UNIVERSITY OF CAMBRIDGE

NATURAL SCIENCES TRIPOS: Part III Experimental and Theoretical Physics
MASTER OF ADVANCED STUDY IN PHYSICS

Tuesday 17 January 2012 14.00 to 16.00

MAJOR TOPICS
Paper 140 (Particle Physics)

Answer **two** *questions only. The approximate number of marks allocated to each part of a question is in the right-hand margin where appropriate. The paper contains 4 sides including this one and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*
You should use a **separate Answer Book** *for each question.*

STATIONERY REQUIREMENTS
2×20-page answer books
Rough workpad
Linear graph paper

SPECIAL REQUIREMENTS
Mathematical formulae handbook
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 Explain the physical motivation for SU(2) flavour symmetry and define isospin. [3]

Assuming SU(2) flavour symmetry, a state consisting of a combination of two u or d quarks decomposes into an isospin triplet

$$|1,-1\rangle = \mathrm{dd}\,, \quad |1,0\rangle = \tfrac{1}{\sqrt{2}}(\mathrm{ud}+\mathrm{du}) \quad \text{and} \quad |1,+1\rangle = \mathrm{uu}\,,$$

and an isospin singlet, $|0,0\rangle = \frac{1}{\sqrt{2}}(\mathrm{ud}-\mathrm{du})$. Derive corresponding isospin states for a system consisting of three u or d quarks. [5]

The flavour and spin components of the wavefunction for a spin-up proton consisting of three coloured quarks can be written

$$|\mathrm{p}\uparrow\rangle = \tfrac{1}{\sqrt{18}}(\, 2\mathrm{u}\!\uparrow\mathrm{u}\!\uparrow\mathrm{d}\!\downarrow - \mathrm{u}\!\uparrow\mathrm{u}\!\downarrow\mathrm{d}\!\uparrow - \mathrm{u}\!\downarrow\mathrm{u}\!\uparrow\mathrm{d}\!\uparrow + \text{cyclic permutations}\,)\,.$$

Derive the equivalent expression for $|\mathrm{p}\uparrow\rangle$ in a model with no colour degrees of freedom, in which the wavefunction can be written as

$$\psi = \phi_{\text{flavour}}\,\chi_{\text{spin}}\,\eta_{\text{space}}\,.$$ [7]

In this colourless model, obtain the SU(3) flavour multiplet structure for the $L = 0$ baryons consisting of combinations of three u, d or s quarks. [4]

In the SU(3)$_c$ colour symmetry of QCD, there are three classes of allowed colour exchange in qq $\to$ qq scattering: $r\,r \to r\,r$, $r\,g \to r\,g$ and $r\,g \to g\,r$. Draw the Feynman diagrams, with the colours clearly labelled, for the three classes of allowed colour exchange in the QCD annihilation process $\mathrm{q}\overline{\mathrm{q}} \to \mathrm{q}'\overline{\mathrm{q}}'$, where q and q′ are quarks of different flavour. [3]

The QCD colour factor for the annihilation process $\mathrm{q}\overline{\mathrm{q}} \to \mathrm{q}'\overline{\mathrm{q}}'$ with colours $i\,\bar{k} \to j\,\bar{l}$ can be written

$$C(i\,\bar{k} \to j\,\bar{l}) = \frac{1}{4}\sum_{a=1}^{8} \lambda^a_{ki}\lambda^a_{jl}\,,$$

where the sum over a corresponds to the sum over the eight gluons represented by the Gell-Mann λ-matrices. Find the individual colour factors for each of the three classes of colour exchange and thus derive the overall colour-averaged colour factor, $\langle |C|^2\rangle$, for the QCD annihilation process $\mathrm{q}\overline{\mathrm{q}} \to \mathrm{q}'\overline{\mathrm{q}}'$. [8]

You may require the following information:
i) The isospin raising operator, $\hat{T}_+$, is defined as

$$\hat{T}_+\,|I,I_3\rangle = \sqrt{I(I+1)-I_3(I_3+1)}\,|I,I_3+1\rangle\;.$$

ii) The Gell-Mann matrices are

$$\lambda_1 = \begin{pmatrix} 0&1&0\\ 1&0&0\\ 0&0&0 \end{pmatrix}, \quad \lambda_4 = \begin{pmatrix} 0&0&1\\ 0&0&0\\ 1&0&0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0&0&0\\ 0&0&1\\ 0&1&0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1&0&0\\ 0&-1&0\\ 0&0&0 \end{pmatrix},$$

$$\lambda_2 = \begin{pmatrix} 0&-i&0\\ i&0&0\\ 0&0&0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0&0&-i\\ 0&0&0\\ i&0&0 \end{pmatrix}, \quad \lambda_7 = \begin{pmatrix} 0&0&0\\ 0&0&-i\\ 0&i&0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1&0&0\\ 0&1&0\\ 0&0&-2 \end{pmatrix}.$$

2 The Super-Kamiokande water Čerenkov detector observes solar neutrinos through the elastic scattering of electron neutrinos from atomic electrons. Draw the two lowest order Feynman diagrams for the process of $\nu_e e^- \to \nu_e e^-$ scattering. [2]

The Lorentz invariant matrix element for the charged current contribution to $\nu_e e^- \to \nu_e e^-$ scattering can be written

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\rho} \left[\overline{u}(p'_e)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_\nu)\right]\left[\overline{u}(p'_\nu)\tfrac{1}{2}\gamma^\rho(1-\gamma^5)u(p_e)\right] .$$

Show that, in the limit where the electron and neutrino masses can be neglected, this can be written

$$M_{fi} = \frac{g_W^2}{2m_W^2} g_{\mu\rho} \left[\overline{u}_\downarrow(p'_e)\gamma^\mu u_\downarrow(p_\nu)\right]\left[\overline{u}_\downarrow(p'_\nu)\gamma^\rho u_\downarrow(p_e)\right] ,$$

where $u_\downarrow$ is a left-handed helicity eigenstate. [5]

Considering only the charged current contribution to $\nu_e e^- \to \nu_e e^-$ scattering, and working in the centre-of-mass frame, express M_{fi} in terms of the centre-of-mass energy, $\sqrt{s}$, and show that the total cross section is given by

$$\sigma(\nu_e e^- \to \nu_e e^-) = \frac{G_F^2 s}{\pi} ,$$

where $G_F/\sqrt{2} = g_W^2/8m_W^2$. [13]

Solar neutrinos detected in Super-Kamiokande are produced primarily from the $^8B \to {}^7Be + e^+ + \nu_e$ process and have a mean energy of approximately 10 MeV. Obtain the value of $\sigma(\nu_e e^- \to \nu_e e^-)$ at this energy, expressing your answer in S.I. units. [3]

The flux of ^8B solar neutrinos at the Earth is expected to be $5 \times 10^6\ \mathrm{cm^{-2}\,s^{-1}}$. Estimate the number of ^8B solar neutrino interactions per day in the Super-Kamiokande detector, of mass 5×10^7 kg. [4]

Briefly explain how solar neutrinos are detected in the Super-Kamiokande experiment and how they are distinguished from the background due to radioactive decays. [3]

[*You may require the following information:*
i) In the limit $E \gg m$, the left- and right-handed helicity spinors for a particle are

$$u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \quad \textit{and} \quad u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix} ,$$

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where $c = \cos(\theta/2)$ and $s = \sin(\theta/2)$ and ϕ is the azimuthal angle.
ii) For spinors ψ and ϕ

$$\begin{aligned}
\overline{\psi}\gamma^0\phi &= \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4\,,\\
\overline{\psi}\gamma^1\phi &= \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1\,,\\
\overline{\psi}\gamma^2\phi &= -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1)\,,\\
\overline{\psi}\gamma^3\phi &= \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2\,.
\end{aligned}$$

iii) The matrix γ^5, which can be written

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

satisfies the anti-commutation relations $\gamma^5\gamma^\mu = -\gamma^5\gamma^\mu$.
iv) In the centre-of-mass frame

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2 s}\langle |M_{fi}|^2\rangle\,.$$

v) $G_\mathrm{F} = 1.166 \times 10^{-5}$ GeV^{-2}, $m_\mathrm{e} = 5.11 \times 10^{-4}$ GeV, $m_\mathrm{p} = 0.94$ GeV and $e = 1.6 \times 10^{-19}$ C.]

3 Write brief notes on **three** of the following topics:

(a) strangeness oscillations; [10]

(b) electron-proton deep inelastic scattering; [10]

(c) the PMNS matrix and its determination; [10]

(d) the Dirac equation and its solutions. [10]

END OF PAPER