UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS
NATURAL SCIENCES TRIPOS: Part III Astrophysics

Tuesday 16 January 2018: 14:00 to 16:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

Answer ***two*** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains four sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You should use a ***separate Answer Book*** *for each question.*

STATIONERY REQUIREMENTS
2 answer books
Rough workpad

SPECIAL REQUIREMENTS
Mathematical Formulae Handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 Suppose that in an extension of the Standard Model there is a fourth generation of heavy leptons comprising a charged lepton, L^-, with a mass of 3.0 TeV, and a heavy neutrino, ν_L, with a mass of 500 GeV. You may assume that these fourth generation leptons have the same couplings to the W, Z and γ as the first three generations.

(a) The L^- would decay via the weak interaction, i.e. $L^- \to e^-\nu_L\bar{\nu}_e$. Draw the leading order Feynman diagram(s) for this decay and, by considering the other possible decays of the W-boson, estimate the branching fraction for $L^- \to \nu_L$ + hadrons. [6]

(b) Heavy charged leptons could be produced in proton-proton collisions via the Drell-Yan process, i.e. the production of L^+L^- through the annihilation of a quark and anti-quark into a photon. Draw the leading order Feynman diagram(s) for this interaction and explain why the cross section is non-zero for proton-proton collisions. [3]

(c) If the squared centre-of-mass energy of the proton-proton collision is s, show that the squared centre-of-mass energy of the qq system is $\hat{s} = x_1x_2s$, where x_1 and x_2 are the fractional momenta carried by the partons involved in the collision. [3]

(d) The QED cross section for $qq \to l^+l^-$, where l is a Standard Model charged lepton, is given by

$$\sigma = \frac{4\pi}{3}\frac{\alpha^2}{s}e_q^2 \times f(s, m_l),$$

where e_q is the quark charge (i.e. $e_u = +2/3$ and $e_d = -1/3$) and $f(s, m_l)$ is a kinematic factor that depends on the lepton mass. Assuming that the $\bar{u}$ and $\bar{d}$ parton distribution functions can be described by a single function, $S(x)$, and neglecting the strange quark contribution, show that the parton model prediction for the $pp \to L^+L^-X$ differential cross section can be written

$$\frac{d^2\sigma}{dx_1dx_2} = f(sx_1x_2, m_L)\frac{2\pi\alpha^2}{81x_1x_2s}\left[9u_V(x_1)S(x_2) + 9u_V(x_2)S(x_1) + 20S(x_1)S(x_2)\right],$$

where $u_V(x)$ is the valence up-quark parton distribution function. Clearly state any assumptions you have made. [10]

(e) Draw a diagram showing the region of x_1 versus x_2 that contributes to the cross section for $pp \to L^+L^-X$ at the LHC operating at $\sqrt{s} = 13$ TeV. [2]

(f) Explaining your reasoning, which of the terms $20S(x_1)S(x_2)$ or $[9u_V(x_1)S(x_2) + 9u_V(x_2)S(x_1)]$, would you expect to dominate in this region? [2]

(g) In the relevant regions of x, the parton distribution functions can be taken to have the approximate forms, $u_V(x) \approx ax^{-\lambda}$ and $S(x) \approx bx^{-\lambda}$. Taking $f(sx_1x_2, m_L) = 1$, and by performing the appropriate integration over x_1 and x_2, obtain an approximate expression for the Drell-Yan cross section for heavy lepton production in terms of α, a, b, λ, m_L and s. [4]

2 Write detailed notes on **one** of the following topics:

(a) parton distribution functions, **or** [30]

(b) SU(3) colour symmetry. [30]

3 The vertex factor for the interaction between a Higgs boson and a fermion is

$$-i\frac{g_W m_f}{2m_W},$$

where g_W is the weak decay constant, and m_W and m_f are the masses of the W-boson and the fermion f, respectively. Write down the matrix element for the decay $H \to f\bar{f}$. [3]

Consider the decay of the Higgs boson in its rest frame where the fermion is produced with polar angle θ and azimuthal angle ϕ. Assuming $m_H \gg m_f$, evaluate the $H \to f\bar{f}$ matrix elements for all four possible combinations of particle and anti-particle helicities and comment on your results. [8]

Given the expression for the decay rate:

$$\frac{d\Gamma}{d\Omega} = \frac{p^*}{32\pi^2 m_H^2}\left\langle |M_{fi}|^2 \right\rangle,$$

where p^* is the centre-of-mass momentum of either final state particle, show that the partial decay width for the Higgs boson to $\tau^+\tau^-$ is

$$\Gamma_\tau = \frac{G_F}{\sqrt{2}}\frac{m_\tau^2 m_H}{4\pi}.$$

[4]

Assuming $m_H = 125$ GeV, and neglecting decays to $\nu\bar{\nu}$, e^+e^-, $\mu^+\mu^-$, $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$, obtain values for the total decay width of the Higgs boson, Γ_H and the branching fraction for $H \to b\bar{b}$. [5]

At a future muon collider operating at the Higgs boson resonance (assumed to be $\sqrt{s} = 125$ GeV), compare the cross section for the process $\mu^+\mu^- \to H \to b\bar{b}$ to the cross section for the QED process $\mu^+\mu^- \to \gamma \to b\bar{b}$ which is $\sigma_{QED} = 4\pi\alpha^2/(9s)$. Comment on the possible advantages and disadvantages of a muon collider compared to an electron-positron collider. [6]

Assuming that a muon collider could operate with fully polarised beams where the helicities of the μ^+ and μ^- can be chosen, explain how one could distinguish between (i) a Higgs boson with a Standard Model scalar coupling to fermions, and (ii) an exotic Higgs boson with 'scalar minus pseudo-scalar' $(1 - \gamma^5)$ couplings. Briefly discuss whether it would be possible to distinguish a Standard Model Higgs boson from one with a pure pseudo-scalar (γ^5) coupling. [4]

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You may make use of the following information:
In the limit $E \gg m$ the Dirac spinors for the helicity eigenstates are

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c_\theta \\ e^{i\phi}s_\theta \\ c_\theta \\ e^{i\phi}s_\theta \end{pmatrix}, u_\downarrow = \sqrt{E}\begin{pmatrix} -s_\theta \\ e^{i\phi}c_\theta \\ s_\theta \\ -e^{i\phi}c_\theta \end{pmatrix}, v_\uparrow = \sqrt{E}\begin{pmatrix} s_\theta \\ -e^{i\phi}c_\theta \\ -s_\theta \\ e^{i\phi}c_\theta \end{pmatrix}, v_\downarrow = \sqrt{E}\begin{pmatrix} c_\theta \\ e^{i\phi}s_\theta \\ c_\theta \\ e^{i\phi}s_\theta \end{pmatrix},$$

where $c_\theta = \cos(\theta/2)$ *and* $s_\theta = \sin(\theta/2)$.
The Dirac matrices are given by

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

$G_F/\sqrt{2} = g_W^2/(8m_W^2)$. *The second and third generation fermion masses are* $m_\mu = 0.106$ *GeV,* $m_\tau = 1.777$ *GeV,* $m_s = 0.1$ *GeV,* $m_c = 1.5$ *GeV,* $m_b = 4.5$ *GeV, and* $m_t = 175$ *GeV. The Fermi constant takes the numerical value* $G_F = 1.166\times10^{-5}$ *GeV*$^{-2}$.
The relativistic Breit-Wigner formula for a resonance of mass m and spin J is

$$\sigma = \frac{4\pi(2J+1)}{m^2}\frac{s\Gamma_i\Gamma_f}{(s-m^2)^2+m^2\Gamma^2},$$

where Γ_i, Γ_f and Γ are the appropriate partial decay widths and the total decay width and s is the square of the center of mass energy.

END OF PAPER