UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS

Monday 17th January 2022 10:00 to 12:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

*Answer **two** questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper has content on 8 sides, including this one, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

*You should use a **separate Answer Book** for each question.*

STATIONERY REQUIREMENTS
2x20-page answer books
Rough workpad

SPECIAL REQUIREMENTS
Mathematical Formulae Handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

The information in this box may be used in any question.

The Pauli-matrices are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The gamma matrix representation of the Part III Particles lecture course was:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = i\gamma^0\gamma^1\gamma^2\gamma^3,$$

which has the following properties:

$$(\gamma^0)^* = \gamma^0,\ \ (\gamma^1)^* = \gamma^1,\ \ (\gamma^2)^* = -\gamma^2,\ \ (\gamma^3)^* = \gamma^3 \ \ \textit{and} \ \ \gamma^2(\gamma^\mu)^* = -\gamma^\mu\gamma^2.$$

Using the above convention, the Part III Particles lecture course defined the following particle and anti-particle spinors:

$$u_\uparrow = N\begin{pmatrix} c \\ e^{i\phi}s \\ \frac{|\boldsymbol{p}|}{E+m}c \\ \frac{|\boldsymbol{p}|}{E+m}e^{i\phi}s \end{pmatrix}, \qquad u_\downarrow = N\begin{pmatrix} -s \\ e^{i\phi}c \\ \frac{|\boldsymbol{p}|}{E+m}s \\ -\frac{|\boldsymbol{p}|}{E+m}e^{i\phi}c \end{pmatrix},$$

$$v_\uparrow = N\begin{pmatrix} \frac{|\boldsymbol{p}|}{E+m}s \\ -\frac{|\boldsymbol{p}|}{E+m}e^{i\phi}c \\ -s \\ e^{i\phi}c \end{pmatrix}, \qquad v_\downarrow = N\begin{pmatrix} \frac{|\boldsymbol{p}|}{E+m}c \\ \frac{|\boldsymbol{p}|}{E+m}e^{i\phi}s \\ c \\ e^{i\phi}s \end{pmatrix}$$

for objects whose three-momentum $\boldsymbol{p}$ *is given by* $|\boldsymbol{p}|(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$ *where* $c = \cos\frac{\theta}{2}$ *and* $s = \sin\frac{\theta}{2}$*. The normalising constant is* $N = \sqrt{E+m}$*.*

$\hbar \approx 1.05\times10^{-34}$ kg m^2/s, $\quad c \approx 3.00\times10^{8}$ m/s, $\quad e \approx 1.60\times10^{-19}$ C.

$m_e = 5.11\times10^{-4}$ GeV. $\quad m_p = m_n = 1.67\times10^{-27}$ kg.

1 Neutrinos produced inside the sun by electron capture in the following process $^7\mathrm{Be} + e^- \to {}^7\mathrm{Li} + \nu_e$ are almost mono-energetic with an energy of $E_\nu = 862$ keV and are produced at a rate of $4.5 \times \pi \times 10^{36}$ per second. The Earth's mean distance from the sun is 150 million km.

The Borexino experiment in Italy is optimised to look for such neutrinos. It observes neutrinos through their elastic scattering $\nu_e e^- \to \nu_e e^-$ with electrons in molecules of an organic solvent called 1,2,4-trimethylbenzene, $C_6H_3(CH_3)_3$ within which a small quantity of a scintillating additive is dissolved. The additive generates scintillation photons in proportion to the lab-frame *kinetic* energy ($\mathrm{KE}_e = E_e - m_e$) of each scattered electron, and it is by this means that scattering events are observed. Many tonnes of solvent are used in the experiment (1 tonne=1000 kg). [*Each of the carbon atoms of the solvent contains six protons and six neutrons.*]

If the above scattering process were mediated exclusively by the W-boson (and if electron and neutrino masses could be neglected) then the total cross section in natural units would be:

$$\sigma(\nu_e e^- \to \nu_e e^-) = \frac{G_F^2 s}{\pi} \qquad (\star)$$

where $\sqrt{s}$ is the centre-of-mass energy and $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$.

(a) Write down in S.I. units a numerical value for the cross section predicted by ($\star$) for ^{7}Be solar neutrinos. [4]

(b) Estimate the number of electrons in one tonne of 1,2,4-trimethylbenzene. Make clear your assumptions and estimate the accuracy of your answer. [4]

(c) Estimate the maximum number of ^{7}Be solar neutrino interactions which occur per day per tonne of the Borexino detector, in the absence of neutrino oscillations, assuming that the cross section for scattering is described by ($\star$). [4]

(d) Describe the angular and energy distributions in the centre-of-mass frame for electrons scattered by neutrinos as a result of charged-current interactions. Include within your answer an algebraic expression for $d\sigma/d\Omega^*$ showing its dependence on the cosine of the angle θ^* between the outgoing electron and the incoming neutrino. [4]

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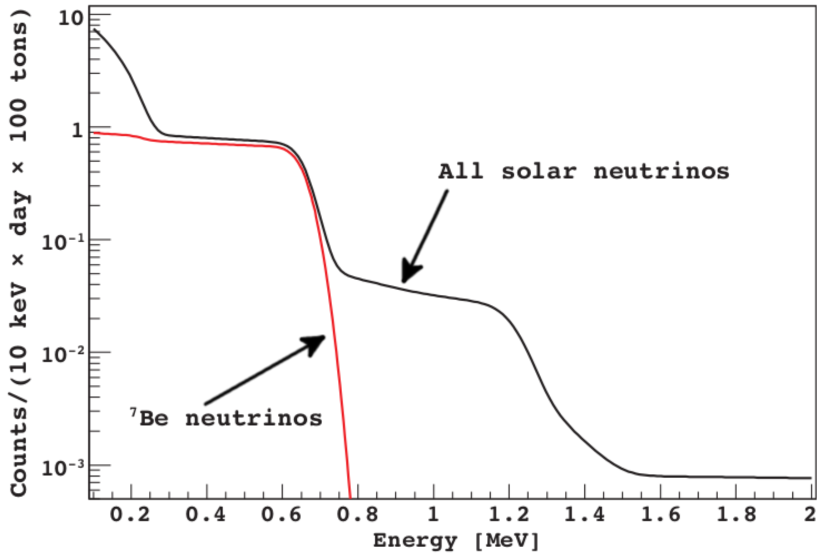
The first Borexino paper to describe observations of ^{7}Be neutrinos (arXiv:0708.2251) noted that:

> "… the recoil electron [kinetic energy distribution has] a rectangular shape with a sharp cut-off edge at 665 keV in the case of ^{7}Be neutrinos (see Fig. 1). The background from the 156 keV β-decay of ^{14}C, intrinsic to the scintillator, limits neutrino observation to [scattered electrons with kinetic] energies above 200 keV."

The figure referred to in the text above shows the following plot:



(e) Check that that $\mathrm{KE}_e^{\mathrm{max}} = 665$ keV is indeed the maximum kinetic energy of an electron scattered by a ^{7}Be solar neutrino by: (i) deriving an algebraic expression for $\mathrm{KE}_e^{\mathrm{max}}$ in terms of m_e and E_ν, and then (ii) confirming that it evaluates to 665 keV. [*You will not be able to neglect the electron mass.*] [4]

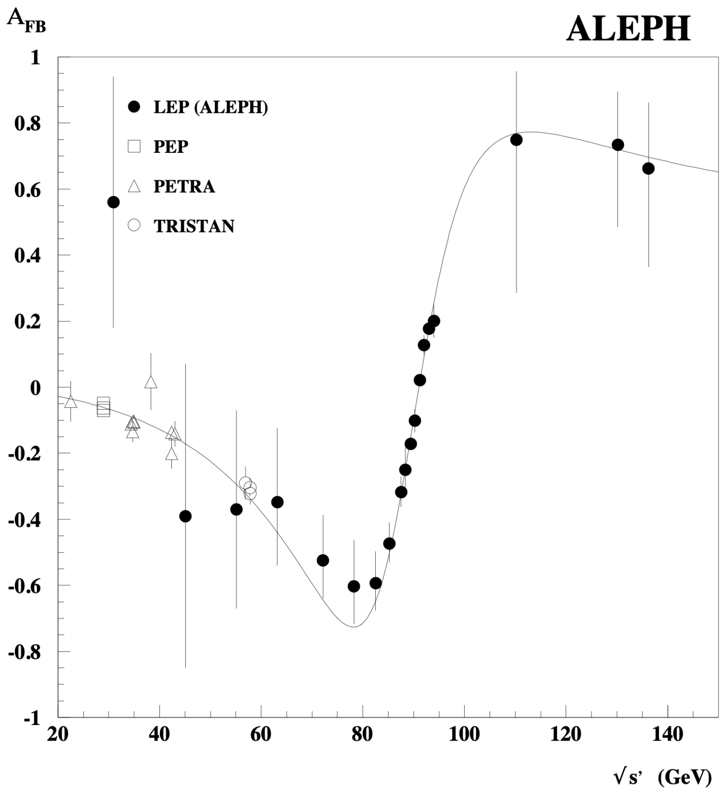
(f) Explain how you could find out whether the Borexino paper's description of the shape of the kinetic energy spectrum of scattered electrons is consistent with your answer to part (d) [*You are not **required** to compute an analytic form for the lab-frame kinetic energy distribution of the electrons, but your answer must provide a clear description of the nature of the computations you would perform.*] [4]

(g) According to the Borexino paper, what fraction of ^{7}Be solar neutrino elastic scattering events are observable, i.e. would have kinetic energies above the β-decay background? [2]

(h) The Borexino experiment reported 49 ± 4 counts/(day · 100 tonne) from neutrino interactions, while 75 ± 4 counts/(day · 100 tonne) were expected without neutrino oscillations. The rate found in part (c) should be significantly larger than either of these two rates. What might be the main cause or causes of the discrepancy? [4]

2 (a) **Forward-Backward Asymmetry of the Z-boson:**

In 1997, one of the LEP experiments (ALEPH) published[1] the following plot of their measurements of the Z-boson's forward-backward asymmetry.



The plot above was captioned:

> 'Measured forward-backward asymmetries of muon-pair production compared with the fit results. … For comparison the measurements at lower energies from [other experiments called] PEP, PETRA and TRISTAN are included.'

The fitted curve in the plot above passes through the point $(m_Z, 0.020)$ where m_Z is the mass of the Z-boson.

(i) From an experimental perspective: how is A_{FB} defined and how are measurements of A_{FB} like those shown in the above plot made? [4]

(ii) From a theoretical perspective, why is A_{FB} worth measuring? What theoretical features of the standard model does it constrain, and how? [8]

(iii) What general conclusions (if any) can be drawn from the ALEPH plot as a whole? What quantitative conclusions (if any) can be drawn from the value of A_{FB} at $\sqrt{s} = m_Z$? What quantitative conclusions about electroweak unification (if any) can we draw from the same data point if it is also assumed that electrons and muons have identical couplings to the Z-boson? [8]

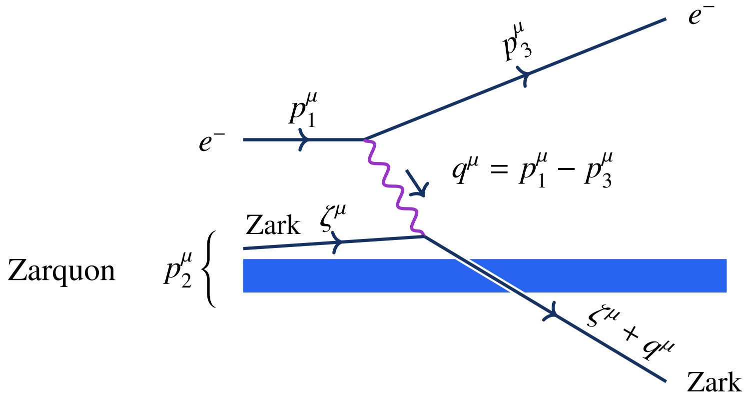
[1] Phys. Lett. B **399** (1997) 329

(b) **The Quark Model of the Hadrons:** The Σ^0, Σ^{0*} and Λ^0 baryons all have the same uds flavour, yet each has a different mass from the other. Why is this? Are there any physical (rather than, say, simply notational or conventional) reasons for them to have different masses? [10]

3 The 'Zarquon' is a hypothetical particle of mass $M > 0$ composed of indestructable 'Zarks'. There are two types of Zark: the 'Fat Zark' and the 'Ferret Zark', with respective masses of $\chi/2$ and $\chi/4$. Every Zarquon contains exactly one Fat Zark and two Ferret Zarks (and nothing else). It is known that if, at some moment in time, a Fat Zark has a three-momentum $\boldsymbol{k}$ in the rest frame of its Zarquon, then at the same moment in time each of the Ferret Zarks has a three-momentum $-\frac{1}{2}\boldsymbol{k}$ in the same frame. Zarquons cannot be polarised, so all directions for $\boldsymbol{k}$ are equally likely.

(a) Write down $|\boldsymbol{k}|$ in terms of M and χ, and then determine the range of values the parameter χ could take (given a value of M). [3]

It is planned to investigate the Zark content of the Zarquon by a series of fixed-target deep inelastic scattering experiments in which a beam of electrons is fired at a Zarquon target as shown:



The probe electron has four-momentum p_1^μ when incoming and p_3^μ when outgoing. The Zarquon has initial four-momentum p_2^μ. The struck Zark has momentum ζ^μ before and $\zeta^\mu + q^\mu$ after the interaction. The rest masses of the Zark and electron are unaffected by their interaction. Assume that the electron mass can be neglected and in the lab frame the momenta p_1^μ, p_2^μ, p_3^μ and ζ^μ take the form:

$$p_1^\mu = \begin{pmatrix} p \\ p \\ 0 \\ 0 \end{pmatrix}, \qquad p_2^\mu = \begin{pmatrix} M \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad p_3^\mu = \begin{pmatrix} E \\ E\cos\theta \\ 0 \\ E\sin\theta \end{pmatrix} \quad \text{and} \quad \zeta^\mu = \begin{pmatrix} \sqrt{m^2 + a^2} \\ -a\cos\alpha \\ a\sin\alpha\cos\delta \\ a\sin\alpha\sin\delta \end{pmatrix}$$

where $p > 0$, $a \geq 0$, $0 \leq \alpha \leq \pi$, $0 \leq \delta < 2\pi$, $0 \leq \theta \leq \pi$ and $E \geq 0$. The value m will be equal to either $\chi/2$ or $\chi/4$ depending on whether the struck object was a Fat Zark or a Ferret Zark.

(b) How will the value of the parameter a depend on whether the struck object was a Fat Zark or a Ferret Zark? What geometrical interpretation can be given to the quantities α and δ ? How is α distributed? [4]

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For the scattering process described, the quantities in the set S

$$S = \{p, M, m, a, \alpha, \delta, \theta\}$$

are not independent of E.

(c) Write down (but do not yet solve) an equation which, if solved, would fix E in terms of the quantities in S. Explain the physical meaning of this constraint. [2]

(d) By solving the constraint just written down (or otherwise) find an expression for E in terms of the quantities in S. [4]

(e) Suggest two physical reasons why an experiment might be unable to make measurements all the way out to $\cos\theta = \pm 1$. [4]

In the rest of this question you may assume that the scattering experiments are conducted only in the regime in which $|\cos\theta| \leq 0.9$ and $10M \ll p$ and that you may simplify expressions by neglecting terms accordingly.

(f) Explain why it is the case that, in the scattering regime just described, your answer for E obtained in part (d) simplifies to

$$E \approx \frac{\sqrt{m^2 + a^2} + a\cos\alpha}{1 - \cos\theta}.$$ [4]

The 'Bjorken x' variable is *defined* by the equation $x = -q^2/(2p_2 \cdot q)$.

(g) To the level of approximation permitted in this scattering regime, show that the 'Bjorken x' variable satisfies

$$x \approx \frac{\sqrt{m^2 + a^2} + a\cos\alpha}{M}.$$ [4]

(h) Determine and then sketch the shape of the parton distribution function $F(x)$ of the Fat Zark. Make sure to show how the key features of $F(x)$ depend on χ and M. [2]

(i) Determine and then sketch the shape of the parton distribution function $f(x)$ of the Ferret Zark. Make sure to show how the key features of $f(x)$ depend on χ and M. [2]

(j) What value would you expect the integral $\int_0^1 (xF(x) + xf(x))\,\mathrm{d}x$ to take and why? [You are not required to evaluate the above integral explicitly.] [1]

END OF PAPER