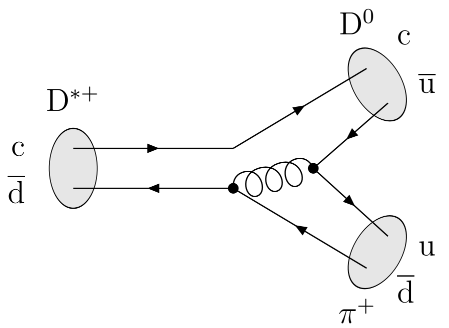
NST Part III Experimental and Theoretical Physics Dr J R Batley

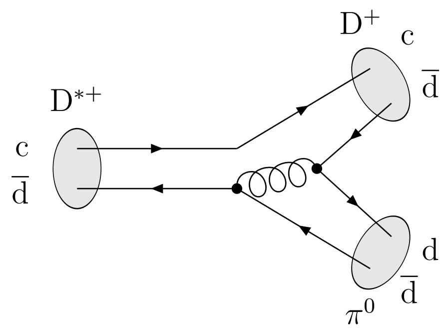
Particle Physics Major Option Exam, January 2003

SOLUTIONS

1. **SU(2) and SU(3)**

(a) Feynman diagrams for $D^{*+} \to D^0\pi^+$ and $D^{*+} \to D^+\pi^0$ decays:





The $\overline{u}$ and $\overline{d}$ antiquarks form an isospin doublet with isospin quantum numbers $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$ and $\left|\frac{1}{2}, \frac{1}{2}\right\rangle$ respectively. The charm quark is an isospin singlet with $|I, I_3\rangle = |0, 0\rangle$. Therefore the D^0 and D^+ mesons, and the D^{*0} and D^{*+} mesons, must each form isospin doublets, and the isospin quantum numbers associated with each decay are:

$$D^{*+} \to D^0\pi^+ : \qquad \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle \to \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle |1, 1\rangle$$
$$D^{*+} \to D^+\pi^0 : \qquad \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle \to \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle |1, 0\rangle$$

The result of combining the state $\left|\frac{1}{2}, \frac{1}{2}\right\rangle$ with the state $|1, 1\rangle$ can be written down by inspection:

$$\left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle = \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle |1, 1\rangle \ .$$

Applying the ladder operator T_- to this equation, using

$$T_- \left|\tfrac{3}{2}, \tfrac{3}{2}\right\rangle = \sqrt{\tfrac{3}{2}\tfrac{5}{2} - \tfrac{3}{2}\tfrac{1}{2}} \left|\tfrac{3}{2}, \tfrac{1}{2}\right\rangle = \sqrt{3} \left|\tfrac{3}{2}, \tfrac{1}{2}\right\rangle$$

$$T_- \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{2}\tfrac{3}{2} - \tfrac{1}{2}(-\tfrac{1}{2})} \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle = \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

$$T_- |1, 1\rangle = \sqrt{1 \times 2 - 1 \times 0}\, |1, 0\rangle = \sqrt{2}\, |1, 0\rangle$$

gives

$$\left|\tfrac{3}{2}, \tfrac{1}{2}\right\rangle = \frac{1}{\sqrt{3}} \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle |1, 1\rangle + \sqrt{\frac{2}{3}} \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle |1, 0\rangle \ .$$

The state orthogonal to this is

$$\left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle = \sqrt{\frac{2}{3}} \left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle |1, 1\rangle - \frac{1}{\sqrt{3}} \left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle |1, 0\rangle \ .$$

In terms of the particles involved in the D^{*+} decays, this can be interpreted as

$$\left|D^{*+}\right\rangle = \sqrt{\frac{2}{3}}\left|D^0\right\rangle\left|\pi^+\right\rangle - \frac{1}{\sqrt{3}}\left|D^+\right\rangle\left|\pi^0\right\rangle \ .$$

Hence:

$$\frac{BR(D^{*+} \to D^0\pi^+)}{BR(D^{*+} \to D^+\pi^0)} = \left(\frac{\sqrt{2/3}}{-\sqrt{1/3}}\right)^2 = 2$$

(b) The only ladder operations on the state $|rr\rangle$ which give non-zero results are:

$$T_-\,|rr\rangle = (|gr\rangle + |rg\rangle) \qquad V_-\,|rr\rangle = (|br\rangle + |rb\rangle) \ .$$

Applying the ladder operators to the state $(|gr\rangle + |rg\rangle)$ then gives (amongst other things)

$$T_-\,(|gr\rangle + |rg\rangle) = 2\,|gg\rangle \qquad V_-\,(|gr\rangle + |rg\rangle) = (|gb\rangle + |bg\rangle) \ .$$

and so on. After normalising the states, we obtain a *sextet*:

$$|rr\rangle \qquad |gg\rangle \qquad |bb\rangle \qquad \frac{1}{\sqrt{2}}(|gr\rangle + |rg\rangle) \qquad \frac{1}{\sqrt{2}}(|br\rangle + |rb\rangle) \qquad \frac{1}{\sqrt{2}}(|gb\rangle + |bg\rangle) \ .$$

which are symmetric under particle interchange.

The states orthogonal to these form a *triplet*:

$$\frac{1}{\sqrt{2}}(|gr\rangle - |rg\rangle) \qquad \frac{1}{\sqrt{2}}(|br\rangle - |rb\rangle) \qquad \frac{1}{\sqrt{2}}(|bg\rangle - |gb\rangle)$$

which are antisymmetric under particle interchange.

The colour coefficients for single-gluon interactions between two quarks of the same colour

$$C(rr \to rr) = C(gg \to gg) = C(bb \to bb) = \frac{1}{3}$$

or of different colour

$$C(rg \to rg) = C(rb \to rb) = C(gr \to gr) = C(gb \to gb) = C(br \to br) = C(bg \to bg) = -\frac{1}{6}$$

$$C(rg \to gr) = C(rb \to br) = C(gr \to rg) = C(gb \to bg) = C(br \to rb) = C(bg \to gb) = \frac{1}{2}$$

were derived in the lectures. For the symmetric states of mixed colour, such as $\psi = 1/\sqrt{2}(gr + rg)$, we have

$$C(\psi \to \psi) = \frac{1}{2}\left[2 \times \frac{-1}{6} + 2 \times \frac{1}{2}\right] = \frac{1}{3} \ .$$

Therefore every state in the sextet has a colour factor $C = 1/3$, corresponding to a *repulsive* potential.

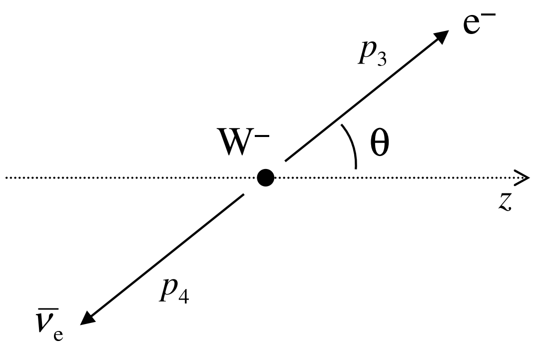
Similarly, for a triplet state such as $\psi = 1/\sqrt{2}(gr - rg)$, we have

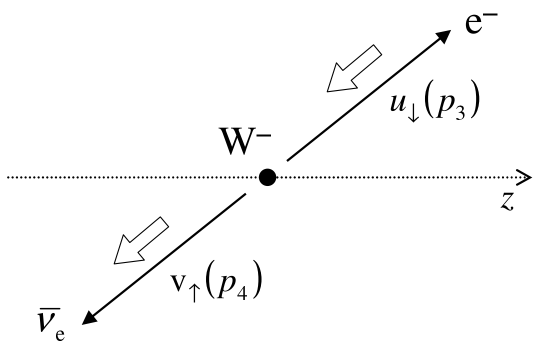
$$C(\psi \to \psi) = \frac{1}{2}\left[2 \times \frac{-1}{6} - 2 \times \frac{1}{2}\right] = -\frac{2}{3}$$

which corresponds to an *attractive* potential.

These results are derived from leading-order perturbation theory (single-gluon exchange) and apply only to high q^2, which means short distances. The long-distance potential is dominated by non-perturbative QCD and is believed to be attractive (confining) only for colour singlet states.

3. **W decay in UA1:**





The matrix element for the decay $W^- \to e^- \overline{\nu}_e$ is given as

$$M_{fi} = \frac{g_W}{\sqrt{2}} \epsilon_\mu(p_1) \overline{u}(p_3) \gamma^\mu \tfrac{1}{2}(1-\gamma^5) v(p_4)$$

This matrix element $(V-A)$ is non-zero only for a left-handed e^- and a right-handed $\overline{\nu}_e$. We must therefore take $u(p_3) = u_\downarrow(p_3)$ and $v(p_4) = v_\uparrow(p_4)$:

$$u_\downarrow(p_3) = u_\downarrow(\theta) = \sqrt{E} \begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \\ \sin\theta/2 \\ -\cos\theta/2 \end{pmatrix} = \sqrt{E} \begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix}$$

$$\overline{u}_\downarrow(p_3) = \sqrt{E}(-s, c, -s, c)$$

$$v_\uparrow(p_4) = v_\uparrow(\theta+\pi) = \sqrt{E} \begin{pmatrix} \sin(\theta+\pi)/2 \\ -\cos(\theta+\pi)/2 \\ -\sin(\theta+\pi)/2 \\ \cos(\theta+\pi)/2 \end{pmatrix} = \sqrt{E} \begin{pmatrix} c \\ s \\ -c \\ -s \end{pmatrix}$$

Since

$$\tfrac{1}{2}(1-\gamma^5) v_\uparrow(p_4) = \tfrac{1}{2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \sqrt{E} \begin{pmatrix} c \\ s \\ -c \\ -s \end{pmatrix} = \sqrt{E} \begin{pmatrix} c \\ s \\ -c \\ -s \end{pmatrix} = v_\uparrow(p_4)$$

the matrix element simplifies to

$$M_{fi} = \frac{g_W}{\sqrt{2}} \epsilon_\mu(p_1) \overline{u}_\downarrow(p_3) \gamma^\mu v_\uparrow(p_4)$$

Operating with each gamma matrix in turn gives

$$\gamma^\mu v_\uparrow(p_4) = \sqrt{e} \left[\begin{pmatrix} c \\ s \\ c \\ s \end{pmatrix}, \begin{pmatrix} -s \\ -c \\ -s \\ -c \end{pmatrix}, \begin{pmatrix} is \\ -ic \\ is \\ -ic \end{pmatrix}, \begin{pmatrix} -c \\ s \\ -c \\ s \end{pmatrix} \right] .$$

The lepton current is therefore

$$\begin{aligned}\overline{u}_\downarrow(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v_\uparrow(p_4) &= E(c^2+s^2-c^2-s^2, s^2-c^2+s^2-c^2, -i(s^2+c^2+s^2+c^2), 4sc)\\ &= 2E(0, s^2-c^2, -i, 2sc)\end{aligned}$$

Using $\sin\theta = 2\sin\theta/2\cos\theta/2 = 2sc$ and $\cos\theta = \cos^2\theta/2 - \sin^2\theta/2 = c^2 - s^2$, this becomes

$$\overline{u}_\downarrow(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, \sin\theta)$$

Taking the scalar product of the electron current with each of the three possible W polarisation vectors gives

$$\begin{aligned}\epsilon_+^\mu: &\quad -\frac{1}{\sqrt{2}}(0,1,i,0)\cdot 2E(0,-\cos\theta,-i,\sin\theta) = -\sqrt{2}E(\cos\theta-1)\\ \epsilon_-^\mu: &\quad \frac{1}{\sqrt{2}}(0,1,-i,0)\cdot 2E(0,-\cos\theta,-i,\sin\theta) = \sqrt{2}E(\cos\theta+1)\\ \epsilon_L^\mu: &\quad (0,0,0,1)\cdot 2E(0,-\cos\theta,-i,\sin\theta) = 2E\cdot -\sin\theta\end{aligned}$$

Therefore

$$\begin{aligned}\epsilon_+^\mu: &\quad |M_{\rm fi}|^2 = g_{\rm W}^2E^2(1-\cos\theta)^2\\ \epsilon_-^\mu: &\quad |M_{\rm fi}|^2 = g_{\rm W}^2E^2(1+\cos\theta)^2\\ \epsilon_L^\mu: &\quad |M_{\rm fi}|^2 = 2g_{\rm W}^2E^2\sin^2\theta\end{aligned}$$

For an unpolarised sample of W bosons, averaging over the three possible spin states gives

$$\langle|M_{\rm fi}|^2\rangle = \frac{1}{3}g_{\rm W}^2E^2\left[(1-\cos\theta)^2+(1+\cos\theta)^2+2\sin^2\theta\right] = \frac{1}{3}g_{\rm W}^2E^2\cdot 4 = \frac{4}{3}g_{\rm W}^2E^2\ .$$

The decay rate is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{p^*}{32\pi^2 m_{\rm W}^2}\langle|M_{\rm fi}|^2\rangle$$

with $p^* = E = m_{\rm W}/2$. Hence,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 m_{\rm W}}\frac{4}{3}g_{\rm W}^2\left(\frac{m_{\rm W}}{2}\right)^2 = \frac{1}{3}\frac{g_{\rm W}^2 m_{\rm W}}{64\pi^2}\ .$$

This is an isotropic decay distribution (as expected since an unpolarised sample of W bosons has no preferred spatial direction) so that integrating over all spatial directions gives a simple factor of 4π:

$$\Gamma(\mathrm{W}^-\to \mathrm{e}^-\overline{\nu}_{\rm e}) = \frac{g_{\rm W}^2 m_{\rm W}}{48\pi}\ .$$

The allowed decays of the W boson are

$$\mathrm{W}^-\to \mathrm{e}^-\overline{\nu}_{\rm e}, \mu^-\overline{\nu}_\mu, \tau^-\overline{\nu}_\tau, \overline{\rm u}{\rm d}, \overline{\rm u}{\rm s}, \overline{\rm u}{\rm b}, \overline{\rm c}{\rm d}, \overline{\rm c}{\rm s}, \overline{\rm c}{\rm b}$$

(the top quark is too heavy) with relative decay rates

$$1, 1, 1, 3|V_{\rm ud}|^2, 3|V_{\rm us}|^2, 3|V_{\rm ub}|^2, 3|V_{\rm cd}|^2, 3|V_{\rm cs}|^2, 3|V_{\rm cb}|^2$$

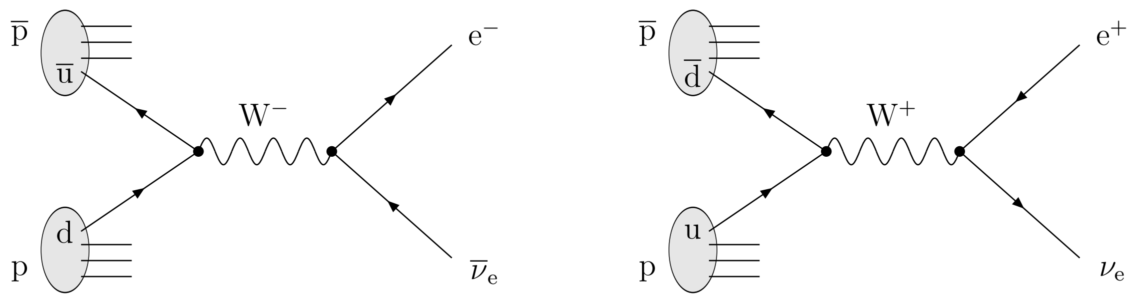
Using unitarity of the CKM matrix:

$$|V_{\rm ud}|^2 + |V_{\rm us}|^2 + |V_{\rm ub}|^2 = 1$$
$$|V_{\rm cd}|^2 + |V_{\rm cs}|^2 + |V_{\rm cb}|^2 = 1$$

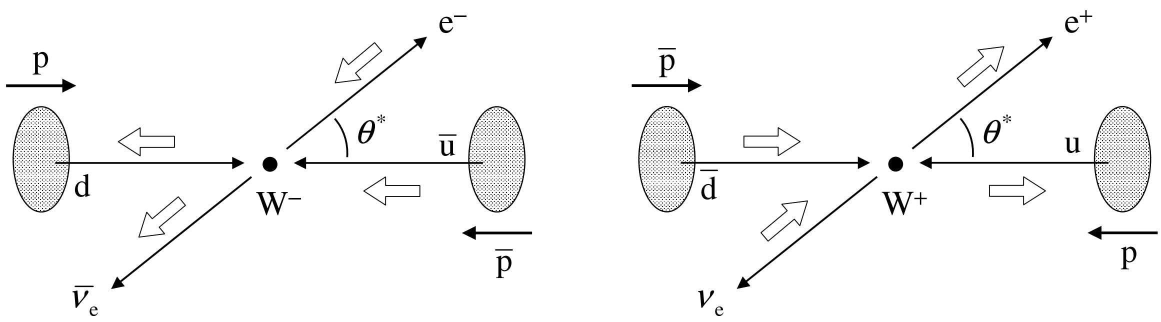
this adds up to 9 units. Hence

$$BR({\rm W}^- \to {\rm e}^-\overline{\nu}_{\rm e}) = \frac{1}{9}$$

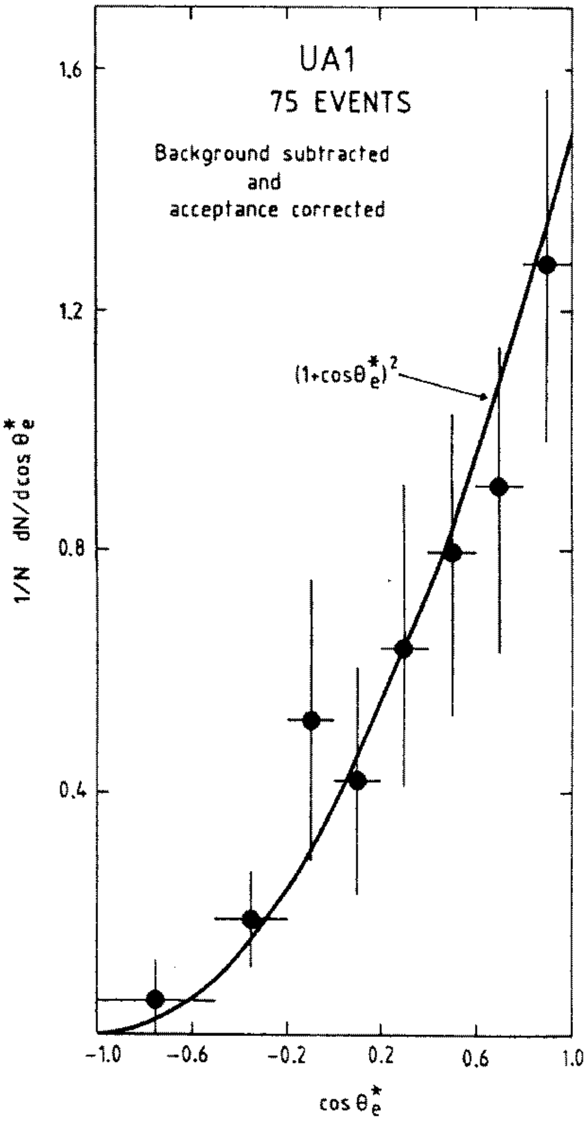
Considering valence quarks only (uud for the proton, $\overline{\rm u}\overline{\rm u}\overline{\rm d}$ for the antiproton), the quark-level processes for W production and decay in $\overline{\rm p}$p collisions are $\overline{\rm u}{\rm d} \to {\rm W}^- \to {\rm e}^-\overline{\nu}_{\rm e}$ and $\overline{\rm d}{\rm u} \to {\rm W}^+ \to {\rm e}^+\nu_{\rm e}$. The corresponding Feynman diagrams are:



In high energy $\overline{\rm p}$p collisions, the quarks, antiquarks and leptons involved can be assumed to be in the relativistic limit. In this limit, because of the $V-A$ structure of charged-current (${\rm W}^\pm$) interactions, only left-handed particles and right-handed antiparticles can interact with ${\rm W}^\pm$ bosons. Hence the d, u, ${\rm e}^-$ and $\nu_{\rm e}$ must be left-handed while the $\overline{\rm u}$, $\overline{\rm d}$, ${\rm e}^+$ and $\overline{\nu}_{\rm e}$ must be right-handed:



(Note that the directions of the p and $\overline{\rm p}$ are reversed between the two diagrams). Hence, in either case, the overlap between the initial and final state spin wavefunctions is maximised for small angles θ^*; the angular distribution of the electron or positron is proportional to $(1+\cos\theta^*)^2$, as seen in the UA1 data:


UA1
75 EVENTS
Background subtracted
and
acceptance corrected
$(1+\cos\theta_e^*)^2$
1/N dN/dcos θ*e
cos θ*e
1.6
1.2
0.8
0.4
-1.0
-0.6
-0.2
0.2
0.6
1.0