Particle Physics Major Option Exam, January 2007

SOLUTIONS

Question 1

Part i) - a total of 4 marks

Use chiral projection operators:

$$P_L = \frac{1}{2}(1-\gamma^5), \quad P_R = \frac{1}{2}(1+\gamma^5)$$

Equate vector and axial-vector couplings to LH and RH couplings

$$\begin{aligned}
\frac{1}{2}\gamma^\mu(c_\mathrm{V} - \gamma^5 c_\mathrm{A}) &= c_\mathrm{L}\gamma^\mu \frac{1}{2}(1-\gamma^5) + c_\mathrm{R}\frac{1}{2}(1+\gamma^5) \\
c_\mathrm{V} - \gamma^5 c_\mathrm{A} &= c_\mathrm{L} + c_\mathrm{R} - (c_\mathrm{L} - c_\mathrm{R})\gamma^5 \\
c_\mathrm{V} = c_\mathrm{L} + c_\mathrm{R} &\qquad c_\mathrm{A} = c_\mathrm{L} - c_\mathrm{R} \\
\Rightarrow \quad c_\mathrm{L} = \frac{1}{2}(c_\mathrm{V} + c_\mathrm{A}) &\qquad c_\mathrm{R} = \frac{1}{2}(c_\mathrm{V} - c_\mathrm{A})
\end{aligned}$$

[4]

Part ii) - a total of 2 marks

Using

$$c_\mathrm{L}^{(e)} = \frac{1}{2}(c_\mathrm{V} + c_\mathrm{A}) = I_W^3 - Q\sin^2\theta_\mathrm{W} = -0.5 - (-1)\sin^2\theta_\mathrm{W} = -0.27$$ [$\frac{1}{2}$]

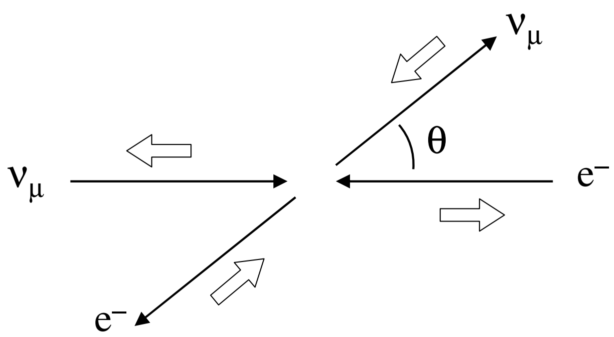
$$c_\mathrm{R}^{(e)} = \frac{1}{2}(c_\mathrm{V} - c_\mathrm{A}) = -Q\sin^2\theta_\mathrm{W} = -(-1)\sin^2\theta_\mathrm{W} = +0.23$$ [$\frac{1}{2}$]

$$c_\mathrm{L}^{(\nu)} = \frac{1}{2}(c_\mathrm{V} = c_\mathrm{A}) = I_W^3 - Q\sin^2\theta_\mathrm{W} = 0.5$$ [$\frac{1}{2}$]

$$c_\mathrm{R}^{(\nu)} = \frac{1}{2}(c_\mathrm{V} - c_\mathrm{A}) = -Q\sin^2\theta_\mathrm{W} = 0$$ [$\frac{1}{2}$]

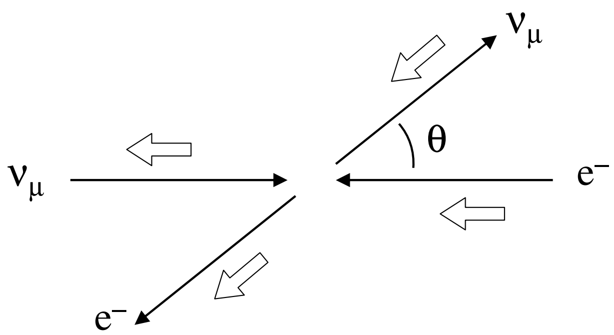
Part iii) - a total of 5 marks

For the LL combination we have:



The total z component of spin is zero. Hence there is no prefered polar angle. [2]

For the LR combination we have:

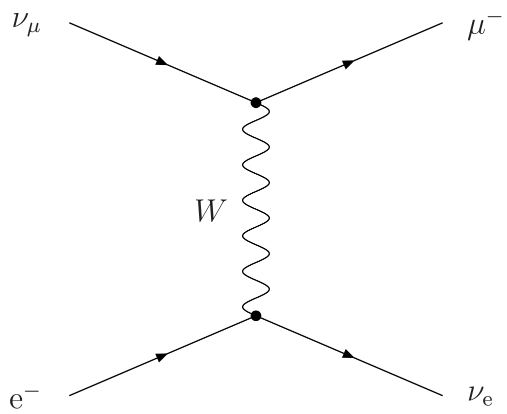


Here the incoming and out-going fermions are in spin-1 states and from the QM properties of spin-1 obtain $(1 + \cos\theta)$ dependence. [2]

The CC weak interaction is pure V-A and hence only couples to LH particles. [1]

Part iv) - a total of 6 marks

Feynman diagram for $\nu_\mu e^- \to \mu^- \nu_e$:



[1]

The matrix element can be obtained from the Feynman rules:

$$-iM_{fi} \;=\; \left[\overline{u}(p_4)\cdot -i\frac{g_W}{\sqrt{2}}\gamma^\nu\frac{1}{2}(1-\gamma^5)\cdot u(p_2)\right]\cdot\left[\frac{-ig_{\mu\nu}}{q^2-m_W^2}\right]\cdot\left[\overline{u}(p_2)\cdot -i\frac{g_W}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)\cdot u(p_1)\right]$$

[2]

Since $E_\nu \ll m_W$ can neglect q^2 in propagator

$$\begin{aligned} M_{fi} &= \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}(p_4)\gamma^\nu\frac{1}{2}(1-\gamma^5)u(p_2)\right]\cdot\left[\overline{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_1)\right] \\ &= \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}(p_4)\gamma^\nu P_L u(p_2)\right]\cdot\left[\overline{u}(p_3)\gamma^\mu P_L u(p_1)\right] \\ &= \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}(p_4)\gamma^\nu u_\downarrow(p_2)\right]\cdot\left[\overline{u}(p_3)\gamma^\mu u_\downarrow(p_1)\right] \end{aligned}$$

[1]

Using the property that helicity is conserved in any interaction invlovling a linear combination of vector and axial-vector currents:

$$M_{fi} \;=\; \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\overline{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)\right]\cdot\left[\overline{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)\right]$$

[1]

Hence can write

$$M = M_{LL} = \frac{g_W^2}{2m_W^2}j_L^{(\nu)}\cdot j_L^{(e)}$$

with currents as given in question.

Part v) - a total of 7 marks Using the spinors for the helicity eigenstates given at the bottom of the questions:

$$u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \qquad u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}$$
$$u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \qquad u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} -c \\ -s \\ c \\ s \end{pmatrix}$$

[2]

Using the expression for the components of the four-vector current and the above spinors immediately obtain

$$\begin{aligned} j_{(\nu)} &= 2E(c, s, -is, c) \\ j_{(e)} &= 2E(c, -s, -is, -c) \end{aligned}$$

[4]

Correctly taking the scalar product:

$$j_{(\nu)} \cdot j_{(e)} = 4E^2(c^2 + s^2 + s^2 + c^2) = 8E^2 = 2s$$

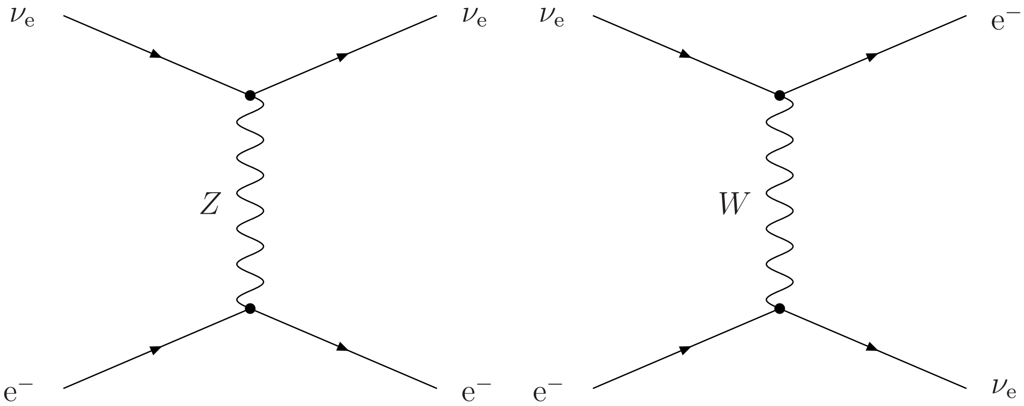
[1]

Which, as required, gives

$$\begin{aligned} M_{LL} &= 2s\frac{g_{\mathrm{W}}^2}{2m_{\mathrm{W}}^2} \\ &= \frac{g_{\mathrm{W}}^2 s}{m_{\mathrm{W}}^2} \end{aligned}$$

Part vi) - a total of 6 marks

Have two Feynman diagrams for $\nu_{\rm e}{\rm e}^- \to {\rm e}^-\nu_{\rm e}$:



[1]

For the process $\nu_\mu{\rm e}^- \to \nu_\mu{\rm e}^-$ only have NC process, whereas for $\nu_{\rm e}{\rm e}^- \to \nu_{\rm e}{\rm e}^-$ have both NC and CC. [1]

For $\nu_{\rm e}{\rm e}^- \to \nu_{\rm e}{\rm e}^-$ process $M_{LL} = M_{LL}^{NC} + M_{LL}^{CC}$, i.e. add the amplitudes. [1]

Need to take ratio of cross sections so require integral over solid angle. [1]

Correct solid angle integration [1]

The calculation. [2]

Part vi) - numerical solution

Using the results given in the question and

$$\frac{g_{\rm Z}}{m_{\rm Z}} = \frac{g_{\rm W}}{m_{\rm W}},$$

for the process $\nu_{\rm e}{\rm e}^- \to \nu_{\rm e}{\rm e}^-$ we have have

$$\begin{aligned}
M_{LR} &= c_{\rm L}^{(\nu)} c_{\rm R}^{(e)} \frac{s g_{\rm Z}^2}{m_Z^2}\frac{1}{2}(1+\cos\theta) \\
&= \frac{1}{2} c_{\rm L}^{(\nu)} c_{\rm R}^{(e)} \frac{s g_{\rm W}^2}{m_W^2}(1+\cos\theta) \\
|M_{LR}|^2 &= \frac{1}{4} c_{\rm L}^{(\nu)2} c_{\rm R}^{(e)2} \frac{s^2 g_{\rm W}^4}{m_W^4}(1+\cos\theta)^2 \\
\int |M_{LR}|^2 {\rm d}\Omega &= \frac{16\pi}{3}\frac{1}{4} c_{\rm L}^{(\nu)2} c_{\rm R}^{(e)2} \frac{s^2 g_{\rm W}^4}{m_W^4} \\
&= \frac{4\pi}{3} c_{\rm L}^{(\nu)2} c_{\rm R}^{(e)2} \frac{s^2 g_{\rm W}^4}{m_W^4}
\end{aligned}$$

For M_{LL} have contributions from both diagrams

$$\begin{aligned}
M_{LL} &= c_{\mathrm{L}}^{(\nu)}c_{\mathrm{L}}^{(e)}\frac{sg_{\mathrm{Z}}^2}{m_Z^2}+\frac{sg_{\mathrm{W}}^2}{m_W^2}\\
&= (1+c_{\mathrm{L}}^{(\nu)}c_{\mathrm{L}}^{(e)})\frac{sg_{\mathrm{W}}^2}{m_W^2}+\frac{sg_{\mathrm{W}}^2}{m_W^2}\\
|M_{LL}|^2 &= (1+c_{\mathrm{L}}^{(\nu)}c_{\mathrm{L}}^{(e)})^2\frac{s^2g_{\mathrm{W}}^4}{m_W^4}\\
\int|M_{LL}|^2\mathrm{d}\Omega &= 4\pi(1+c_{\mathrm{L}}^{(\nu)}c_{\mathrm{L}}^{(e)})^2\frac{s^2g_{\mathrm{W}}^4}{m_W^4}
\end{aligned}$$

For the process $\nu_\mu\mathrm{e}^-\to\nu_\mu\mathrm{e}^-$ only have NC process:

$$\begin{aligned}
\int|M_{LR}|^2\mathrm{d}\Omega &= \frac{4\pi}{3}c_{\mathrm{L}}^{(\nu)2}c_{\mathrm{R}}^{(e)2}\frac{s^2g_{\mathrm{W}}^4}{m_W^4}\\
\text{and}\quad \int|M_{LL}|^2\mathrm{d}\Omega &= 4\pi(c_{\mathrm{L}}^{(\nu)}c_{\mathrm{L}}^{(e)})^2\frac{s^2g_{\mathrm{W}}^4}{m_W^4}
\end{aligned}$$

[Note added by Christopher Lester in 2016. Alasdair McNab (a Part III student) has pointed out to me that there are errors in both the printed tripos paper for the last part of this question, and in the worked solutions provided by the original examiner. The original question asked for people to show that the states ratio of cross sections was 1/6 when it is not, and furthermore the worked answer contained a proof that was flawed in other ways. A corrected answer follows, together with the original answer supplied by Dr (now Prof) Thomson.]

Incorrect answer from Thomson:

The ratio of cross-sections is:

$$\begin{aligned}
\frac{\sigma(\nu_\mu\mathrm{e}^-\to\nu_\mu\mathrm{e}^-)}{\sigma(\nu_{\mathrm{e}}\mathrm{e}^-\to\nu_{\mathrm{e}}\mathrm{e}^-)} &= \frac{(1+c_{\mathrm{L}}^{(e)}c_{\mathrm{L}}^{(\nu)})^2+\frac{1}{3}(c_{\mathrm{L}}^{(e)}c_{\mathrm{L}}^{(\nu)})^2}{(c_{\mathrm{L}}^{(e)}c_{\mathrm{L}}^{(\nu)})^2+\frac{1}{3}(c_{\mathrm{L}}^{(e)}c_{\mathrm{L}}^{(\nu)})^2}\\
&= \frac{(1-0.27)^2+\frac{1}{3}0.23^2}{0.27^2+\frac{1}{3}0.23^2}\\
&= 6.08.
\end{aligned}$$

Corrected answer from McNab:

The ratio of cross-sections is:

$$\begin{aligned}
\frac{\sigma(\nu_\mu\mathrm{e}^-\to\nu_\mu\mathrm{e}^-)}{\sigma(\nu_{\mathrm{e}}\mathrm{e}^-\to\nu_{\mathrm{e}}\mathrm{e}^-)} &= \frac{(c_{\mathrm{L}}^{(\nu)}c_{\mathrm{L}}^{(e)})^2+\frac{1}{3}(c_{\mathrm{L}}^{(\nu)}c_{\mathrm{R}}^{(e)})^2}{(1+c_{\mathrm{L}}^{(\nu)}c_{\mathrm{L}}^{(e)})^2+\frac{1}{3}(c_{\mathrm{L}}^{(\nu)}c_{\mathrm{R}}^{(e)})^2}\\
&= \frac{(0.5\cdot-0.27)^2+\frac{1}{3}(0.5\cdot0.23)^2}{(1+0.5\cdot-0.27)^2+\frac{1}{3}(0.5\cdot0.23)^2}\\
&= 0.0301\\
&\approx \frac{1}{33}
\end{aligned}$$

[2]

(20 December 2016)

Question 2

Part (a)

(i) Neglecting the electron mass ($p_1^2 = p_3^2 = 0$) we have

$$\begin{aligned}
q^2 &= (p_1 - p_3)^2 \\
-Q^2 &= p_1^2 + p_2^2 - 2p_1.p_3 \\
-Q^2 &= -2(E_1, 0, 0, E_1).(E_3, E_1 \sin\theta, 0, E_3 \cos\theta) \\
Q^2 &= 2E_1E_3(1 - \cos\theta) \\
Q^2 &> 0
\end{aligned}$$

[3]

(ii) The final state hadronic system must contain at least one baryon, hence $M_X^2 > m_p^2$:

$$\begin{aligned}
(p_2 + q)^2 &\geq m_p^2 \\
p_2^2 + 2p_2.q + q^2 &\geq m_p^2 \\
m_p^2 + 2p_2.q - Q^2 &\geq m_p^2 \\
-Q^2 &\geq -2p_2.q \\
Q^2 &\leq 2p_2.q \\
\therefore \quad x = \frac{Q^2}{2p_2.q} &\leq 1
\end{aligned}$$

[3]

(iii) In the Lab.frame

$$\begin{aligned}
y &= \frac{p_2.q}{p_2.p_1} \\
&= \frac{(m_p, 0, 0, 0).(p_1 - p_3)}{E_1 m_p} \\
&= \frac{m_p(E_1 - E_3)}{m_p E_1} \\
&= 1 - E_3/E_1 \\
\therefore \quad & 0 < y < 1
\end{aligned}$$

[3]

(iv) Working in the centre-of-mass frame, $p_1 = (E, 0, 0, E)$, $p_2 = (E, 0, 0, -E)$ and $p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$

$$
\begin{aligned}
y &= \frac{p_2.q}{p_2.p_1} \\
&= \frac{p_2.(p_1 - p_3)}{2E^2} \\
&= \frac{p_2.(0,0,0,E(1-\cos\theta^*))}{2E^2} \\
&= \frac{1}{2}(1-\cos\theta^*)
\end{aligned}
$$

[3]

(v) Working in the infinite momentum frame, neglect masses and transvese momenta. Let ξ be the fraction of the protons momentum carried by the quark. After the collision the struck quark's four momentum is $q + \xi p_2$:

$$
\begin{aligned}
(q+\xi p_2)^2 &= q^2 + 2\xi p_2.q + \xi^2 p_2^2 \\
&\approx q^2 + 2\xi p_2.q \\
\xi &= -\frac{q^2}{2p_2.q} \\
&= \frac{Q^2}{2p_2.q} \\
&= x
\end{aligned}
$$

[3]

Part (b)

part i) - a total of 7 marks

Feynman diagram is just the QED process $q\bar{q} \to \mu^+\mu^-$. [1]

The cross-section is non-zero for protons due to the sea anti-quarks which arise from gluon splitting within the proton. [1]

Let the colliding protons have energy E in the C.o.M. frame. The four-momenta of the colliding quark and anti-quark are $p_1 = (x_1E, 0, 0, x_1E)$ and $p_2 = (x_2E, 0, 0, -x_2E)$. The centre-of-mass energy of the $q\bar{q}$ collision is $\hat{s}$:

$$
\begin{aligned}
\hat{s} &= (p_1+p_2)^2 \\
&= (x_1+x_2)^2E^2 - (x_1-x_2)^2E^2 \\
&= 4x_1x_2E^2 \\
&= x_1x_2s
\end{aligned}
$$

[2]

The number quarks in proton 1 with momentum fraction between x_1 and x_1+dx_1 is $f_q(x_1)\mathrm{d}x_q$. Hence for a particular quark flavour in proton 1 colliding with the appropriate falvour anti-quark in proton 2

$$d^2\sigma = \frac{1}{3}\sigma(\hat{s})f_q(x_1)f_{\overline{q}}(x_2)\mathrm{d}x_1\mathrm{d}x_2$$ [2]

Note the factor of 1/3 arises because a quark can only annihilate with an anti-quark of the same colour (2 of the above 3 marks are given for this).

Can also have the same interaction between an anti-quark in proton 1 and a quark in proton 2

$$d^2\sigma = \frac{1}{3}\sigma(\hat{s})(f_q(x_1)f_{\overline{q}}(x_2) + f_q(x_2)f_{\overline{q}}(x_1)\mathrm{d}x_1\mathrm{d}x_2$$ [1]

Summing over quarks and using the above expression for $\hat{s}$ gives:

$$\frac{d^2\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = \frac{4\pi\alpha^2}{9x_1x_2s}\sum_q e_q^2[f_q(x_1)f_{\overline{q}}(x_2) + f_q(x_2)f_{\overline{q}}(x_1)]$$ [1]

Finally, considering only u and d quarks

$$\frac{d^2\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = \frac{4\pi\alpha^2}{91x_1x_2s}\{4[u(x_1)\overline{u}(x_2) + u(x_2)\overline{u}(x_1)] + d(x_1)\overline{d}(x_2) + d(x_2)\overline{d}(x_1)\}$$ [1]

part ii) - a total of one mark Direct probe of anti-quark sea parton distribution functions. [1]

part iii) - a total of four marks

At low Q^2 mainly see annihilation from sea quarks (dominate at low x). Would expect the same sea quark distributions in pi^+/pi^- and hence the ratio is one. At high Q^2 mainly see annihilation of pion valence quarks. Hence cross-sections in ratio of squares of anti-quark charges giving a ratio of one quarter. [2] [2]

Question 3

Marks will be awarded for the following points

a) The role of QCD colour symmetry in the quark model of hadrons

- Mention of SU(3)
- Colour charges $r, g, b + \overline{r}, \overline{g}, \overline{b}$
- Mention of Gell-Mann matrices
- Drawing “triangle” of colour isospin and hypercharge for quarks.
- Correctly locating quarks on diagram
- Drawing anti-quark “triangle”
- Mentioning colour confinement
- Relating confinement to colour singlet states
- Colour singlets have $I_3^C = 0$ and $Y^C = 0$
- Mention of ladder operators and relation to colour singlets
- Indicating how to combine representations (2 marks)
- $3 \times 3 \times 3 = 1 + 8 + 8 + 10$
- $3 \times \overline{3} = 8 + 1$
- Giving colour wave functions for $q\overline{q}$ or qqq
- Colour singlets only exist for $q\overline{q}$ and qqq
- mention qqq colour singlet is anti-symmetric
- mention of overall symmetry of wave-function
- overall structure and clarity of argument (2 marks)

Each of the above points is worth half a mark out of the ten total awarded. There are two marks available for overall clarity of the discussion + hitting all the main points.

b) CP Violation in the decays of neutral kaons to pions

- C and P symmetries
- Strong/Flavour Eigenstates $K^0(d\bar{s})$ and $\overline{K}^0(s\bar{d})$ (2 marks)
- Strong eigenstates clearly not CP eigenstates
- CP eigenstates: $K_1 = \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0)$ with CP +1 and $K_2 = \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0)$ with CP -1 (1+1 mark for correct form)
- Mixing via box diagram
- Neutral kaons decay via weak interaction.
- If weak interaction conserves CP then expect decays from CP eigenstates
- Can show decays to $\pi\pi$ are CP even and $\pi\pi\pi$ CP odd (2 marks + 2 for showing this)
- CP odd decay suppressed by phase space - long-lived
- would expect to see only $\pi\pi\pi$ decays at long distance from production
- Experimentally observe small fraction 10^{-3} of $\pi\pi$ decays, i.e. CP violation (2 marks)
- Therefore observed states not quite CP eigenstates: $K_S \propto K_1 + \epsilon K_2$ and $K_L \propto K_2 - \epsilon K_1$
- Also have CP violation in decay, but ϵ'/ϵ small

A good second class mark (6.5-7.0/10) will include the main points above in some form. For a first class mark (¿7/10) require clarity in description of eigenstates and relation to decay/propagation.

c) CP Violation in the decays of neutral kaons to pions

- v. brief intro to neutrino oscillations
- $\overline{\nu}_{\mathrm{e}}$
- energies $\sim 1\,\mathrm{MeV}$
- detected by inverse beta reaction + Feynman diagram
- signal double coincidence detect annihilation photons + neutron (2 marks)
- scintillator detectors + brief description (2 marks)
- decription of oscillation signal including mention of two Δm^2 scales, dependence on baseline, dependence on mixing angles + diagram (4 marks)
- naming experiments : CHOOS, Kamland
- brief experimental detail (2 marks)
- CHOOZ negative result sets limit on θ_{13}
- KamLand positive results compatible with solar neutrino
- KamLand + SNO gives precise measurement of Δm^2_{12} and θ_{12}

d) Experimental Tests of the Electroweak Theory

Answers should include a brief discussion of relations between EW paramrters, measurement of the W and Z masses + measurements of weak mixing angle from asymmetries. Marks for:

- mentioning EW sector of SM fixed by three parameters
- relation to higgs mass/top mass
- Z mass from peak of BW resonance
  distortion due to ISR
  biases due to tidal distortions and TGV
- W mass from direct reconstruction
  can't use cross section as not resonant
  mention decays and what is measured
- Mixing angle from asymmetries
  relation to parity violation
  relation to couplings
- AFB
  diagram
- ALR
  polarized beams
- Knew top mass before discovered
- Higgs mass prediction