UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS

Monday 18th January 2021 10:00 to 12:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

Answer ***two*** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper has content on five sides, including this one, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You should use a ***separate Answer Book*** *for each question.*

STATIONERY REQUIREMENTS
2x20-page answer books
Rough workpad

SPECIAL REQUIREMENTS
Mathematical Formulae Handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

The information in this box may be used in any question.

For a particle of mass m the two-body decay width is given by

$$\Gamma = \frac{|\boldsymbol{p}^*|}{32\pi^2 m^2} \int |M|^2 d\Omega$$

if $|\boldsymbol{p}^*|$ *denotes the magnitude of the momentum either decay product in the centre of mass frame.*

The Pauli-matrices are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The representation of gamma matrices used in the Part III Particles lecture course was

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix},$$

which has the following properties:

$$(\gamma^0)^* = \gamma^0, \quad (\gamma^1)^* = \gamma^1, \quad (\gamma^2)^* = -\gamma^2, \quad (\gamma^3)^* = \gamma^3 \quad \textit{and} \quad \gamma^2(\gamma^\mu)^* = -\gamma^\mu\gamma^2.$$

Using the above convention, the Part III Particles lecture course defined the following particle and anti-particle spinors:

$$u_\uparrow = N \begin{pmatrix} c \\ e^{i\phi}s \\ \frac{|\boldsymbol{p}|}{E+m}c \\ \frac{|\boldsymbol{p}|}{E+m}e^{i\phi}s \end{pmatrix}, \qquad u_\downarrow = N \begin{pmatrix} -s \\ e^{i\phi}c \\ \frac{|\boldsymbol{p}|}{E+m}s \\ -\frac{|\boldsymbol{p}|}{E+m}e^{i\phi}c \end{pmatrix},$$

$$v_\uparrow = N \begin{pmatrix} \frac{|\boldsymbol{p}|}{E+m}s \\ -\frac{|\boldsymbol{p}|}{E+m}e^{i\phi}c \\ -s \\ e^{i\phi}c \end{pmatrix}, \qquad v_\downarrow = N \begin{pmatrix} \frac{|\boldsymbol{p}|}{E+m}c \\ \frac{|\boldsymbol{p}|}{E+m}e^{i\phi}s \\ c \\ e^{i\phi}s \end{pmatrix}$$

for objects whose three-momentum $\boldsymbol{p}$ *is given by* $|\boldsymbol{p}|(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$ *where* $c = \cos\frac{\theta}{2}$ *and* $s = \sin\frac{\theta}{2}$. *The normalising constant is* $N = \sqrt{E+m}$.

1 Experimentally, leptonic decays of charged pions are seen to involve muons far more frequently than electrons:

$$\frac{\Gamma\left[\pi^- \to e^-\bar{\nu}_e\right]}{\Gamma\left[\pi^- \to \mu^-\bar{\nu}_\mu\right]} \approx (1.3 \pm 0.1) \times 10^{-4}.$$

(a) Why are lepton currents of the form $\bar{\Psi}\gamma^\mu\Phi$ and $\bar{\Psi}\gamma^\mu\gamma^5\Phi$ called vector and axial currents, respectively? [2]

(b) What parts of the weak interaction are termed 'maximally parity violating', and why? [1]

(c) Explain how the weak interaction's 'vector minus axial' coupling can explain the aforementioned observation concerning charged pion decay rates. [*You are not expected to numerically re-derive the actual value of the ratio given in the question – you may simply outline the key issues.*] [4]

(d) Why would a 'vector plus axial' variant of the weak interaction make the same quantitative prediction for the pion decay rates? [1]

(e) Derive predictions for the ratio $\frac{\Gamma\left[\pi^- \to e^-\bar{\nu}_e\right]}{\Gamma\left[\pi^- \to \mu^-\bar{\nu}_\mu\right]}$ which would apply if it were the case that the weak interaction instead had currents of the form $\bar{\Psi}\Phi$ (or, if you prefer, $\bar{\Psi}\gamma^5\Phi$). Leave your answer in terms of the parameters m_e, m_μ, m_π and quantities derived from them. No marks are available for a numerical estimate of the ratio, only the functional form is desired. [10]

Putting the weak interaction entirely to one side, and focusing only on QCD, suppose now that the u, d and s quarks existed with all their usual quantum numbers, except that they had spin zero.

(f) Discuss the resulting spectrum of hadrons and their properties. You should specifically consider the possible J^P values of the meson multiplets, and the J^P value and multiplicity of the lightest baryon multiplet, and whether or not the resulting spectra are compatible with those we see for normal (*i.e.* fermionic) quarks. [*Bosons have the same parity as antibosons.*] [12]

2 In the simplified Feynman rules given in the lecture course, the propagators for the photon and W-boson were given as

$$-\frac{ig_{\mu\nu}}{q^2} \quad \text{and} \quad -\frac{i\left(g_{\mu\nu} - q^\mu q^\nu/m_W^2\right)}{q^2 - m_W^2}$$

respectively. Both have a denominator of the form $q^2 - m^2$.

(a) Comment on the circumstances in which it might be reasonable to replace $q^2 - m^2$ in the denominator of a propagator with $q^2 - m^2 + im\Gamma$, explaining what Γ might mean in this context. [4]

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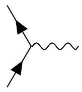
Now suppose that there exist Bogus universes containing only electrons, positrons, muons, antimuons and Bogons, and that interactions are described by a theory called Quantum Bogodynamics (or QBD for short). Suppose that QBD is identical to QED except that: (i) photons are replaced by Bogons, (ii) there are two types of Bogon instead of one type of photon, and (iii) in some universes Bogons can be massive ($0 \le m_1 \le m_2$). Furthermore, suppose that the coupling strengths e_1 and e_2 for the two types of Bogon need neither be equal nor have the same sign. In short, you may assume that in any Bogus universe the QBD Feynman rules have propagator and vertex factors for the k^{th} type of Bogon as follows:

 $= \dfrac{-ig_{\mu\nu}}{q^2 - m_k^2 + im_k\Gamma}$,  $= ie_k\gamma^\mu$

where $\Gamma \ge 0$ is a non-negative constant.

(b) In a universe in which $0 < m_1 = m_2$, $0 < \Gamma \ll m_1$ and $e_1 e_2 > 0$, would it be possible for Bogus physicists looking at $e^+e^- \to \mu^+\mu^-$ data to determine that there are two types of Bogon rather than one? Would anything change if we were to consider instead $e_1 e_2 < 0$? [4]

Suppose there are two Bogus universes A and B, and that in each of these universes $0 < m_1 < m_2$, $\Gamma = 0$ and e_1 is equal to a non-zero constant e. Further suppose that Universe A has $e_2 = 0$ while Universe B has $e_2 = e$.

(c) For which range(s) of $\sqrt{s}$ (if any) is the tree-level cross section for the process $e^+e^- \to \mu^+\mu^-$ bigger in Universe A than in Universe B, and for which range(s) of $\sqrt{s}$ (if any) is the reverse true? [16]

A 'Bogus e^+e^- Collider' previously only able to reach centre of mass energies of up to $\sqrt{s} = 9m_2$ is upgraded to allow it also to access the region $9m_2 < \sqrt{s} < 10m_2$. A team of Bogus physicists (who believe themselves to live in Universe B) find that the $e^+e^- \to \mu^+\mu^-$ cross section measured by the collider in the new energy region appears to undershoot the theoretical predictions made by their (previously reliable) Bogus Standard Model.

(d) What conclusion(s) might these physicists draw from the new data? [3]

A funding crisis in physics prohibits the construction of further energy upgrades to the Bogus e^+e^- Collider.

(e) What would you recommend the aforementioned physicists should do to get the most out of the machine they already have? [3]

3 The second handout of this year's lecture course derived many properties of spinors and the Dirac equation assuming the usual '3D' Minkowski spacetime having one time and three space dimensions. Which of those properties would remain the same, and which would change (and how) assuming instead a '2D' spacetime having two space dimensions in addition to time? [30]

Credit will be given for the quality of the arguments which relate specifically to the 2D case, and the degree to which they convey to the marker the sense that the candidate understands the physics and concepts underlying the Dirac equation, spinors and fermions. No credit will be given for merely reporting what happens in 3D, though comparisons between 2D and 3D are encouraged if they help to explain important features of the 2D spinors.

Candidates may wish to structure an answer around some of the following questions, though no candidate is required to give answers to all of them, and no candidate is forbidden from discussing other questions which they feel are relevant:

1. What are the number, dimensions and required commutation or anticommutation relations of the smallest α and β matrices that a 2D Dirac Hamiltonian should use?
2. Does it remain sensible to create γ matrices from α and β matrices, and if so in what way?
3. Does the Dirac equation take a new form in 2D ?
4. Does it remain beneficial to create v-spinors in addition to u-spinors?
5. Does the 2D theory predict both particles and anti-particles?
6. Do states still carry intrinsic spin angular momentum?
7. What explicit form (or forms) might a 2D spinor take for a particle of mass m having energy E and momentum (p_x, p_y) within the spatial two-space?
8. Certain 3D-spinor wave functions change sign when subjected to 2π rotations about certain axes. Is there anything analogous for spinors in 2D ?

END OF PAPER