UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS
NATURAL SCIENCES TRIPOS: Part III Astrophysics

Tuesday 15 January 2019: 14:00 to 16:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

*Answer **two** questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains eight sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

*You should use a **separate Answer Book** for each question.*

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
|---|---|
| 2x20-page answer books | Mathematical Formulae Handbook |
| Rough workpad | Approved calculator allowed |

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 It is an experimental fact that there exist Ultra-High Energy (UHE) cosmic rays with energies many orders of magnitude larger than the energies at even the most powerful current particle colliders. Due to the almost perfect vacuum of intergalactic space UHE cosmic rays can travel huge distances without any interaction. It is known however, that once the cosmic rays reach a certain threshold energy they start interacting with the photon cosmic background radiation (CMB). This is known as the Greisen-Zatsepin-Kuzmin (GZK) limit which effectively implies that there is an upper cutoff in the energies of the cosmic rays that can be observed.

(a) In the above context, consider the reaction

$$p + \gamma \longrightarrow n + \pi^+ .$$

Derive the threshold proton energy required for the above reaction to take place (in Earth's reference frame); you may assume that the colliding proton and photon travel in exactly opposite directions (i.e. a head-on collision). You may use the following masses:

$$m_p = 938.3\,\text{MeV}, \quad m_n = 939.6\,\text{MeV}, \quad m_{\pi^+} = 139.6\,\text{MeV},$$

and that the temperature of the CMB radiation is $T_{\text{CMB}} = 2.7\,\text{K}$ which corresponds to $E_\gamma = 0.23 \times 10^{-3}\text{eV}$.

[5]

(b) Imagine that the same astrophysical phenomenon that produces the UHE proton cosmic rays can also produce UHE electron neutrinos $\nu_e$. Since, besides photons, the CMB is also expected to contain a large density of neutrino-antineutrino pairs of all flavors, by considering the reaction

$$\nu_e + \bar{\nu}_e \longrightarrow Z,$$

show that a GZK-like limit should exist also for neutrinos. What is the value of the threshold neutrino energy (assuming head-on collisions, as above)? Assume all neutrinos are exactly massless. You may also use that $E_{\bar{\nu},CMB} = 0.17 \times 10^{-3}$ eV (which is slightly different than the CMB photon energy above) and $m_Z = 91\,\text{GeV}$.

[5]

(c) Comment on your result in light of the fact that the Planck mass is $M_{\text{Planck}} \approx 10^{18}\,\text{GeV}$. [2]

(d) Draw all Leading Order Feynman diagrams contributing to the process

$$\nu_e + \bar{\nu}_e \longrightarrow \nu_e + \bar{\nu}_e .$$

[4]

(e) Considering all neutrinos as exactly massless and having 4-momenta as follows: $\nu_e(p_1) + \bar{\nu}_e(p_2) \longrightarrow \nu_e(p_3) + \bar{\nu}_e(p_4)$ write the matrix element for each diagram. Show that all matrix elements are of the form

$$M = \frac{c}{q^2 - m^2} j_1 \cdot j_2 ,$$

for some constant factor $c$, momentum $q$, mass $m$ and currents $j_{1,2}$. Write them down explicitly, paying particular attention to the helicities of the fermions. [7]

(f) Write an explicit parametrization of the four 4-momenta involved in the scattering process.

Using the generic expressions for massless Dirac spinors of definite helicity provided below, write explicitly all spinors, $\psi$, that appear in the above currents but not the conjugated ones $\bar{\psi}$.

Compute explicitly the $\mu = 0$ components (i.e. the time-components) of all the currents derived above. You may give the *final* results for the case when $\phi = 0$. [7]

[*You may make use of the following information:*
*A generic massless 4-vector p can be parametrized as follows:*

$$p = E\,(1, \hat{p})\ ,$$

*where* $\hat{p}$ *is a 3-vector of unit length:* $\hat{p} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$.
*In the massless limit* $E \gg m$ *Dirac spinors for some of the helicity eigenstates are:*

$$u_{\downarrow} = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix}, v_{\uparrow} = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix},$$

*where* $c = \cos(\theta/2)$ *and* $s = \sin(\theta/2)$.]

2 The natural way to directly measure the top-quark coupling to the Higgs boson ($h$) at the LHC is via the process:

$$pp \longrightarrow t\bar{t}h\,.$$

(a) Draw all Leading Order Feynman diagrams for the subprocess

$$q\bar{q} \longrightarrow t\bar{t}h\,,$$

assuming that:

$$m_u = m_d = m_s = m_c = m_b = 0\,.$$

Do not consider any loop-induced couplings. Do not consider diagrams involving photons, $W^{\pm}$ or $Z$ bosons.

[4]

(b) Compute the overall colour factor for the contribution to the cross-section (not just the matrix elements) for the subprocess

$$q\bar{q} \longrightarrow t\bar{t}h\,.$$

In doing so you may utilize either the Gell-Mann matrices given below or the following property of the Gell-Mann matrices:

$$\mathrm{tr}\left(\lambda^a \lambda^b\right) = 2\delta^{ab}\,.$$

In the calculation one has to sum/average over the colour of all final/initial state particles, as appropriate. [6]

(c) Some models of beyond the Standard Model physics predict the existence of an extra Higgs boson (denoted $H$). Assume $H$ has exactly the same quantum numbers as the Standard Model Higgs boson $h$, the same couplings to the Standard Model particles as $h$, does not couple to $h$, however its mass is different $m_H \neq m_h$.

Suggest and very briefly describe a strategy for the search for such a particle at the LHC operating at c.m. energy of 13 TeV. While the mass $m_H$ is unknown, theoretical arguments suggest it may be around 20 GeV. What are the possible decay channels for such a state? Without detailed calculations briefly comment on the differences with respect to the Standard Model Higgs. Assume that:

$$m_b = 5\,\mathrm{GeV}, \;\; m_u = m_d = m_s = m_c = 0, \;\; m_e = m_\mu = m_\tau = 0\,.$$

You may find useful the following results for masses and decay widths:

$$m_h = 120\,\mathrm{GeV}, \;\; m_t = 173\,\mathrm{GeV}, \;\; m_W = 80\,\mathrm{GeV}, \;\; m_Z = 91\,\mathrm{GeV}\,,$$

$$\Gamma_t = 1.5\,\mathrm{GeV}, \;\; \Gamma_W = 2\,\mathrm{GeV}, \;\; \Gamma_Z = 2.5\,\mathrm{GeV}\,.$$

You may need to consider the Breit-Wiger resonance formula which is derived from the propagator of an unstable particle (with mass $m$ and total width $\Gamma$):

$$\frac{1}{q^2 - m^2 + im\Gamma}\,.$$

[5]

(d) Consider the ratio:

$$R = \frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)},$$

at Leading Order but in a theory where the (spin 0) $W$ bosons couple to fermions through the pseudo-scalar coupling:

$$\bar{\psi}\gamma^5\phi .$$

Write down the matrix element for the process $\pi^- \to \mu^- \bar{\nu}_\mu$ in terms of basis spinors. Work in the limit $q^2 << m_W^2$ where $q$ is the momentum of the off-shell $W$ boson and assume the hadronic current reads $\bar{u}\gamma^5 v = f_\pi m_{\pi^-}$ for some constant $f_\pi$ which has the same value for all $\pi^-$ decays. [2]

Define the 1-to-2 decay kinematics for the process $\pi^- \to \mu^- \bar{\nu}_\mu$ in the $\pi^-$ rest frame and use it to write down explicitly all spinors relevant for the muon and the antineutrino. Show that in this frame the antineutrino and muon energies take the following form:

$$E_\nu = \frac{m_{\pi^-}^2 - m_\mu^2}{2m_{\pi^-}}, \quad E_\mu = \frac{m_{\pi^-}^2 + m_\mu^2}{2m_{\pi^-}}.$$

[3]

Calculate explicitly the matrix element for the process $\pi^- \to \mu^- \bar{\nu}_\mu$ for all possible combinations of basis spinors. [Separate the terms that depend on the muon mass $m_\mu$; the $m_\mu$-independent terms could be combined into an overall factor which is of no interest.] Show that all non-vanishing muon and antineutrino spinor combinations are of the form:

$$\bar{u}\gamma^5 v \sim (1 + x)\sqrt{E_\nu(E_\mu + m_\mu)},$$

where $u$ and $v$ are two basis spinors and $x = E_\nu/(E_\mu + m_\mu)$. [5]

With the help of the general expression:

$$\Gamma = \frac{p^*}{32\pi^2 m_{\pi^-}^2} \int < |M_{fi}|^2 > d\Omega,$$

where $< |M_{fi}|^2 >$ is the corresponding spin summed/averaged matrix element squared and:

$$p^* = \frac{m_{\pi^-}^2 - m_\mu^2}{2m_{\pi^-}},$$

show that in this model $R = O(1)$. [$m_{\pi^-} = 139.6\,\text{MeV}$ and $m_\mu = 105.7\,\text{MeV}$; the electron mass may be neglected whenever this is possible]. [4]

Compare with the corresponding value in the Standard Model ($R = O(10^{-4})$ based on the Standard Model coupling $\bar{\psi}\gamma^\mu(1 - \gamma^5)\phi$) and briefly comment on the main reason for such a difference. [1]

(TURN OVER

[*A general basis for massive Dirac spinors for (anti)fermions with 4-momentum p is:*

$$u_1 = N\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}, u_2 = N\begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}, v_1 = N\begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, v_2 = N\begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix},$$

*where:* $N = \sqrt{E+m}$, $p^2 = m^2$ *and* $E = p^0$.
*The following result might be helpful:*

$$\gamma^0\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}.$$

*The Gell-Mann matrices are given by:*

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \; \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \; \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

]

3 Answer the following questions:

(a) State the 3 Sakharov conditions. Which outstanding problem in physics are they trying to address? [5]

(b) We know that weak eigenstates like $\nu_e, \nu_\mu$ can oscillate, i.e. $P(\nu_e \to \nu_\mu) \neq 0$ at any time $t > 0$ if at $t = 0$ we have a pure $\nu_e$ state.

Show that mass eigenstates like $e, \mu$ do not oscillate i.e. $P(e \to \mu) = 0$ at any time $t$. [You may neglect the existence of $\tau$-flavour.] [3]

Show that any linear combination of mass eigenstates is also preserved with time (in the sense that the relative contribution of a given mass eigenstate in that combination is constant with time). [You may neglect the existence of $\tau$-flavour.] [3]

(c) Consider the following two scattering processes (which are of Charged Current Deep Inelastic Scattering type):

$$\nu_\mu M \longrightarrow \mu^- + X\,,$$
$$\bar{\nu}_\mu M \longrightarrow \mu^+ + X\,,$$

where $M$ is assumed to be a stable meson and $X$ represents the resulting hadronic final state. Assume $M$ has the following flavour structure:

$$M = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}\,.$$

[One can think of $M$ as being the neutral pion $\pi^0$ when the scattering processes above happen on a time scale much shorter than the life-time of $\pi^0$.] Throughout this problem ignore any other flavours besides $u$ and $d$ and work in the limit $q^2 \ll m_W^2$ where $q$ is the momentum of the off-shell $W$ boson.

Identify all partonic reactions contributing to the above processes. Comment on the helicities of the fermions entering each matrix element. [3]

Assume that the partonic matrix element for each one of the contributing partonic processes is of the form:

$$M_{fi} = C \times \hat{s} \times f(\cos\theta)\,,$$

where $\hat{s}$ is the square of the total energy available to the partonic reaction, $f$ is a polynomial that depends on the process and $C$ is a constant factor which also depends on the process. Derive the expression for the function $f$ in all partonic reactions based on your expectation for the angular dependence of a system of two fermions with a given total angular momentum $J = 0, \pm 1$. [5]

From the following parton-level relations:

$$\sigma_{\nu q} = \sigma_{\bar{\nu}\bar{q}}\,, \quad \sigma_{\bar{\nu} q} = \sigma_{\nu\bar{q}}\,,$$
$$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}\,, \quad \sigma_{\nu q}/\sigma_{\bar{\nu} q} = 3\,,$$

(TURN OVER

where $q$ denotes any type of quark, determine the corresponding constant factor $C$ for each one of the contributing partonic processes. You may use the following relations: $G_F/\sqrt{2} = g_W^2/(8m_W^2)$ as well as that in the centre of mass frame:

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 \hat{s}} < |M_{fi}|^2 > ,$$

where $< |M_{fi}|^2 >$ is the spin summed/averaged matrix element squared for the corresponding process. [5]

Assuming that at high energies the collision is described in terms of partonic densities $q^M(x)$ and that $q^M(x) = \bar{q}^M(x)$ for all quark flavours while $u^M(x) \neq d^M(x)$, show that the cross-sections for the two processes are equal, i.e.

$$\sigma(\nu_\mu M) = \sigma(\bar{\nu}_\mu M) .$$

[5]

Why is it plausible to expect that $q^M(x) = \bar{q}^M(x)$ for the meson M? [1]