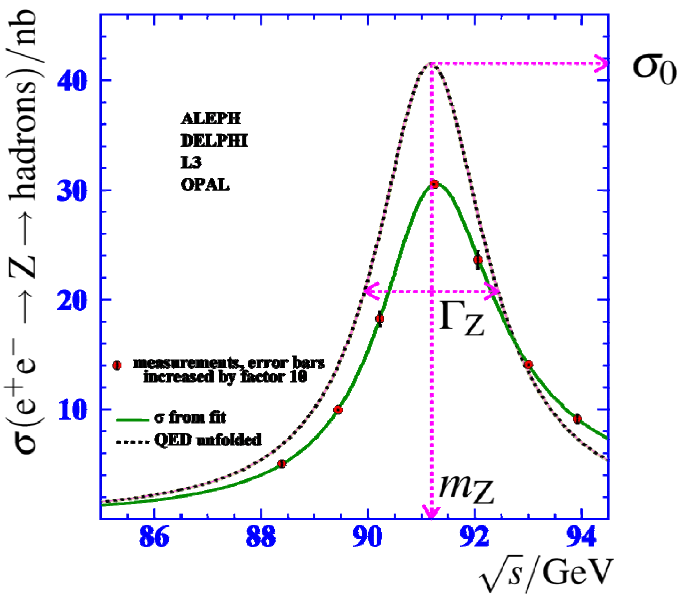
Particle Physics

Michaelmas Term 2009
Prof Mark Thomson

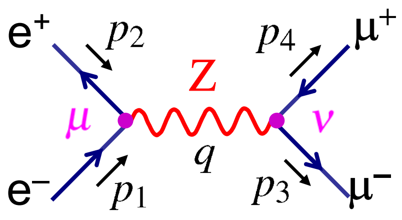


Handout 14 : Precision Tests of the Standard Model

The Z Resonance

★ **Want to calculate the cross-section for** $e^+e^- \to Z \to \mu^+\mu^-$

•**Feynman rules for the diagram below give:**



e⁺e⁻ vertex: $\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \frac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

μ⁺μ⁻ vertex: $\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \frac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)$

$$\Rightarrow \quad -iM_{fi} = [\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].\frac{-ig_{\mu\nu}}{q^2 - m_Z^2}.[\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]$$

$$\Rightarrow \quad \boxed{M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].[\overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]}$$

★ **Convenient to work in terms of helicity states by explicitly using the Z coupling to LH and RH chiral states (ultra-relativistic limit so helicity = chirality)**

$$\tfrac{1}{2}(c_V - c_A\gamma^5) = c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

LH and RH projections operators

hence $c_V = (c_L + c_R)$, $c_A = (c_L - c_R)$

and
$$\begin{aligned}\tfrac{1}{2}(c_V - c_A\gamma^5) &= \tfrac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5) \\ &= c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)\end{aligned}$$

with $c_L = \frac{1}{2}(c_V + c_A)$, $c_R = \frac{1}{2}(c_V - c_A)$

★ **Rewriting the matrix element in terms of LH and RH couplings:**

$$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1+\gamma^5)u(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)v(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1+\gamma^5)v(p_4)]$$

★ **Apply projection operators remembering that in the ultra-relativistic limit**

$$\tfrac{1}{2}(1-\gamma^5)u = u_\downarrow; \quad \tfrac{1}{2}(1+\gamma^5)u = u_\uparrow, \quad \tfrac{1}{2}(1-\gamma^5)v = v_\uparrow, \quad \tfrac{1}{2}(1+\gamma^5)v = v_\downarrow$$

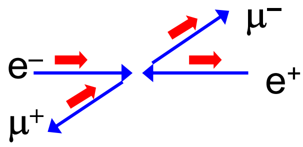
$$\Rightarrow \quad M_{fi} = -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu u_\uparrow(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu v_\downarrow(p_4)]$$

★ **For a combination of V and A currents,** $\overline{u}_\uparrow\gamma^\mu v_\uparrow = 0$ **etc, gives four orthogonal contributions**

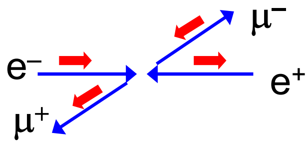
$$\Rightarrow \quad \boxed{\begin{aligned}-\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}&[c_L^e \overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)] \\ &\times[c_L^\mu \overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]\end{aligned}}$$

★ **Sum of 4 terms**

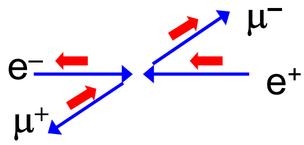
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu}[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



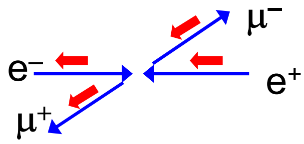
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu}[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu}[\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu}[\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state <u>particles</u>

★ **Fortunately we have calculated these terms before when considering**

$e^+e^- \to \gamma \to \mu^+\mu^-$ **giving:** **(pages 137-138)**

$$[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] = s(1+\cos\theta)$$ **etc.**

★ Applying the QED results to the Z exchange with gives:

$$\frac{e^2}{q^2} \rightarrow \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

where $q^2 = s = 4E_e^2$

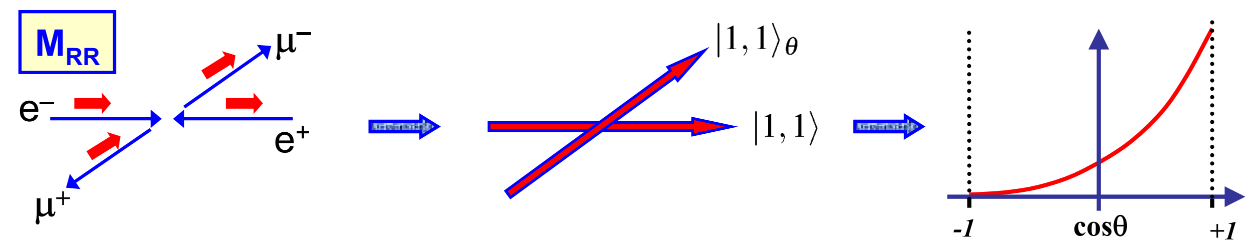
$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1+\cos\theta)^2$$

$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1-\cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1-\cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1+\cos\theta)^2$$

★ As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.



The Breit-Wigner Resonance

★ Need to consider carefully the propagator term $1/(s - m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson

★ To do this need to account for the fact that the Z boson is an unstable particle

•For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

•For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \qquad \text{with} \qquad \tau = \frac{1}{\Gamma_Z}$$

•Equivalent to making the replacement

$$m \rightarrow m - i\Gamma/2$$

★In the Z boson propagator make the substitution:

$$m_Z \rightarrow m_Z - i\Gamma_Z/2$$

★ Which gives:

$$(s - m_Z^2) \longrightarrow [s - (m_Z - i\Gamma_Z/2)] = s - m_Z^2 + i m_Z \Gamma_Z + \tfrac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + i m_Z \Gamma_Z$$

where it has been assumed that $\Gamma_Z \ll m_Z$

★ Which gives

$$\left| \frac{1}{s - m_Z^2} \right|^2 \rightarrow \left| \frac{1}{s - m_Z^2 + i m_Z \Gamma_Z} \right|^2 = \frac{1}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2}$$

★ **And the Matrix elements become**

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2 \qquad \textbf{etc.}$$

★ **In the limit where initial and final state particle mass can be neglected:**

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

(page 31)

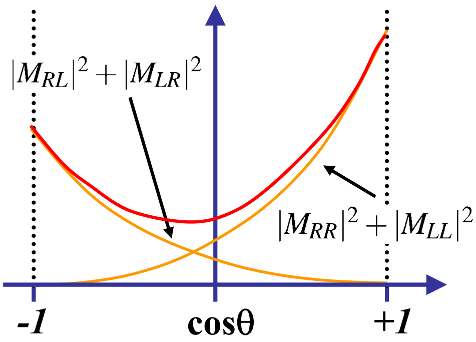
★ **Giving:**

$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

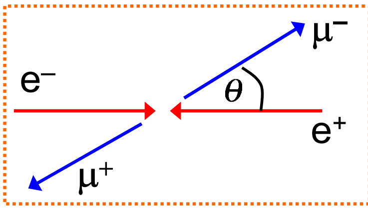
$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$



★ **Because $|M_{LL}|^2+|M_{RR}|^2 \neq |M_{LR}|^2+|M_{RL}|^2$, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).**



Cross section with unpolarized beams

★ **To calculate the total cross section need to sum over all matrix elements and average over the initial spin states. Here, assuming unpolarized beams (i.e. both e^+ and both e^- spin states equally likely) there a four combinations of initial electron/positron spins, so**

$$\langle|M_{fi}|^2\rangle = \frac{1}{2}\cdot\frac{1}{2}\cdot(|M_{RR}|^2+|M_{LL}|^2+|M_{LR}|^2+|M_{RL}|^2)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\frac{g_Z^4 s^2}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times\{[(c_R^e)^2(c_R^\mu)^2+(c_L^e)^2(c_L^2)^2](1+\cos\theta)^2 + [(c_L^e)^2(c_R^\mu)^2+(c_R^e)^2(c_L^2)^2](1-\cos\theta)^2\}$$

★ **The part of the expression {…} can be rearranged:**

$$\{\ldots\} = [(c_R^e)^2+(c_L^e)^2][(c_R^\mu)^2+(c_L^\mu)^2](1+\cos^2\theta) + 2[(c_R^e)^2-(c_L^e)^2][(c_R^\mu)^2-(c_L^\mu)^2]\cos\theta \qquad \textbf{(1)}$$

and using $c_V^2+c_A^2 = 2(c_L^2+c_R^2)$ **and** $c_V c_A = c_L^2+c_R^2$

$$\{\ldots\} = \frac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta$$

★ Hence the complete expression for the unpolarized differential cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\langle|M_{fi}|^2\rangle$$

$$= \frac{1}{64\pi^2}.\frac{1}{4}.\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times$$

$$\left\{\tfrac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta\right\}$$

★ Integrating over solid angle $d\Omega = d\phi d(\cos\theta) = 2\pi d(\cos\theta)$

$\int_{-1}^{+1}(1+\cos^2\theta)d(\cos\theta) = \int_{-1}^{+1}(1+x^2)dx = \frac{8}{3}$ and $\int_{-1}^{+1}\cos\theta d(\cos\theta) = 0$

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]}$$

★ Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions

$$(c_V^f)^2+(c_A^f)^2$$

Connection to the Breit-Wigner Formula

★ Can write the total cross section

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2]}$$

in terms of the Z boson decay rates (partial widths) from page 473 (question 26)

$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^e)^2+(c_A^e)^2]$ and $\Gamma(Z\to\mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}[(c_V^\mu)^2+(c_A^\mu)^2]$

$$\Rightarrow \quad \sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

★ Writing the partial widths as $\Gamma_{ee} = \Gamma(Z\to e^+e^-)$ etc., the total cross section can be written

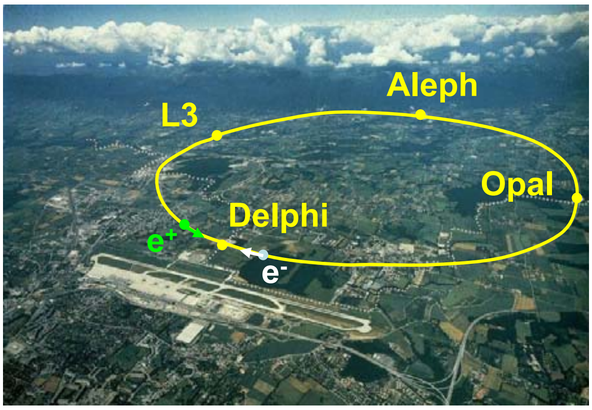
$$\boxed{\sigma(e^+e^-\to Z\to f\bar{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff}} \qquad (2)$$

where f is the final state fermion flavour:

(The relation to the non-relativistic form of the part II course is given in the appendix)

Electroweak Measurements at LEP

★The **Large Electron Positron (LEP) Collider at CERN (1989-2000)** was designed to make precise measurements of the properties of the Z and W bosons.



- 26 km circumference accelerator straddling French/Swiss boarder
- Electrons and positrons collided at 4 interaction points
- 4 large detector collaborations (each with 300-400 physicists):
  - ALEPH,
  - DELPHI,
  - L3,
  - OPAL

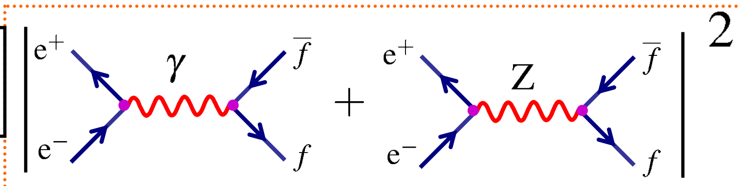
Basically a large Z and W factory:

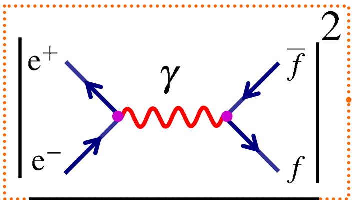
★ **1989-1995:** Electron-Positron collisions at $\sqrt{s}$ = 91.2 GeV
- 17 Million Z bosons detected

★ **1996-2000:** Electron-Positron collisions at $\sqrt{s}$ = 161-208 GeV
- 30000 W^+W^- events detected

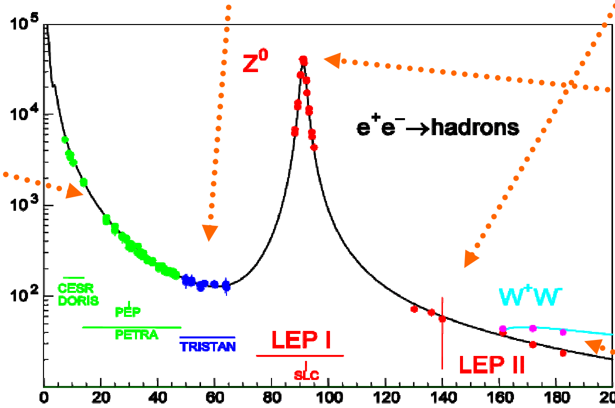
e^+e^- Annihilation in Feynman Diagrams

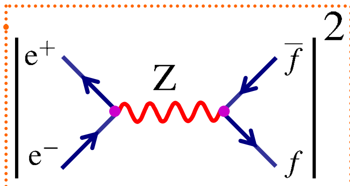
In general e^+e^- annihilation involves both photon and Z exchange : + interference





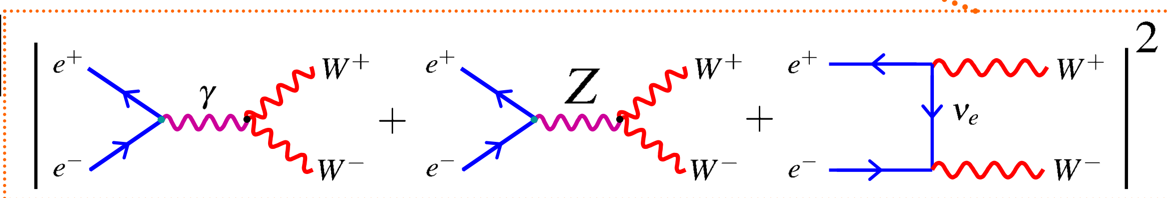
Well below Z: photon exchange dominant





At Z resonance: Z exchange dominant

High energies: WW production



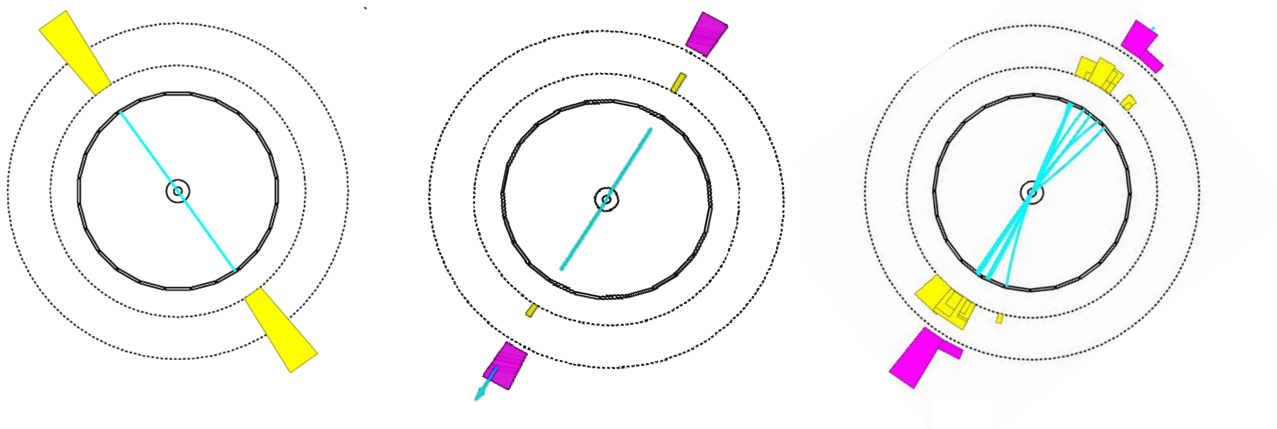
Cross Section Measurements

★ **At Z resonance mainly observe four types of event:**

$$e^+e^- \to Z \to e^+e^- \qquad e^+e^- \to Z \to \mu^+\mu^- \qquad e^+e^- \to Z \to \tau^+\tau^-$$
$$e^+e^- \to Z \to q\bar{q} \to \text{hadrons}$$

★ **Each has a distinct topology in the detectors, e.g.**

$$e^+e^- \to Z \to e^+e^- \qquad e^+e^- \to Z \to \mu^+\mu^- \qquad e^+e^- \to Z \to \text{hadrons}$$



★ **To work out cross sections, first count events of each type**

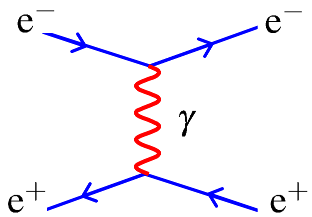
★ **Then need to know "integrated luminosity" of colliding beams, i.e. the relation between cross-section and expected number of interactions**

$$N_{\text{events}} = \mathscr{L}\sigma$$

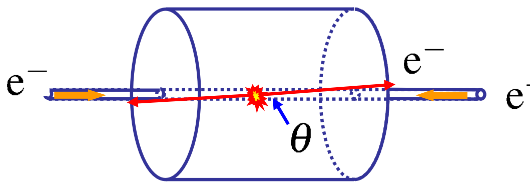
★ **To calculate the integrated luminosity need to know numbers of electrons and positrons in the colliding beams and the exact beam profile**

- very difficult to achieve with precision of better than 10%

★ **Instead "normalise" using another type of event:**



- **Use the QED Bhabha scattering process**
- **QED, so cross section can be calculated very precisely**
- **Very large cross section – small statistical errors**
- **Reaction is very forward peaked – i.e. the electron tends not to get deflected much**



$$\frac{d\sigma}{d\Omega} \propto \frac{1}{q^4} \propto \frac{1}{\sin^4\theta/2} \quad \Rightarrow \quad \boxed{\frac{d\sigma}{d\theta} \propto \frac{1}{\theta^3}}$$

Photon propagator **e.g. see handout 5**

- **Count events where the electron is scattered in the very forward direction**

$$N_{\text{Bhabha}} = \mathscr{L}\sigma_{\text{Bhabha}} \Rightarrow \mathscr{L}$$

σ_{Bhabha} **known from QED calc.**

★ **Hence all other cross sections can be expressed as**

$$\boxed{\sigma_i = \frac{N_i}{N_{\text{Bhabha}}}\sigma_{\text{Bhabha}}} \Rightarrow$$

Cross section measurements Involve just event counting !

Measurements of the Z Line-shape

★ Measurements of the Z resonance lineshape determine:

- m_Z : peak of the resonance
- Γ_Z : FWHM of resonance
- Γ_f : Partial decay widths
- N_ν : Number of light neutrino generations

★ Measure cross sections to different final states versus C.o.M. energy $\sqrt{s}$

★ Starting from

$$\sigma(e^+e^- \to Z \to f\overline{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \qquad (3)$$

maximum cross section occurs at $\sqrt{s}=m_Z$ with peak cross section equal to

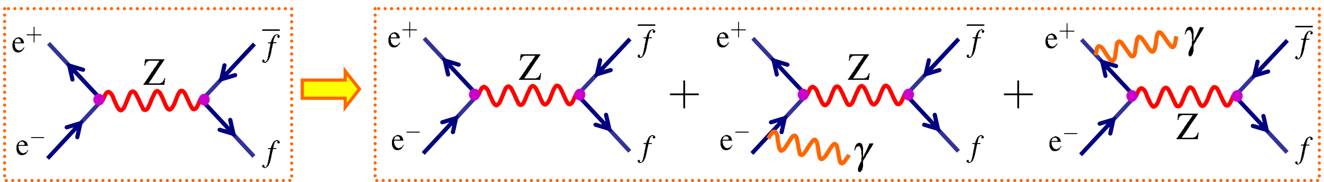
$$\boxed{\sigma^0_{f\overline{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}}$$

★ Cross section falls to half peak value at $\sqrt{s} \approx m_z \pm \frac{\Gamma_Z}{2}$ which can be seen immediately from eqn. (3)

★ Hence $\Gamma_Z = \frac{\hbar}{\tau_Z} = \text{FWHM of resonance}$

★ In practise, it is not that simple, QED corrections distort the measured line-shape

★ One particularly important correction: initial state radiation (ISR)



★ Initial state radiation reduces the centre-of-mass energy of the e^+e^- collision

$$e^+ \xrightarrow{E} \quad \xleftarrow{E} e^- \qquad \sqrt{s}=2E$$

becomes

$$\xrightarrow{E} \quad \xleftarrow{E-E_\gamma} \qquad \sqrt{s'} \approx 2E\left(1-\frac{E_\gamma}{2E}\right)$$

★ Measured cross section can be written:

$$\sigma_{\text{meas}}(E) = \int \sigma(E')f(E',E)\text{d}E'$$

Probability of e+e- colliding with C.o.M. energy E' when C.o.M energy before radiation is E

★ Fortunately can calculate $f(E',E)$ very precisely, just QED, and can then obtain Z line-shape from measured cross section

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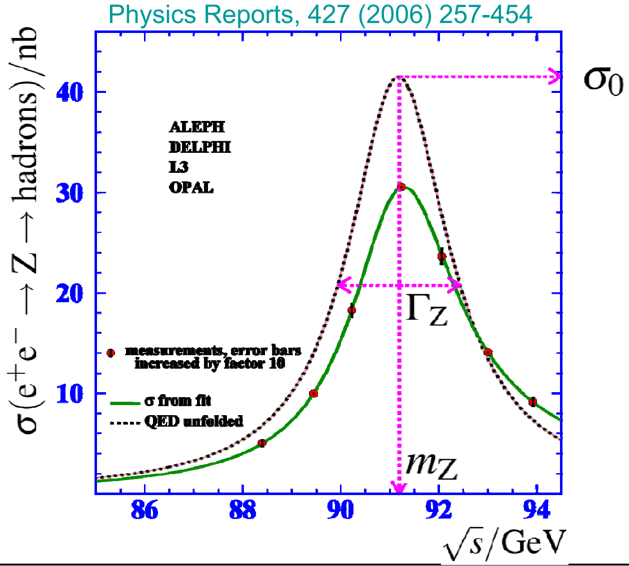


★ In principle the measurement of m_Z and Γ_Z is rather simple: run accelerator at different energies, measure cross sections, account for ISR, then find peak and FWHM

$$\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}} \qquad \boxed{\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}}$$

★ 0.002 % measurement of m_Z !

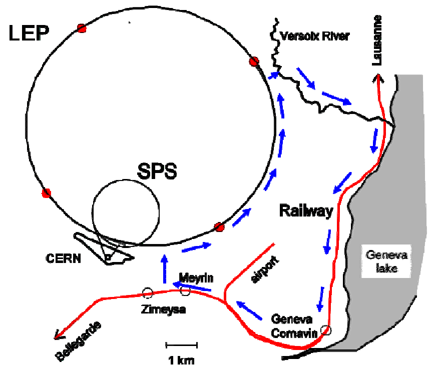
★ To achieve this level of precision – need to know energy of the colliding beams to better than 0.002 % : sensitive to unusual systematic effects…

Moon:

- As the moon orbits the Earth it distorts the rock in the Geneva area very slightly !
- The nominal radius of the accelerator of 4.3 km varies by ±0.15 mm
- Changes beam energy by ~10 MeV : need to correct for tidal effects !

Trains:

- Leakage currents from the TGV railway line return to Earth following the path of least resistance.
- Travelling via the Versoix river and using the LEP ring as a conductor.
- Each time a TGV train passed by, a small current circulated LEP slightly changing the magnetic field in the accelerator
- LEP beam energy changes by ~10 MeV



Number of generations

★ Total decay width measured from Z line-shape: $\Gamma_Z = 2.4952 \pm 0.0023\,\mathrm{GeV}$

★ If there were an additional 4th generation would expect $Z \to \nu_4 \overline{\nu}_4$ decays even if the charged leptons and fermions were too heavy (i.e. > $m_Z/2$)

★ Total decay width is the sum of the partial widths:

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\mathrm{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

★ Although don't observe neutrinos, $Z \to \nu\overline{\nu}$ decays affect the Z resonance shape for **all** final states

★ For all other final states can determine partial decay widths from peak cross sections:

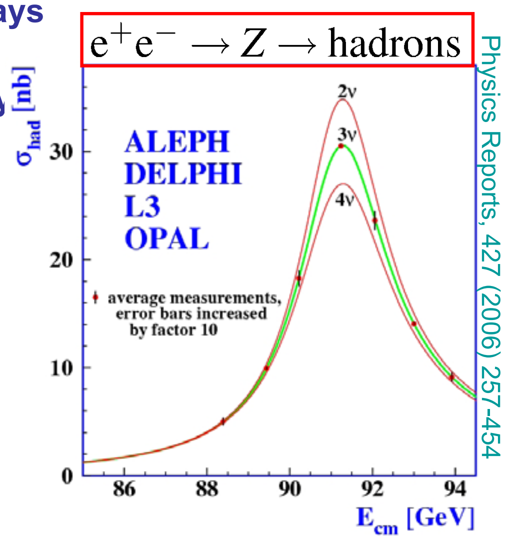
$$\sigma^0_{f\overline{f}} = \frac{12\pi}{m_Z^2}\frac{\Gamma_{ee}\Gamma_{ff}}{\Gamma_Z^2}$$

★ Assuming lepton universality:

$$\Gamma_Z = \underbrace{3\Gamma_{\ell\ell}}_{\text{measured from peak cross sections}} + \underbrace{\Gamma_{\mathrm{hadrons}}}_{\text{measured from peak cross sections}} + \underbrace{\boxed{N_\nu}}_{\text{}}\,\underbrace{\Gamma_{\nu\nu}}_{\text{calculated, e.g. question 26}}$$

measured from Z lineshape | measured from peak cross sections | calculated, e.g. question 26

$$\Rightarrow \boxed{N_\nu = 2.9840 \pm 0.0082}$$



★ **ONLY 3 GENERATIONS** (unless a new 4th generation neutrino has very large mass)

Forward-Backward Asymmetry

★ On page 495 we obtained the expression for the differential cross section:

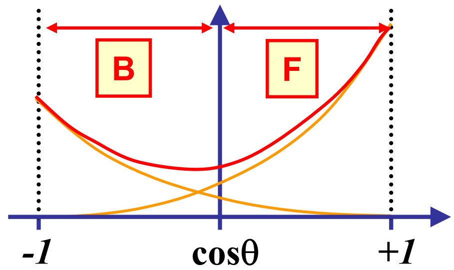
$$\langle|M_{fi}|\rangle^2 \propto [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2](1+\cos^2\theta)+[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2]\cos\theta$$

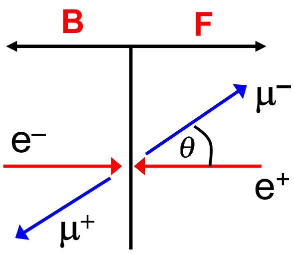
★ The differential cross sections is therefore of the form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \kappa\times[A(1+\cos^2\theta)+B\cos\theta] \qquad \left\{ \begin{aligned} A &= [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2] \\ B &= [(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2] \end{aligned} \right.$$

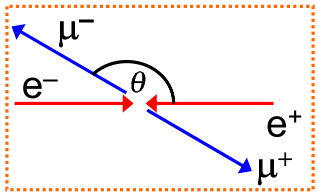
★ Define the **FORWARD** and **BACKWARD** cross sections in terms of angle incoming electron and out-going particle

$$\boxed{\sigma_F \equiv \int_0^1 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta} \qquad \boxed{\sigma_B \equiv \int_{-1}^0 \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta}$$



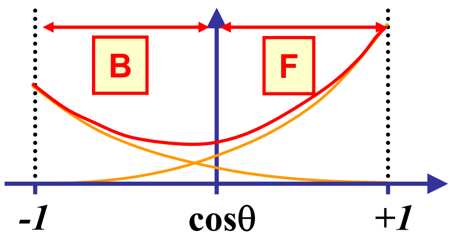


e.g. "backward hemisphere"



★ The level of asymmetry about **cosθ=0** is expressed in terms of the Forward-Backward Asymmetry

$$A_{\mathrm{FB}} = \frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B}$$



• Integrating equation (1):

$$\sigma_F = \kappa\int_0^1[A(1+\cos^2\theta)+B\cos\theta]\mathrm{d}\cos\theta = \kappa\int_0^1[A(1+x^2)+Bx]\mathrm{d}x = \kappa\left(\frac{4}{3}A+\frac{1}{2}B\right)$$

$$\sigma_B = \kappa\int_{-1}^0[A(1+\cos^2\theta)+B\cos\theta]\mathrm{d}\cos\theta = \kappa\int_0^1[A(1+x^2)+Bx]\mathrm{d}x = \kappa\left(\frac{4}{3}A-\frac{1}{2}B\right)$$

★ Which gives:

$$A_{\mathrm{FB}} = \frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4}\left[\frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2}\right]\cdot\left[\frac{(c_L^\mu)^2-(c_R^\mu)^2}{(c_L^\mu)^2+(c_R^\mu)^2}\right]$$

★ This can be written as

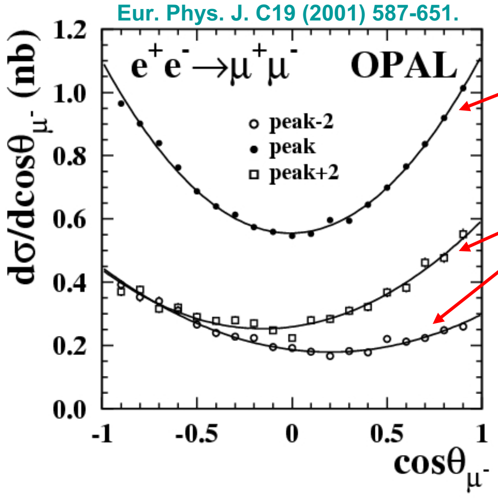
$$\boxed{A_{\mathrm{FB}} = \frac{3}{4}A_eA_\mu} \quad \text{with} \quad \boxed{A_f \equiv \frac{(c_L^e)^2-(c_R^e)^2}{(c_L^e)^2+(c_R^e)^2} = \frac{2c_V^fc_A^f}{(c_V^f)^2+(c_A^f)^2}} \qquad (4)$$

★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

Measured Forward-Backward Asymmetries

★ Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g. $e^+e^- \to Z \to \mu^+\mu^-$

OPAL Collaboration,
Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_W \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \to \gamma \to \mu^+\mu^-$ leads to a larger asymmetry

★LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

★To relate these measurements to the couplings uses $A_{\mathrm{FB}} = \frac{3}{4}A_e A_\mu$

★ In all cases asymmetries depend on A_e

★ To obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4}A_e^2$ (also see Appendix II for A_{LR})

Determination of the Weak Mixing Angle

★ From LEP : $A_{FB}^{0,f} = \frac{3}{4}A_e A_f$

★ From SLC : $A_{LR} = A_e$

$\Big\}\ A_e, A_\mu, A_\tau, \ldots$

Putting everything together

$$A_e = 0.1514 \pm 0.0019$$
$$A_\mu = 0.1456 \pm 0.0091$$
$$A_\tau = 0.1449 \pm 0.0040$$

includes results from other measurements

with
$$A_f \equiv \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} = 2\frac{c_V/c_A}{1+(c_V/c_A)^2}$$

★ Measured asymmetries give ratio of vector to axial-vector Z coupings.

★ In SM these are related to the weak mixing angle

$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q\sin^2\theta_W}{I_W^3} = 1 - \frac{2Q}{I_3}\sin^2\theta_W = 1 - 4|Q|\sin^2\theta_W$$

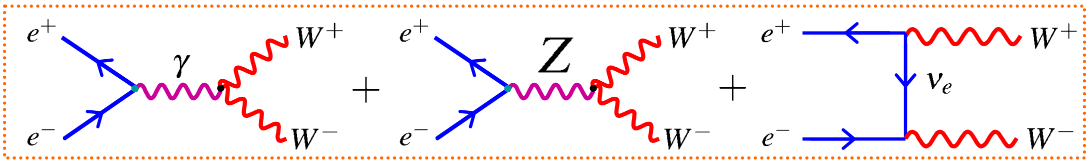
★ Asymmetry measurements give precise determination of $\sin^2\theta_W$

$$\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$$

W^+W^- Production

★ From 1995-2000 LEP operated above the threshold for W-pair production

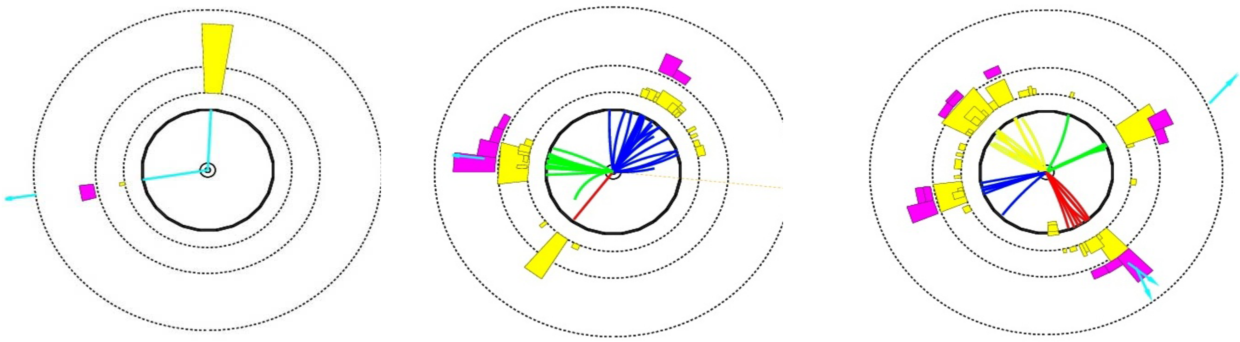
★ Three diagrams "CC03" are involved



★ W bosons decay (p.459) either to leptons or hadrons with branching fractions:

$$Br(W^- \to \text{hadrons}) \approx 0.67 \qquad Br(W^- \to e^-\overline{\nu}_e) \approx 0.11$$
$$Br(W^- \to \mu^-\overline{\nu}_\mu) \approx 0.11 \qquad Br(W^- \to \tau^-\overline{\nu}_\tau) \approx 0.11$$

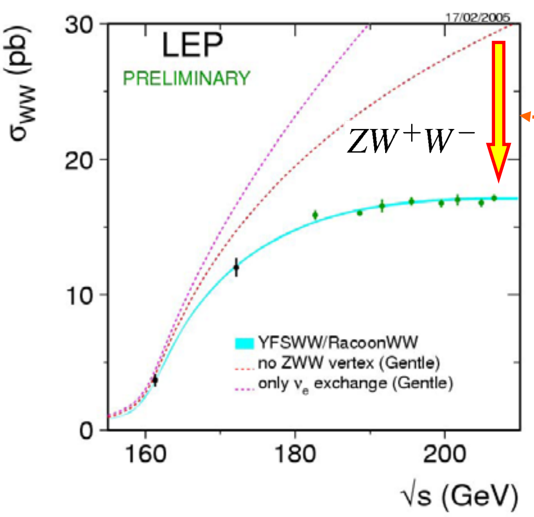
★ Gives rise to three distinct topologies



$\boxed{W^+W^- \to \ell^+\nu\ell^-\overline{\nu}}$ $\boxed{W^+W^- \to q\overline{q}\ell\nu}$ $\boxed{W^+W^- \to q\overline{q}q\overline{q}}$

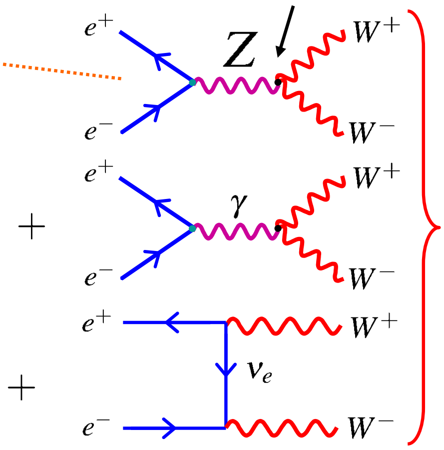
$e^+e^- \to W^+W^-$ Cross Section

★ Measure cross sections by **counting events** and normalising to low angle Bhabha scattering events



★ Data consistent with SM expectation

★ Provides a direct test of ZW^+W^- vertex



★ Recall that without the Z diagram the cross section violates unitarity

★ Presence of Z fixes this problem

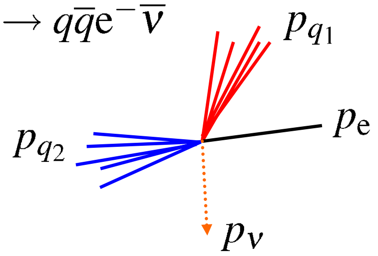
W-mass and W-width

★ Unlike $e^+e^- \to Z$, the process $e^+e^- \to W^+W^-$ is not a resonant process

⟹ Different method to measure W-boson Mass

•Measure energy and momenta of particles produced in the W boson decays, e.g.

$$W^+W^- \to q\bar{q}e^-\bar{\nu}$$

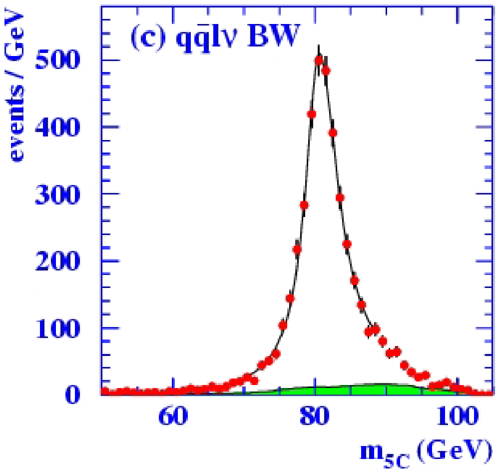


- Neutrino four-momentum from energy-momentum conservation !

$$p_{q_1} + p_{q_2} + p_e + p_\nu = (\sqrt{s}, 0)$$

- Reconstruct masses of two W bosons

$$M_+^2 = E^2 - \vec{p}^2 = (p_{q_1} + p_{q_2})^2$$
$$M_-^2 = E^2 - \vec{p}^2 = (p_e + p_\nu)^2$$



$$\approx \frac{1}{2}(M_+ + M_-)$$

★ Peak of reconstructed mass distribution gives

$$\boxed{m_W = 80.376 \pm 0.033\,\text{GeV}}$$

★ Width of reconstructed mass distribution gives:

$$\boxed{\Gamma_W = 2.196 \pm 0.083\,\text{GeV}}$$

Does not include measurements from Tevatron at Fermilab

The Higgs Mechanism

(For proper discussion of the Higgs mechanism see the Gauge Field Theory minor option)

★ In the handout 13 introduced the ideas of gauge symmetries and electroweak unification. However, as it stands there is a small problem; this only works for massless gauge bosons. Introducing masses in any naïve way violates the underlying gauge symmetry.

★The Higgs mechanism provides a way of giving the gauge bosons mass

★ In this handout motivate the main idea behind the Higgs mechanism (however not possible to give a rigourous treatment outside of QFT). So resort to analogy:

Analogy:

- Consider Electromagnetic Radiation propagating through a plasma
- Because the plasma acts as a polarisable medium obtain "dispersion relation"

From IB EM:

$$n^2 = 1 - \frac{n_e e^2}{\varepsilon_0 m_e \omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

n = refractive index
ω = angular frequency
ω_p = plasma frequency

- Because of interactions with the plasma, wave-groups only propagate if they have frequency/energy greater than some minimum value

$$E > E_0 = \hbar\omega_p$$

- Above this energy waves propagate with a group velocity $v_g = \dfrac{c^2}{v_p} = nc$

- Dropping the subscript and using the previous expression for n

$$v^2 = c^2 n^2 = c^2\left(1 - \frac{\hbar^2\omega_p^2}{\hbar^2\omega^2}\right) = c^2\left(1 - \frac{E_0^2}{E^2}\right)$$

- Rearranging gives

$$\frac{E_0^2}{E^2} = 1 - \frac{v^2}{c^2} \quad \Rightarrow \quad \boxed{E = E_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \gamma m c^2}$$

with $m = E_0/c^2$

- **Massless photons propagating through a plasma behave as massive particles propagating in a vacuum !**

The Higgs Mechanism

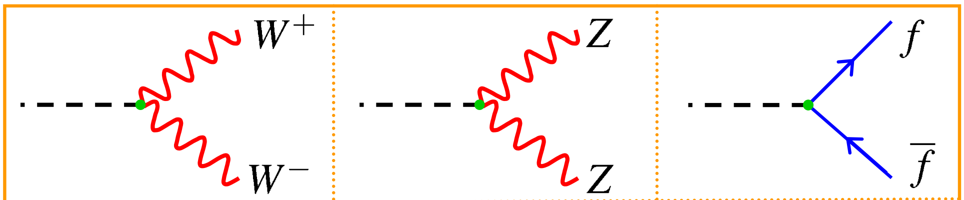
★ Propose a scalar (spin 0) field with a **non-zero vacuum expectation value** **(VEV)**

Massless Gauge Bosons propagating through the vacuum with a non-zero Higgs VEV correspond to massive particles.

★ The Higgs is **electrically neutral** but carries **weak hypercharge of 1/2**
★ The photon does not couple to the Higgs field and remains massless
★ The W bosons and the Z couple to weak hypercharge and become massive

★ The Higgs mechanism results in absolute **predictions** for **masses of gauge bosons**
★ In the SM, fermion masses are also ascribed to interactions with the Higgs field
- however, here no prediction of the masses – **just put in by hand**

Feynman Vertex factors:



$ig_W m_W g^{\mu\nu}$	$ig_Z m_Z g^{\mu\nu}$	$-i\frac{g_W}{2m_W}m_f$

★ Within the SM of Electroweak unification with the Higgs mechanism:

⇒ **Relations between standard model parameters**

$$\boxed{m_W = \left(\frac{\pi\alpha_{em}}{\sqrt{2}G_F}\right)^{\frac{1}{2}}\frac{1}{\sin\theta_W}} \qquad \boxed{m_Z = \frac{m_W}{\cos\theta_W}}$$

★ Hence, if you know **any three** of : $\alpha_{em}, G_F, m_W, m_Z, \sin\theta_W$ predict the other two.

Precision Tests of the Standard Model

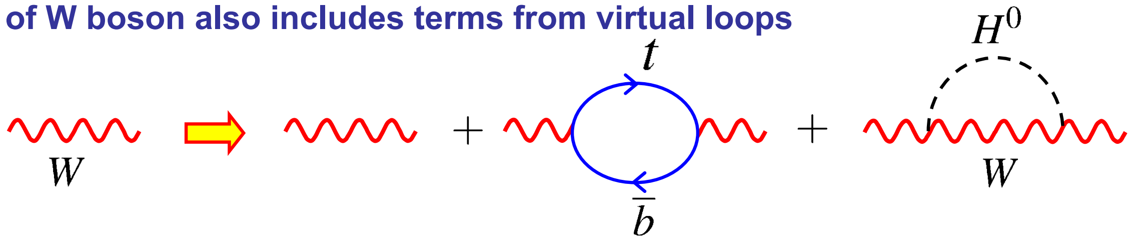
★ From LEP and elsewhere have precise measurements – can test predictions of the Standard Model !

•e.g. predict: $\boxed{m_W = m_Z \cos\theta_W}$ measure $\boxed{m_Z = 91.1875 \pm 0.0021\,\mathrm{GeV}}$ $\boxed{\sin^2\theta_W = 0.23154 \pm 0.00016}$

•Therefore expect: $\boxed{m_W = 79.946 \pm 0.008\,\mathrm{GeV}}$ but measure $\boxed{m_W = 80.376 \pm 0.033\,\mathrm{GeV}}$

★ Close, but not quite right – but have only considered lowest order diagrams

★ Mass of W boson also includes terms from virtual loops



$$\boxed{m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)}$$

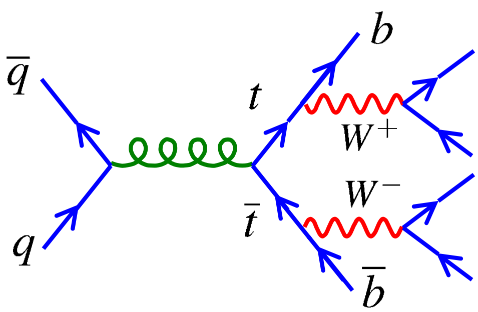
★ Above "discrepancy" due to these virtual loops, i.e. by making very high precision measurements become sensitive to the masses of particles inside the virtual loops !

The Top Quark

★ From virtual loop corrections and precise LEP data can predict the top quark mass:

$$\boxed{m_t^{\mathrm{loop}} = 173 \pm 11\,\mathrm{GeV}}$$

★ In 1994 top quark observed at the Tevatron proton anti-proton collider at Fermilab – with the predicted mass !



★ The top quark almost exclusively decays to a bottom quark since

$$|V_{tb}|^2 \gg |V_{td}|^2 + |V_{ts}|^2$$

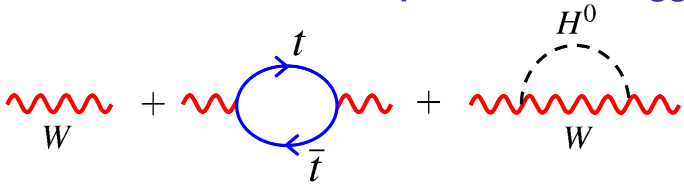
★ Complicated final state topologies:

$$t\bar{t} \to b\bar{b}q\bar{q}q\bar{q} \to 6\ \mathrm{jets}$$
$$t\bar{t} \to b\bar{b}q\bar{q}\ell\nu \to 4\ \mathrm{jets} + \ell + \nu$$
$$t\bar{t} \to b\bar{b}\ell\nu\ell\nu \to 2\ \mathrm{jets} + 2\ell + 2\nu$$

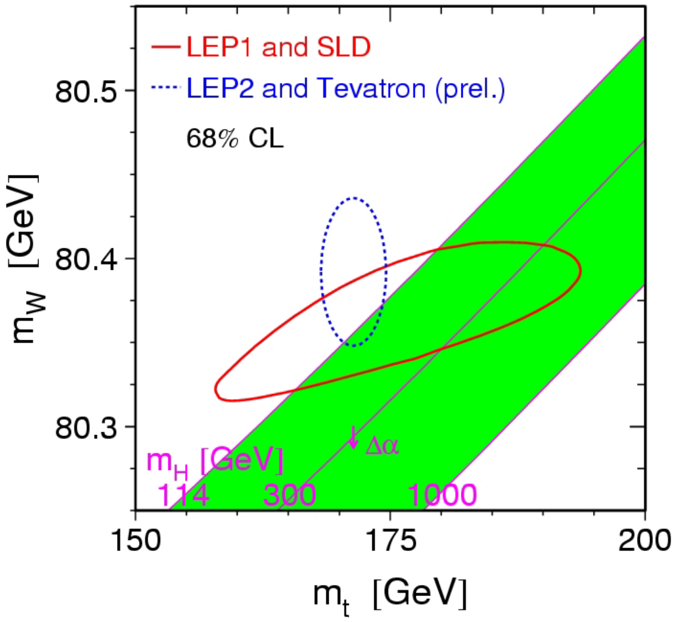
★ Mass determined by direct reconstruction (see W boson mass)

$$\boxed{m_t^{\mathrm{meas}} = 174.2 \pm 3.3\,\mathrm{GeV}}$$

★ But the W mass also depends on the Higgs mass (albeit only logarithmically)



$$m_W = m_W^0 + a m_t^2 + b \ln\left(\frac{m_H}{m_W}\right)$$



★ Measurements are sufficiently precise to have some sensitivity to the Higgs mass

★ Direct and indirect values of the top and W mass can be compared to prediction for different Higgs mass

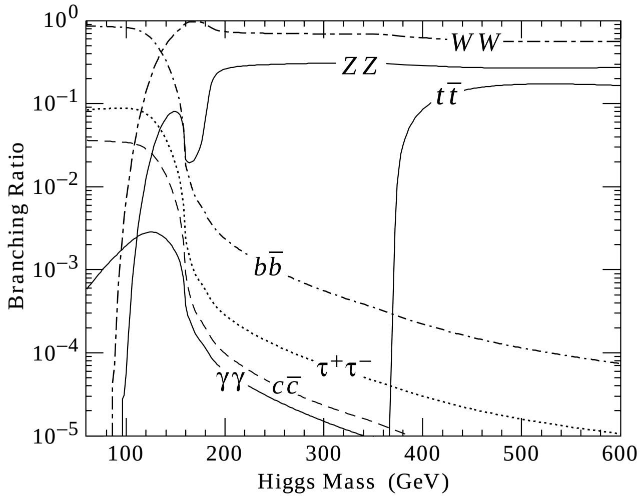
- **Direct:** W and top masses from direct reconstruction
- **Indirect:** from SM interpretation of Z mass, θ_W etc. and

★ Data favour a light Higgs:

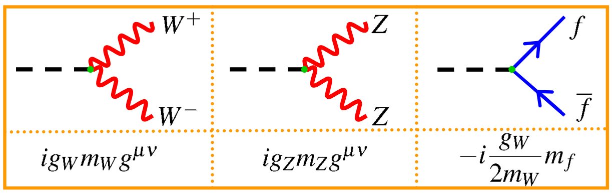
$$\Rightarrow \quad \boxed{m_H < 200\,\text{GeV}}$$

Hunting the Higgs

★ The Higgs boson is an essential part of the Standard Model – but does it exist ?
★ Consider the search at LEP. Need to know how the Higgs decays



- Higgs boson couplings proportional to mass

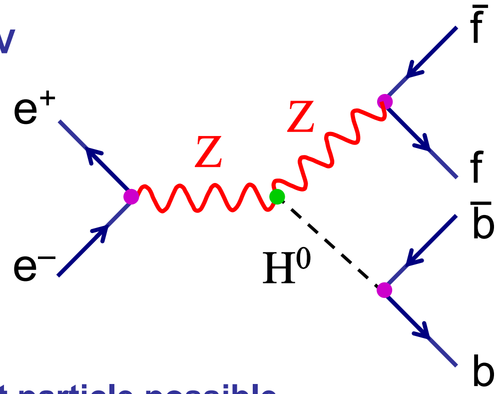


- Higgs decays predominantly to heaviest particles which are energetically allowed (Question 30)

$m_H < 2m_W$	mainly	$H^0 \to b\bar{b}$ + approx 10% $H^0 \to \tau^+\tau^-$
$2m_W < m_H < 2m_t$	almost entirely	$H^0 \to W^+W^-$ + $H^0 \to ZZ$
$m_H > 2m_t$	either	$H^0 \to W^+W^-$, $H^0 \to ZZ$, $H^0 \to t\bar{t}$

A Hint from LEP ?

- ★ LEP operated with a C.o.M. energy upto 207 GeV
- ★ For this energy (assuming the Higgs exists) the main production mechanism would be the "Higgsstrahlung" process

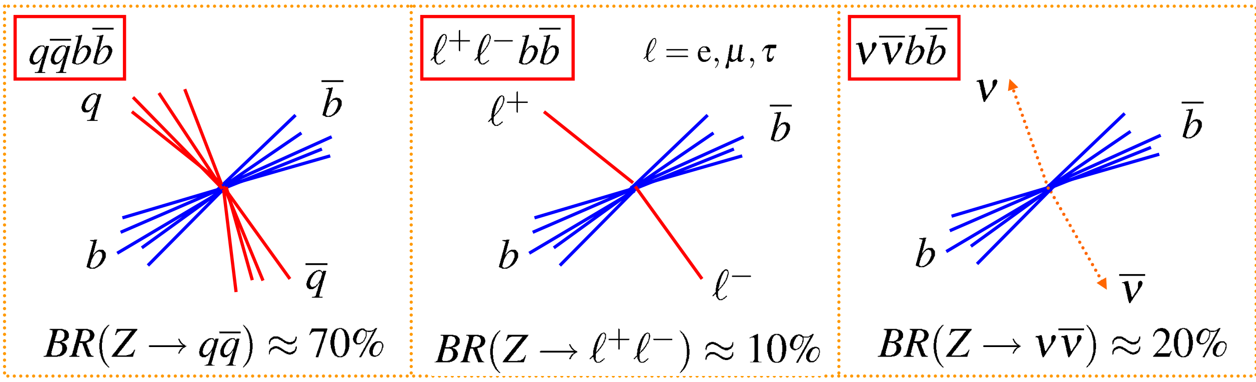


- ★ Need enough energy to make a Z and H; therefore could produce the Higgs boson if

$$m_H < 207\,\mathrm{GeV} - m_Z$$

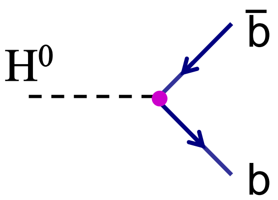
i.e. if $$m_H < 116\,\mathrm{GeV}$$

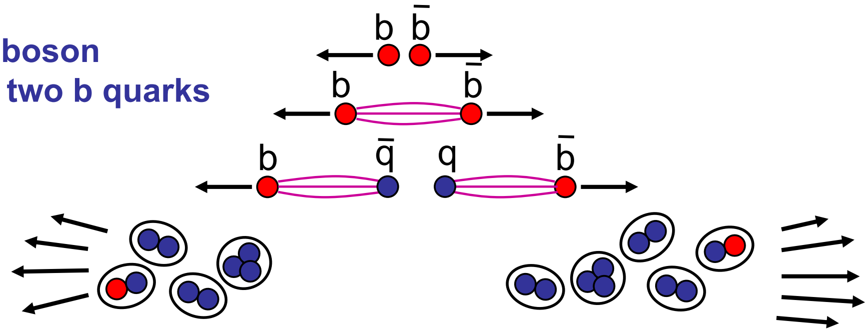
- ★ The Higgs predominantly decays to the heaviest particle possible
- ★ For $m_H < 116\,\mathrm{GeV}$ this is the b-quark (not enough mass to decay to WW/ZZ/tt)



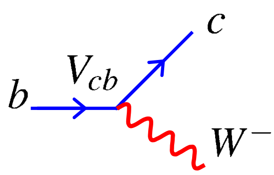
Tagging the Higgs Boson Decays

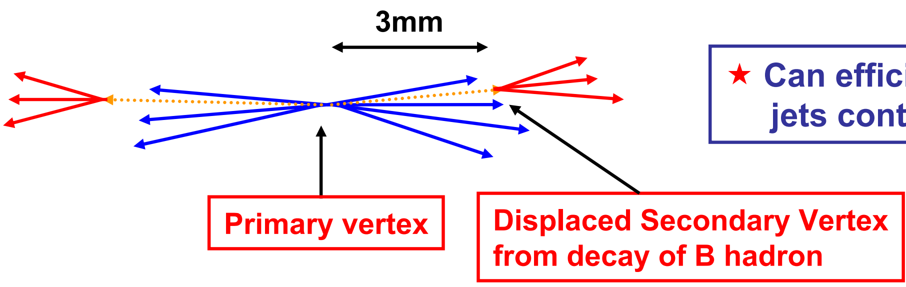
- ★ One signature for a Higgs boson decay is the production of two b quarks





- ★ Each jet will contain one b-hadron which will decay weakly
- ★ Because V_{cb} is small $(V_{cb} \approx 0.04)$ hadrons containing b-quarks are relatively long-lived
- ★ Typical lifetimes of $\tau \sim 1 \times 10^{-12}\,\mathrm{s}$
- ★ At LEP b-hadrons travel approximately **3mm** before decaying

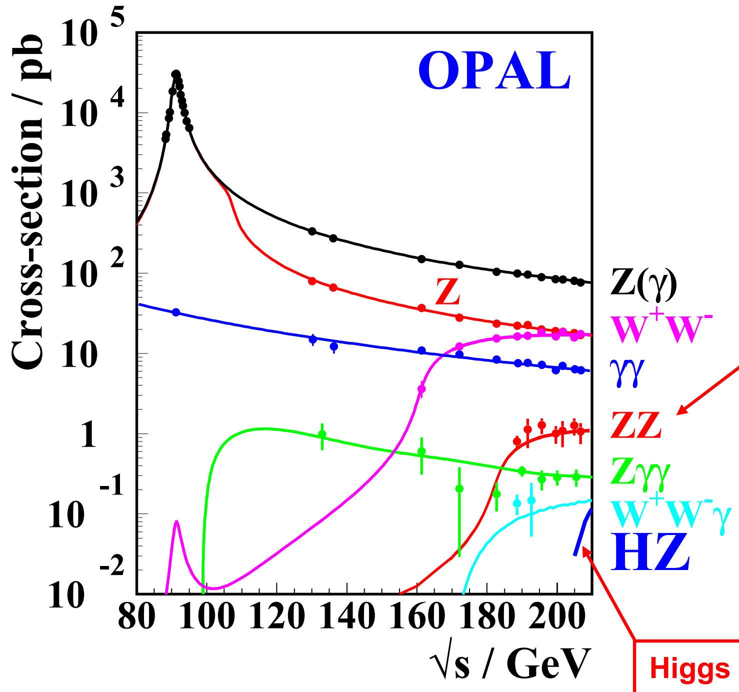




★ Can efficiently identify jets containing b quarks

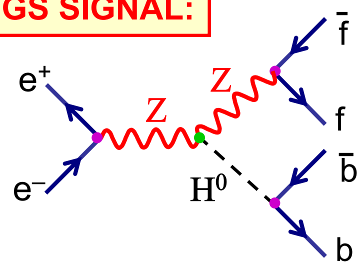
★ Clear experimental signature, but small cross section, e.g. for $m_H \approx 115\,\mathrm{GeV}$ would only produce a few tens of $e^+e^- \rightarrow H^0$ events at LEP

★ In addition, there are large "backgrounds"

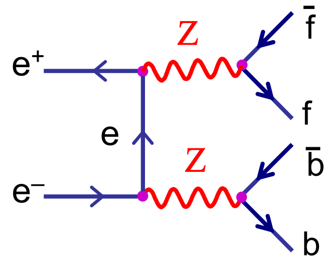


Higgs production cross section (m_H=115 GeV)

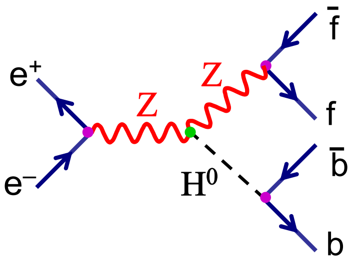
HIGGS SIGNAL:



MAIN BACKGROUND:

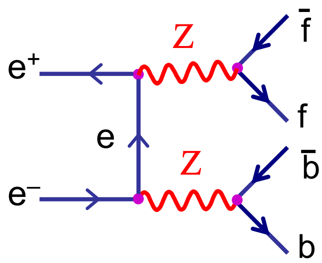


★ The only way to distinguish



$\boxed{m_{b\bar{b}} = m_H}$

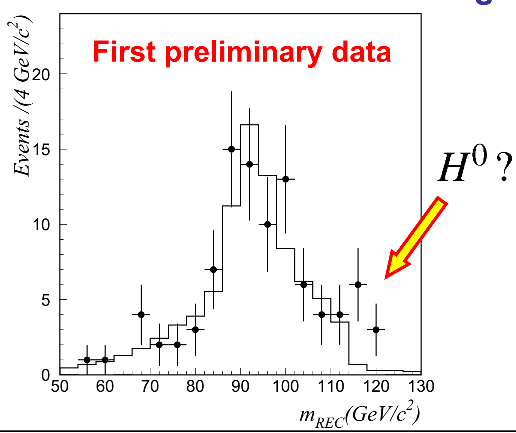
from



$\boxed{m_{b\bar{b}} = m_Z}$

is the from the invariant mass of the jets from the boson decays

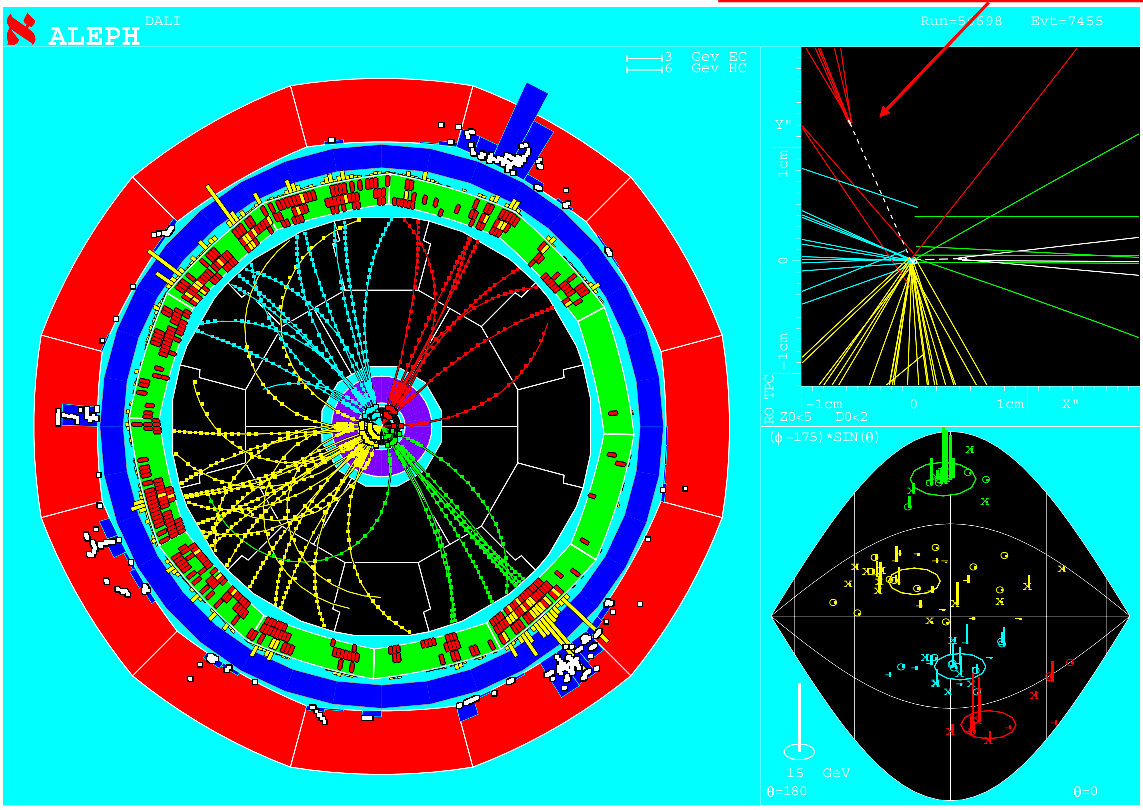
★ In 2000 (the last year of LEP running) the ALEPH experiment reported an excess of events consistent with being a Higgs boson with mass 115 GeV



- ALEPH found 3 events which were high relative probability of being signal
- L3 found 1 event with high relative probability of being signal
- OPAL and DELPHI found none

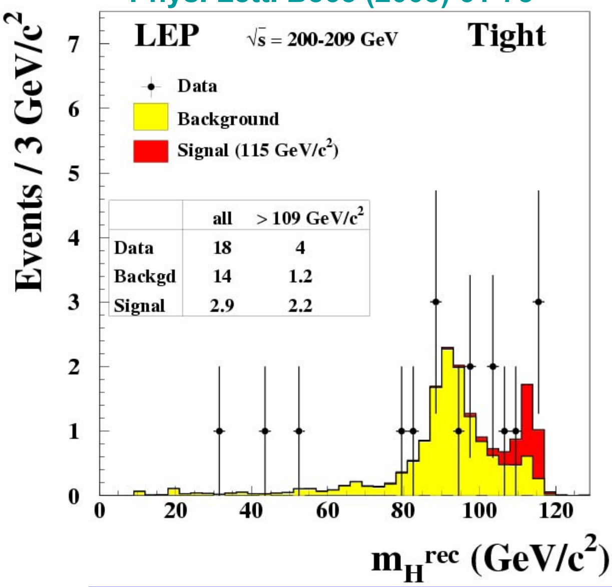
Example event:

Displaced vertex from b-decay



Combined LEP Results

Phys. Lett. B565 (2003) 61-75



- ★ Final combined LEP results fairly inconclusive
- ★ A hint rather than strong evidence…
- ★ All that can be concluded:

$$\boxed{m_H > 114\,\text{GeV}}$$

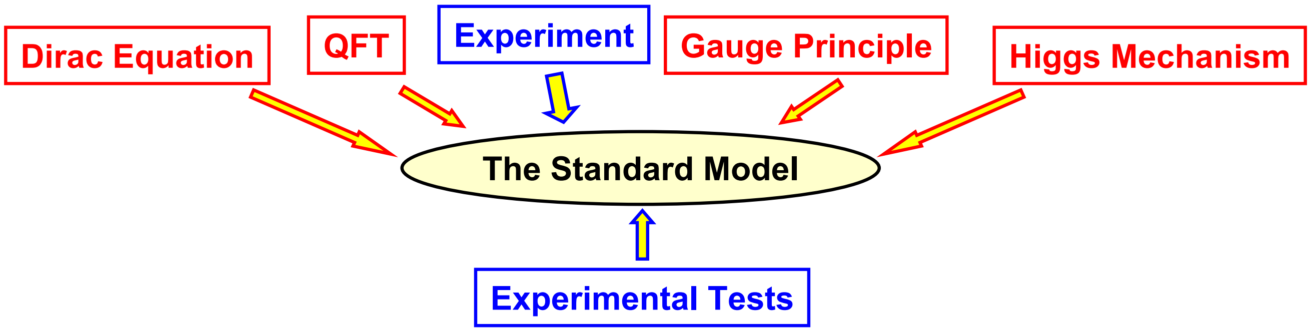
The Higgs boson remains the missing link in the Standard Model

★The LHC will take first physics data in early **2010**
★ If the Higgs exists it will be found ! (although may take a few years)

★The SM will then be complete…

Concluding Remarks

- ★ In this course (I believe) we have covered almost all aspects of modern particle physics (and to a fairly high level)
- ★ The Standard Model of Particle Physics is one of the great scientific triumphs of the late 20th century
- ★ Developed through close interplay of experiment and theory



- ★ Modern experimental particle physics provides many precise measurements. and the **Standard Model** successfully describes all current data !
- ★ Despite its great success, we should not forget that it is just a model; a collection of beautiful theoretical ideas cobbled together to fit with experimental data.
- ★ There are many issues / open questions…

The Standard Model : Problems/Open Questions

- ★ The Standard Model has too many free parameters:

$$m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, m_e, m_\mu, m_\tau, m_d, m_s, m_b, m_u, m_c, m_t$$

$$\theta_{12}, \theta_{13}, \theta_{23}, \delta \quad + \quad \lambda, A, \rho, \eta \qquad e, G_F, \theta_W, \alpha_S \qquad m_H, \theta_{CP}$$

- ★ Why three generations ?
- ★ Why $SU(3)_c \times SU(2)_L \times U(1)$?
- ★ Unification of the Forces
- ★ Origin of CP violation in early universe ?
- ★ What is Dark Matter ?
- ★ Why is the weak interaction V-A ?
- ★ Why are neutrinos so light ?
- ★ Does the Higgs exist ? + gives rise to huge cosmological constant
- ★ Ultimately need to include gravity

★ **Over the last 25 years particle physics has progressed enormously.**

In the next 10 years we will almost certainly have answers to some of the above questions – maybe not the ones we expect…

The End

Appendix I: Non-relativistic Breit-Wigner

★ For energies close to the peak of the resonance, can write $\sqrt{s} = m_Z + \Delta$

$$s = m_Z^2 + 2m_Z\Delta + \Delta^2 \approx m_Z^2 + 2m_Z\Delta \qquad \text{for} \quad \Delta \ll m_Z$$

so with this approximation

$$\begin{aligned}(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2 \approx (2m_Z\Delta)^2 + m_Z^2\Gamma_Z^2 &= 4m_Z^2(\Delta + \tfrac{1}{4}\Gamma_Z^2) \\ &= 4m_Z^2[(\sqrt{s}-m_Z)^2 + \tfrac{1}{4}\Gamma_Z^2]\end{aligned}$$

★ Giving: $\sigma(e^+e^- \to Z \to f\bar{f}) \approx \dfrac{3\pi}{m_Z^4}\dfrac{s}{(\sqrt{s}-m_Z)^2 + \frac{1}{4}\Gamma_Z^2}\Gamma_e\Gamma_f$

★ Which can be written:

$$\sigma(E) = \frac{g\lambda_e^2}{4\pi}\frac{\Gamma_i\Gamma_f}{(E-E_0)^2 + \frac{1}{4}\Gamma^2} \qquad (3)$$

$\Gamma_i,\ \Gamma_f$: are the partial decay widths of the initial and final states

$E,\ E_0$: are the centre-of-mass energy and the energy of the resonance

$g = \frac{(2J_Z+1)}{(2S_e+1)(2S_e+1)}$ is the spin counting factor $g = \frac{3}{2\times 2}$

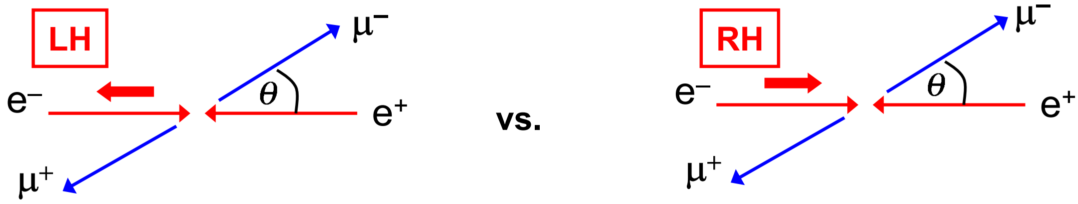
$\lambda_e = \frac{2\pi}{E}$: is the Compton wavelength (natural units) in the C.o.M of either initial particle

★ This is the **non-relativistic** form of the **Breit-Wigner** distribution first encountered in the part II particle and nuclear physics course.

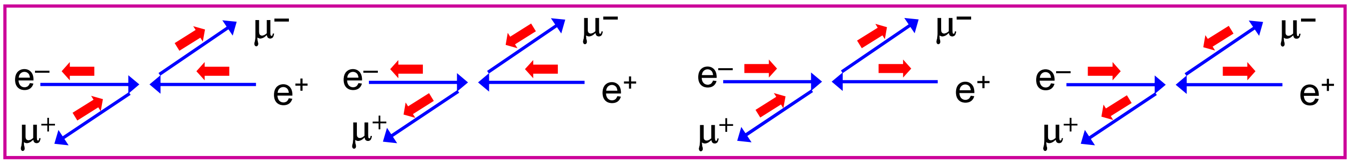
Appendix II: Left-Right Asymmetry, A_{LR}

★ At an e^+e^- linear collider it is possible to produce polarized electron beams e.g. SLC linear collider at SLAC (California), **1989-2000**

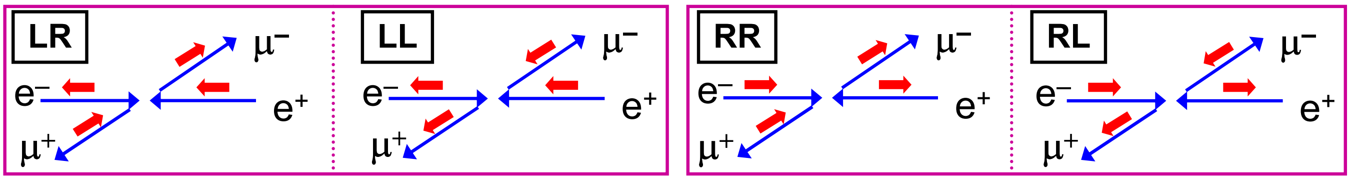
★ Measure cross section for any process for LH and RH electrons separately



- At LEP measure total cross section: sum of 4 helicity combinations:



- At SLC, by choosing the polarization of the electron beam are able to measure cross sections **separately** for LH / RH electrons



★ Averaging over the two possible polarization states of the positron for a given electron polarization:

$$\langle|M_L|\rangle^2 = \tfrac{1}{2}(|M_{LL}|^2 + |M_{LR}|^2) \qquad \langle|M_R|\rangle^2 = \tfrac{1}{2}(|M_{RL}|^2 + |M_{RR}|^2)$$

$$\Rightarrow \quad \boxed{\sigma_L = \tfrac{1}{2}(\sigma_{LR} + \sigma_{LL})} \qquad \boxed{\sigma_R = \tfrac{1}{2}(\sigma_{RR} + \sigma_{RL})}$$

★ Define cross section asymmetry:

$$\boxed{A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}}$$

★ Integrating the expressions on page 494 gives:

$$\sigma_{LL} \propto (c_L^e)^2(c_L^\mu)^2 \quad \sigma_{LR} \propto (c_L^e)^2(c_R^\mu)^2 \quad \sigma_{RL} \propto (c_R^e)^2(c_L^\mu)^2 \quad \sigma_{RR} \propto (c_R^e)^2(c_R^\mu)^2$$

$$\Rightarrow \quad \sigma_L \propto (c_L^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2] \qquad \sigma_R \propto (c_R^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2]$$

$$\boxed{A_{LR} = \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = A_e}$$

★ Hence the Left-Right asymmetry for any cross section depends only on the couplings of the electron