UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS

Monday 16th January 2023 10:00 to 12:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

Answer ***two*** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper has content on six sides, including this one, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You should use a ***separate Answer Book*** *for each question.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2x20-page answer books	Mathematical Formulae Handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

The information in this box may be used in any question.

The Pauli-matrices are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The gamma matrix representation of the Part III Particles lecture course was:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = i\gamma^0\gamma^1\gamma^2\gamma^3,$$

which has the following properties:

$$(\gamma^0)^* = \gamma^0, \ (\gamma^1)^* = \gamma^1, \ (\gamma^2)^* = -\gamma^2, \ (\gamma^3)^* = \gamma^3 \ \textit{and} \ \gamma^2(\gamma^\mu)^* = -\gamma^\mu\gamma^2.$$

Using the above convention, the Part III Particles lecture course defined the following particle and anti-particle spinors:

$$u_\uparrow = N\begin{pmatrix} c \\ e^{i\phi}s \\ \frac{|\boldsymbol{p}|}{E+m}c \\ \frac{|\boldsymbol{p}|}{E+m}e^{i\phi}s \end{pmatrix}, \qquad u_\downarrow = N\begin{pmatrix} -s \\ e^{i\phi}c \\ \frac{|\boldsymbol{p}|}{E+m}s \\ -\frac{|\boldsymbol{p}|}{E+m}e^{i\phi}c \end{pmatrix},$$

$$v_\uparrow = N\begin{pmatrix} \frac{|\boldsymbol{p}|}{E+m}s \\ -\frac{|\boldsymbol{p}|}{E+m}e^{i\phi}c \\ -s \\ e^{i\phi}c \end{pmatrix}, \qquad v_\downarrow = N\begin{pmatrix} \frac{|\boldsymbol{p}|}{E+m}c \\ \frac{|\boldsymbol{p}|}{E+m}e^{i\phi}s \\ c \\ e^{i\phi}s \end{pmatrix}$$

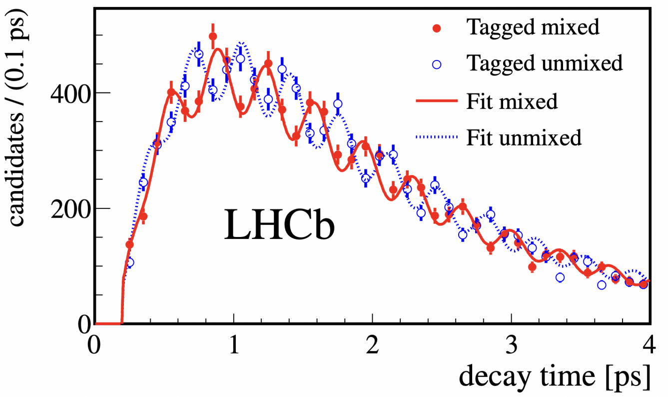
for objects whose three-momentum $\boldsymbol{p}$ *is given by* $|\boldsymbol{p}|(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$ *where* $c = \cos\frac{\theta}{2}$ *and* $s = \sin\frac{\theta}{2}$. *The normalising constant is* $N = \sqrt{E+m}$.

$\hbar \approx 1.05 \times 10^{-34}$ kg m^2/s, $\quad c \approx 3.00 \times 10^8$ m/s, $\quad e \approx 1.60 \times 10^{-19}$ C.

$m_e = 5.11 \times 10^{-4}$ GeV. $\quad m_p = m_n = 1.67 \times 10^{-27}$ kg.

1 A B_s^0 meson contains an anti bottom quark together with a strange quark and so both it and its antiparticle are referred to as strange b-mesons. In 2013 the LHCb Collaboration published a paper[1] describing evidence for $B_s^0 - \overline{B}_s^0$ oscillations for strange b-mesons produced in proton-proton collisions at the Large Hadron Collider.

(a) Draw a Feynman diagram indicating the process(es) most likely to allow $B_s^0 - \overline{B}_s^0$ mixing. [5]



A figure from the LHCb paper is reproduced above. The filled-circle data points labelled 'Tagged mixed' count events in which the experimenters believe that a strange b-meson was produced with one flavour but decayed with another (i.e. $B_s^0 \to \overline{B}_s^0$ or $\overline{B}_s^0 \to B_s^0$), while the open circles labelled 'Tagged unmixed' count events in which the experimenters believe that a strange b-meson was produced and decayed without changing flavour (i.e. $B_s^0 \to B_s^0$ or $\overline{B}_s^0 \to \overline{B}_s^0$). In both cases, the 'candidate' counts are the number of events which were observed to have 'decay times' (i.e. times between birth and death) which lie in the relevant bin of the histogram.

(b) How might the members of the LHCb collaboration have been able to determine the flavour of the strange b-mesons at production? [8]

(c) How might the experimenters have been able to determine the flavour of the strange b-mesons at decay? [6]

(d) Explain why, for positive decay times t, and in the absence of any oscillations or detector considerations, one might expect the probability density $p(t)$ for B_s^0 decay times to be proportional to

$$\Gamma_s e^{-\Gamma_s t} \cosh\left(\frac{\Delta\Gamma_s}{2} t\right)$$

where Γ_s is the B_s^0 decay width and $\Delta\Gamma_s$ the decay width difference between the light and heavy mass eigenstates. [6]

(e) The experimentally observed data shown in the LHCb plot do not look much like the function we have just derived. What are the likely causes of the differences you can see, and what do they tell us? [5]

[1]Precision measurement of the $B_s^0 - \bar{B}_s^0$ oscillation frequency with the decay $B_s^0 \to D_s^- \pi^+$, [arXiv:1304.4741]

(TURN OVER

2 In the deep inelastic scattering of an electron with four-momentum p_1 by a proton with four-momentum p_2, the following Lorentz invariant variables can be defined:

$$Q^2 \equiv -q^2; \qquad x \equiv \frac{Q^2}{2p_2 \cdot q}; \qquad y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$$

where $q = p_1 - p_3$ and p_3 is the four-momentum of the scattered electron. Neglecting the mass of the electron:

(a) show that $Q^2 > 0$; [3]

(b) by considering the invariant mass of the final state hadronic system, or otherwise, determine the range of values which x can take. Comment on the physical significance of x, and comment on the interpretation which can be attached to events with $x = 1$; [6]

(c) deduce the nature of the limiting case or condition in which y relates to the centre-of-mass-frame scattering angle $\theta^\star$ of the electron via the formula:

$$y = \frac{1}{2}(1 - \cos\theta^\star)\,.$$ [4]

The so-called 'Drell-Yan' process is the production of lepton pairs ($\ell^+\ell^-$) in hadron-hadron collisions through the annihilation of a quark and an anti-quark into a photon.

(d) Draw the Feynman diagram for this process and explain why the cross section is non-zero for proton-proton collisions. [2]

The cross section for $q\bar{q}$ annihilation is

$$\sigma = \frac{4\pi}{3}\frac{\alpha^2}{\hat{s}}e_q^2$$

where e_q is the quark charge (i.e. $e_u = +\frac{2}{3}$ and $e_d = -\frac{1}{3}$ and $\hat{s}$ is the quark-anti-quark energy in their centre-of-mass frame.

(e) Neglecting any strange quark contributions, show that the parton model prediction for the $pp \to \mu^+\mu^- X$ differential cross section can be written in the form:

$$\frac{d^2\sigma}{dx_1 dx_2} = \left\{A\,[u(x_1)\bar{u}(x_2) + u(x_2)\bar{u}(x_1)] + B\left[d(x_1)\bar{d}(x_2) + d(x_2)\bar{d}(x_1)\right]\right\}$$

in which s is the centre of mass energy of the proton-proton system, and in which A and B are two real constants which you should determine. Your answer should make clear what the functions $u(x)$, $d(x)$, $\bar{u}(x)$ and $\bar{d}(x)$ represent, and to what x_1 and x_2 refer. [10]

Drell-Yan production of muon paris has been studied in pion collisions with carbon targets. Carbon (C) contains an equal number of protons and neutrons.

(f) If the invariant mass of the observed $\mu^+\mu^-$ system is Q^2, explain why the ratio ρ defined by

$$\rho = \frac{\sigma(\pi^+\mathrm{C} \to \mu^+\mu^- X)}{\sigma(\pi^-\mathrm{C} \to \mu^+\mu^- X)}$$

might approach unity for small Q^2. Furthermore: what might you expect it to approach to as Q^2 approaches s? [5]

3 The Super-Kamiokande water Čerenkov detector observes solar neutrinos through the elastic scattering of electron neutrinos from atomic electrons.

(a) Draw the two lowest order Feynman diagrams for the process of $\nu_e e^- \to \nu_e e^-$ scattering. [2]

The Lorentz invariant matrix element for the charged current contribution to $\nu_e e^- \to \nu_e e^-$ scattering can be written

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\rho} \left[\bar{u}(p_e')\gamma^\mu \frac{1}{2}(1-\gamma^5)u(p_{\nu_e})\right]\left[\bar{u}(p_{\nu_e}')\gamma^\rho \frac{1}{2}(1-\gamma^5)u(p_e)\right].$$

(b) Show that, in the limit where the electron and neutrino masses can be neglected, the only scattering processes to have non-zero matrix elements are between left-handed helicity particles, and that for those the matrix element can be written as follows:

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\rho} \left[\bar{u}_\downarrow(p_e')\gamma^\mu u_\downarrow(p_{\nu_e})\right]\left[\bar{u}_\downarrow(p_{\nu_e}')\gamma^\rho u_\downarrow(p_e)\right]$$

where $u_\downarrow$ is a left-handed helicity eigenstate. [5]

(c) Considering only the charged current contribution to $\nu_e e^- \to \nu_e e^-$ scattering, and working in the centre-of-mass frame, express M_{fi} in terms of the centre-of-mass energy, $\sqrt{s}$, and show that the total cross section is given by

$$\sigma(\nu_e e^- \to \nu_e e^-) = \frac{G_F^2 s}{\pi},$$

where $G_F/\sqrt{2} = g_W^2/8m_W^2$. [10]

(TURN OVER

Solar neutrinos detected in Super-Kamiokande are produced primarily from the $^{8}\mathrm{B} \to {}^{7}\mathrm{Be} + e^{+} + \nu_e$ process and have a mean energy of approximately 10 MeV.

(d) Obtain the value of $\sigma(\nu_e e^- \to \nu_e e^-)$ at this energy, expressing your answer in S.I. units. [3]

The flux of ^{8}B solar electron neutrinos at the Earth is expected to be $2.3 \times 10^{10}\ \mathrm{m^{-2}s^{-1}}$.

(e) Neglecting all interactions other than the charged current interaction, estimate the number of ^{8}B solar electron neutrino interactions per day in the Super-Kamiokande detector, of mass 5×10^{7} kg. [4]

(f) Briefly explain how solar neutrinos are detected in the Super-Kamiokande experiment and how they are distinguished experimentally from the background due to radioactive decays. [6]

[*You may make use of the following pieces of information which make use of the gamma-matrix conventions adopted by the lecture course:*
i) For spinors ψ and ϕ:

$$\begin{aligned}
\bar{\psi}\gamma^0\phi &= \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4,\\
\bar{\psi}\gamma^1\phi &= \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1,\\
\bar{\psi}\gamma^2\phi &= -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1),\\
\bar{\psi}\gamma^3\phi &= \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2,
\end{aligned}$$

ii) With starred quantities being defined in the centre-of-mass frame, the differential cross section for a two-to-two scattering process satisfies:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*} = \frac{1}{64\pi^2 s}\frac{|\boldsymbol{p}_f^*|}{|\boldsymbol{p}_i^*|}\left\langle\left|M_{fi}\right|^2\right\rangle.$$

iii) $G_F = 1.166\times10^{-5}\ \mathrm{GeV^{-2}}$, $m_e = 5.11\times10^{-4}$ GeV, $m_p = 0.94\ \mathrm{GeV} = 1.67\times10^{-27}$ kg *and* $e = 1.6\times10^{-19}$ C.]

END OF PAPER