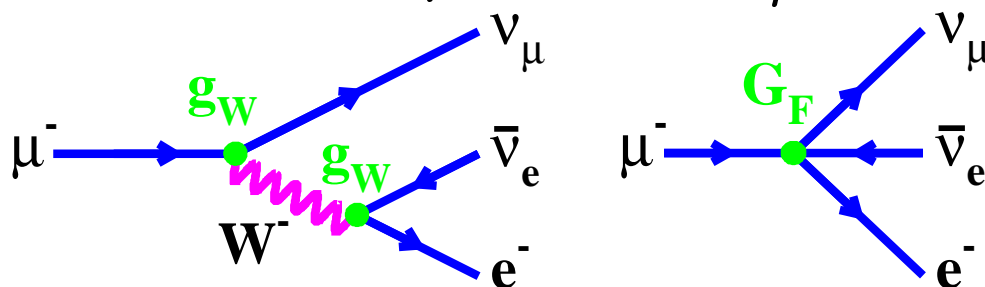
BR = 6 %

but NOT ALWAYS !



## Weak Leptonic Decays

- ★ Muons are fundamental leptons ( $m_\mu \approx 206m_e$ ).
- ★ Electro-magnetic decay  $\mu^- \rightarrow e^- \gamma$  **IS NOT** observed; the EM interaction does not change flavour.
- ★ Only the **WEAK** charged current changes flavour.
- ★ Muons decay weakly :  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$



As  $m_\mu^2 \ll M_W^2 \Rightarrow$  can use **FERMI** theory to calculate decay width (analogous to  $\beta$  decay).

FERMI theory gives decay width proportional to  $m_\mu^5$  (Sargent Rule):

However more complicated phase space integration (previously neglected kinetic energy of recoiling nucleus)

$$\text{gives } \Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_F^2}{192\pi^3} m_\mu^5$$

- ★ Muon mass and lifetime known with high precision.

$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$$

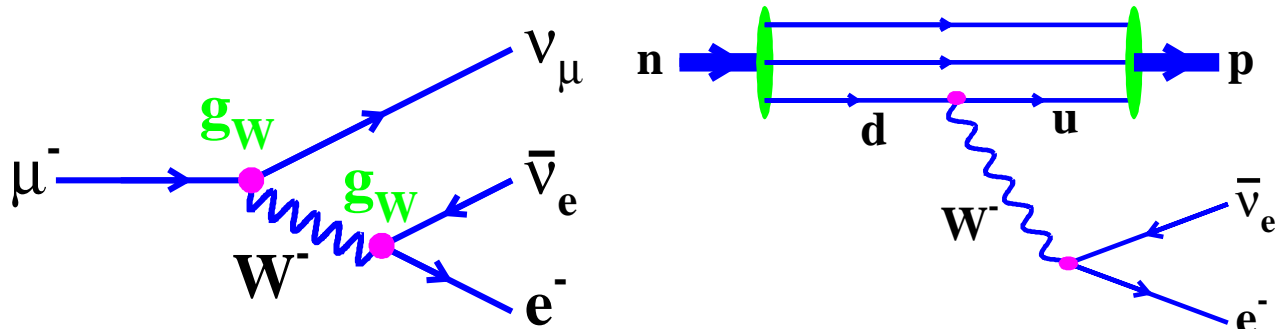
- ★ Use muon decay to fix strength of **WEAK** interaction  $G_F$

$$G_F = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

- ★  $G_F$  is one of the best determined **fundamental** quantities in particle physics.

## Universality of Weak Coupling

Can compare  $G_F$  measured from  $\mu^-$ -decay with that obtained from  $\beta$ -decay



From muon decay measure:

$$G^\mu = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

From  $\beta$ -decay measure:

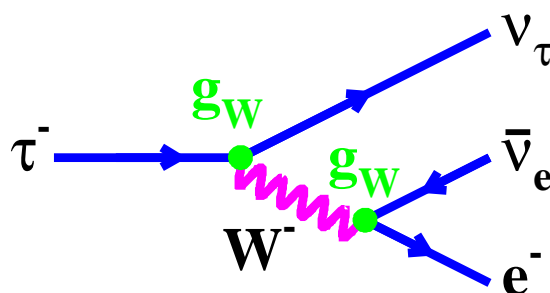
$$G^\beta = (1.136 \pm 0.003) \times 10^{-5} \text{ GeV}^{-2}$$

Taking ratio gives

$$\frac{G^\beta}{G^\mu} = 0.974 \pm 0.003$$

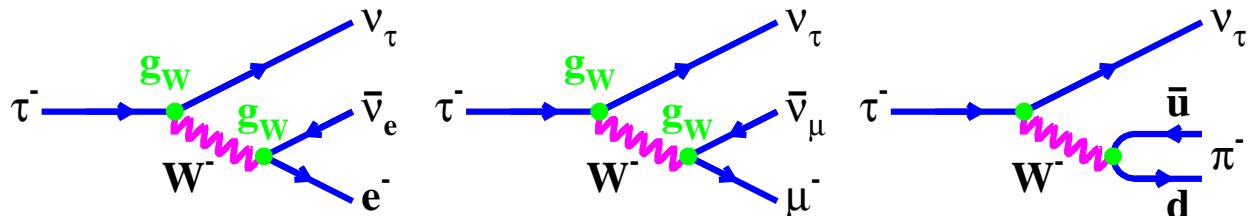
Conclude that the strength of the weak interaction is almost the same for muons/electrons as for up/down quarks and we'll shortly come back to the origin of this difference ( $\cos \theta_c$ )

Can also test universality of the WEAK interaction in  $\tau$ -decays, e.g.



# Tau Decays

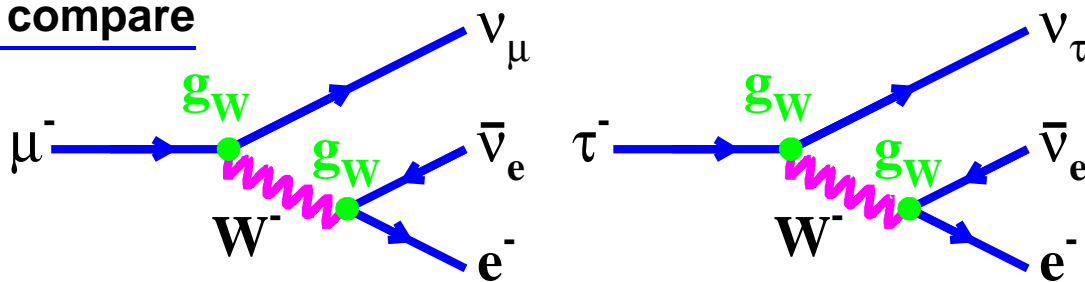
The  $\tau$  mass is relatively large,  $m_\tau = 1.777 \text{ GeV}$ ,  
 and as  $m_\tau > \{m_\mu, m_\pi, m_\rho, \dots\}$   
 there a number of possible tau decay modes, e.g.



**Tau Branching Fractions:**

- ★  $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$  (17.8 ± 0.1 %)
- ★  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  (17.3 ± 0.1 %)
- ★  $\tau^- \rightarrow \text{hadrons}$  (64.7 ± 0.2 %)

First compare



$$\frac{1}{\tau_\mu} = \Gamma_{\mu \rightarrow e} = \frac{G_F^2}{192\pi^3} m_\mu^5$$

$$\frac{1}{\tau_\tau} = \frac{1}{Br(\tau \rightarrow e)} \Gamma_{\tau \rightarrow e} = \frac{1}{0.178} \frac{G_F^2}{192\pi^3} m_\tau^5$$

If universal strength of WEAK interaction expect

$$\frac{\tau_\tau}{\tau_\mu} = 0.178 \frac{m_\mu^5}{m_\tau^5}$$

$m_\mu, m_\tau, \tau_\mu$  are all precisely measured

Using:  $m_\mu = 105.658 \text{ MeV}$

$m_\tau = (1777.0 \pm 0.3) \text{ MeV}$

$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$

gives a PREDICTION of

$$\tau_\tau = 2.91 \pm 0.01 \times 10^{-13} \text{ s}$$

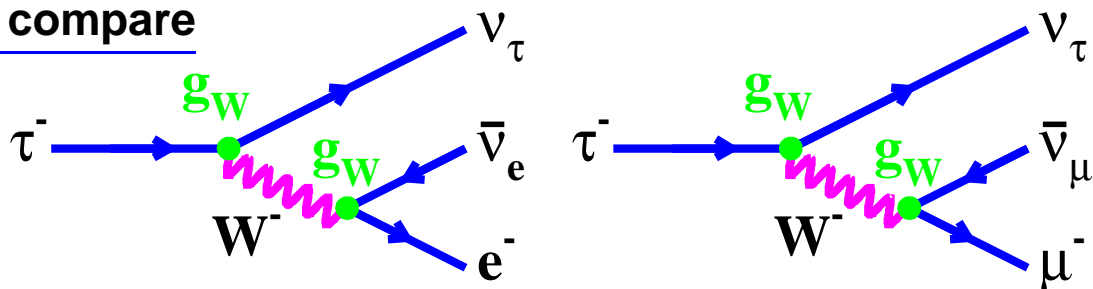
compare to MEASURED VALUE:

$$\tau_\tau = 2.91 \pm 0.01 \times 10^{-13} \text{ s}$$

Consistent with the predicted value, *i.e.*

★ Same **WEAK CC** coupling for  $\mu$  and  $\tau$ .

Also compare



IF same couplings expect:

$$\frac{Br(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)}{Br(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} = 0.9726$$

(the small difference is due to the slight reduction in phase space due to the non-negligible muon mass)

The observed ratio

$$0.974 \pm 0.005$$

is consistent with the prediction **0.9726**

★ Same **WEAK CHARGED CURRENT** coupling for  $e, \mu$  and  $\tau \rightarrow$  **LEPTON UNIVERSALITY**

(see Question 9 on the problem sheet)

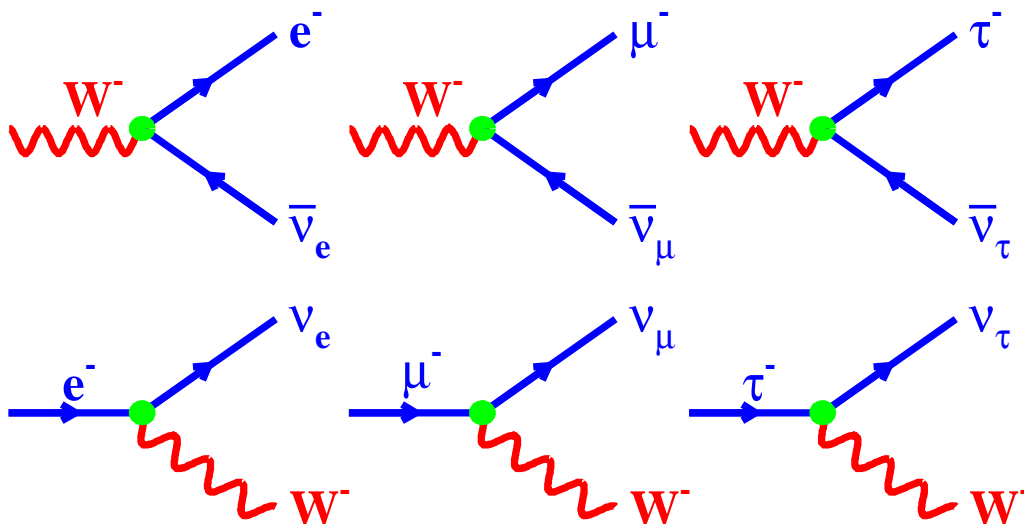
## W Leptonic Couplings

### Standard Model W Boson Couplings

- ★ In the Standard Model the 'charge' of the WEAK interactions is called WEAK ISOSPIN.
- ★ Leptons are represented in Doublets

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

- ★ W-bosons only 'couple' particles **within** a doublet.
- ★ e.g. no  $W e^- \nu_\mu$  coupling.

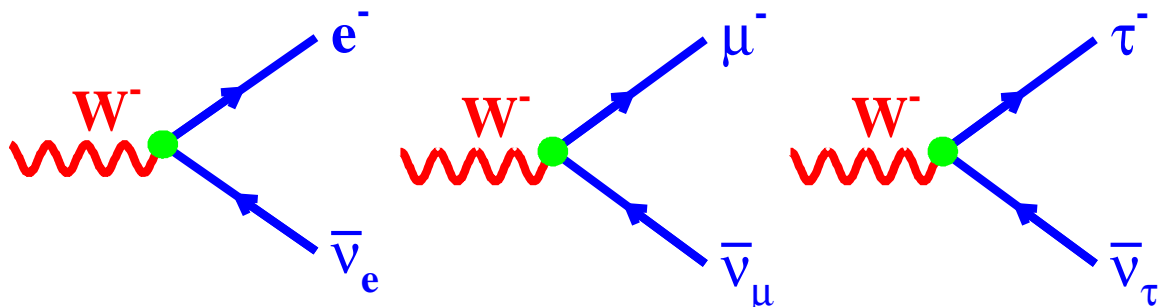


- ★ UNIVERSAL COUPLING STRENGTH

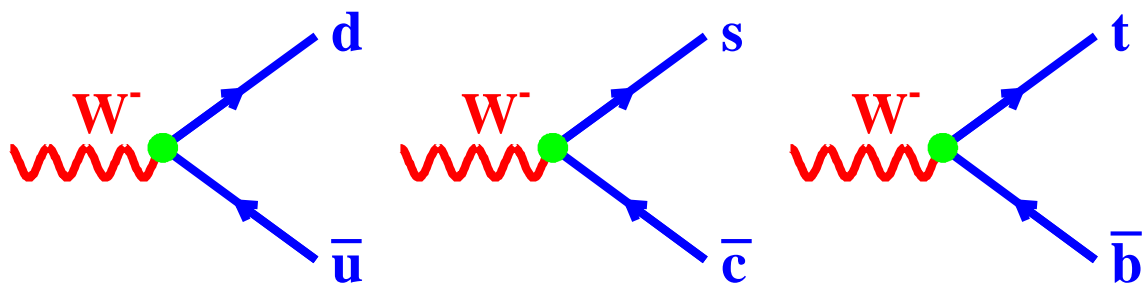
$$\frac{g_w}{\sqrt{2}}$$

## Weak Interactions of Quarks

In the Standard Model, the leptonic weak couplings take place within generation,



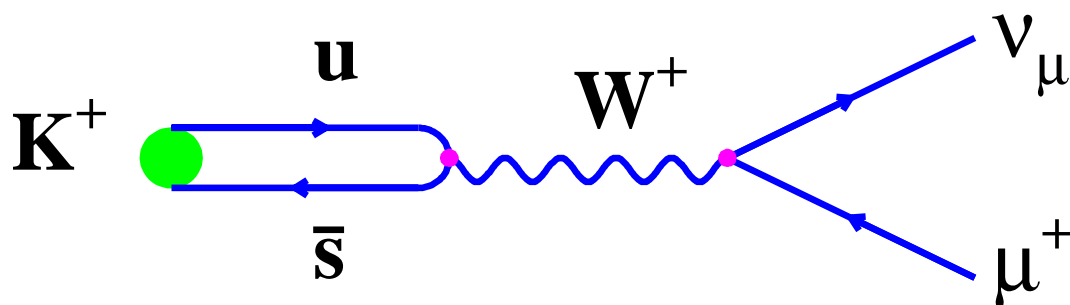
Natural to expect same Pattern for QUARKS *i.e.*



Unfortunately its not that simple !

### Example

The decay  $K^+ (u\bar{s}) \rightarrow \mu^+ \nu_\mu$  suggests a  $W^+ u\bar{s}$  coupling



# Cabibbo Mixing Angle

## Four-Flavour Quark Mixing

- ★ the states which take part in the WEAK interaction are ORTHOGONAL combinations of the states of definite flavour (d,s)
- ★ For 4-flavours,  $\{ d, u, s \text{ and } c \}$ , the mixing can be described by a single parameter: the **CABIBBO ANGLE**  $\theta_c$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

**Couplings become:**

$$\begin{array}{ccc} \begin{array}{c} d' \\ \swarrow \\ W^- \\ \searrow \\ \bar{u} \end{array} & = & \begin{array}{c} d \\ \swarrow \\ W^- \\ \searrow \\ \bar{u} \end{array} \quad \begin{array}{c} s \\ \swarrow \\ W^- \\ \searrow \\ \bar{u} \end{array} \\ \cos \theta_c d + \sin \theta_c s & & g_W \cos \theta_c \quad g_W \sin \theta_c \end{array}$$

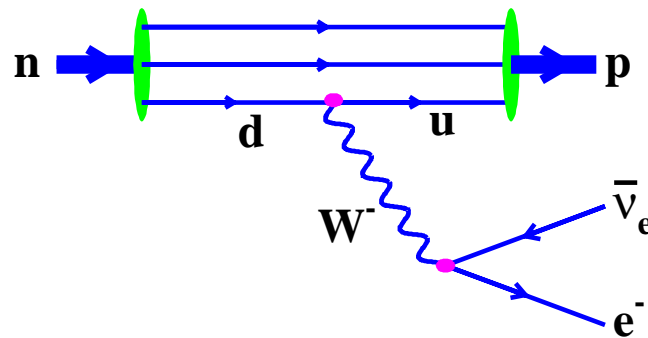
$$\begin{array}{ccc} \begin{array}{c} s' \\ \swarrow \\ W^- \\ \searrow \\ \bar{c} \end{array} & = & \begin{array}{c} s \\ \swarrow \\ W^- \\ \searrow \\ \bar{c} \end{array} \quad \begin{array}{c} d \\ \swarrow \\ W^- \\ \searrow \\ \bar{c} \end{array} \\ \cos \theta_c s - \sin \theta_c d & & g_W \cos \theta_c \quad -g_W \sin \theta_c \end{array}$$

**EXPERIMENTALLY:**

$$\theta_c = 13^\circ$$

## EXAMPLE: Nuclear Beta Decay

Recall:

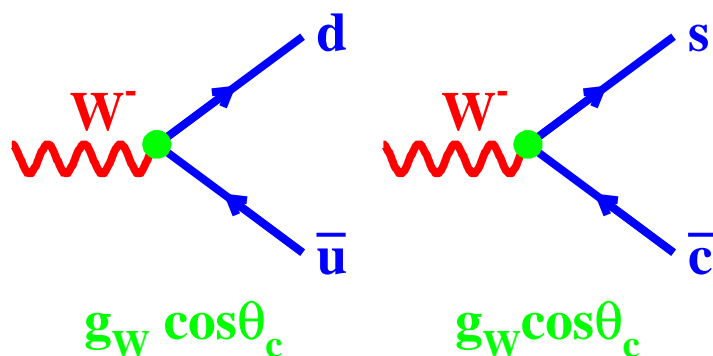


$$G_{\mu} = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$$

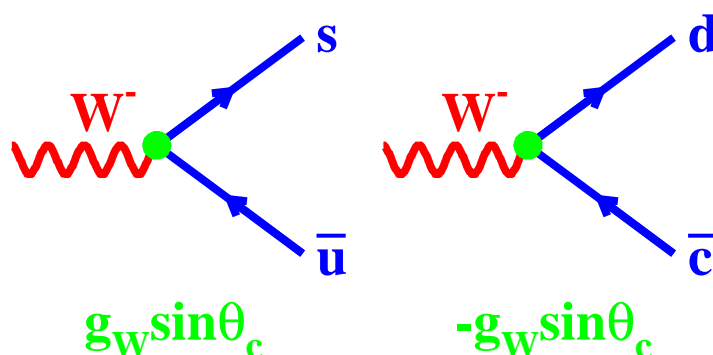
$$G_{\beta} = (1.136 \pm 0.003) \times 10^{-5} \text{ GeV}^{-2}$$

- ★ Strength of  $ud$  coupling  $\propto g_w \cos \theta_c$
- ★  $(G_{\beta})^2 \propto |M|^2 \propto \cos^2 \theta_c$
- ★ Hence expect  $G_{\beta} = \cos \theta_c G_{\mu}$
- ★ It works,  $1.16632 \times \cos 13^{\circ} = 1.136$

**Cabibbo Favoured :  $|M|^2 \propto \cos^2 \theta_c$**

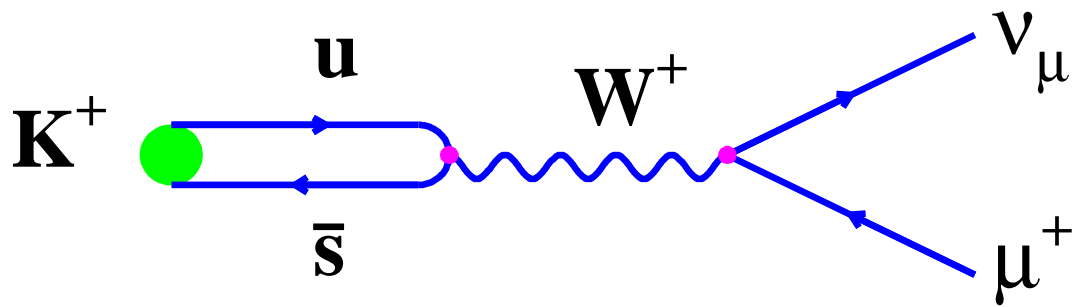


**Cabibbo Suppressed :  $|M|^2 \propto \sin^2 \theta_c$**





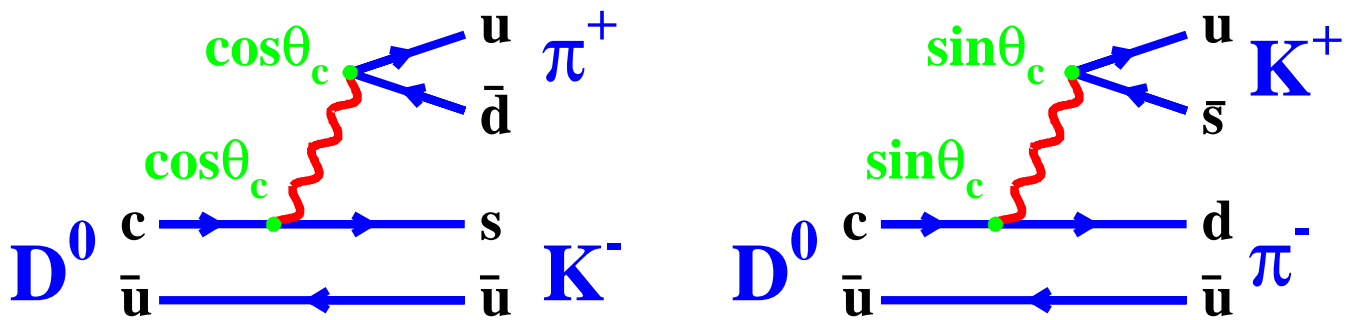
EXAMPLE:  $K^+ \rightarrow \mu^+ \nu_\mu$



$u\bar{s}$  coupling  $\Rightarrow$  Cabibbo suppressed

$$|M|^2 \propto \sin^2 \theta_c$$

EXAMPLE:  $D^0 \rightarrow K^- \pi^+$ ,  $D^0 \rightarrow K^+ \pi^-$



Expect

$$\frac{\Gamma(D^0 \rightarrow K^+ \pi^-)}{\Gamma(D^0 \rightarrow K^- \pi^+)} = \frac{\sin^4 \theta_c}{\cos^4 \theta_c} \approx 0.0028$$

Measure  $0.0038 \pm 0.0008$

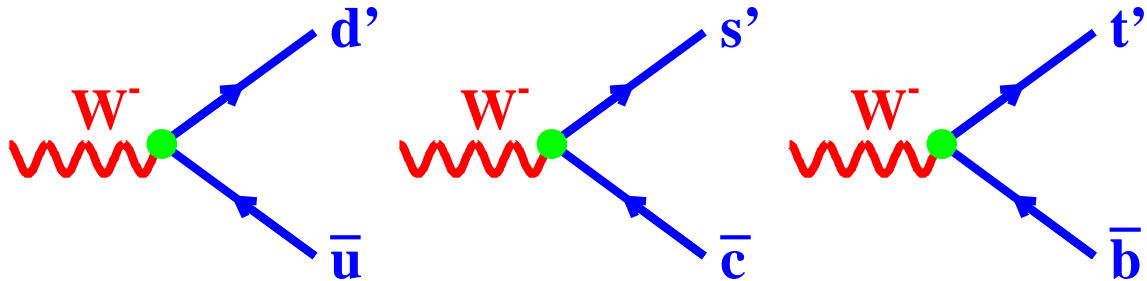
$D^0 \rightarrow K^+ \pi^-$  is DOUBLY Cabibbo suppressed

(see Question 8 on the problem sheet)

# CKM Matrix

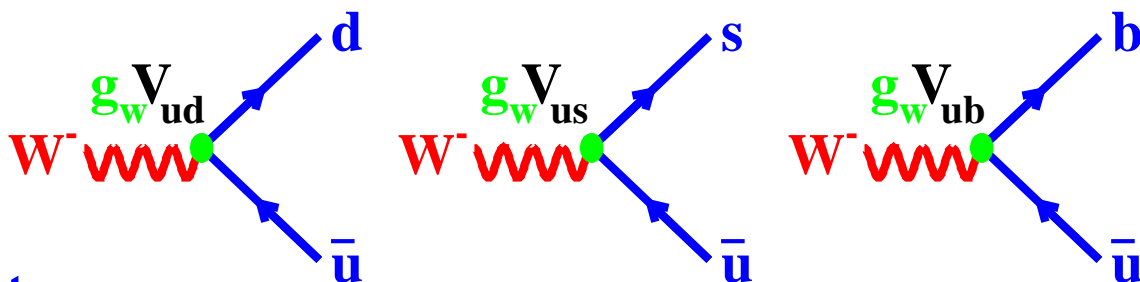
## Cabibbo-Kobayashi-Maskawa Matrix

### Extend to 3 generations



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

### Giving couplings



### Note

$$V_{ckm} \approx \begin{pmatrix} \cos \theta_c & \sin \theta_c & 0.01 \\ -\sin \theta_c & \cos \theta_c & 0.05 \\ 0.01 & -0.05 & 1 \end{pmatrix}$$

### sometimes written

$$V_{ckm} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & \lambda^3 \\ -\lambda & 1 - \frac{\lambda^2}{2} & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

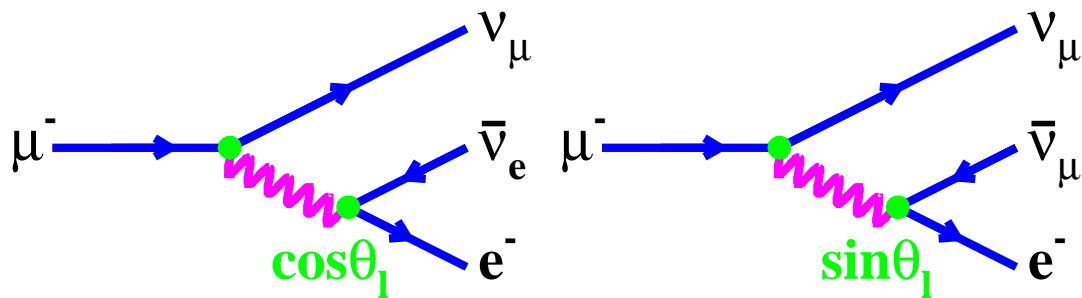
with  $\lambda = \sin \theta_c$

(see Question 10 on the problem sheet)

## Lepton Mixing Matrix ?

Natural to ask if there is an equivalent of the CKM Matrix for leptons.

### HYPOTHETICAL EXAMPLE:



The neutrinos are unobserved, (*i.e.* don't distinguish the different neutrino final states). Consequently the amplitude for  $\mu^- \rightarrow e^- \nu \bar{\nu}$

$$|M|^2 \propto g_w^2 (\cos^2 \theta_l + \sin^2 \theta_l)$$

★ In the quark sector, mass differences between quarks (and the hadrons they form) allow us to distinguish the different final states

See Handout VIII for the evidence that there is **MIXING** in the lepton sector

# Summary

## WEAK INTERACTION (CHARGED-CURRENT)

- ★ Parity violated due to the HELICITY structure of the interaction
- ★ Force mediated by massive W-bosons,  $M_W = 80.4 \text{ GeV}$
- ★ Intrinsically stronger than EM interaction
- ★ Universal coupling to quarks and leptons
- ★ Quarks take part in the interaction as mixtures of the flavour eigenstates
- ★  $G_F = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2}$  from muon decay

## ELECTROWEAK UNIFICATION - next handout

- ★ Neutral Current WEAK interaction -  $Z^0$
- ★ Unification of WEAK and EM forces

## APPENDIX: VECTOR-AXIAL VECTOR (V–A)

### NON-EXAMINABLE

In the DIRAC equation the WEAK interaction vertex has the form **VECTOR – AXIAL-VECTOR**

$$\gamma^\mu (1 - \gamma^5)$$

Consider Dirac spinors for a **particle** traveling along the  $z$ -axis

$$u_R = N \begin{pmatrix} 1 \\ 0 \\ p \\ (E+m) \\ 0 \end{pmatrix} \quad u_L = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ -p \\ (E+m) \end{pmatrix}$$

The WEAK interaction matrix element looks like

$$\langle \bar{u}_R | \gamma^\mu (1 - \gamma^5) | u_L \rangle$$

it has the form **VECTOR** ( $\gamma^\mu$ ) minus **AXIAL-VECTOR**  $\gamma^\mu \gamma^5$

In matrix form:

$$\begin{aligned} 1 - \gamma^5 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \end{aligned}$$

Consider the effect of the interaction on **LH** and **RH** spinors:

$$(1 - \gamma^5)u_R = N \begin{pmatrix} 1 - \frac{p}{(E+m)} \\ 0 \\ -1 + \frac{p}{(E+m)} \\ 0 \end{pmatrix}$$

$$(1 - \gamma^5)u_L = N \begin{pmatrix} 0 \\ 1 + \frac{p}{(E+m)} \\ 0 \\ -1 - \frac{p}{(E+m)} \end{pmatrix}$$

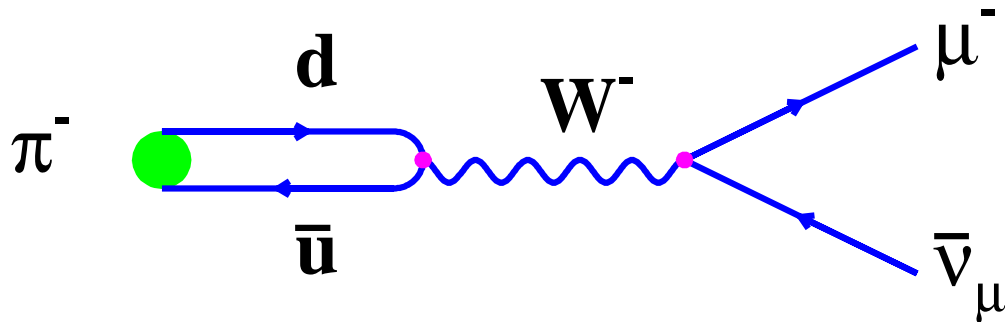
Massless limit,  $m \rightarrow 0$ ,  $p \rightarrow E$ :

$$(1 - \gamma^5)u_R = 0$$

- ★ For massless particles, the form of the interaction projects out **LH** particle states, i.e. only LH-particles take part in the WEAK interaction.
- ★ For massive particles, the form of the interaction **preferentially** projects out **LH** particles.

**IF** neutrinos were massless (**which is not quite the case**), the WEAK couplings of RH neutrinos and LH anti-neutrinos would be zero, and if the  $\nu_R$  and  $\bar{\nu}_L$  state exist they would only experience the gravitational interaction !

## EXAMPLE: $\pi^\pm$ Decay

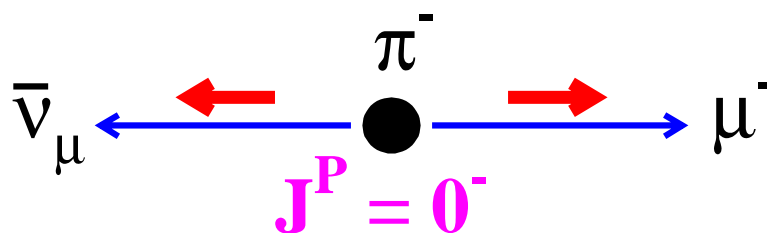


**Charged Pion decay branching fractions:**

- ★  $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$                       99.9877 %
- ★  $\pi^- \rightarrow e^- \bar{\nu}_e$                         0.0123 %

Naïvely might expect slightly larger branching fraction for  $\pi^- \rightarrow e^- \bar{\nu}_e$  due to phase space !

Consider Spin/Helicity



- ★ Conservation of angular momentum  $\Rightarrow$  muon and neutrino spins in opposite directions.  
**SAME HELICITY**
- ★ Neutrinos massless,  $\therefore$  only **RH** anti-neutrino takes part in **WEAK** interaction
- ★ therefore  $\mu^-$  is also right-handed
- ★ IF massless, e.g.  $m_\mu = 0$ , the **WEAK** Matrix element would be exactly zero

$$(1 - \gamma^5)u_R = N \begin{pmatrix} 1 - \frac{p}{(E+m)} \\ 0 \\ -1 + \frac{p}{(E+m)} \\ 0 \end{pmatrix}$$

“Wrong-Handed” ME (zero for  $m = 0$ )

$$M \propto f_{wrong} = \frac{1}{2} \left( 1 - \frac{p}{E+m} \right)$$

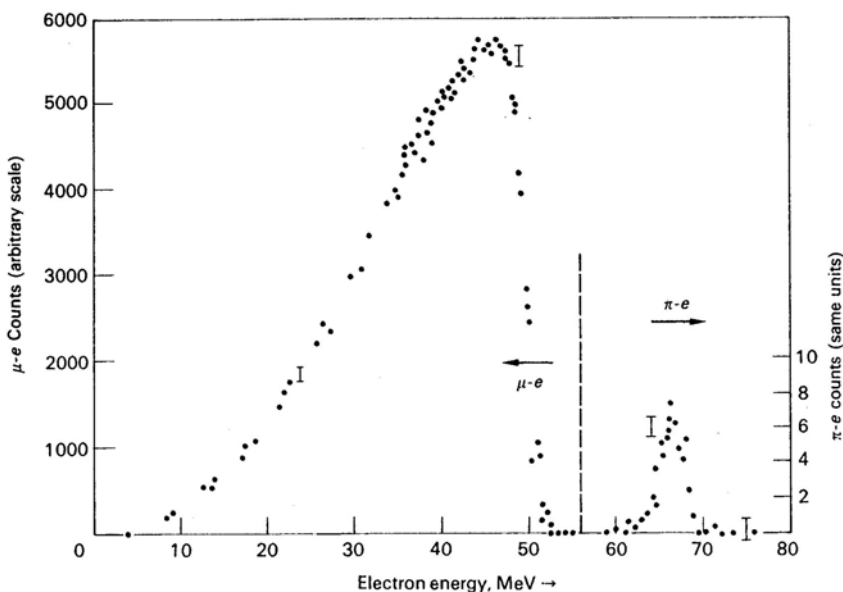
$\pi^- \rightarrow$	$p_{lept}$	$E_{lept}$	$f_{wrong}$
$\mu^- \bar{\nu}_\mu$	30 MeV	110 MeV	<b>0.43</b>
$e^- \bar{\nu}_e$	70 MeV	70 MeV	<b>0.0035</b>

★  $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$  is non-relativistic

★ Decay  $\pi^- \rightarrow e^- \bar{\nu}_e$  suppressed relative to  $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$ :  $\left(\frac{0.0035}{0.43}\right)^2 \approx 6 \times 10^{-5}$

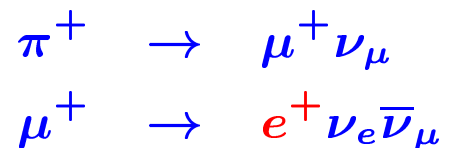
★ Once phase-space taken into account:

$$\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = 1.23 \times 10^{-4}$$



Positron decay spectrum from  $\pi^+$  decays.

Large Peak



Small Peak

