NATURAL SCIENCES TRIPOS Part III

Tuesday 17 January 2006 2:00pm to 3:30pm

EXPERIMENTAL AND THEORETICAL PHYSICS (4)
Particle Physics

Answer **two** *questions only. The approximate number of marks allotted to each part of a question is indicated in the right hand margin where appropriate. The paper contains* **4** *sides (including this one) and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

STATIONERY REQUIREMENTS
20 Page Answer Book
Rough Work Pad
Metric Graph Paper

SPECIAL REQUIREMENTS
Mathematical Formulae Handbook

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 Write brief accounts of **two** of the following:

(a) electron-nucleon elastic scattering; [15]

(b) isospin and SU(3) flavour symmetry; [15]

(c) experimental tests of electroweak unification; [15]

(d) CP violation in the neutral kaon system. [15]

2 Draw the leading-order, quark-level Feynman diagram for the decay $\tau^- \to \pi^- \nu_\tau$ of the τ lepton. [2]

The Lorentz-invariant matrix element M_{fi} for $\tau^- \to \pi^- \nu_\tau$ decay is given by

$$M_{\mathrm{fi}} = \sqrt{2} G_{\mathrm{F}} f_\pi V_{\mathrm{ud}}\ j.p_4\,,$$

where G_{F} is the Fermi constant, f_π is a constant, V_{ud} is a CKM matrix element, $j.p_4$ is the scalar product of the lepton current $j^\mu = \overline{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)u(p_1)$ with the π^- four-momentum p_4, and p_1 and p_3 are the four-momenta of the τ^- and ν_τ respectively.

Working in the rest frame of the τ^-, with the τ^- spin fully polarised in the $+z$ direction, show that (up to an arbitrary overall phase factor) the lepton current is given by

$$j^\mu = \sqrt{2m_\tau p^*}(-\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}, -\mathrm{i}\cos\frac{\theta}{2}, \sin\frac{\theta}{2})\,,$$

where m_τ is the τ^- mass, and the ν_τ has four-momentum $p_3^\mu = (p^*, p^*\sin\theta, 0, p^*\cos\theta)$. Hence show that the magnitude of M_{fi} is

$$|M_{\mathrm{fi}}| = 2G_{\mathrm{F}} f_\pi |V_{\mathrm{ud}}| m_\tau \sqrt{m_\tau p^*}\sin\frac{\theta}{2}\ .$$ [8]

Explain, with the aid of a diagram, why M_{fi} vanishes when $\theta = 0$. Write down an equivalent expression for $|M_{\mathrm{fi}}|$ when the τ^- spin is fully polarised in the $-z$ direction. [3]

Given that the rest-frame differential decay rate is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{p^*}{32\pi^2 m_\tau^2}|M_{\mathrm{fi}}|^2\ ,$$

show that the partial width for the decay of an unpolarised sample of τ^- leptons is

$$\Gamma(\tau^- \to \pi^-\nu_\tau) = \frac{G_{\mathrm{F}}^2 f_\pi^2}{16\pi}|V_{\mathrm{ud}}|^2 m_\tau^3 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2\ ,$$

where m_π is the π^- mass. [5]

The mean lifetime of the τ^- is 2.91×10^{-13} s. Taking $f_\pi = m_\pi$, and making a suitable approximation for the value of V_{ud}, estimate the branching ratio of the $\tau^- \to \pi^- \nu_\tau$ decay. [4]

Explain qualitatively why experimental studies of atmospheric neutrino interactions provide evidence for $\nu_\mu \to \nu_\tau$ oscillations, and indicate how the values of the oscillation parameters Δm^2 and $\sin^2 2\theta$ can be inferred. Summarise briefly what can be deduced about the values of neutrino masses and the structure of the PMNS matrix from neutrino oscillation experiments. (Detailed descriptions of experiments, and precise numerical values are not expected.) [8]

You may require the following information:

The π^- and τ^- masses are 139.6 *MeV and* 1.777 *GeV, respectively.*

$G_{\mathrm{F}} = 1.166 \times 10^{-5}$ *GeV*$^{-2}$.

$\hbar = 6.582 \times 10^{-25}$ *GeV s* .

The helicity eigenstate spinors $u_\uparrow(p)$ and $u_\downarrow(p)$ for a particle of mass m and four-momentum $p^\mu = (E, p\sin\theta\cos\phi, p\sin\theta\sin\phi, p\cos\theta)$ are

$$u_\uparrow(p) = \sqrt{E+m}\begin{pmatrix} \cos\frac{\theta}{2} \\ e^{\mathrm{i}\phi}\sin\frac{\theta}{2} \\ \frac{p}{E+m}\cos\frac{\theta}{2} \\ \frac{p}{E+m}e^{\mathrm{i}\phi}\sin\frac{\theta}{2} \end{pmatrix}, \qquad u_\downarrow(p) = \sqrt{E+m}\begin{pmatrix} -\sin\frac{\theta}{2} \\ e^{\mathrm{i}\phi}\cos\frac{\theta}{2} \\ \frac{p}{E+m}\sin\frac{\theta}{2} \\ -\frac{p}{E+m}e^{\mathrm{i}\phi}\cos\frac{\theta}{2} \end{pmatrix}.$$

The Dirac matrices are given by

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \; \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \; \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ -\mathrm{i} & 0 & 0 & 0 \end{pmatrix},$$

$$\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \; \gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

(TURN OVER

3 Top quark pairs are produced in hadron-hadron collisions predominantly via the processes $q\overline{q} \to t\overline{t}$ and $gg \to t\overline{t}$. Draw (in total) three leading order QCD Feynman diagrams for these processes. [4]

Outline in general terms how the total cross-section for $t\overline{t}$ production can be calculated in terms of integrals over appropriate parton distribution functions, and explain how these distribution functions can be determined experimentally. [6]

The QCD vertex factor associated with the interaction of a quark and a gluon is proportional to $\frac{1}{2}\lambda^a_{ij}$, where $i, j = 1, 2, 3$ (or r,g,b) are colour indices, and λ^a is an SU(3) λ-matrix. List the allowed colour configurations for the process $q\overline{q} \to t\overline{t}$, and determine the values of the corresponding colour factors, such as $C(r\overline{r} \to r\overline{r})$, contributing to the matrix element. [7]

Hence determine the colour factor associated with the $q\overline{q} \to t\overline{t}$ component of the overall cross-section for $t\overline{t}$ production in high energy hadron-hadron collisions. [3]

A $t\overline{t}$ pair is produced in a proton-antiproton interaction at the Tevatron collider, where proton and antiproton beams of energy 0.98 TeV collide head-on. The t quark is emitted at right-angles to the direction of each beam and has a momentum of 40 GeV. The $\overline{t}$ antiquark is emitted at an angle of 50° to the incoming proton direction. The total momentum of the $t\overline{t}$ system is directed along the incoming proton direction. Calculate the momentum fractions x_1 and x_2 of the partons which interacted to produce the $t\overline{t}$ pair. What would be the values of x_1 and x_2 if this $t\overline{t}$ pair were to be produced instead in a proton-proton collision at the LHC, where proton beams of energy 7 TeV will interact head-on? [8]

[*The top quark has a mass of* 175 *GeV.*]

What is the most likely nature of the partons producing the $t\overline{t}$ pair at the Tevatron and at the LHC? [2]

[*The standard representation of the SU(3) λ-matrices is*

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$
]

END OF PAPER