Particle Physics Major Option Exam, January 2005

SOLUTIONS

1. **CP Violation:**

The processes $\overline{\rm p}{\rm p} \to {\rm K}^+\pi^-\overline{\rm K}^0$ and $\overline{\rm p}{\rm p} \to {\rm K}^-\pi^+{\rm K}^0$ are allowed by strangeness conservation, but $\overline{\rm p}{\rm p} \to {\rm K}^+\pi^-{\rm K}^0$ and $\overline{\rm p}{\rm p} \to {\rm K}^-\pi^+\overline{\rm K}^0$ are forbidden. Hence the charge of the ${\rm K}^\pm$ or $\pi^\pm$ can be used to tag the initial ${\rm K}^0$ or $\overline{\rm K}^0$ flavour.

The $\pi^+\pi^-$ final state from a neutral kaon decay must have $L = 0$, and hence the parity is

$$P = P_\pi.P_\pi.(-1)^L = +1 \ .$$

Charge conjugation C on a $\pi^+\pi^-$ system has the same effect as parity P. Hence $C = P = +1$, and $CP = +1$.

The mass eigenstates

$$|{\rm K_S}\rangle \propto |{\rm K_1}\rangle + \epsilon\,|{\rm K_2}\rangle\,, \qquad |{\rm K_L}\rangle \propto |{\rm K_2}\rangle + \epsilon\,|{\rm K_1}\rangle$$

evolve as

$$|{\rm K_S}(t)\rangle = |{\rm K_S}\rangle\,\theta_{\rm S}(t) = |{\rm K_S}\rangle\,e^{-im_{\rm S}t-\Gamma_{\rm S}t/2}, \qquad |{\rm K_L}(t)\rangle = |{\rm K_L}\rangle\,\theta_{\rm L}(t) = |{\rm K_L}\rangle\,e^{-im_{\rm L}t-\Gamma_{\rm L}t/2}$$

The strangeness eigenstates ${\rm K}^0$ and $\overline{\rm K}^0$ are

$$\left|{\rm K}^0\right\rangle \propto |{\rm K_L}\rangle + |{\rm K_S}\rangle\,, \qquad \left|\overline{\rm K}^0\right\rangle \propto |{\rm K_L}\rangle - |{\rm K_S}\rangle \ .$$

A state which is initially pure ${\rm K}^0$ therefore evolves with time as

$$\begin{aligned}\left|{\rm K}^0(t)\right\rangle &\propto |{\rm K_L}\rangle\,\theta_{\rm L}(t) + |{\rm K_S}\rangle\,\theta_{\rm S}(t)\\ &\propto (|{\rm K_2}\rangle + \epsilon\,|{\rm K_1}\rangle)\,\theta_{\rm L} + (|{\rm K_1}\rangle + \epsilon\,|{\rm K_2}\rangle)\,\theta_{\rm S}\\ &\propto |{\rm K_1}\rangle\,(\theta_{\rm S} + \epsilon\theta_{\rm L}) + |{\rm K_2}\rangle\,(\theta_{\rm L} + \epsilon\theta_{\rm S}) \ . \end{aligned} \qquad (1)$$

Assuming that CP violation in the decay process itself can be neglected, the $\pi\pi$ final state, with $CP = +1$, arises entirely from the $CP = +1$ eigenstate ${\rm K_1}$. Hence

$$\begin{aligned}\Gamma({\rm K}^0_{t=0} \to \pi\pi) \propto \left|\left\langle {\rm K_1}\middle|{\rm K}^0(t)\right\rangle\right|^2 &\propto |\theta_{\rm S} + \epsilon\theta_{\rm L}|^2\\ &= \left|e^{-im_{\rm S}t-\Gamma_{\rm S}t/2} + \epsilon.e^{-im_{\rm L}t-\Gamma_{\rm L}t/2}\right|^2\\ &= e^{-\Gamma_{\rm S}t} + |\epsilon|^2 e^{-\Gamma_{\rm L}t} + 2|\epsilon|e^{-(\Gamma_{\rm L}+\Gamma_{\rm S})t/2}\cos(\Delta m.t - \phi)\end{aligned}$$

where $\Delta m \equiv m_{\rm L} - m_{\rm S}$, $\epsilon \equiv |\epsilon|e^{i\phi}$, and we have used the complex number relation $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2{\rm Re}(z_1 z_2^*)$. Hence

$$\boxed{\Gamma({\rm K}^0_{t=0} \to \pi\pi) \propto \left[e^{-\Gamma_{\rm S}t} + |\epsilon|^2 e^{-\Gamma_{\rm L}t} + 2|\epsilon|e^{-(\Gamma_{\rm L}+\Gamma_{\rm S})t/2}\cos(\Delta m.t - \phi)\right]} \qquad (2)$$

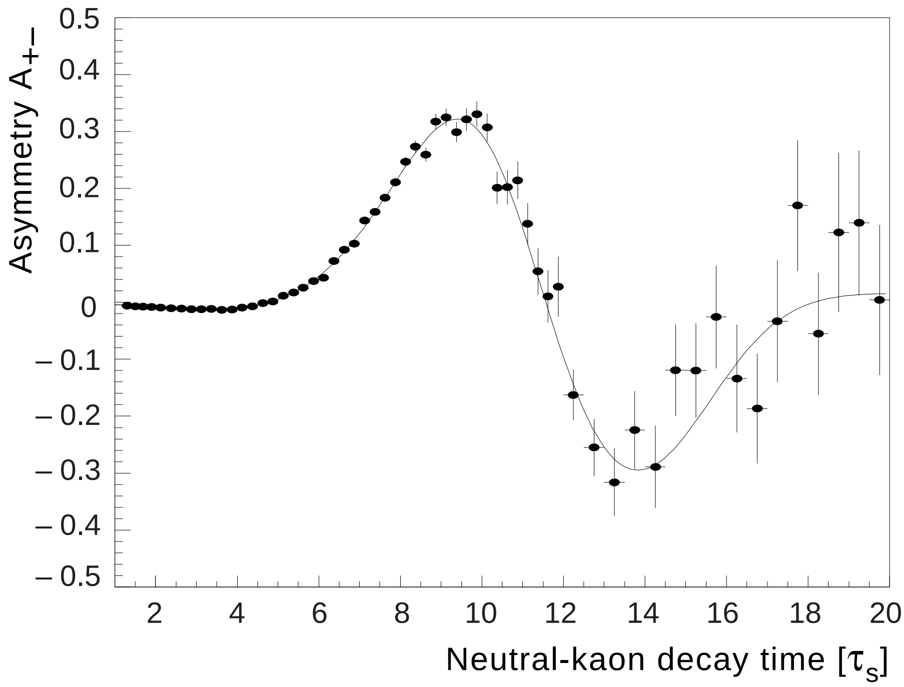
Similarly, for a beam which is initially in a pure $\overline{\mathrm{K}}^0$ state,

$$\boxed{\Gamma(\overline{\mathrm{K}}^0_{t=0} \to \pi\pi) \propto \left[e^{-\Gamma_{\mathrm{S}} t} + |\epsilon|^2 e^{-\Gamma_{\mathrm{L}} t} - 2|\epsilon| e^{-(\Gamma_{\mathrm{L}}+\Gamma_{\mathrm{S}})t/2} \cos(\Delta m.t - \phi)\right]} \tag{3}$$

Since $\Gamma_{\mathrm{S}} = 1/\tau_{\mathrm{S}}$, the asymmetry A_{+-} is

$$A_{+-} \equiv \frac{\Gamma(\overline{\mathrm{K}}^0_{t=0} \to \pi\pi) - \Gamma(\mathrm{K}^0_{t=0} \to \pi\pi)}{\Gamma(\overline{\mathrm{K}}^0_{t=0} \to \pi\pi) + \Gamma(\mathrm{K}^0_{t=0} \to \pi\pi)} \approx \frac{-2|\epsilon| e^{t/2\tau_{\mathrm{S}}} \cos(\Delta m.t - \phi)}{1 + |\epsilon|^2 e^{t/\tau_{\mathrm{S}}}} \ . \tag{4}$$

From the Figure, the asymmetry A_{+-} is zero at $t/\tau_{\mathrm{S}} \approx 11.5$. The cosine term must vanish at



this point, and, because of the overall minus sign in the expression for A_{+-}, must be *increasing* with time. Therefore $\Delta m.t - \phi = 3\pi/2$ (not $\pi/2$), giving

$$\phi = \Delta m.t - \frac{3\pi}{2} \approx \frac{(3.5 \times 10^{-12}\,\mathrm{MeV}) \times 11.5 \times (0.9 \times 10^{-10}\,\mathrm{s})}{(6.58 \times 10^{-22}\,\mathrm{MeV.\,s})} - \frac{3\pi}{2} \approx 0.793 \quad (\approx 45.4^\circ) \ .$$

The asymmetry reaches a maximum value $A_{+-} \approx 0.32$ at $t/\tau_{\mathrm{S}} \approx 9.5$. Neglecting the term $|\epsilon|^2 e^{9.5}$ (≈ 0.05) in the denominator and making the approximation $\cos(\Delta m.t - \phi) \approx -1$ at the maximum gives

$$|\epsilon| \approx \frac{0.32}{2e^{(9.5)/2}} \approx 1.4 \times 10^{-3} \ .$$

This estimate of $|\epsilon|$ can be improved (not necessary for exam purposes) by making a more accurate estimate of the value of the cosine term at the maximum using

$$\Delta m.t - \phi \approx \frac{(3.5 \times 10^{-12}\,\mathrm{MeV}) \times 9.5 \times (0.9 \times 10^{-10}\,\mathrm{s})}{(6.58 \times 10^{-22}\,\mathrm{MeV.\,s})} - 0.793 \approx 3.75 \ ,$$

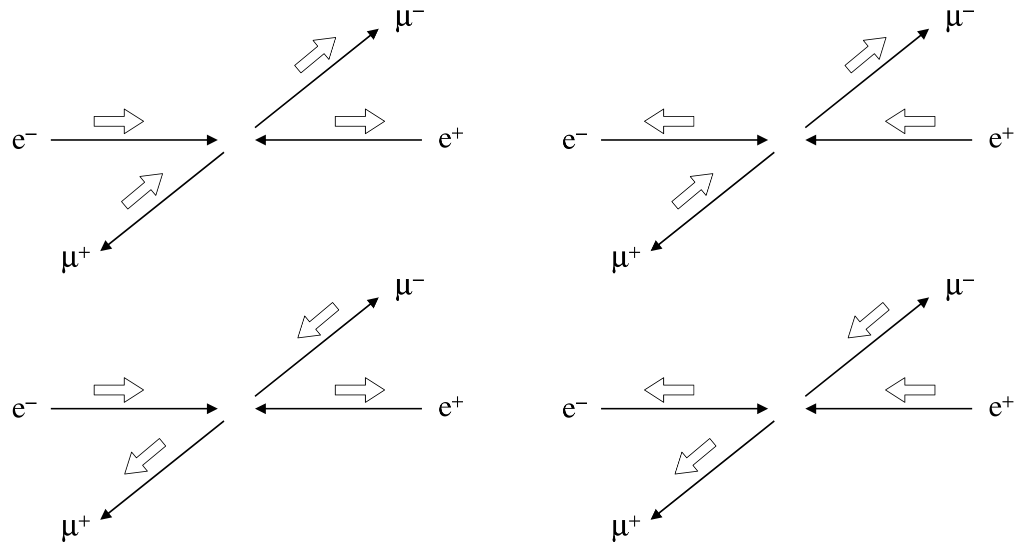
which gives $\cos(\Delta m.t - \phi) \approx -0.818$ and hence $|\epsilon| \approx 1.7 \times 10^{-3}$. If, in addition, the full denominator is used, we have (at the maximum)

$$0.32 \approx \frac{-2|\epsilon|e^{(9.5)/2} \times (-0.818)}{1 + |\epsilon|^2 e^{9.5}} ,$$

and hence a quadratic equation for $|\epsilon|$, $(13360)|\epsilon|^2 - (591)|\epsilon| + 1 = 0$, which improves the estimate to $|\epsilon| \approx 1.8 \times 10^{-3}$.

3. **The Z resonance:**

The $Z^0 f\bar{f}$ interaction, proportional to $c_L^f \gamma^\mu(1-\gamma^5) + c_R^f \gamma^\mu(1+\gamma^5)$, is a mixture of $V - A$ and $V + A$ interactions. For *any* mixture of V and A currents, *particle* helicity is conserved in the massless limit (helicity conservation). Hence, for $e^+e^- \to f\bar{f}$ scattering, the electron and positron must have *opposite* helicity, as must the fermion and antifermion. The allowed spin configurations are therefore (for the particular case that the outgoing fermion is a μ^-):



For the RL configuration, with a right-handed electron and a left-handed fermion (bottom left diagram above), the cross section must vanish for $\theta = 0$ by angular momentum conservation. The overlap of initial and final spin states gives a factor $(1 - \cos\theta)^2$ in the differential cross section. The electron vertex gives a factor c_R^e in the matrix element and the fermion vertex a factor c_L^f. Similar considerations apply to the other diagrams, with angular momentum conservation requiring the cross section to vanish for $\theta = 0$ for the RL and LR configurations, and for $\theta = \pi$ for RR and LL. Overall, the differential cross sections are:

$$\frac{d\sigma_{RR}}{d\Omega} \propto (c_R^e)^2 (c_R^f)^2 (1+\cos\theta)^2$$
$$\frac{d\sigma_{LL}}{d\Omega} \propto (c_L^e)^2 (c_L^f)^2 (1+\cos\theta)^2$$
$$\frac{d\sigma_{LR}}{d\Omega} \propto (c_L^e)^2 (c_R^f)^2 (1-\cos\theta)^2$$
$$\frac{d\sigma_{RL}}{d\Omega} \propto (c_R^e)^2 (c_L^f)^2 (1-\cos\theta)^2 \ .$$

The integrals of the $(1 \pm \cos\theta)^2$ angular distributions are the same for all four cases, so that the total cross sections are in the ratio

$$\sigma_{RR} \propto (c_R^e)^2 (c_R^f)^2, \quad \sigma_{RL} \propto (c_R^e)^2 (c_L^f)^2, \quad \sigma_{LR} \propto (c_L^e)^2 (c_R^f)^2, \quad \sigma_{LL} \propto (c_L^e)^2 (c_L^f)^2 \ .$$

The total cross sections σ_L and σ_R for producing a final fermion which is left-handed or right-handed, respectively, in unpolarised e^+e^- collisions are

$$\sigma_L = \tfrac{1}{2} \cdot \tfrac{1}{2} (\sigma_{LL} + \sigma_{RL}) \propto \left[(c_L^e)^2 + (c_R^e)^2\right] (c_L^f)^2 \propto (c_L^f)^2$$
$$\sigma_R = \tfrac{1}{2} \cdot \tfrac{1}{2} (\sigma_{LR} + \sigma_{RR}) \propto \left[(c_L^e)^2 + (c_R^e)^2\right] (c_R^f)^2 \propto (c_R^f)^2 \ .$$

The ratio of cross sections is therefore $\sigma_{\rm L} : \sigma_{\rm R} = (c_{\rm L}^{\rm f})^2 : (c_{\rm R}^{\rm f})^2$.

The average polarisation of the final state fermion is therefore

$$\boxed{P = \frac{\sigma_{\rm R} - \sigma_{\rm L}}{\sigma_{\rm R} + \sigma_{\rm L}} = \frac{(c_{\rm R}^{\rm f})^2 - (c_{\rm L}^{\rm f})^2}{(c_{\rm R}^{\rm f})^2 + (c_{\rm L}^{\rm f})^2} = -A_f} \ .$$

($P = +1$ if the fermion is always right-handed, $P = -1$ if the fermion is always left-handed, $-1 < P < +1$ in general).

Since the tau has $I_W^{(3)} = -\frac{1}{2}$ and $Q = -1$, we have

$$c_{\rm L}^{\tau} = -\tfrac{1}{2} + \sin^2\theta_{\rm W}, \qquad c_{\rm R}^{\tau} = \sin^2\theta_{\rm W}\ ,$$

and hence

$$\frac{(-\frac{1}{2} + \sin^2\theta_{\rm W})^2 - (\sin^2\theta_{\rm W})^2}{(-\frac{1}{2} + \sin^2\theta_{\rm W})^2 + (\sin^2\theta_{\rm W})^2} = 0.1465\ .$$

This gives a quadratic equation for $x \equiv \sin^2\theta_{\rm W}$:

$$2 \times (0.1465)x^2 + (1 - 0.1465)x - \tfrac{1}{4}(1 - 0.1465) = 0$$

which can be solved to give $\sin^2\theta_{\rm W} \approx -3.14$ or $\sin^2\theta_{\rm W} \approx 0.232$, the latter obviously being the correct result.