UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Physics
MASTER OF ADVANCED STUDY IN PHYSICS
NATURAL SCIENCES TRIPOS: Part III Astrophysics

Tuesday 17 January 2017: 14:00 to 16:00

MAJOR TOPICS
Paper 1/PP (Particle Physics)

Answer ***two*** *questions only. The approximate number of marks allocated to each part of a question is indicated in the right-hand margin where appropriate. The paper contains four sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You should use a ***separate Answer Book*** *for each question.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2x20-page answer books	Mathematical Formulae Handbook
Rough workpad	Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

V7.4

1 The Mandelstam variables s, t and u for $2 \to 2$ scattering processes are defined as $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$, where p_1 and p_2 are incoming four-momenta and p_3 and p_4 are outgoing four-momenta. Within this question you may neglect the masses of all incoming and outgoing particles.

(a) Show that s, t and u are not independent by considering $s + t + u$. [2]

(b) Find s, t and u in terms of p and θ, where p is the magnitude of the three-momentum of p_1 in the centre-of-mass frame, and θ is the angle between the spatial parts of p_1 and p_3 in that same frame. [4]

Ten scattering processes (numbered 0 to 9) and six quantities (denoted A to F) are listed in the following tables.

0	$e^-\mu^- \to e^-\mu^-$	5	$e^-\mu^+ \to e^+\mu^-$
1	$e^-e^+ \to \mu^-\mu^+$	6	$e^-e^- \to \mu^-\mu^-$
2	$e^-e^- \to e^-e^-$	7	$e^-_R\mu^-_R \to e^-\mu^-$
3	$e^-e^+ \to e^-e^+$	8	$e^-_R\mu^-_L \to e^-\mu^-$
4	$e^-\mu^+ \to e^-\mu^+$	9	$e^-_R e^+_R \to \mu^-\mu^+$

A	$\frac{u^2}{t^2}$
B	$\frac{s^2+u^2}{t^2} + \frac{2s^2}{tu} + \frac{s^2+t^2}{u^2}$
C	$\frac{t^2+u^2}{s^2}$
D	$\frac{s^2+u^2}{t^2}$
E	$\frac{s^2}{t^2}$
F	$\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}$

Each of the quantities A to F represents the square of the modulus of the tree-level QED spin-averaged matrix element $\langle |M|^2 \rangle$ of one or more of the processes 0 to 9, after omission of any overall factors that do not depend on s, t or u. For each process the particle names are given in the order corresponding to momentum labels $p_1 p_2 \to p_3 p_4$.

(c) Determine which (if any) of the processes in 0 to 9 are 'allowed' at tree-level in QED, and which (if any) are correspondingly 'not allowed'. [1]

(d) For each 'allowed' process in 0 to 9, in turn:

(i) draw all the tree-level QED Feynman diagrams for that process;

(ii) identify the quantity in A to F which represents the s, t and u dependence of $\langle |M|^2 \rangle$ for that process, explaining your reasoning; and

(iii) qualitatively sketch the $\cos\theta$ dependence of $\langle |M|^2 \rangle$.

[*Hint: Answers to part (ii) need not contain lengthy mathematical proofs or deriavations from first-principles where simpler arguments can be found.*] [23]

2 Write detailed notes on **one** of the following topics:

(a) Baryon wave functions, **or** [30]

(b) Experiments that provide evidence for neutrino oscillations. [30]

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3 In electron-proton scattering, the Lorentz invariant quantities:

$$s = (p_1 + p_2)^2, \;\; Q^2 = -q^2 = -(p_1 - p_3)^2, \;\; x = \frac{Q^2}{2p_2.q} \;\; \text{and} \;\; y = \frac{p_2.q}{p_1.p_2},$$

are defined in terms of p_1 and p_2, the four-momenta of the initial-state electron and proton respecively, and p_3, the four-momentum of the scattered electron. Neglecting the Q^2 dependence of the structure functions, F_1^{ep} and F_2^{ep}, the differential cross section for electron-proton deep inelastic scattering can be written as

$$\frac{\mathrm{d}^2\sigma^{ep}}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{ep}(x)}{x} + y^2 F_1^{ep}(x)\right].$$

(a) For the case where the proton is at rest, express s, Q^2, x and y in terms of the proton mass, m_p, the electron scattering angle θ in the lab frame, and the energies of the incoming and scattered electron, E_1 and E_3. [4]

(b) In the parton model, show that x can be interpreted as the fraction of the proton's momentum carried by the struck quark in a frame where the proton has infinite momentum. Explain any assumptions made. [4]

(c) The differential cross section for electron-quark scattering can be written as

$$\frac{\mathrm{d}\sigma^{eq}}{\mathrm{d}Q^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$

where e_q is the charge of the quark. Using the parton model, including contributions from the light quarks (u, d, s) only, show that

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)],$$

where $u(x)$, $d(x)$ and $s(x)$ are the up-, down- and strange-quark parton distribution functions for the proton. Obtain a similar expression for the electron-neutron structure function $F_2^{en}(x)$. [6]

(d) Stating clearly any assumptions made, show that

$$\int_0^1 \frac{[F_2^{ep}(x) - F_2^{en}(x)]}{x}\mathrm{d}x = \frac{1}{3} + \frac{2}{3}\int_0^1 [\bar{u}(x) - \bar{d}(x)]\mathrm{d}x$$

and comment on the consequences of the observed value being 0.24 ± 0.03. [6]

(e) In the parton model for neutrino-nucleon scattering the structure functions are

$$F_2^{\nu p}(x) = 2x[d(x) + s(x) + \bar{u}(x)] \qquad \text{and} \qquad F_2^{\nu n}(x) = 2x[u(x) + \bar{s}(x) + \bar{d}(x)].$$

Assuming $s(x) = \bar{s}(x)$, obtain an expression for $xs(x)$ in terms of the structure functions for neutrino- and electron-nucleon scattering, $F_2^{\nu N}(x) = \frac{1}{2}(F_2^{\nu p}(x) + F_2^{\nu n}(x))$ and $F_2^{eN}(x) = \frac{1}{2}(F_2^{ep}(x) + F_2^{en}(x))$. [7]

(f) Provide possible physical explanations for why $\bar{d}(x) > \bar{u}(x) > \bar{s}(x)$. [3]

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