

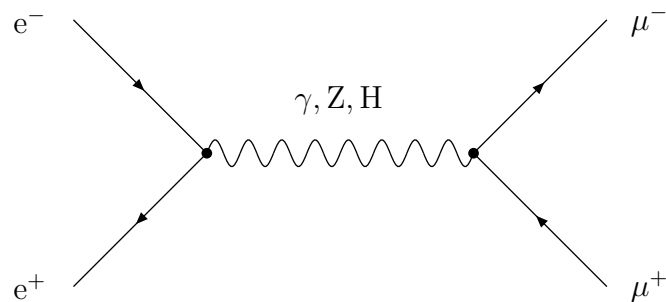
# Particle Physics Major Option Exam, January 2008

## SOLUTIONS

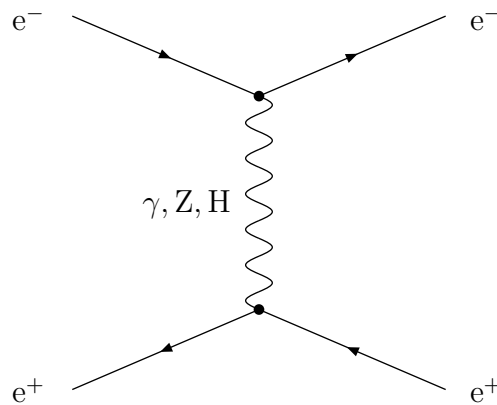
### Question 1

**Part i)** Draw the lowest order Standard Model Feynman diagrams for the process  $e^+e^- \rightarrow \mu^+\mu^-$  and the additional diagram(s) for  $e^+e^- \rightarrow e^+e^-$ . Discuss the relative importance of the different diagrams at  $\sqrt{s} = m_Z$ .

For  $e^+e^- \rightarrow \mu^+\mu^-$  have three possible s-channel diagrams:



For  $e^+e^- \rightarrow e^+e^-$  have three possible t-channel diagrams:



[1]

At  $\sqrt{s} = m_Z$ , the s-channel Z diagram dominates, although the t-channel  $e^+e^- \rightarrow e^+e^-$  diagram is important for small electron scattering angles (i.e. small  $q^2$ ). The Higgs diagrams are negligible due to smallness of electron mass (i.e. Higgs-electron coupling).

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**Part ii)** The forward-backward asymmetry is defined as  $A_{FB} = (\sigma_F - \sigma_B)/(\sigma_F + \sigma_B)$ , where  $\sigma_F = \sigma(\cos\theta > 0)$  and  $\sigma_B = \sigma(\cos\theta < 0)$ . Explain: a) why  $A_{FB}$  for  $e^+e^- \rightarrow e^+e^-$  is different from that for  $e^+e^- \rightarrow \mu^+\mu^-$  and b) why for centre-of-mass energies in the range  $\sqrt{s} = m_Z \pm \Gamma_Z$ ,  $A_{FB}$  for  $e^+e^- \rightarrow \mu^+\mu^-$  depends strongly on  $\sqrt{s}$ .

a) For electrons  $A_{FB}$  has a large contribution from the  $s$ -channel photon exchange diagram which results in a large asymmetry, i.e. many more electrons produced in the forward direction. [2]

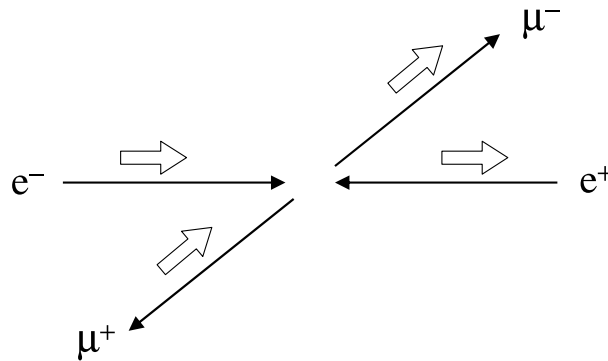
b) Away from resonance interference with the  $\gamma$  exchange diagram leads to a strong energy-dependence. [1]

**Part iii)** The matrix elements for the process  $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$  at  $\sqrt{s} = m_Z$  are:

$$\begin{aligned} |M_{RR}|^2 &= \kappa(c_R^e)^2(c_R^\mu)^2(1 + \cos\theta)^2, & |M_{LL}|^2 &= \kappa(c_L^e)^2(c_L^\mu)^2(1 + \cos\theta)^2, \\ \text{and } |M_{RL}|^2 &= \kappa(c_R^e)^2(c_L^\mu)^2(1 - \cos\theta)^2, & |M_{LR}|^2 &= \kappa(c_L^e)^2(c_R^\mu)^2(1 - \cos\theta)^2, \end{aligned}$$

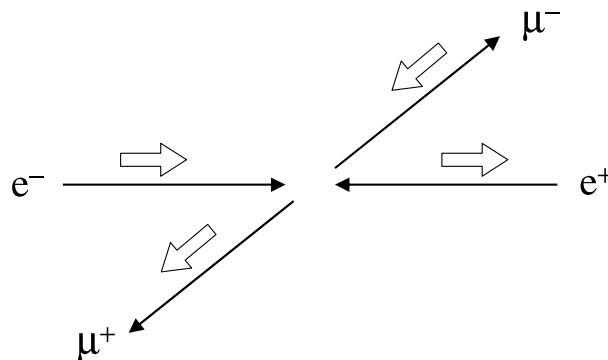
where  $\kappa = g_Z^4 m_Z^2 / \Gamma_Z^2$ , and  $g_Z c_L$  and  $g_Z c_R$  are the coupling strengths of the  $Z$  to left- and right-handed particles. Draw diagrams indicating the helicities of the initial- and final-state particles for the matrix elements  $M_{RR}$  and  $M_{RL}$  and explain clearly why only four of the possible sixteen helicity combinations give non-zero matrix elements.

For the RR combination we have:



[1.5]

For the RL combination we have:



[1.5]

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The interaction is of the form  $\frac{1}{2}\gamma^\mu(c_V - \gamma^5 c_A)$  and for any combination of vector/axial-vector couplings the chiral nature of the interaction and the fact that chiral states correspond to helicity states for ultra-relativistic particles only certain helicity combinations contribute. For example:

$$\begin{aligned} (1 - \gamma^5)\gamma^\mu(1 - \gamma^5) &= (1 - \gamma^5)(1 + \gamma^5)\gamma^\mu \\ &= (1 - \gamma^5\gamma^5)\gamma^\mu = 0 \end{aligned}$$

[2]

**Part iv)** For unpolarised electrons and positrons, the differential cross section for  $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$  can be written in the form  $d\sigma/d\Omega = A(1 + \cos^2\theta) + B \cos\theta$ . Using  $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \langle |M|^2 \rangle$ , find expressions for  $A$  and  $B$  in terms of the  $Z$  couplings to left- and right-handed particles and show that

$$\sigma(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-) = \frac{g_Z^4}{48\pi\Gamma_Z^2} [(c_R^e)^2 + (c_L^e)^2] [(c_R^\mu)^2 + (c_L^\mu)^2].$$

Assuming lepton universality,  $c_L^e = c_L^\mu = c_L$  and  $c_R^e = c_R^\mu = c_R$ , use the measurement of  $\sigma(e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-) = 2.00 \times 10^{-37} \text{ m}^2$  to obtain a value for  $c_R^2 + c_L^2$ .

Starting from

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \langle |M|^2 \rangle$$

Summing over all possible diagrams and averaging over four possible initial states.

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \frac{1}{64\pi^2 s} (|M_{RR}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{LL}|^2)$$

Using the matrix elements given:

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{1}{256\pi^2 m_Z^2} (|M_{RR}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{LL}|^2) \\ &= \frac{g_Z^4 m_Z^2}{256\pi^2 m_Z^2 \Gamma_Z^2} \{ ((c_R^e)^2 (c_R^\mu)^2 + (c_R^e)^2 (c_L^\mu)^2 + (c_L^e)^2 (c_R^\mu)^2 + (c_L^e)^2 (c_L^\mu)^2) (1 + \cos^2\theta) \\ &\quad + (2(c_R^e)^2 (c_R^\mu)^2 - 2(c_R^e)^2 (c_L^\mu)^2 - 2(c_L^e)^2 (c_R^\mu)^2 + 2(c_L^e)^2 (c_L^\mu)^2) \cos\theta \} \\ &= A(1 + \cos^2\theta) + B \cos\theta, \end{aligned}$$

with

$$\begin{aligned} A &= \frac{g_Z^4}{256\pi^2 \Gamma_Z^2} [(c_R^e)^2 + (c_L^e)^2] [(c_R^\mu)^2 + (c_L^\mu)^2] \\ B &= \frac{2g_Z^4}{256\pi^2 \Gamma_Z^2} [(c_L^e)^2 - (c_R^e)^2] [(c_L^\mu)^2 - (c_R^\mu)^2] \end{aligned}$$

[4]

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To obtain total cross-section integrate over solid angle

$$\begin{aligned}
\sigma &= \int A(1 + \cos^2 \theta) + B \cos \theta d\Omega \\
&= 2\pi \int_{-1}^{+1} A(1 + \cos^2 \theta) + B \cos \theta d(\cos \theta) \\
&= 2\pi A \left[ x + \frac{x^3}{3} \right]_{-1}^{+1} \\
&= \frac{16}{3} \pi A \\
&= \frac{16}{3} \pi \frac{g_Z^4}{256\pi^2 \Gamma_Z^2} [(c_R^e)^2 + (c_L^e)^2] [(c_R^\mu)^2 + (c_L^\mu)^2] \\
&= \frac{g_Z^4}{48\pi \Gamma_Z^2} [(c_R^e)^2 + (c_L^e)^2] [(c_R^\mu)^2 + (c_L^\mu)^2]
\end{aligned}$$

With lepton universality and converting cross section to natural units

$$\begin{aligned}
\sigma &= \frac{g_Z^4}{48\pi \Gamma_Z^2} (c_R^2 + c_L^2)^2 \\
(c_R^2 + c_L^2)^2 &= \frac{48\pi \Gamma_Z^2}{g_Z^4} \frac{2.00 \times 10^{-37}}{(0.197 \times 10^{-15})^2} \\
&= 0.0152 \\
c_R^2 + c_L^2 &= 0.123
\end{aligned}$$

**Part v)** For the process  $e^+e^- \rightarrow Z \rightarrow \mu^+\mu^-$  obtain an expression for  $A_{FB}$  in terms of  $c_L$  and  $c_R$ . Taking  $A_{FB} = 0.017$  and the result you obtained for  $c_R^2 + c_L^2$ , determine values for  $|c_L|$  and  $|c_R|$ .

$$\begin{aligned}
\sigma_F &= 2\pi \int_0^1 A(1 + \cos^2 \theta) + B \cos \theta d(\cos \theta) \\
&= 2\pi \left[ Ax + A\frac{x^3}{3} + B\frac{x^2}{2} \right]_0^{+1} \\
&= 2\pi \left( \frac{4}{3}A + \frac{1}{2}B \right) \\
\text{similarly } \sigma_B &= 2\pi \left( \frac{4}{3}A - \frac{1}{2}B \right) \\
A_{FB} &= \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3B}{8A} \\
&= \frac{3(c_L^2 - c_R^2)^2}{8(c_L^2 + c_R^2)^2}
\end{aligned}$$

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Using the the measured value of 0.017 and  $c_R^2 + c_L^2 = 0.123$  gives

$$\begin{aligned}(c_L^2 - c_R^2)^2 &= 0.0007 \\ c_L^2 - c_R^2 &= \pm 0.026\end{aligned}$$

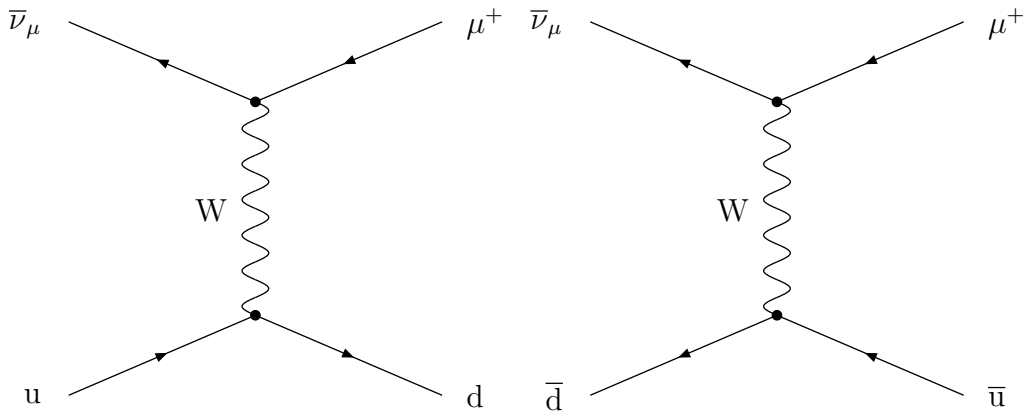
Two possible solutions,  $|c_L| = 0.27$  and  $|c_R| = 0.22$  or  $|c_L| = 0.22$  and  $|c_R| = 0.27$ . Full marks given for either solution. [4]

**Part vi)** Discuss briefly how  $|c_L|$  and  $|c_R|$  are determined for the different lepton flavours when universality is not assumed.

By measuring asymmetries/cross-sections for electrons can determine couplings to electrons, i.e. measure  $\mathcal{A}_e$ . Using this the other cross section and asymmetry measurements give  $\mathcal{A}_\mu$ . Can also use  $A_{LR}$  to get  $\mathcal{A}_e$ . [2]

## Question 2

**Part i)** Draw Feynman diagrams for the possible  $\bar{\nu}_\mu$  charged-current weak interactions with the constituents of the proton assuming that the only u, d,  $\bar{u}$ , and  $\bar{d}$  are present.



The diagrams involving a down or anti-up quark are forbidden by charge conservation (i.e. wrong type of W involved).

**Part ii)** The differential cross sections for the charged-current weak interactions of high energy  $\bar{\nu}_\mu$  with quarks/anti-quarks are:

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega^*} = \frac{G_F^2 \hat{s}}{16\pi^2} (1 + \cos\theta^*)^2, \quad \text{and} \quad \frac{d\sigma_{\bar{\nu}\bar{q}}}{d\Omega^*} = \frac{G_F^2 \hat{s}}{4\pi^2}$$

where  $\theta^*$  is the polar angle of the final-state  $\mu^+$  in the centre-of-mass frame. Explain the angular dependences of these cross sections.

**Part iii)** In  $\bar{\nu}_\mu$  deep-inelastic scattering,  $y$  is defined as  $y \equiv (p_2 \cdot q) / p_2 \cdot p_1$ , where  $p_1$  and  $p_2$  are the respective four-momenta of the  $\bar{\nu}_\mu$  and the struck quark, and  $q$  is the four momentum of the virtual W-boson. Neglecting particle masses, show that

$$y = \frac{1}{2}(1 - \cos\theta^*), \quad \frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{G_F^2 \hat{s}}{\pi} (1 - y)^2, \quad \text{and} \quad \frac{d\sigma_{\bar{\nu}\bar{q}}}{dy} = \frac{G_F^2 \hat{s}}{\pi},$$

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where  $\hat{s}$  is the centre-of-mass energy of the neutrino–quark system.

Working in the centre-of-mass frame and neglecting particle masses,  $p_1 = (E, 0, 0, E)$ ,  $p_2 = (E, 0, 0, -E)$  and  $p_3 = (E, E \sin \theta^*, 0, E \cos \theta^*)$

$$\begin{aligned} y &= \frac{p_2 \cdot q}{p_2 \cdot p_1} \\ &= \frac{p_2 \cdot (p_1 - p_3)}{2E^2} \\ &= \frac{p_2 \cdot (0, 0, 0, E(1 - \cos \theta^*))}{2E^2} \\ &= \frac{1}{2}(1 - \cos \theta^*) \end{aligned}$$

To calculate differential cross sections in terms of  $y$ , start from

$$\frac{d\sigma}{dy} = \frac{d\sigma}{d\Omega^*} \frac{d\Omega^*}{dy}$$

No azimuthal dependence, so integrate of  $\phi$

$$\frac{d\Omega^*}{dy} = 2\pi \sin \theta^* \frac{d\theta^*}{dy}$$

Using  $y = \frac{1}{2}(1 - \cos \theta^*)$

$$\begin{aligned} \frac{dy}{d\theta^*} &= \frac{1}{2} \sin \theta^* \\ \text{gives } \frac{d\Omega^*}{dy} &= 4\pi \end{aligned}$$

From which it immediately follows that

$$\begin{aligned} \frac{d\sigma_{\bar{\nu}_\mu \bar{q}}}{dy} &= \frac{G_F^2 \hat{s}}{\pi}, \\ \text{and } \frac{d\sigma_{\bar{\nu}_\mu q}}{dy} &= \frac{G_F^2 \hat{s}}{4\pi} (1 + \cos \theta^*)^2 \\ \frac{d\sigma_{\bar{\nu}_\mu q}}{dy} &= \frac{G_F^2 \hat{s}}{\pi} (1 - y)^2. \end{aligned}$$

**Part iv)** Many neutrino experiments employ detectors made of iron which contains an equal number of neutrons and protons. By considering the  $\bar{\nu}_\mu$  interactions with protons and neutrons in terms of the parton distribution functions for the proton,  $u(x)$ ,  $d(x)$ ,  $\bar{u}(x)$  and  $\bar{d}(x)$ , show that

$$\frac{d\sigma_{\bar{\nu}_\mu N}}{dy} \equiv \frac{1}{2} \left( \frac{d\sigma_{\bar{\nu}_\mu n}}{dy} + \frac{d\sigma_{\bar{\nu}_\mu p}}{dy} \right) = \frac{G_F^2}{2\pi} s [f_{\bar{q}} + (1 - y)^2 f_q],$$

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where  $s$  is the centre-of-mass energy of the neutrino–nucleon system, and  $f_q$  and  $f_{\bar{q}}$  are the fractions of the momentum of the nucleon carried by the quarks and anti-quarks respectively.

For anti-neutrino proton scattering we have

$$\frac{d\sigma_{\bar{\nu}_\mu p}}{dy} = \frac{G_F^2}{\pi} \hat{s} (1-y)^2 (u(x)dx + \bar{d}(x)dx)$$

Using  $\hat{s} = xs$

$$\frac{d\sigma_{\bar{\nu}_\mu p}}{dy} = \frac{G_F^2}{\pi} s (1-y)^2 (xu(x)dx + x\bar{d}(x)dx)$$

Similarly for anti-neutrino neutron scattering

$$\begin{aligned} \frac{d\sigma_{\bar{\nu}_\mu n}}{dy} &= \frac{G_F^2}{\pi} s (1-y)^2 (xu_n(x)dx + x\bar{d}_n(x)dx) \\ \frac{d\sigma_{\bar{\nu}_\mu n}}{dy} &= \frac{G_F^2}{\pi} s (1-y)^2 (xd(x)dx + x\bar{u}(x)dx) \end{aligned}$$

Giving

$$\begin{aligned} \frac{d\sigma_{\bar{\nu}_\mu N}}{dy} &\equiv \frac{1}{2} \left( \frac{d\sigma_{\bar{\nu}_\mu n}}{dy} + \frac{d\sigma_{\bar{\nu}_\mu p}}{dy} \right) = \frac{G_F^2}{2\pi} s \int_0^1 x(u+d)(1-y)^2 + x(\bar{u} + \bar{d})dx \\ &= \frac{G_F^2}{2\pi} s (f_q(1-y)^2 + f_{\bar{q}}) \end{aligned}$$

**Part v)** For a beam of 100 GeV  $\bar{\nu}_\mu$ , the total  $\bar{\nu}_\mu$  charged-current deep-inelastic nucleon cross section is measured to be

$$\sigma_{\bar{\nu}_\mu N} = \frac{1}{2} (\sigma_{\bar{\nu}_\mu p} + \sigma_{\bar{\nu}_\mu n}) = 3.4 \times 10^{-41} \text{ m}^2$$

and the mean value of  $y$  is measured to be 0.34. Use these results to determine  $f_q$  and  $f_{\bar{q}}$ . [9]

Integrating the above expression

$$\begin{aligned} \sigma &= \frac{G_F^2}{2\pi} s \int_0^1 (f_q(1-y)^2 + f_{\bar{q}}) dy \\ &= \frac{G_F^2}{2\pi} s \left( f_{\bar{q}} + \frac{1}{3} f_q \right) \end{aligned}$$

In terms of the laboratory frame neutrino energy  $s = 2m_N E_\nu$  so

$$\sigma = \frac{G_F^2}{\pi} m_N E_\nu \left( f_{\bar{q}} + \frac{1}{3} f_q \right)$$

Using the values given find

$$\begin{aligned} \left( f_{\bar{q}} + \frac{1}{3} f_q \right) &= \frac{\pi}{0.94 \times 100 G_F^2} \frac{3.4 \times 10^{-41}}{(0.197 \times 10^{-15})^2} \\ &= 0.215 \end{aligned}$$

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The mean value of  $y$  is given by:

$$\begin{aligned}\bar{y} &= \frac{\int y f_{\bar{q}} + y(1-y)^2 f_q dy}{\int f_{\bar{q}} + y(1-y)^2 f_q} \\ &= \frac{\frac{1}{2} f_{\bar{q}} + \frac{1}{12} f_q}{f_{\bar{q}} + \frac{1}{3} f_q}\end{aligned}$$

The denominator can be obtained from  $(f_{\bar{q}} + \frac{1}{3} f_q) = 0.215$  obtained from the cross section:

$$\frac{1}{2} f_{\bar{q}} + \frac{1}{12} f_q = 0.34 \times 0.215$$

Combining this with the expression from the cross section and solving the simultaneous equation gives  $f_q = 0.41$  and  $f_{\bar{q}} = 0.08$ .

### Question 3

Write brief notes on **three** of the following:

(a) *Electron-proton elastic scattering;* [10]

(b) *The proton wave-function. You should include a discussion of the reasons for the symmetries of the different parts of the wave-function;* [10]

(c) *The differences in the methods for detecting for  $\bar{\nu}_e$  from nuclear reactors,  $\nu_e$  from the sun, and atmospheric  $\nu_\mu$ . You should include a brief discussion of relevant energy thresholds for the different reactions;* [10]

(d) *CP violation in the Standard Model.* [10]

Answers in the form of a logically ordered bullet-pointed list are acceptable. Diagrams and simple calculations should be included where appropriate.

#### a) Electro-proton elastic scattering

The main points are:

- Elastic - proton remains intact
- Virtual photon interacts with proton as a whole (i.e. coherently)
- Only one independent variable - scattering angle fully determines kinematics, i.e. ( $x = 1$ )
- Rutherford scattering is non-relativistic recoilless limit
- Mott scattering electron relativistic, no recoil.
- Both Mott and Rutherford scattering purely electric interaction
- Charge distribution described by form factor

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- Form factor is FT of charge distribution
- At relativistic energies with proton recoil Rosenbluth formula
- Both electric term and magnetic term
- Experimentally  $G_E$  and  $G_M$  show that magnetic and electric distributions are the same using anomalous magnetic moment of
- Proton has rms radius of 1 fm
- Discussion of experimental measurement at low energy
- High energy measure  $G_M$
- Due to form factor elastic scattering cross-section falls away rapidly with  $q^2$ .

## b) The proton wave-function.

- Baryon wave-functions have flavour, colour, spin, and space parts
- Overall anti-symmetric (fermions)
- Space part symmetric ( $L=0$ )
- Colour confinement requires quarks be in a colour singlet state
- Colour singlet =  $\frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr)$  is anti-symmetric under particle exchange
- Thus, flavour x spin is symmetric
- Spin and isospin are  $SU(2)$  symmetries
- Combine two particles in  $SU(2)$  (either spin or isospin) to symmetric triplet and anti-sym singlet
- Combination of three particles gives 4 symmetric states and two mixed symmetry states.
- Mixed sym states are either symmetric or anti-symmetric under interchange of particles 1 & 2, but no overall symmetry
- symmetric spin states correspond to spin=3/2 i.e. Delta etc
- spin-half wave-function is linear combination of  $MS(\text{spin}) \times MS(\text{flavour})$  and  $MA(\text{spin}) \times MA(\text{flavour})$

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### c) Neutrino Detection

- Reactor/solar neutrinos  $\sim 1$  MeV (nuclear physics)
- Atmospheric neutrinos  $\sim 1$  GeV
- CC or NC interactions
- CC interactions with atomic electrons or nuclei
- High CC thresholds for reactions with atomic electrons ( $E_{\nu} \geq 11$  GeV for  $\nu_{\mu}$ ) - extra marks for derivation
- Lower CC thresholds for interactions with nucleons - extra marks for derivation
- Solar neutrinos:
  - Cerenkov radiation to detect elastic scattering of electrons
  - Radiochemical experiments
  - SNO uses D<sub>2</sub>O to simultaneously detect CC, ES, and NC reactions (extra marks for brief discussion)
- Reactor neutrinos:
  - Reactors produce large flux of  $\bar{\nu}_e$
  - detector via inverse  $\beta$ -reaction  $\bar{\nu}_e + p \rightarrow e^+ + n$
  - Low energy so large background, use coincidence in time of annihilation photons and photon from neutron capture
  - Mention K2K or CHOOZ
- Atmospheric neutrinos
  - High energy so easier to detect muon/electron
  - Cerenkov rings in Super-Kamiokande

## d) CP Violation in the SM

- Universe is matter dominated - no evidence of regions of anti-matter (lack of annihilation photons at matter–anti-matter boundary)
- To obtain small excess of anti-matter require CP violation at level of  $10^9 + 1$  baryons to every  $10^9$  anti-baryons in early universe.
- Describe parity.
- Describe charge conjugation.
- Assuming CPT, CP violation implies violation of T.
- In SM two place where CP arises: PMNS matrix and CKM matrix.
- CKM and PMNS matrices are unitary.
- For three generations can have a complex phase which gives CP violation
- Not possible for two generations.
- CP violation observed in kaon system
- Describe main features of CP violation in kaons
  - CP eigenstates
  - CP even decays to  $\pi\pi$  and CP odd decays to  $\pi\pi\pi$
  - CP states roughly correspond to KS and KL
  - At long distance have pure KL beam
  - But KL observed to decay to  $\pi\pi$  at level of 0.1 %
  - explained by CP violation in mixing
- CP violation enters in box diagrams because  $V_{ij} \neq V_{ij}^*$
- CP violation in SM not sufficient to explain baryon dominated universe

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