UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
NATURAL SCIENCES TRIPOS: Part III Astrophysics
MASTER OF ADVANCED STUDY IN PHYSICS
MASTER OF ADVANCED STUDY IN ASTROPHYSICS

Tuesday 14th January 2020: 14:00 to 16:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

*Answer **two** questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains four sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

*You should use a **separate Answer Book** for each question.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2x20-page answer books	Mathematical Formulae Handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

(7th January 2020)

1 Consider the decay $\pi^+ \to \mu^+\nu_\mu$ where the π^+ has quark content $u\bar{d}$.

(a) Draw a Feynman diagram for this decay. [3]

(b) Draw a diagram showing the momentum and spin directions of the outgoing particles in the centre-of-mass frame, explaining clearly the reasons for your choice of spin state. [3]

(c) The lepton current for the final state can be written as

$$\bar{u}(p_3)\gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) v(p_4)$$

where p_3 is the four-momentum of the ν_μ and p_4 is that of the μ^+. Forms for the γ-matrices and spinors can be found at the end of the question. Show that the magnitude of the lepton current is proportional to

$$\frac{(E+m-p)\sqrt{p}}{\sqrt{E+m}}$$

where E, m and p are the energy, the mass and the magnitude of the three-momentum of the μ^+. You are not required to compute the components of the current. [6]

(d) Given that the two-body decay rate is

$$\Gamma = \frac{p}{8\pi m_\pi^2}\left\langle |M|^2 \right\rangle,$$

where m_π is the mass of the pion and M is the matrix element for the decay, estimate the ratio

$$\frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)}.$$

You may assume that the momentum of the muon emitted in the pion decay is 30 MeV while that of the electron is 70 MeV. Their respective masses are 106 MeV and 0.511 MeV. [6]

(e) Explain how measurements of the decay

$$^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^\star + e^- + \bar{\nu}_e$$

can be used to show that the laws of nature are not symmetric under parity. [8]

(f) To what extent do forward-backward asymmetries measured at LEP test for the presence (or absence) of parity as a symmetry of the Standard Model? [4]

[*The gamma matrix and spinor conventions used in the lecture course were:*

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3,$$

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$u_\uparrow = N \begin{pmatrix} \hat{c} \\ e^{i\phi}\hat{s} \\ \alpha\hat{c} \\ \alpha e^{i\phi}\hat{s} \end{pmatrix}, u_\downarrow = N \begin{pmatrix} -\hat{s} \\ e^{i\phi}\hat{c} \\ \alpha\hat{s} \\ -\alpha e^{i\phi}\hat{c} \end{pmatrix}, v_\uparrow = N \begin{pmatrix} \alpha\hat{s} \\ -\alpha e^{i\phi}\hat{c} \\ -\hat{s} \\ e^{i\phi}\hat{c} \end{pmatrix}, v_\downarrow = N \begin{pmatrix} \alpha\hat{c} \\ \alpha e^{i\phi}\hat{s} \\ \hat{c} \\ e^{i\phi}\hat{s} \end{pmatrix}$$

where $N = \sqrt{E+m}$, $\hat{c} = \cos\left(\frac{\theta}{2}\right)$, $\hat{s} = \sin\left(\frac{\theta}{2}\right)$, $\alpha = \frac{|\boldsymbol{p}|}{E+m}$, *and where* θ *and* ϕ *are the usual spherical angles (polar and azimuthal respectively) and where* E, $\boldsymbol{p}$ *and* m *are the energy, momentum and mass of the particle (or antiparticle).*]

2 Suppose that after Brexit the UK takes back control of the laws of physics from Europe by abolishing the use of the colour green in QCD. This change makes post-Brexit QCD a two-colour $SU(2)$ symmetry based on red (not white) and blue, rather than on the three-colour $SU(3)$ symmetry which the UK used throughout its membership of the European Union.

(a) Determine which 'Brexit mesons' and 'Brexit baryons' (or their nearest equivalents) could exist by constructing any important colour, flavour and spin wave-functions. Categorise the expected 'Brexit hadrons' by type (meson/baryon), spin, and the multiplets they inhabit. Compare the main similarities and differences between the pre- and post-Brexit structure of hadrons. [*Each of the three instructions above (there is one per sentence) carries approximately eight marks. It is not necessary to address the issues in the order listed. You need only consider light quark types: u, d and s.*] [24]

(b) Are any four or five quark-and/or-antiquark states allowed by the pre-Brexit Standard Model? What sort of four or five quark-and/or-antiquark states could be seen after Brexit? [6]

3

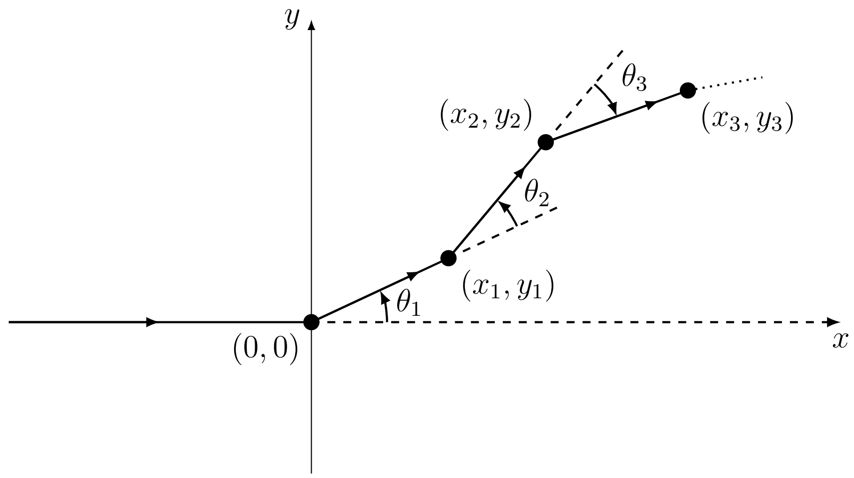
(a) How do cosmic rays form in the upper atmosphere? [2]

(b) Comment on the relative sizes of the sea-level fluxes of down-going ($\downarrow$) and up-going ($\uparrow$) atmospheric neutrinos and anti-neutrinos of all flavours. Make clear your assumptions. [6]

(c) Describe the Super-Kamiokande experiment and explain which flavours of atmospheric neutrino (and antineutrino) you might expect it to be able to see. Justify your answer with kinematic calculations where necessary. [8]

(d) What have Super-Kamiokande atmospheric neutrino results told us about the neutrinos of the Standard Model? [2]

Now consider a high energy cosmic ray muon in a two-dimensional universe which is filled with concrete in the region $x \geq 0$ and contains vacuum elsewhere. The muon starts on the negative x-axis and enters the concrete at the origin where it immediately scatters through a random angle θ_1 as illustrated below. It then travels for a distance δl before scattering again through another random angle θ_2. This process repeats for a total of N scatterings, over which time the muon will have travelled a path-length $D = N\delta l$ in the concrete. The first three scatterings are shown in the diagram. Every angle θ_i may be assumed to be independent of the others, and each may be assumed to be drawn from a Normal Distribution with mean 0 and variance K. You may assume that the individual scattering angles and the net scattering angle are all small ($\ll 1$). The position of the comsic ray after it has travelled a distance δl beyond the n-th scatter is denoted (x_n, y_n).



(e) Show that

$$\text{Var}(y_N) = \alpha \frac{D^3}{\delta l}\left(1 + O\left(\frac{\delta l}{D}\right)\right)$$

for some constant α, which you should determine. [*You may use, without proof, the identity* $\sum_{k=1}^{n} k^2 = \frac{1}{6}n(n+1)(2n+1)$, *and may also assume that for independent random variables* $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$.] [6]

(f) Without lengthy calculations, describe how (if at all) you would expect the above answer to change in a three-dimensional universe. [2]

(g) What message might the last two results send to an engineer hoping to use cosmic ray muons for high-resolution concrete tomography? [2]

(h) How might such an engineer exploit the fact that not all cosmic ray muons have the same energy to improve the resolution of a concrete scanner? [2]

(7th January 2020)